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# Elastic-Plastic Stability and Imperfection Sensitivity of the Euler Column

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Over the years plastic postbuckling and imperfection sensitivity of the Euler column have been investigated by among others the founder of asymptotic plastic theory Hutchinson, see (Hutchinson 1974), and by Tvergaard & Needleman (1982).

In this study semi-asymptotic expansions for handling the elastic-plastic stability problems of the Euler column are developed. The method utilizes the fact that the equilibrium path associated with the lowest plastic bifurcation of a geometrically perfect realization of the structure eventually approaches the reduced modulus equilibrium. This is utilized to enhance Hutchinson's plastic asymptotic expansion (Hutchinson 1974) in advanced post-bifurcation by application of asymptotic trial functions which are better suited to comply with this fact than regular polynomials. An important advantage of this is that the reduced modulus solution may be determined more accurately over the range in which the plastic maximum load of the geometrically imperfect version of the column is located. As also the equilibrium of the geometrically imperfect structure approaches reduced modulus load equilibrium, a similar method is used to establish the equilibrium in that case.

The applicability of the method for the geometrically perfect case is demonstrated by three examples which all show excellent approximation of the postbuckling path in the area around maximum load. For one of these examples the asymptotic method for the imperfect case is used to determine equilibria of the geometrically imperfect Euler column. Good estimates of the maximum load were found for small to moderate imperfection amplitudes, while the point of initiating unloading was determined less accurately. However, when the enhanced method of predicting the equilibrium before linear elastic unloading initiates proposed by Christensen & Byskov (2008), we get excellent approximation of the entire equilibrium of the imperfect column even for large imperfection amplitudes.

## References

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