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Role of topology in compensated magnetic systems

Special Collection: Emerging Materials in Antiferromagnetic Spintronics

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ABSTRACT

Topology plays a crucial and multifaceted role in solid state physics, leading to a remarkable array of newly investigated materials and phenomena. In this Perspective, we provide a brief summary of well-established model materials with a particular focus on compensated magnets and highlight key phenomena that emerge due to the influence of topology in these systems. The overview covers various magneto-transport phenomena, with a particular focus on the extensively investigated anomalous magneto-transport effects. Furthermore, we look into the significance of topology in understanding elementary magnetic excitations, namely magnons, where the role of topology gained considerable attention from both theoretical and experimental perspectives. Since electrons and magnons carry energy, we explore the implications of topology in combined heat and spin transport experiments in compensated magnetic systems. At the end of each section, we highlight intriguing unanswered questions in this research direction. To finally conclude, we offer our perspective on what could be the next advancements regarding the interaction between compensated magnetism and topology.

I. INTRODUCTION

Topological materials offer an exciting platform to explore exotic quantum states, such as the quantum anomalous Hall effect (AHE),[1–4] and high-energy physics phenomena, an example being the chiral anomaly,[5] in condensed matter systems. Generally, the relationship between topology and magnetism is a thriving area of research in condensed matter physics, as evidenced by a growing body of literature on the topic.[6–9] In particular, compensated magnets change the symmetry in topological materials, leading to intriguing experimental consequences.[10–12] By *compensated magnetic systems*, we mean all collinear, non-collinear, and non-coplanar antiferromagnets as well as the recently identified class of collinear compensated magnets with anisotropic spin split band structured–dubbed altermagnets.[13–18] Please note that while ferrimagnets also exhibit a temperature dependent magnetization compensation point, we mainly excluded them from this Perspective.

Compensated magnetic materials are not only of academic interest but also hold great promise for practical applications. This is partly caused by their potential for high integration densities and their high operating speeds up to the THz regime. In addition, many compensated magnets are compatible with semiconductor technology, which holds a lot of promise for unconventional computer architectures, such as bio-inspired (neuromorphic) computing[19] and magnon-based[20–22] and skyrmion-based electronics.[23]

This Perspective is structured into the introductory Secs. II and III introducing the nomenclature and typical experimental techniques. In Sec. IV, we discuss the workhorse material systems with an emphasis on electric transport. In Sec. V, we discuss the topology of magnetic excitations (magnons) in compensated magnets, before discussing anomalous transverse spin and heat transport in Sec. VI. We conclude with an outlook in Sec. VII.

II. REAL SPACE AND K-SPACE TOPOLOGIES

The effort to understand magnetic and topological properties continuously opens new research directions that link magnetic properties and topology. In this context, it is important to distinguish between the real space and momentum (*k*-space) topologies and the corresponding Berry curvature.

In real space, atoms are arranged in three-dimensions, often in regular fashion in the form of a crystal. The electronic properties of materials can be described by the distribution of electrons and the interactions between them. The real space topology refers to the topology of the physical properties of the material in real space. For example, certain textures of the magnetization, e.g., skyrmions,[24] can be associated with a winding number. This integer winding number can be used to distinguish, for example, skyrmions from antiskyrmions and is often called topological charge. Electrons traveling through such magnetic textures pick up a non-vanishing Berry phase because the texture acts as a time-dependent exchange field in the frame of the moving electron. See Refs. 25 and 26 for a review on skyrmions.

In contrast, *k*-space refers to the space of wavevectors, which are related to the momentum of electrons. In *k*-space, the electronic properties of a crystal are described by the electronic bands, and the *k*-space topology refers to the topology of the electronic wave function of a certain band in momentum space. The integral of the *k*-space Berry phase over the whole Brillouin zone can be non-zero if the time reversal symmetry in the band structure is broken by the magnetic order.

The interplay of the magnetic order and topology can be viewed from several perspectives. In Fig. 1, we provide several examples of material classes that can be considered "topological" and contain compensated magnets. In Sec. IV, we present representatives of each class. The magnetic ordering can lead to non-trivial real space topology, and it is typically discussed in context of skyrmions and non-coplanar antiferromagnets. The non-trivial *k*-space topology is often associated with topological insulators, Weyl and Dirac semimetals, and these band structure properties are compatible with the compensated magnetic ordering.

Topological insulators are materials whose bulk volume has an energy gap between inverted valence and conduction bands, which leads to conductive surface states.[27] In Dirac materials, double degenerate bands touch at specific points in momentum space, which causes a fourfold degenerate point, which is protected by symmetry.[28] Weyl semimetals serve as the counterpart to Dirac semimetals in the absence of either inversion $\mathcal{P}$ or time-reversal $\mathcal{T}$ symmetry.[29] These semimetals feature Weyl points that act as monopoles of momentum space Berry curvature and are connected by topologically protected Fermi arcs. The band dispersions of Dirac and Weyl semimetals near the band touching points are visualized in Fig. 2. In addition to materials where the bands only touch in discrete points, there are also nodal line semimetals where the non-degenerate conduction and valence bands touch along the closed loops in momentum space.[30] Non-trivial topological properties in *k*-space can also arise from the non-collinear ordering of compensated magnets.

The real space and *k*-space topologies and the corresponding Berry curvature are often treated separately; however, in certain cases, both contribute to the overall Berry curvature and the related experimental observations.

III. DIRECT AND INDIRECT EXPERIMENTAL OBSERVATIONS

Various techniques have been used to image topological features in real space, such as Lorentz Transmission Electron Microscopy (LTEM),[32] Magnetic Force Microscopy (MFM),[33] or Magneto-Optical Kerr Effect (MOKE),[34] although not all techniques bear the lateral resolution to look into the exact configuration of the topological object. The *k*-space topology is typically visualized by angular resolved photoemission spectroscopy (ARPES).[35] However,

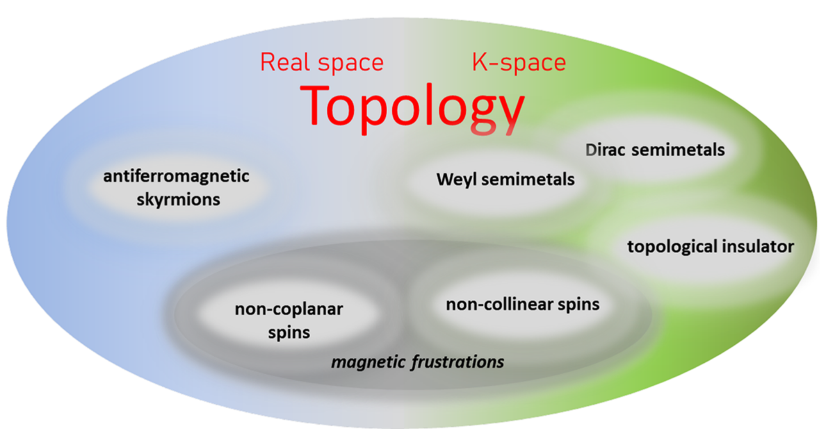

FIG. 1. Examples of classes of compensated magnetic materials where the real or *k*-space topology plays an important role.

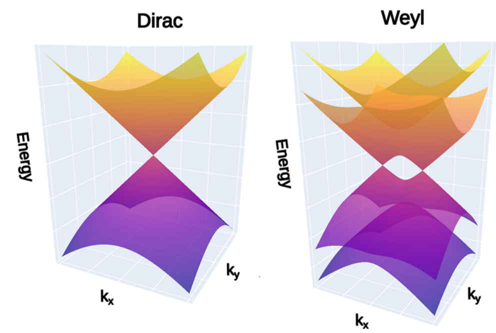

FIG. 2. Energy dispersion of electronic bands near the characteristic touching points in Dirac and Weyl semimetals. The dispersion is obtained from Eq. (2) in Ref. 31.

there are indirect experimental indications of non-trivial topology apart from direct visualizations, for example transport techniques, which shall be discussed in the next paragraphs, starting with the Hall effect.

The topological Hall effect (THE) and the intrinsic anomalous Hall effect (AHE) are two well-known effects that arise from particular material properties in real and momentum space, respectively. These phenomena can be understood as the real space and reciprocal space limits of the generalized phase-space Berry phases of conduction electrons. Several review papers have extensively covered the Hall effects.[36,37] Although THE and AHE have been treated separately in the past, recent studies have shown that when the size of the real space topological object becomes comparable to the mean free path, the crossover from the real to momentum space Berry curvature occurs.[38] This is particularly true in the case of small skyrmions. Moreover, in certain materials with strong spin–orbit coupling, comparable to the exchange splitting and smoothly varying spin texture, Berry phase accumulation in both real space and k-space can occur.[39]

The discussed unification of the anomalous and topological Hall effects was recently performed by Verma *et al.* using a semi-classical approach that includes all phase-space Berry curvatures.[40] They showed that the Hall resistivity is the sum of an anomalous term arising from the momentum-space curvature and a topological term related to the real-space curvature. The thermoelectric counterpart of the AHE—the anomalous Nernst effect (ANE)—is an equally important tool for the characterization of topological magnets especially because the intrinsic anomalous Nernst effect reflects the Berry curvature near the Fermi energy as discussed in more detail in Sec. VI.

A particularly interesting member of the family of Hall effects is the quantum anomalous Hall effect. This effect occurs in topological materials with the non-trivial k-space topology.[1–4] Here, chiral states can carry dissipationless electrical current along the sample edges (even in zero magnetic field), causing a quantized transversal resistance in the absence of external magnetic fields and vanishing longitudinal resistivity due to spontaneously broken time-reversal symmetry.[4] Two additional phenomena that relate to the emergence of chiral currents are the chiral[5] and mixed axial–gravitational anomalies.[41] They originate in the breakdown of chiral symmetry related to the non-trivial k-space topology and the mentioned chiral currents. In experiments, the chiral anomaly is visible as a negative longitudinal magnetoresistance when magnetic and electrical fields are applied in parallel.[42,43] Then, the data are strongly anisotropic and typically have a linear dependency for small magnetic fields. However, the unambiguous identification of the chiral anomaly is notoriously difficult because several effects can also lead to similar signals.[44–46]

One open issue is how to unambiguously attribute the measured signals to the THE.[47] Various artifacts can look similar to the THE: For example, if multiple types of carriers participate in the transport, the ordinary Hall effect can exhibit a non-linear behavior with regard to the applied, external magnetic field.[48] Nevertheless, the THE is often extracted assuming a linear relationship between the AHE and the magnetization of the investigated sample, which is not without problems especially in compensated magnets with a vanishing net magnetic moment. Another complication is the potential presence of two different contributions to the hysteresis loop with a distinct coercivity and an opposite sign, which can lead to signals reminiscent of the THE.[49,50]

Many open questions come with magneto-optical techniques. So far, we only discussed dc magneto-conductive effects, such as the anomalous Hall effect, and these normally have a magneto-optical counterpart. Consequently, topological contributions are expected to occur in magneto-optical experiments as well, e.g., arising from the non-zero spin chirality.[51] In addition, some of the magneto-optical phenomena are predicted to be quantized in topological compensated magnets.[52] Since light can interact with magnetic excitations, such as magnons, magneto-optical experiments enable new experimental techniques, which will be discussed in more detail in Sec. V.

IV. TOPOLOGICAL MATERIALS AND RELATED ELECTRONIC TRANSPORT

Materials that combine the topological properties and magnetic order are intensively studied for at least a decade,[9] and high throughput calculations were employed to identify promising material candidates.[53] Compensated topological magnets are, in particular, interesting.[10] In the following, we present several well-studied model compounds, which belong to the systems mentioned in Fig. 1. We discuss interesting consequences of the interplay between compensated magnetic order and topology. Our model compounds of choice are depicted in Fig. 3. We start with compensated magnets that show non-trivial k-space topology.

In the case of Dirac semimetals, the occurrence of double degenerate band crossings relies on the preservation of combined $\mathcal{PT}$ symmetry. Within magnetically ordered systems, this unique property is only found in antiferromagnetic materials. Among these materials, orthorhombic CuMnAs as depicted in Fig. 3 has garnered attention[54] as a noteworthy example, displaying an antiferromagnetic behavior at room temperature.[55] Interestingly, the protection of the Dirac point's symmetry can be toggled on and off by reorienting the magnetic order vector. This reorientation has been predicted to yield significant magnetotransport responses.[12] A similar phenomenon occurs in the Dirac nodal line metal $MnPd_2$, wherein the reorientation of magnetic moments allows for a transition between the symmetry-protected degenerate state and the gapped state of the nodal line.[56]

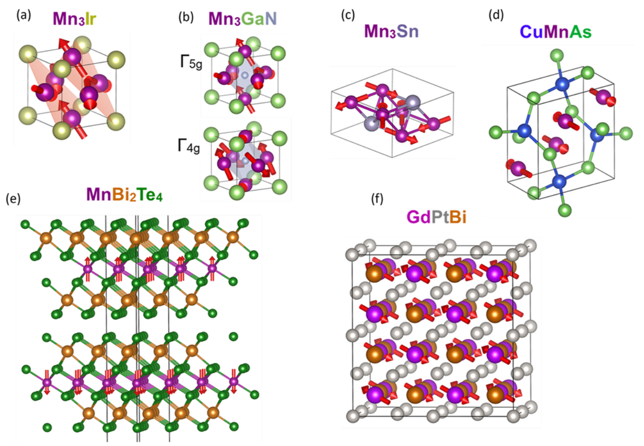

FIG. 3. Model materials with compensated magnetic order where topological properties play an important role. (a)–(f) Crystallographic unit cells of these materials together with their magnetic order.

Magnetic materials without $\mathcal{PT}$ symmetry can form Weyl semimetals, and many of them are ferromagnetic.[57,58] Here, we focus only on those with compensated magnetic order. The Weyl nodes can have a large impact on the magnetotransport properties even in the presence of other metallic bands around the Fermi energy. For example, in non-collinear antiferromagnets, such as Mn_3Sn (cf. Fig. 3) and Mn_3Ge, a link between the Weyl nodes and the anomalous transport signal was found.[59–63] Mn_3Sn and Mn_3Ge are also examples of materials, where both the topological Hall effect (THE) and anomalous Hall effect (AHE) can coexist. On the one hand, the momentum space Berry curvature arises from the Weyl nodes and symmetry-breaking triangular structure, which leads to the AHE. On the other hand, pressure[64] or magnetic field[65,66] induced canting of the moments can give rise to non-zero scalar spin chirality (which measures the solid angle spanned by three neighboring spins) and generate real space Berry curvature. Furthermore, in Mn_3Sn, the THE was also found to be linked to the non-coplanar magnetic structure in domain walls formed during field reversal.[67] The scalar spin chirality was also linked to a large spontaneous Hall response in the non-coplanar antiferromagnetic van der Waals materials $CoTa_3S_6$ and $CoNb_3S_6$.[68]

The Mn_3X-family of compounds remains central to the research focused on non-collinear antiferromagnets, and a variety of new phenomena were already demonstrated. Despite the vanishing magnetization, a sizable anomalous Hall effect was predicted[69,70] and measured.[59,61] In analogy to the anomalous Hall effect, also the optical and thermal counterparts of this anomalous transport effect, the magneto-optical Kerr effect and anomalous Nernst effect, were found[71–73] and will be discussed in Sec. VI. In addition, the magnetic spin Hall effect, chiral anomaly, or antiferromagnetic tunnel magnetoresistance could be first identified in the Mn_3X-family.[42,74–76] The AHE was also confirmed in other non-collinear antiferromagnet Mn_3Ir, which is also depicted in Fig. 3.[77]

Another family of compensated magnets with a predicted large anomalous Hall[78,79] response are the antiperovskites (Mn_3XN with X = Ga, Zn, Ag, Sn, or Ni), and experimental confirmation exists for Mn_3NiN[80] and Mn_3SnN.[81] However, the Weyl points are not the prime source of the large anomalous response in this family of compounds. Instead, the Berry curvature induced through the spin–orbit coupling between the occupied and unoccupied states dominates in these compounds.[82,83] Interestingly, in the Mn_3XN compounds, two magnetic structures related by 90° rotation of the magnetic moments can exist, of which only the one denoted Γ_{4g} shows an anomalous Hall effect (cf. Fig. 3).

In $Mn(Bi_{1-x}Sb_x)_2Te_4$, a Weyl state was induced by applying a magnetic field, which transformed the antiferromagnetic order to a ferromagnetic one.[84] Another representative of non-centrosymmetric Weyl semimetals is CeAlSi in which the non-collinear magnetic structure enables the formation of two kinds of domain walls with a distinct topology and, therefore, the possibility to tune the Weyl cones by controlling the magnetic order.[85]

Strong anomalous transport responses related to Weyl fermions were also found in the family of half Heusler compounds RPtBi, where R represents a rare earth element, such as Yb, Nd, or Gd.[86,87] GdPtBi as depicted in Fig. 3 is a well-studied, zero gap semimetal, which undergoes a band inversion under the application of an external magnetic field, leading to the formation of Weyl nodes. In addition, magnetotransport experiments in GdPtBi show the chiral anomaly,[88,89] which is explained by an interplay between the topological band structure features and many-body interactions.[90]

One of the most notable consequences of non-trivial topology is the emergence of the quantized version of the anomalous Hall effect, known as the quantum anomalous Hall effect. The initial experimental observation was reported in the ferromagnetic topological insulator $Cr_{0.15}(Bi_{0.1}Sb_{0.9})_{1.85}Te_3$,[91] which required sub-kelvin temperatures to manifest the effect. To enhance the temperature range, the magnetically compensated compound $MnBi_2Te_4$ as depicted in Fig. 3 was developed by incorporating a MnTe bilayer into the non-magnetic topological insulator Bi_2Te_3. The objective was to introduce a substantial energy gap in the topological surface states, resulting in a robust quantum anomalous Hall effect.[92] Experiments employing angle-resolved photoemission spectroscopy revealed the gap formation of surface states resembling Dirac cones due to the presence of magnetic order.[93] As anticipated, the quantum anomalous Hall effect was, indeed, observed up to 6.5 K[2] under the influence of a saturating magnetic field. Promisingly, compounds such as $MnSb_2Te_4$, which exhibit a doubled magnetic ordering temperature, are projected to further extend these limits in the future,[94] provided the bulk conductance can be effectively minimized.[95]

We shift our focus to materials exhibiting a non-trivial real-space topology. Magnetic skyrmions are magnetic textures with a topologically non-trivial winding such that the magnetization covers the entire sphere in spin space. Skyrmions are actively discussed both from a fundamental perspective and in terms of their potential applications; for a dedicated review, see Ref. 23. Skyrmions are found, for example, in non-centrosymmetric materials,[24,96] such as MnSi,[97] FeGe,[98] and GdFeCo.[99] A variety of textures, differing, for example, in their topological charge (skyrmion vs antiskyrmion) and their detailed spin rotation directions (Bloch vs Néel skyrmion), have been reported both theoretically and experimentally.[100,101]

Skyrmions do not only appear in the form of a regular array called a skyrmion lattice but also isolated, where they behave like a particle.[102] They can be moved by an electrical current[103] or a heat gradient.[104] Furthermore, the skyrmion Hall effect, corresponding to a transversal deflection of the skyrmion under an electrical bias due to the magnus force, has been observed.[105,106] The topological Hall effect is sometimes interpreted as an inverse effect to the skyrmion Hall effect.[47] Although most of the skyrmions are observed in ferromagnets, antiferromagnetic materials are increasingly studied theoretically[107–111] and, recently, also experimentally.[112,113] In particular, synthetic antiferromagnets are a platform to engineer antiferromagnetic skyrmions.[114–117]

In antiferromagnetic materials, the magnetic order vector (Néel vector) can also wind while the magnetization remains compensated as illustrated in Fig. 4, leading to a non-trivial real space topology. The antiferromagnetic skyrmions, apart from the general advantages of compensated magnets, have also the specific advantage that the perpendicular deflection of current driven skyrmions can be suppressed. This skyrmion Hall effect is a major hurdle for skyrmion based devices, mostly in case of narrow wires. It was shown that the current driven skyrmion motion in antiferromagnetic materials can completely suppress the skyrmion Hall effect.[114] In addition, the topological spin Hall effect[110,118] due to the antiferromagnetic skyrmions is also expected to be more robust than that of the ferromagnetic counterpart.[119]

There are a couple of open issues in this field. One challenge lies in accurately determining the precise magnetic ground state of compensated magnets. This difficulty arises because for compensated magnetic materials, there are often various magnetic structures which are separated only by small energy barriers and otherwise very similar global properties. Another open issue is the expansion of the family of magnetically compensated materials with a non-trivial topology. While some of the materials mentioned in the literature are promising, there is a push for advancements in terms of achieving higher critical temperatures and reducing reliance on critical or toxic elements.

V. TOPOLOGY AND MAGNONS IN COMPENSATED SYSTEMS

We now direct our attention to the excitation of magnetically ordered ground states. The fundamental excitation in compensated or uncompensated magnets is known as a magnon. It is worth noting that the concept of topology is not limited to electronic systems, as magnetic insulators can also exhibit non-trivial magnon topology.[122] Both the momentum-space (band structure) topology and real-space topology concepts can be extended to magnons with suitable adaptations. It is crucial to emphasize here that magnons are non-conserved bosons. As a result, unlike electronic systems whose ground state can be topological, topological magnons are always finite-energy excitations above a trivial ground state.[123] While the low-energy physics of magnons is, therefore, topologically trivial, topological features can be expected when the magnet is excited at higher energies, either coherently or thermally.

As a model example, we introduce the topological magnon insulator or magnon Chern insulator.[123–127] Similar to electronic Chern insulators, topological magnon insulators exhibit spectral gaps that support chiral edge states [see Fig. 5(a)]. Once coher-

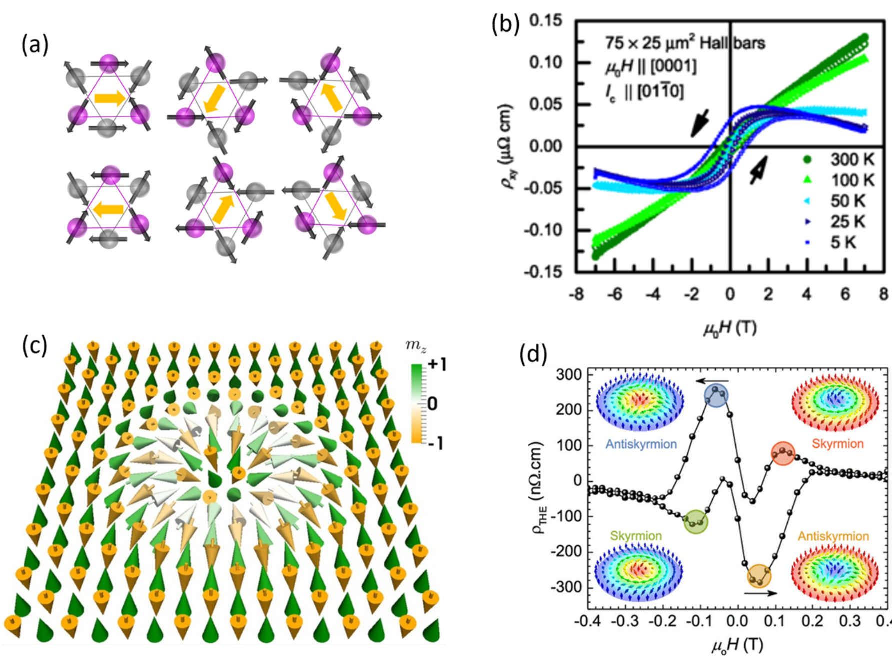

FIG. 4. Example of the k-space and real space topologies and related electronic transport. (a) Possible magnetic domains in the Mn_3Sn Weyl semimetal. (b) Anomalous Hall effect arising from the k-space topology in the Mn_3Sn Weyl semimetal. Taken from Ref. 120. (c) Antiferromagnetic skyrmion taken from Ref. 115. (d) Topological Hall effect associated with the skyrmion phase. Taken from Ref. 121.

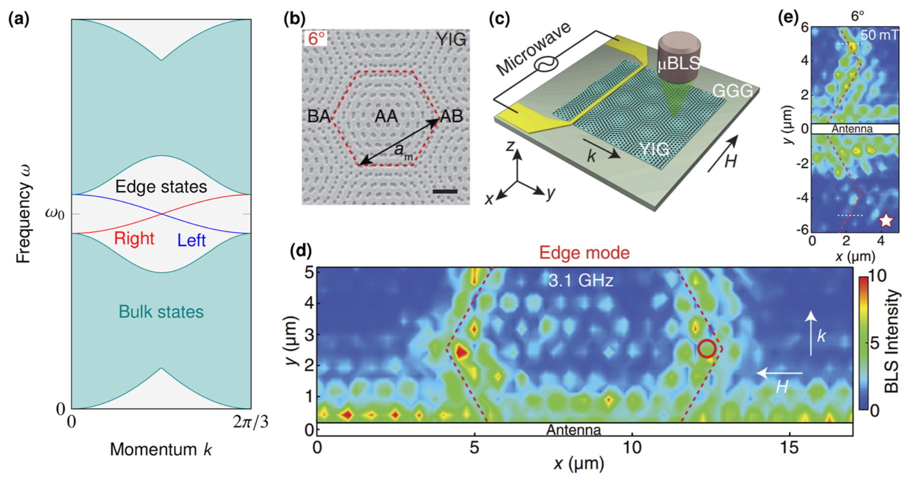

FIG. 5. Topological magnon insulators. (a) Representative magnon spectrum of a magnon Chern insulator, with a gap between two bulk bands. This gap hosts chiral edge states, which propagate unidirectionally along the edges of the sample. (b)–(e) Experimental evidence for chiral edge magnons in antidot lattices. (b) Antidot lattice made of ferrimagnetic yttrium iron garnet (YIG). (c) Experimental setup with a microwave antenna and microfocused Brillouin light scattering (BLS). (d) Spatially resolved BLS intensity at an excitation frequency in the bulk bandgap, showing only edge magnons. (e) Evidence of strong nonreciprocity of the edge mode. Panels (b)–(e) were taken from Ref. 129.

ently excited, these edge magnons circulate around the edges of the sample, bypassing defects due to their robustness to elastic backscattering. Since chiral edge states naturally break the time-reversal symmetry, topological magnon insulators require magnetic point groups compatible with ferromagnetism.[128] Importantly, a net ferromagnetic moment itself is not required. Thus, as long as a compensated magnetic texture breaks enough magnetic point group symmetries to be compatible with ferromagnetism, the magnetic excitations can exhibit chiral edge states. An example of this scenario is the family of kagome antiferromagnets with compensated vector chiral order.[128]

Chiral edge magnons are proposed as coherent information channels in the field of magnon spintronics, which aims to use magnons in next-generation computing paradigms.[130] Since magnons do not carry charge, magnonic computation is not associated with Joule heating, allowing for low-power computation. Topological protection of chiral edge magnons could considerably enhance the lifetime of magnetically encoded signals, allowing for special topological magnon interferometers,[22,131] and proposals for fault-tolerant magnonic devices have been made.[123] Currently, magnonic technologies rely heavily on ferromagnets, whose stray fields complicate downscaling. Therefore, the realization of topological magnons in magnets with compensated order is an exciting prospect for magnonics.

Beyond chiral edge states in topological magnon insulators, there are other topological boundary states in materials with non-trivial magnon topology. Collinear compensated magnets can exhibit symmetries that enforce a Kramers-like degeneracy of magnons, leading to Z_2 topological magnon phases with helical edge states.[132–134] In three dimensions, the bulk magnetic excitation spectrum can exhibit Weyl magnons with magnon surface arcs both in ferromagnets[135] and in compensated magnets.[136] Furthermore, there can be nodal-line magnons with drumhead surface states.[137,138]

Experimentally, the direct observation of magnon topology is very challenging. Nevertheless, there are many methods that provide insights into magnon properties, and the obtained results can then be correlated with magnon topology. The well-known methods to investigate the magnon band structure are inelastic scattering techniques utilizing neutrons and photons.

Inelastic neutron scattering[139] provides high momentum and energy resolution. It is particularly well-suited for studying bulk single crystals, which typically have a large volume and high quality. However, growing such crystals can be challenging for certain materials. Nonetheless, inelastic neutron scattering has been instrumental in identifying potential topological magnon bandgaps in the bulk spectrum of various materials. For example, in the field-polarized phases of Cu(1,3-benzenedicarboxylate),[140] CrI_3,[141] $CrGeTe_3$, and $CrSiTe_3$,[142] inelastic neutron scattering studies have revealed the presence of topological magnon bandgaps. Furthermore, evidence for Dirac[143–145] and Weyl magnons[146] has also been observed. While these examples primarily involve field-polarized magnetic materials, compensated magnets such as Cu_3TeO_6 and $CoTiO_3$ have also been shown to exhibit Dirac nodal line magnons.[139,147–149]

Other inelastic light scattering techniques, such as resonant inelastic x-ray scattering,[150,151] Raman scattering,[152,153] and Brillouin light scattering (BLS),[129] allow us to probe thin film materials and provide μm spatial resolution. Recently, microfocused Brillouin light scattering has been used to provide direct evidence for chiral edge states of classical dipolar spin waves in nanostructured antidot magnonic crystals made of yttrium iron garnet (YIG)[129] [see Figs. 5(b)–5(e)]. These optical detection techniques can be combined with ultrafast excitation schemes to probe non-equilibrium coherent magnon states. With regard to antiferromagnetic magnons, the progress in THz spectroscopy allows for resonant excitation and detection of magnon modes.

At GHz frequencies, nitrogen vacancy centers in diamond in combination with a scanning probe approach provide a high spatial resolution and enable sensing of magnon excitations.[154–156] In addition, magnon transport experiments allow for exploring the role of magnon topology. In particular, all-electrical magnon transport experiments utilizing charge-to-spin current conversion processes for the injection and detection of magnon spin currents provide access to the chirality of magnon modes.[157–159] In such experiments, the two antiferromagnetic magnon modes with an opposite chirality give rise to rich electronics inspired phenomena, such as the magnon Hanle effect,[160,161] and pave the way for devices related to magnon topology.

Generally speaking, the experimental and unambiguous detection of magnon topology is very challenging[144] and further advances and new developments are needed to push into this direction. Potential ideas encompass spin-polarized scanning tunneling microscopy,[162] magnon interference tunneling spectroscopy,[163] and long-distance spin-qubit coupling.[164] Moreover, it is possible to utilize spin or thermal Hall transport experiments, which we discuss in Sec. VI.

It is important to highlight that spin textures in real space can also give rise to magnon topology. Such spin textures can be, for example, topological, regular skyrmion lattices[126,165,166] but also artificial magnonic crystals.[123,129,167] An important experimental perspective is the engineering and control of magnon topology down to ultrafast timescales. In this regard, tuning of spin textures by external parameters, such as the external magnetic field or strain, may allow for *in situ* control of magnon topology, which should be the focus of future experiments. Another important aspect is the investigation of magnon topology away from thermal equilibrium and novel functionalities generated by non-equilibrium states in regard to magnon topology guided by theoretical predictions.[168–171]

VI. TOPOLOGY AND THERMAL DEGREE OF FREEDOM

Topology also plays a prominent role in the interaction between spin and heat degrees of freedom, which is exploited in the field of spin caloritronics.[172] An excellent illustration of this is the anomalous Nernst effect (ANE), which is the thermal counterpart to the anomalous Hall effect. In the ANE, a transversal voltage is induced by a thermal gradient rather than an electrical voltage. Initially, the ANE was thought to occur exclusively in ferromagnetic materials. Recent research has revealed its increase due to the non-trivial topology of the band structure in ferromagnetic Weyl semimetals.[173,174] In addition, its presence in several non-collinear antiferromagnets[71,175–177] is also shown in Fig. 6. The identification of intrinsic contributions associated with the topology of the band structure has sparked a renewed interest in the anomalous Nernst effect.[178] This growing attention underscores the significance of topology in comprehending and harnessing the anomalous Nernst effect. The advantage of the spin-caloritronic approach is that thermal gradients can be easily applied in various geometries with respect to the Néel vector of an antiferromagnet as illustrated in Fig. 6. The thermal gradient can also be applied only locally, which enabled us to employ it as a magnetic microscopy tool.[73,176]

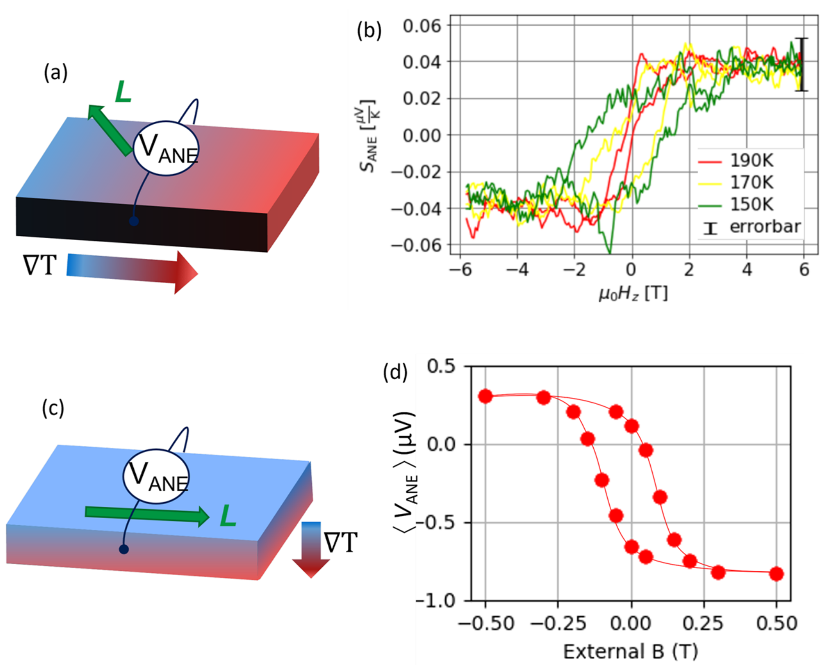

FIG. 6. Example of the spin-caloritronic phenomena in topological antiferromagnets. Various geometries of the thermal gradient allow for the detection of anomalous Nernst effect in materials with various magnetic anisotropies—for example, Mn_3Sn with the Néel vector in the sample plane or Mn_3NiN with a component of the Néel vector in the out-of-sample-plane geometry. (a) Thermal gradient applied in the sample plane, Neel vector has out-of-plane component and the anomalous Nernst signal is detected.

Furthermore, there are other magneto-thermo-electronic phenomena that have received less exploration. For instance, the application of a thermal gradient can break the chiral symmetry in topologically non-trivial materials, resulting in a thermal chiral anomaly. This phenomenon has been demonstrated in non-magnetic topological insulators[179] and in the topological compensated magnet GdPtBi.[88] Real-space topology can also give rise to a topological Nernst effect, which has been observed in various systems.[180]

In magnetic insulators, a temperature inbalance also causes a heat flow because charge-neutral quasiparticles (phonons, magnons, etc.) carry energy. Similar to electrons, magnons experience an anomalous velocity due to a Berry curvature in momentum space, leading to intrinsic transverse transport,[124,181–183] or scatter off magnetic textures with non-trivial real-space Berry curvature, leading to skew-scattering type transverse transport.[184,185] A net transverse particle flow is necessarily accompanied by a heat flow, leading to a thermal Hall effect,[124,182,183] which was measured for several ferromagnets[181,186,187] and explored in theory for antiferromagnets in a magnetic field.[188–190] Since the symmetry analysis of response tensors is agnostic to the microscopic carriers, the symmetry requirements for the electronic anomalous Hall effect also apply to the thermal Hall effect of magnons. Therefore, as long as the magnetic point group is compatible with ferromagnetism, a thermal Hall effect can be expected despite compensated order.[128] An example is again the compensated kagome antiferromagnets with vector chiral order. Their magnonic thermal Hall conductivity depends crucially on the spin rotation angle, a finding that could explain the experimental thermal Hall data of Ca and Cd kapellasite at low temperatures.[166,191]

The thermal Hall effect in magnetic insulators is a useful indicator of a magnon spectrum with broken time-reversal symmetry (which could support topological magnons). Nevertheless, it is experimentally difficult to unambiguously relate thermal Hall signals to a particular type of quasiparticle, because all charge-neutral quasiparticles respond to a temperature gradient, in particular phonons. While magnons are expected to reflect the time-reversal symmetry breaking of the magnetic order most directly, phonons also pick up signatures of this breaking due to spin–lattice coupling. Consequently, in addition to an all-magnon contribution to the thermal Hall effect, magnon–phonon hybridization[192–199] and magnon–phonon scattering[200,201] are expected to play a major role, potentially overshadowing any contribution from a magnon Berry curvature. Moreover, recent experimental results show a discrepancy between the detected thermal Hall conductivities[202,203] and those predicted by theory,[204,205] with the theory based on intrinsic Berry curvature effects overestimating the Hall-type transport. The main shortcoming of the Berry curvature theory is its restriction to the free-particle limit, where many-body interactions between quasiparticles are neglected. The validity of the free-particle approximation must be evaluated on a case-by-case basis, but, in general, it becomes questionable at higher temperatures when the thermally induced quasiparticle density grows. While first steps toward an inclusion of many-body effects have been taken,[201] a rigorous many-body theory of the thermal Hall effect remains to be developed.

Due to the finite anomalous velocity arising from the Berry curvature, magnons are also expected to exhibit a spin Nernst effect,[182] which is a transverse spin current as a response to an applied temperature gradient. Importantly, the magnonic spin Nernst effect can occur in compensated textures that do not support a thermal Hall effect. A simple material example is collinear antiferromagnets on the honeycomb lattice, whose magnon spin-Berry curvature gives rise to a finite spin Nernst signal, as predicted theoretically in Refs. 206 and 207. Experimental data consistent with a spin Nernst effect in $MnPS_3$ was presented in Ref. 208. The microscopic picture of the magnonic spin Nernst effect in collinear antiferromagnets is that the two opposite magnon spin species get deflected in opposite transverse directions. Therefore, while there is no net transverse particle current, there is a transverse spin current. Magnons in non-collinear compensated textures are proposed to exhibit particularly rich transverse spin transport phenomena, both intrinsic due to a spin Berry curvature[209] and extrinsic due to scattering[210] and Ref. 211 (magnetic spin Nernst effect). A magnon spin Nernst effect in antiferromagnets may also arise from spin-dependent skew scattering at antiferromagnetic skyrmion defects[212] and from magnetoelastic coupling.[196,213,214] Additional experimental confirmation of the magnonic spin Nernst effect is still pending.

VII. OUTLOOK

The interplay between topology and compensated magnetic order encompasses a vast research area, with numerous unresolved questions. One of the most prominent challenges is to push the quantum anomalous Hall effect to higher temperatures, surpassing the reliance on temperatures below 4.2 K (liquid helium), since this advancement would significantly enhance a chance for a practical application. However, there are numerous other tasks that lie ahead of us. From a fundamental perspective, there is a need for a deeper understanding of the connections between the real space and *k*-space topologies. This understanding is intrinsically linked to the development of a unified theory that can encompass both the anomalous Hall effect and the topological Hall effect. When considering the inclusion of thermal degrees of freedom, a natural question arises: can a similar unified theory be applicable to both the anomalous Nernst effect and the topological Nernst effect in topologically non-trivial materials?

Another open issue pertains to the general symmetry constraints of ANE and AHE, which do not necessarily need to be the same. This is due to the fact that the magnetothermal transport tensor and the conductivity tensor transform differently.[215,216] However, it is worth noting that experimental evidence for a material exhibiting the ANE without the AHE remains elusive. Similarly, the ANE could, in principle, reflect topological features in the band structure near the Fermi level that are absent in the AHE.[217]

An unambiguous way to connect THE-like features in magneto-transport experiments and topological structures, such as skyrmions, would be the simultaneous measurement of both. One way to achieve this could be *in situ* transport experiments in a Lorentz transmission electron microscope.[218] Currently, many researchers rely on the alignment between THE data and micromagnetic simulations, which not only requires the knowledge of numerous material parameters. In addition, a correlation of magnetotransport data with imaging techniques is problematic if not obtained from identical samples. The contentious issue of subtracting transport and magnetometry data can be partially mitigated by measuring both THE and its thermal counterpart, the topological

Nernst effect, in a single device.[180] However, this approach relies on the measurability of the topological Nernst effect and entails increased experimental effort.

Visualizing any solitons on compensated magnets poses a significant challenge, as many techniques that are effective for studying ferromagnets are not applicable in this case. Although there has been progress in this direction,[219] further work is needed because compensated magnetic order is believed to play a vital role in the formation of exotic skyrmion states, such as quantum skyrmions.[220] To image spin textures in compensated magnets, advanced magnetic imaging techniques must be employed, and the spin-caloritronic approach can be a helpful tool. However, there is still much work to be performed. First, we need to comprehensively describe the spin-caloritronic phenomena in these materials. The magnetic microscopy based on spin-caloritronic effects was so far utilizing mostly the anomalous Nernst effect, but many other phenomena may be used for spatial imaging of the magnetic properties. Different symmetry of these effects measured locally can then be used to disentangle various contributions. The spatial resolution of the scanning thermal gradient microscopy will be required to use a more localized thermal gradient, such as scanning near-field optical microscope (SNOM)[221] or a heated AFM tip.[222]

When it comes to exploring new materials, it would be intriguing to investigate alternatives to orthorhombic CuMnAs, where the topological properties can be tuned by the magnetic order. Another exciting path is magnetic van der Waals systems where the electronic properties can be tuned by dimensionality and twisted stacking. Can the number of layers control the topology of the electronic bands? Such topological magnetic van der Waals systems might be a great playground to explore.

An entirely unexplored realm lies in the interplay between the real-space and *k*-space topologies in the context of altermagnetism, which is an emerging class of compensated magnets.[13,223–227] In altermagnets, the non-magnetic atoms at non-centrosymmetric positions in the crystal structure lead to a breaking of time reversal symmetry in the band structure. These materials exhibit anisotropic alternating spin splitting in both momentum and real space. The interplay between the *k*-space topology and altermagnetism is discussed in Ref. 228, with a particular emphasis on the presence of spin-degenerate nodal surfaces that may become Berry curvature hotspots when spin–orbit coupling is considered. Great advantage of altermagnets is that techniques such as magneto-optical Kerr effect (MOKE), anomalous Nernst effect microscopy, and other methods that were previously believed to be absent in collinear magnets could potentially be used to visualize altermagnets.

As far as magnons are concerned, further research on topological magnons in compensated magnets is highly desired because the compensated magnetic insulators are prime candidates for fast and dissipation free magnonics. Besides the identification of novel topological magnon materials[229,230] and their unique signatures in experiments, current research is aimed at understanding the effects of many-body interactions, which are very different from electronic systems. For example, recent theoretical work has uncovered the important role of non-number preserving interactions at zero temperature.[231–236] Although non-collinear antiferromagnets can support topological chiral edge magnons,[128] the noncollinearity leads to detrimental spontaneous magnon decays.[237] This generic many-body phenomenon is not suppressed by non-trivial free-particle topology,[231] and chiral edge magnons will decay.[238] Although magnons in collinear compensated magnets will also suffer from decays, the decay rates are expected to be smaller than in non-collinear magnets because they result from higher-order interaction processes. Consequently, collinear magnetism could be a way to stabilize topological magnons. Thus, altermagnets are a promising platform to combine collinearity (for more stable magnons), a compensated moment (for high integration and fast dynamics), and chiral edge states (for topological transport and backscattering immunity). However, magnon topology and thermal Hall responses in altermagnets have not been addressed so far.

The first predictions regarding electronic thermal Hall effects and the anomalous Nernst effect in metallic altermagnets have only emerged very recently.[239] To date, there have been no reports regarding these effects in insulating altermagnets. From an experimental standpoint, it is imperative to employ enhanced techniques for detecting both the anomalous Nernst effect[180] and thermal Hall effects.[240]

In summary, we discussed magneto-thermal transport phenomena as well as excitations in compensated magnetic systems. In particular, the many intriguing open questions regarding the interaction between compensated magnetism and topology shall encourage researchers to further explore this exciting and rapidly evolving field with a potential impact on both fundamental research and technological advancements.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Helena Reichlova: Conceptualization (equal); Writing – original draft (equal). **Dominik Kriegner**: Conceptualization (equal); Writing – original draft (equal). **Alexander Mook**: Conceptualization (equal); Writing – original draft (equal). **Matthias Althammer**: Conceptualization (equal); Writing – original draft (equal). **Andy Thomas**: Conceptualization (equal); Writing – original draft (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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