Accurate Derivation of Lossy Equivalent Circuit of Waveguide External Coupling

***Abstract* — A novel technique to derive the lossy equivalent circuit of waveguide external couplings of narrowband cavity filters with arbitrary cavity cross section and arbitrary coupling geometry is presented in this contribution. The technique makes use of a reduced CAD model to obtain the element values of the equivalent lossy circuit. Therefore the contribution of the external coupling structures to the total dissipated power and total stored energy of the filter can be derived and a clear separation between resonators and couplings is established. A first degree filter has been designed, simulated, and manufactured. Its lossy equivalent circuit has been extracted and comparisons with simulation and measurement show excellent agreement.**

***Index Terms* — Band pass filters, circuit optimization, coupling circuits, microwave filters, passive circuits**

I. INTRODUCTION

Today most demanding applications require microwave components capable of supporting more and more power with very low loss dissipation.

Commonly, only canonical shapes such as rectangular and circular geometries are used in the design of waveguide filters. However, with the advent of the new 3D manufacturing techniques and the maturing of the traditional ones novel freedom is available at low cost in the choice of the internal geometry and to apply geometry optimization techniques to reduce loss dissipation.

However, full-wave geometry optimization of a complete waveguide filter is not feasible due to the extreme complex geometries expected to be obtained and the very long simulation time entailed to it. Instead, cavities and coupling structures can be individually optimized and then combined to yield a final improved filter RF response. To that end, a suitable segmentation to find the loss and energy storage contributions of the different cavities and coupling structures in a filter is needed.

This paper is focused on the derivation of a lossy equivalent circuit of external waveguide couplings for the characterisation of the loss contribution in the coupling apertures to the overall dissipation losses.

Previous publications [1]-[3] centered their work in the characterization of the external coupling with the goal to derive the unloaded Q factor of the associated cavity. In [1] and [2] a lossy equivalent circuit is proposed of an external coupling, but the loading of cavity due to the coupling structure was not considered. Thus, the coupling aperture needed to be small so frequency and unloaded Q of the cavity were not strongly affected. Other workers [3] overcome this limitation, but the coupling aperture is assumed to be lossless. On the other hand, Miraftab and Yu [4] have proposed an equivalent circuit of lossy couplings based on [5]. However, the paper is centered on advanced synthesis techniques and their goal is to characterize lossy couplings obtained by predistortion techniques. Losses due to material finite conductivity are neglected.

The different and novel formulation presented here allows to obtain an equivalent circuit of external coupling including losses due to material finite conductivity and loading of the cavity due to the coupling aperture. The technique described here can be applied to any arbitrarily shaped coupling structure and waveguide cross section. Therefore, the value of the approach presented is that it can be used to optimize the shapes of external couplings individually, with the aim at reducing the loss contribution of these elements in a given filter structure.

Measured and simulated results of a first degree filter confirm the excellent accuracy obtained with the new equivalent circuit for the loss prediction of practical waveguide external couplings.

II. EQUIVALENT CIRCUIT MODEL

The proposed model for the derivation of the equivalent circuit of external couplings consists of a single resonator coupled to the input feed source and output load as shown in Fig. 1 where a symmetry plane SS' along the direction of propagation can be observed. The corresponding equivalent circuit is that of Fig. 2, where one half of the circuit has been omitted due to the symmetry of the model.

The quantity l_{EXT} represents the input line, the coupling is defined by the circuit between the planes AA' to BB', and l_{RES} is the halved resonator by the symmetry plane. $Z_S = R_S + jX_S$ and $Z_P = R_P + jX_P$ are the four unknowns to be found. $Z_{in,TL}$, Z_{in} and Z_L are the impedances seen along the circuit whereas Z_C and γ_C are the characteristic impedance and propagation constant of the line. If canonical shapes such as rectangular or circular geometries are used as input line and resonator, Z_C and γ_C can be analytically calculated as in [5]. However, they can be directly extracted from full-wave simulation when no analytical expressions are available (for arbitrary cross section geometries). The impedance at the symmetry plane can either be $Z_{SS'} = 0$ or $Z_{SS'} = \infty$ depending on whether an electric (odd mode) or a magnetic (even mode) wall is used, respectively.

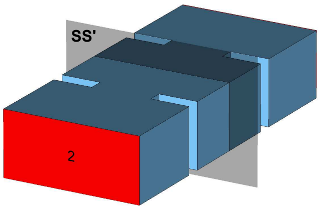

Fig. 1. Complete waveguide external coupling CAD model.

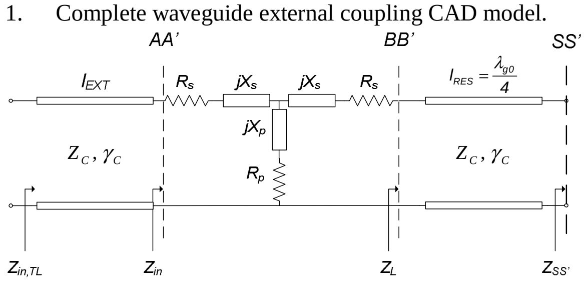

Fig. 2. Suggested lossy equivalent circuit of waveguide external coupling.

The procedure to find the unknowns of the coupling circuit can be divided in three stages: first, the effects of the input line must be de-embedded; second, an analytical equation for the input impedance from the AA' plane must be found; and third, the input impedance of the model at two frequency points must be calculated from full-wave and used as solutions for the analytical equation to find the unknowns.

To de-embed the effects of the feeding line, its length must be known. The length given in the waveguide model cannot be used due to the loading effects of the waveguide coupling over the line. Hence the corrected length of the external line will be derived using the phase of the S_{11} parameter. A signal traveling through the circuit in Fig. 2 will have a total phase shift of

$$\varphi(S_{11}) = 2\varphi(l_{EXT}) + 2\varphi(coupling) + \\ +2\varphi\left(\frac{\lambda_{g0}}{4}\, resonator\right) + \varphi(SS') = 2\varphi(l_{EXT}) + \varphi(SS') \quad (1)$$

since the phase shift introduced by the coupling and the halved resonator must be $\pi/2$ rad and $-\pi/2$ rad, respectively, if low loss is assumed. $\varphi(SS')$ is the phase shift of an electric (180º) or a magnetic wall (0º). The length of the external line can then be calculated as

$$l_{EXT} = \frac{\varphi(l_{EXT})}{\beta_0} = \frac{\lambda_{g0}}{2\pi}\varphi(l_{EXT}) = \frac{\lambda_{g0}}{2\pi}\frac{\varphi(S_{11}) - \varphi(SS')}{2} \quad (2)$$

where $\varphi(S_{11})$ is the unwrapped phase of the reflection parameter from the full-wave simulation.

The analytical input impedance of the circuit in Fig. 2 can be calculated as

$$z_{in} = R_S + R_p + j(X_S + X_p) - \\ - \frac{(R_p + jX_p)^2}{R_S + R_p + j(X_S + X_p) + \mathrm{Re}(z_L) + j\,\mathrm{Im}(z_L)} \quad (3)$$

where R_S, X_S, R_p and X_p are the four unknowns to be found, and z_{in} is a normalized impedance that can be obtained by standard transmission line impedance transformations [5] as

$$z_{in} = \frac{Z_{in}}{Z_C} = \frac{Z_{in,TL} - Z_C \tanh(\gamma_C l_{EXT})}{Z_C - Z_{in,TL} \tanh(\gamma_C l_{EXT})} \quad (4)$$

where $Z_{in,TL}$ is the input impedance of the CAD model in Fig. 1 calculated from full-wave simulation.

If an electric wall is placed at the symmetry plane SS', z_L can be obtained as in (5), whereas if a magnetic wall is placed instead, z_L is obtained as in (6).

$$z_L = \frac{Z_L}{Z_C} = \tanh(\gamma_C l) \quad (5)$$

$$z_L = \frac{Z_L}{Z_C} = \frac{1}{\tanh(\gamma_C l)} \quad (6)$$

To find the equivalent circuit of the coupling, three approaches are discussed in this work. The first approach only uses the information of the model in Fig. 1 when an electric wall is placed at SS'. Likewise, for the second approach, the equivalent circuit is obtained only with the information of the model in Fig. 1 when a magnetic wall is placed at SS'. Finally, the third approach computes the equivalent circuit of the external coupling from the model in Fig. 1 when both, an electric and a magnetic wall, are used.

For the first two cases, a system of equations can be obtained combining (3) with (5) or (6) depending on whether an electric or a magnetic wall is being used in the model, respectively. For one frequency point, two equations can be obtained from (3): $\mathrm{Re}(z_{in})$ and $\mathrm{Im}(z_{in})$. Hence, at least two frequency points are needed to find the four unknowns. Two equations, (10) and (11), will be extracted from the center frequency (ω_0) and two more equations, (12) and (13), from a frequency point (ω_S) near resonance.

For the third approach, the input impedance (3) of the circuit in Fig. 2 must be obtained when an electric wall and a magnetic wall are placed at SS'. Combining these input impedances, the impedance parameters of the complete model with no symmetry planes are derived as

$$z_{11} = \frac{z_{in,MW} + z_{in,EW}}{2} \quad (1.7)$$

$$z_{21} = \frac{z_{in,MW} - z_{in,EW}}{2} \quad (1.8)$$

and its input impedance as

$$z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + 1} \tag{1.9}$$

Again, two equations can be obtained from (1.9): the real and the imaginary parts. Therefore, two frequency points are also needed in this approach to find the four unknowns.

In all cases, the second frequency point should be chosen near resonance, where the frequency dependence of the waveguide coupling model and the equivalent circuit are very similar. The nonlinear system of four equations and four unknowns can be finally solved numerically.

In lumped elements circuits, the even and odd mode theory for symmetric circuits must be fully applied since information about the whole circuit is shared between the modes. However, when the even and odd mode theory is applied in a lossy transmission line, both modes have information about the whole circuit and thus, they can be studied separately. However, for this application it is expected that the odd mode will not yield the right coupling element values since $Z_{SS'} = 0$ and $z_L \gg Z_S$ and therefore, Z_S is *hidden* in z_L and the system does not converge to the right solution. This is not the case for the even mode, where $Z_{SS'} = \infty$ and $z_L \sim Z_S$ and a valid solution is expected to be found. A valid solution is also expected if the even and odd mode theory is applied completely.

III. Experimental Results

A first degree filter shown in Fig. 3 in WR187 waveguide with a center frequency of 4.5 GHz and with -10 dB couplings has been designed and manufactured to verify the technique.

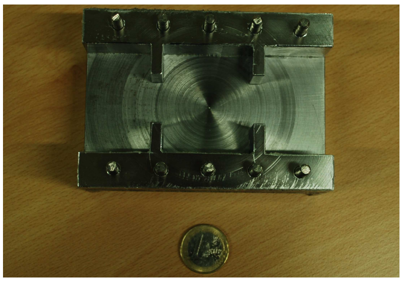

Fig. 3. Manufactured prototype

The material used in the fabrication was a steel alloy with unknown electrical properties. The electrical conductivity in the simulation was optimized to match the center insertion loss of the measured data. The equivalent circuit was then obtained using the three outlined procedures.

Table 1 lists all the information extracted from the full-wave simulation where $f_S = f_0 + 10$ MHz ($f_S/f_0 = 1.0022$) has been chosen. The resulting circuit values when only the even mode is used are collected in Table 2.

TABLE 1
EXTERNAL COUPLING PARAMETERS

$Z_{in,TL,even}(\omega_0)$	*277.48 + j357.26 Ω*
$Z_{in,TL,even}(\omega_S)$	*426.81 + j585.17 Ω*
$Z_{in,TL,odd}(\omega_0)$	*14.57 - j620.65 Ω*
$Z_{in,TL,odd}(\omega_S)$	*14.20 - j606.60 Ω*
$\varphi(S_{11,even}(\omega_0))$	*-258.785º*
$\varphi(S_{11,odd}(\omega_0))$	*-80.835054º*

$$\mathrm{Re}\left(z_{in}\big|_{\omega_0}\right) = R_1 - \frac{\left(R_p^2 - X_p^2\right)\cdot\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_0}\right)\right) + 2\cdot R_p\cdot X_p\cdot\left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_0}\right)\right)}{\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_0}\right)\right)^2 + \left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_0}\right)\right)^2} \tag{10}$$

$$\mathrm{Im}\left(z_{in}\big|_{\omega_0}\right) = X_1 + \frac{\left(R_p^2 - X_p^2\right)\cdot\left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_0}\right)\right) - 2\cdot R_p\cdot X_p\cdot\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_0}\right)\right)}{\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_0}\right)\right)^2 + \left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_0}\right)\right)^2} \tag{11}$$

$$\mathrm{Re}\left(z_{in}\big|_{\omega_S}\right) = R_1 - \frac{\left(R_p^2 - X_p^2\right)\cdot\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_S}\right)\right) + 2\cdot R_p\cdot X_p\cdot\left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_S}\right)\right)}{\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_S}\right)\right)^2 + \left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_S}\right)\right)^2} \tag{12}$$

$$\mathrm{Im}\left(z_{in}\big|_{\omega_S}\right) = X_1 + \frac{\left(R_p^2 - X_p^2\right)\cdot\left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_S}\right)\right) - 2\cdot R_p\cdot X_p\cdot\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_S}\right)\right)}{\left(R_1 + \mathrm{Re}\left(z_L\big|_{\omega_S}\right)\right)^2 + \left(X_1 + \mathrm{Im}\left(z_L\big|_{\omega_S}\right)\right)^2} \tag{13}$$

TABLE 2
FINAL EQUIVALENT LOSSY CIRCUIT VALUES FROM EVEN MODE

Section	Parameter	Value
TL Resonator	*Length*	$\lambda_{g0}/2$ *m*
	$Z_0(\omega_0)$	$1\ \Omega$
	$\alpha(\omega_0)$	$241\cdot10^{-3}$ *Nepers/m*
	$\beta(\omega_0)$	*67.147 rad/s*
Coupling 1	R_s	*0.2889 mΩ*
	L_s	*-0.1696 H·rad*
	R_p	*2.882 mΩ*
	L_p	*0.1634 H·rad*

Fig. 4 and Fig. 5 show the full-wave simulation, the measured data and the response from the equivalent circuit obtained using the three different approaches. As expected, a good convergence is not observed when only an electric wall is used to obtain the input impedance of the model under study. On the other hand, excellent agreement between the full-wave simulation, the response from the equivalent circuit, and the measured data is obtained both in magnitude and in phase when the other two approaches are used (only even mode and full even-odd mode).

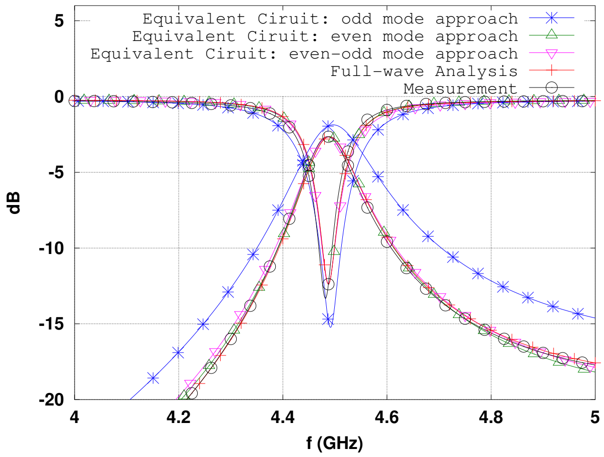

Fig. 4. Equivalent circuit, simulated and measured RF responses in magnitude

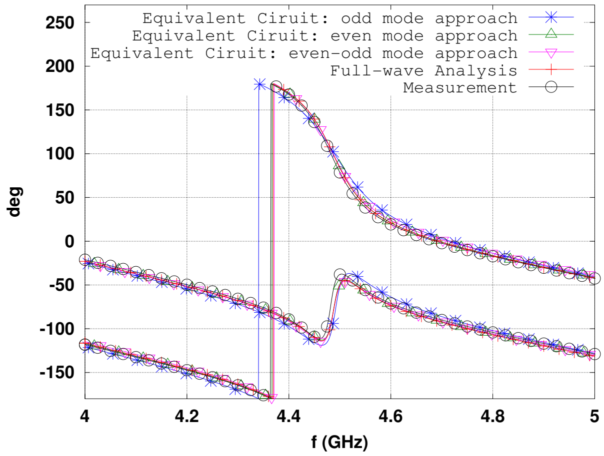

Fig. 5. Equivalent circuit, simulated and measured RF responses in phase

A study on how the second frequency point affect the accuracy of the results has been carried out for the second approach (only even mode) and it is summarized in Table 3. Good results are obtained when $f_S \leq f_0 \pm 50$ MHz ($f_S/f_0 = 1 \pm .0111$).

TABLE 3
EXTRACTED PARAMETERS FROM SIMULATION, MEASUREMENT AND EQUIVALENT CIRCUIT FOR DIFFERENT F_S

Data from	f0 (GHz)	BW (MHz)	IL (dB)	GD (ns)
Simulation	*4.4896*	*103.2*	*-2.649*	*3.437*
Mesaurement	*4.485*	*103.75*	*-2.649*	*3.493*
EC ($f_S = f_0 + 150$ *MHz*)	*4.4904*	*117.6*	*-2.655*	*3.051*
EC ($f_S = f_0 + 50$ *MHz*)	*4.4898*	*108.6*	*-2.647*	*3.287*
EC ($f_S = f_0 + 10$ *MHz*)	*4.4898*	*104.4*	*-2.643*	*3.395*
EC ($f_S = f_0 + 2$ *MHz*)	*4.4898*	*103.8*	*-2.643*	*3.418*
EC ($f_S = f_0 - 2$ *MHz*)	*4.4898*	*103.2*	*-2.642*	*3.430*
EC ($f_S = f_0 - 10$ *MHz*)	*4.4898*	*102.6*	*-2.642*	*3.453*
EC ($f_S = f_0 - 50$ *MHz*)	*4.4898*	*99*	*-2.637*	*3.574*
EC ($f_S = f_0 - 150$ *MHz*)	*4.4898*	*89.4*	*-2.625*	*3.924*

VII. CONCLUSION

A technique to derive the equivalent circuit of a lossy external coupling has been presented. A first degree filter has been designed and manufactured. Excellent agreement is reported between the simulated full-wave response, its derived equivalent circuit response and the measured data. The technique was found robust for a wide range of second frequency points (f_S). The theory is currently being used to optimize the shape of external couplings in order to reduce losses in waveguide filters.

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