The Initial Stage of Neck Formation in an X-Pinch

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Abstract. A model is proposed to describe the initial stage of neck formation in an X-pinch that proceeds in three stages: the electrical explosion of metal wires that generates the X-pinch; the expansion of the wire material that occurs due to an excess of the gas-kinetic pressure over the pressure of the magnetic field. The model allows one to predict the minimum rate of current rise at which the formation of a "hot spot" in an X-pinch is possible. The minimum current rise rate is determined by the thermodynamic parameters of the wires at a critical point; it is of the order of 1 kA/ns.

1. Introduction

In the 80s of the last century, the method of production of high-temperature dense plasmas by electrically exploding crossed wires was proposed at P. N. Lebedev Physical Institute (Russia) [1]. As this version of wire load consists of wires crossed as if they form of the letter "X", it has received the name X-pinch. In X-pinch experiments, two or more metal wires 10–50 μm in diameter are used, through which a current pulse of amplitude from 100 kA to several megaamperes with a rise time of 100–500 ns is passed [2–7]. In the wire crossing region, a plasma hot spot of several micrometers in size forms, which lifetime is about 1 ns. The parameters of this type of X-ray sources make them irreplaceable for use in projection radiography of various short-lived physical objects. In particular, we used X-pinches in studying the thermal instabilities [8–10] that develop in electrically exploded conductors.

Notwithstanding that X-pinches and their applications have been the topic of a rather large body of research, not nearly all features of X-pinches are now well studied. In particular, the criterion for the rise rate of the current passing through an X-pinch that was derived from experimental data [4],

$$\frac{dI}{dt} > (1-1{,}5)\,\frac{kA}{ns}, \qquad (1)$$

has not yet been substantiated. This criterion is very important, as in the experiments where the current rise rate was lower than that obeying (3), no hot spot formed irrespective of the current amplitude and, hence, a bright radiation source was not produced.

2. The Initial stage of Neck Formation

Let us consider the processes that occur in an X-pinch produced by the explosion of two or more crossed wires. In this case, the mass per unit length of the X-pinch is given by the formula $m_0 = \frac{Nm_w}{\cos\varphi}$, where N is the number of wires, φ is the angle of inclination of the wires to the X-pinch axis, m_w is the mass per unit length of an individual wire, r_0 is the initial radius of a wire, ρ_0 is the density of the wire metal under normal conditions, and φ is the angle of inclination of a wire to the z-axis. For analysis, it is convenient to use the effective initial radius of a wire $R_0 = \sqrt{\frac{m_0}{\pi\rho_0}}$.

The process of formation of a neck in an X-pinch is assumed to proceed in three stages.

Stage I is the electrical explosion of the wires. At this stage, the wires melt and further heat up to the point where they cease to exhibit metallic conduction [11,12]. Denote the duration of this stage by t_1.

Stage II is the expansion of the wire material caused by the excess of the gas-kinetic pressure over the magnetic field pressure immediately after the explosion. The expansion proceeds until the magnetic pressure becomes higher than the gas-kinetic pressure due to the current rise [13]. Denote the duration of this stage, from the explosion to the cessation of expansion, by t_2.

Stage III is the compression of the wire material which is accompanied by the outflow of matter from the neck region. At the final stage of the neck formation, a hot spot forms whose dimensions are less than the length of the neck [3]. The radiation pulse arises at the hot spot when the mass is almost completely lost. Denote the duration of this stage, from the onset of compression to the occurrence of a radiation pulse, by t_3.

The time of the overall process, from the onset of current passage to the occurrence of an X-ray pulse, is the sum of the times of these stages: $t_f = t_1 + t_2 + t_3$.

Let us consider details of the first two stages.

Stage I, the electrical explosion of the wires

During the electrical explosion the wire material passes through all phases from a condensed state to plasma. The explosion of the metal occurs near a critical point [11] at which the magnetic pressure fails to restrain the expanding material. The critical point corresponds to the point of the phase diagram at which the lines separating the liquid, the gas, and the two-phase region meet. The parameters of the material at the critical point (the temperature T_{cr}, the pressure p_{cr}, and the density ρ_{cr}) for some metals are given in Table I.

The explosion time can be defined in terms of a quantity, called the integral of the specific action of current, as [14, 15]

$$h = \int_0^{t_1} j^2(t)dt , \qquad (2)$$

where $j = \frac{I}{S}$ is the current density in a wire, I is the current flowing through the pinch, and $S = N\pi r_0^2 = \frac{m_0}{\rho_0}\cos\varphi$ is the total cross-sectional area of the wires. Values of the integral of specific current action for some metals are given in Table I.

For a linear rate of current rise, the time dependence of the current density can be described as $j(t)=\frac{\rho_0}{m_0\cos\varphi}\frac{dI}{dt}t$. Substituting this expression into the integral (2), we obtain an estimate for the explosion time:

$$t_1=\left[\frac{3hm_0^2\cos^2\varphi}{\rho_0^2}\left(\frac{dI}{dt}\right)^{-2}\right]^{1/3}. \tag{3}$$

As the explosion of a wire occurs near the critical point and the density at the critical point, ρ_{cr} , is a factor of 3–4 less than the normal density of the metal, ρ_0 , the radial dimension of the wire increases about twofold by the time of its explosion.

According to (3), for the typical parameters of X-pinches (the effective wire radius $R_0 = 2\times10^{-3}$ cm, $\cos\varphi=35^0$ and the current rise rate $\frac{dI}{dt}=1\frac{kA}{ns}$) the explosion time is estimated as $t_1\approx$ 5.3 ns for Fe, 5.7 ns for Au and Al, 5.8 ns for W, and 7.5 ns for Cu.

Stage II, the expansion of the wires

Let us estimate the wire expansion time. This quantity can be estimated only roughly because of the complexity of the processes occurring at this stage. The expansion starts after the explosion of the wire; that is, at the time t_1, and stops at the time t_1+t_2 when the magnetic field pressure becomes higher than the gas-kinetic pressure. As the explosion occurs near the critical point, the parameters of the wire material during expansion can be characterized by its thermodynamic parameters at the critical point. Therefore, the condition for the wire material to stop expanding can be written as

$$\frac{B^2(t)}{8\pi}\approx ap_{cr}, \tag{4}$$

where $B(t)=\frac{2I(t)}{cR(t)}$ is the magnetic field at the boundary of the wire, R (t) is the effective wire radius, c is the velocity of light in vacuum, a > 1 is the dimensionless coefficient (hereinafter, a = 2). As mentioned, the pinch radius at the wire crossing by the time of explosion is about $2R_0$; therefore, the time dependence of the wire radius can be approximated by the relation $R(t)\approx 2R_0+vt$, where v is the velocity of expansion of the wire. The characteristic expansion velocity can be estimated as the thermal velocity of atoms at a temperature equal to the temperature of the metal at the critical point; that is, as $v\approx\sqrt{\frac{2kT_{cr}}{m_i}}$, where k is Boltzmann's constant, m_i is the atomic mass, and T_{cr} is the temperature at the critical point.

Substituting the expressions for the wire radius and for the magnetic field at the wire boundary into (4), in view of , we obtain an estimate for the wire expansion time:

$$t_2\approx\frac{4cR_0\sqrt{\pi p_{cr}}-t_1\frac{dI}{dt}}{\frac{dI}{dt}-\left(\frac{dI}{dt}\right)_{\min}}, \tag{5}$$

where $\left(\frac{dI}{dt}\right)_{\min}=\sqrt{\frac{8\pi c^2p_{cr}kT_{cr}}{m_i}}$.

As it follows from the expression (5), for the current rise rate equal to $\left(\frac{dI}{dt}\right)_{\min}$ the wire expansion time tends to infinity; hence, this value can be used as an estimate of the minimum current rise rate at which the stage of expansion can go to the stage of compression. The value of $\left(\frac{dI}{dt}\right)_{\min}$ is determined only by the parameters of the metal; the values of $\left(\frac{dI}{dt}\right)_{\min}$ for some metals are given in Table I. The values of this quantity for all metals appear to be close to each other because of the closeness of the critical pressures and temperatures. As it can be seen from Table I, to satisfy the criterion (1); that is, to create conditions for the formation of a neck in an X-pinch, the rate of current rise should be greater than $\left(\frac{dI}{dt}\right)_{\min}$, and the criterion (1) can be written as

$$\frac{dI}{dt} > (2-3)\cdot\left(\frac{dI}{dt}\right)_{\min} . \tag{1a}$$

As the rate of current rise in X-pinches is always greater than $\left(\frac{dI}{dt}\right)_{\min}$, expression (5) can be rewritten as

$$t_1 + t_2 \approx 4cR_0\sqrt{\pi\rho_{cr}}\left(\frac{dI}{dt}\right)^{-1} . \tag{6}$$

Table I. Parameters of metals [14].

Article I. Metal	Article I u	Article I u	Article l	Ag	Ni	Fe	W	Ti	Mo	Zn
h, 10^9 $A^2 \cdot s/cm^4$	4.1	1.8	1.8	2.8	1.9	1.4	1.85[14]	0.8	–	–
ρ_{cr}, g/cm^3	2.39	5.68	0.64	2.93	2.19	2.03	4.85[16]	1.13 [16]	3.18	2.29
T_{cr}, eV	0.72	0.77	0.69	0.61	0.89	0.83	1.38[16]	0.75[16]	1.39	0.275
p_{cr}, kbar	7.46	6.1	4.47	4.5	9.12	8.25	11.8 [16]	4.78[16]	12.63	2.63
$\left(\frac{dI}{dt}\right)_{\min}$, kA/ns	0.45	0.24	0.54	0.26	0.58	0.55	0.47	0.43	0.66	0.17

The expression (6) allows one to estimate the net time of the wire explosion and expansion. For the typical X-pinch parameters (effective wire radius $R_0 = 2\times10^{-3}$ cm and current rise rate $\frac{dI}{dt} = 1\frac{kA}{ns}$) the net time of explosion and expansion is estimated as $t_1 + t_2 \approx 12.9$ ns for Fe, 11.1 ns for Au, 9.7 ns for Al, 15.4 ns for W, and 12.2 ns for Cu. Comparing these values with the explosion time estimated above by (3), we see that for the given X-pinch parameters the expansion times t_2 are comparable to the explosion times t_1 and $t_1 + t_2 << t_3 < t_f$.

3. Conclusion

The model allows one to find the minimum current rise rate at which X-pinch necking is possible. The minimum current rise rate is determined by the dynamics of expansion of the wire material and depends on the wire parameters (pressure and temperature) at a critical point, the point in the phase diagram where the lines separating the liquid, the gas, and the two-phase (liquid plus vapour) region meet. As for all metals the critical pressures and temperatures are close to each other, the minimum current rise rates are almost the same for them (about 1 kA/ns).

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