



**AALBORG UNIVERSITY**  
DENMARK

**Aalborg Universitet**

## **On model reduction of hybrid systems**

Shaker, Hamid Reza; Wisniewski, Rafal

*Publication date:*  
2009

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Shaker, H. R., & Wisniewski, R. (2009). *On model reduction of hybrid systems*. Abstract from CICADA workshop on Hybrid Systems and Model Reduction, Manchester, United Kingdom.  
[http://www.mims.manchester.ac.uk/events/workshops/HSMoR09/HSMoR09\\_files/HSMoR09.pdf](http://www.mims.manchester.ac.uk/events/workshops/HSMoR09/HSMoR09_files/HSMoR09.pdf)

### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### **Take down policy**

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# ON MODEL REDUCTION OF HYBRID SYSTEMS

H. R. Shaker, R. Wisniewski

Hybrid Systems Group, Section for Automation and Control, Aalborg University, Aalborg, Denmark

The ever-increasing need for accurate mathematical modelling of physical as well as artificial processes for simulation and control leads to models of high complexity. The complexity appears as high order describing dynamical system or complex nonlinear structure. This problem demands efficient computational prototyping tools to replace such complex models by approximate simpler models, which are capable of capturing dynamical behaviour and preserving essential properties of the complex one. Due to this fact model reduction methods have become increasingly popular over the last two decades. On the other hand, most of the methods that are proposed so far for control and analysis of hybrid and switched systems are suffering from high computational burden when dealing with large-scale dynamical systems. Because of the weakness of nonlinear model reduction techniques and due to pressing needs for efficient analysis and control of large-scale dynamical hybrid and switched systems; it is essential to study model reduction of hybrid and switched systems in particular. This fact has motivated the researchers in hybrid systems to study model reduction. To the best of our knowledge just few research works on model reduction of hybrid systems have been reported in the literature [1]-[4]. Therefore this problem is still largely open. We will start the talk with giving a brief walk-through the previous methods, their advantages and disadvantages concerning computations and conservatism and also from system and control theoretic technical viewpoint. We will proceed with the classification of hybrid and switched systems and we will mainly concentrate on switched systems with autonomous switching and state-based switched systems. The generalized gramian framework for model reduction of switched systems with autonomous switches will be presented then. This framework is based on the generalized gramians instead of gramians [5]. In order to compute the generalized gramians, one should solve Lyapunov inequalities instead of Lyapunov equations. This method is used to devise a technique for structure preserving model reduction methods in [6]. We first show that the generalized method in [5] can be extended to various gramian based reduction methods. We have modified the original method in [5] to avoid numerical instability and to achieve more numerical efficiency by building Petrov-Galerkin projection based on generalized gramians. A method based on the balanced model reduction within frequency bound will be presented in this framework. It is applied for model reduction of switched system by solving system of Lyapunov inequalities to find common generalized gramian. We show that the proposed framework for model reduction of switched system is stability preserving. The numerical results followed by a brief discussion on feasibility and error bound of the method will be presented. Latter, we will sketch a stability preserving framework which is based on local (generalized) gramians instead of common generalized gramian.

We will move to another category of switched systems for the rest of the talk. One of the most important classes of hybrid systems which has been studied extensively in the literature is a class of piecewise affine systems. It is very general and equivalent to other classes of hybrid systems e. g. mixed logical dynamical systems, linear complementary systems, and maxmin-plus-scaling systems. To our knowledge the only available study in the context of reduction of affine systems in the literature is the work done by Habets and Schuppen [2] which has considered the problem of the exact reduction due to non-observability. Our presented work is a generalization of the method in [2]. It is easy to show that in our method if we restrict our attention just to reduction due to non-observability the method also provides the same results as [2]. The technique which will be presented is based on the transformation of affine dynamical systems inside the cells to a new structure and it can be applied to both exact reduction and also approximate model reduction. The information regarding local input-output behaviour and also switching information are embedded in this structure. In this framework both controllability and observability of the affine system within the polytopes are considered for reduction purpose. Numerical results show that in general for exact reduction this framework works well but in the case of approximate reduction some other issues should be taken into account such as stability preservation. Although the accuracy of the method within the cell is quite well, depending on the dynamics outside of the cell it might happen that the approximation in the neighbourhood and outside of the cell is not satisfactory. These problems need further investigation and research to be done. We will conclude the talk with the direction for future research and some ideas for further modifications and generalizations.

## References:

- [1] E. Mazzi, A. S. Vincentelli, A. Balluchi, and A. Bicchi. "Hybrid system model reduction" IEEE Int. Conf. on Decision and Control, 2008.
- [2] Luc C.G.J.M. Habets and Jan H. van Schuppen. Reduction of affine systems on polytopes. In Proceedings of the International Symposium MTNS ,2002.
- [3] H. Gao, J. Lam, C. Wang "Model simplification for switched hybrid systems" Systems & Control Letters, Vol. 55, No. 12. 2006, pp. 1015-1021.
- [4] Y. Chahlaoui , "Model reduction of switched dynamical systems", 4<sup>th</sup> Conference on Trends in Applied mathematics in Tunisia, Algeria and Morocco., Morocco, Kenitra,2009.
- [5] G. E. Dullerud and E G. Paganini. A course in robust control theory : a convex approach. Springer, New York, 2000.
- [6] Li Li and F. Paganini, "Structured Coprime Factor Model Reduction based on LMIs", Automatica, Vol.41, pp:145-151, 2005.