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A solution to plane problems of micropolar elasticity

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A SOLUTION TO PLANE PROBLEMS OF MICROPOLAR ELASTICITY Analysis of the mechanical properties of engineering materials with microstructure generally requires modification of the concept of a simple material. A number of sophisticated averaging techniques have been developed to model these materials with microstructure within the framework of classical continuum mechanics. However, the nature of a discrete microstructure in some materials presents a significantproblem in modeling these materials as continua in certain situations. Of particular concern are those situations in which severe deformation occurs within highly localized regions of the material where the size scale of the deformation region is of the same order as that of the microstructure. One approach which recognizes specific microstructure is the theory of micropolar materials. In this model, a material point within the body undergoes the usual displacement from its original position, and in addition, the material point is assumed to undergo a rotation which is independent of the displacement.This model obviously encounters severe philosophical difficulties if considered as representative of a continuum. It does, however, reflect the mechanics of a discrete structure where, for example, the forces and moments in a beam connecting two masses requires information about the relative rotations as well as the displacements of the two masses. Under these conditions, a characteristic length is associated with the material, and the resulting stress and strain tensors are generally non-symmetric. In two-dimensional material models these microstructures are often represented by hexagonal and equilateral trianglegeometries which exhibit three-fold symmetry in the plane. For two-dimensional linearly elastic micropolar materials, this three-fold symmetry condition requires that both the stresses and couplestresses be isotropic in the plane. In this work we obtain a general solution to the field equations of plane micropolar elasticity in terms of three potential functions. Two of these functions are analytic, and the third function satisfies the modified homogeneous Helmholtz equation. Representations for the complex displacement vector, the complex stress combinations, the complex couple stress, the rotation, and the resultant complex force on a contour are presented in terms of these potential functions. Solutions for the infinite plane with either a stress and couple stress free or displacement and rotation free circular boundary and loaded in the far field by a uniaxial stress have been obtained. These circular boundary problems do not provide large displacement gradients and the micropolar effects are quite small. We have also investigated problems in which the loading induces large localized displacement gradients for which the micropolar effects are significant. These include the classical crack problem and the half space loaded uniformly over a finite length of the boundary.

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