



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Modal Estimation of Civil Structures Subject to Ambient and Harmonic Excitation

Andersen, Palle; Brincker, Rune; Ventura, Carlos; Cantieni, Reto

*Published in:*

Conference Proceedings : IMAC-XXVI : A Conference & Exposition on Structural Dynamics

*Publication date:*

2008

*Document Version*

Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Andersen, P., Brincker, R., Ventura, C., & Cantieni, R. (2008). Modal Estimation of Civil Structures Subject to Ambient and Harmonic Excitation. In Conference Proceedings : IMAC-XXVI : A Conference & Exposition on Structural Dynamics Society for Experimental Mechanics.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Modal Estimation of Civil Structures Subject to Ambient and Harmonic Excitation

Palle Andersen

Structural Vibration Solutions A/S, NOVI Science Park,  
Niels Jernes Vej 10, DK-9220 Aalborg East, Denmark

Rune Brincker

Department of Civil Engineering, Aalborg University,  
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

Carlos Ventura

Department of Civil Engineering, The University of British Columbia,  
6250 Applied Science Lane, Vancouver, B.C, V6T-1Z4, Canada

Reto Cantieni

RCI Dynamics,  
Raubbühlstrasse 21B, CH-8600 Dübendorf, Switzerland

## Nomenclature

$\gamma$	Kurtosis
$y(t)$	Measured response vector
$n_y$	Number of response channels
$f_j$	$j$ th discrete frequency
$H_j$	Harmonic indicator
$m_j$	Median of Kurtosis
$G_{yy}(f)$	Spectral density of measured response at the $f$ th discrete frequency
$n_p$	Number of Projection Channels
$S_{i,j}$	$i$ th singular value of $G_{yy}$ at the $j$ th discrete frequency
$X_{i,j}$	$i$ th normalized singular value of $G_{yy}$ at the $j$ th discrete frequency

## ABSTRACT

In this paper addresses the problems of separating structural modes and harmonics arising from sinusoidal excitation. Though the problem is mostly know in mechanical engineering applications such as rotating machinery, some civil engineering applications experiences the same challenges. A robust and fast harmonic detection procedure is presented and illustrated on a civil engineering case.

## 1 Introduction

One of the major advantages of testing civil engineering structures compared to mechanical structures is that the ambient excitation nearly always is broad-banded and multiple input. This makes the response measurements obtained from such structures extremely suitable for all popular estimation algorithms in Operational Modal Analysis. They all rely on the assumptions that the input forces are derived from Gaussian white noise and are exciting in multiple points.

Mechanical engineering structures such as engines and other structures having rational part tend to be much more difficult to handle for most algorithms, especially if the sinusoidal forces have more energy than the ambient excitation. In this case the structural modes typically are weakly excited and sometimes they are more or less drowning in the noise. To account for this the measurement systems used must a high measurement range to be able to catch the weak structural response and at the same time prevent clipping from the strong sinusoidal forces.

However, even with good measurements system it is impossible to prevent the harmonics from the sinusoidal excitation to appear in the acquired data, which means that also the modal estimation algorithms must be able to handle the presence of harmonics. Further, it turns out that the presence of harmonics not only is limited to mechanical application, there is a range of cases where civil engineers have to face the harmonics presence as well. Large structures like gravity dams have rotating parts in terms of the turbines, production facilities in cement and mining industry have large rotating parts as well and bell towers exhibits sinusoidal excitation during ringing with the bells.

In this paper, we will present a fast algorithm for detection of harmonics originating from sinusoidal excitation. The technique consists of two steps; first a fast search for potential harmonics is performed in frequency domain. In step two a statistical assessment of the potential harmonics is made to determine if they are in fact harmonics. In the following, step 2 is described first running over all discrete frequencies between DC and the Nyquist frequency. After that we describe how to optimize the algorithm by the introduction of step 1.

When the harmonics are detected, the information is fed to the modal estimation algorithm, enabling it to account for the harmonic presence. The harmonic detection approach will be demonstrated on a civil engineering case; A gravity dam.

## 2 Testing for Harmonics at Spectral Frequencies

### 2.1 The Central Limit Theorem

According to the central limit theorem the distribution of the response of a structural system subject to multiple random inputs will tend to a Gaussian distribution as the number of independent input goes to infinity. If the distribution of the different inputs have a bell shaped distribution indicating that most amplitudes will be close to their mean value, which is typical for wind and wave loading, then only a few number of inputs are necessary for the response to become approximately Gaussian. However, if the input on the other hand is dominated by amplitudes far from the mean value, which is the case of a sinusoidal excitation, then it takes much more inputs before the structural response will turn Gaussian, Wirsching et al. [1].

Therefore, testing of the shape of Probability Density Function (PDF) of the measured response is an effective way to detect if a few sinusoidal excitation forces are presents. Especially, if the response is examined in narrow frequency intervals, it is possible to obtain information about which intervals that are dominated by harmonics and which are not.

### 2.2 Testing PDF's Shape using Kurtosis

There are a numerous ways to test if sampled data has a specific PDF or not, like the  $\chi^2$ -test, Papoulis [2], most in some ways based on the sampled mean value  $\mu$  and the sampled standard deviation  $\sigma$ . Here, we will use the fact that the Kurtosis  $\gamma$  of a the  $n_y \times 1$  dimensional vector  $y(t)$  of measured response, defined as

$$\gamma = \frac{E[(y(t) - \mu)^4]}{\sigma^4} \quad (1)$$

for Gaussian distributed data with zero mean value and unit variance, is equal to 3. The Kurtosis for a sinusoidal data with zero mean value, unit variance and a random phase is on the other hand always 1.5.

### 2.3 The Basic Testing Algorithm

In practice we have to be able to test the probability density function in several frequency intervals, characterized by their center frequency  $f_j$ , and in several measurement channels  $y_i(t)$ , for  $i = 1$  to  $n_y$ . The output of the test algorithm should an indicator  $H_j$ , for  $j = 1$  to  $n_f$ .  $H_j$  is a function of the center frequency  $f_j$ , where  $j$  is all center frequencies we like to test between DC and the Nyquist frequency setting DC to  $j = 1$  and the Nyquist frequency to  $j = n_f$ . The indicator should be 1 at center frequencies where a harmonic is present and otherwise 0.

The algorithm used here contains the following steps:

1. Normalize each measurement channels  $y_i(t)$  to zero-mean and unit variance using sampled mean and variance.
2. For all center frequencies  $f_j$  of interest, perform a narrow band-pass filtering around  $f_j$ .
3. Calculate the Kurtosis  $\gamma_{ij}$  for the at  $f_j$  band-pass filtered signal  $y_i(t)$ .
4. For each center frequency  $f_j$  calculate the median value  $m_j$  of the Kurtosis  $\gamma_j$  over all measurement channels  $y_i(t)$ . This median is a robust measure for the mean value used to account for possible outliers due to noise etc.
5. For each center frequency  $f_j$  assess if  $m_j$  deviates significantly from 3. If so set  $H_j$  equal to 1, and otherwise 0.

This algorithm has been tested on a series of real data cases and proven efficient, Jacobsen et al. [3],[4].

### 3 Optimizing Search for Possible Harmonics

The major drawback of the basic algorithm for harmonic detection is that it becomes rather time consuming in case of many measurement channels  $n_y$  and when testing many frequencies  $n_f$ . It would be desirable to find ways of reducing both  $n_y$  and  $n_f$ .

#### 3.1 Reducing the Number of Measurement Channels

Since the spectral density matrix  $G_{yy}(f)$  of the measured response at some discrete frequency  $f$  typically consist of much more columns than there are modes participating at that frequency, many of the columns of are linear dependent upon each other resulting in a rank deficiency of the spectral matrices. For system with many measurement channels  $n_y$  there is therefore typically a substantial amount of redundancy, indicating that it might not be necessary to actually Kurtosis test on all channels. The subset of channels we will test is called the Projection Channels in the following and the number of projection channels denoted  $n_p$ .

In case of multiple testing using multiple setups, where a certain amount of sensors are kept at the same locations, while the rest are moved from one measurement setup to another, a good initial choice of projection channels is to choose them as the reference channels. The quality of this choice can be verified by applying the Singular Value Decomposition (SVD) to the spectral densities matrices  $G_{yp}(f)$ ,

$$USV^H = G_{yp}(f) \quad (2)$$

where index  $y$  indicate the measurement vector  $y(t)$  and  $p$  the subset of channels selected as the projection channels. The matrices  $G_{yp}(f)$  and  $S$  both have dimension  $n_y \times n_p$ .  $S$  is diagonal matrix consisting of  $n_p$  singular values.

By plotting the singular values for all the frequencies all modes in the projection channels will be revealed. If the last plotted singular value forms a horizontal line over the frequency band of interest, and if the other singular values display a good mode separation, then the choice is fine. If not then other and / or more projection channels should be included. If more projection channels are needed, the channels to look for should contain as much new information as possible about the system compared to the channels already selected. This evaluation can be performed using a simple analysis of the correlation coefficients between the difference measurement channels. Figure 3.1 display a poor choice of projection channels for a system, whereas figure 3.2 shows an appropriate choice.

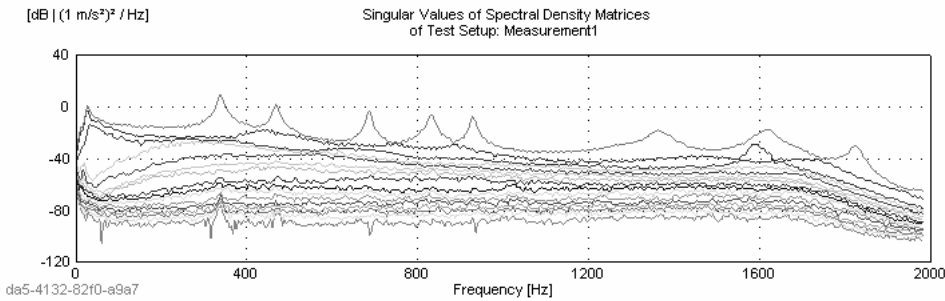


Figure 3.1: 16 projection channels were chosen resulting in 16 singular values per frequency. All the lower singular values being completely horizontal indicates a substantial amount of redundant information at all frequencies.

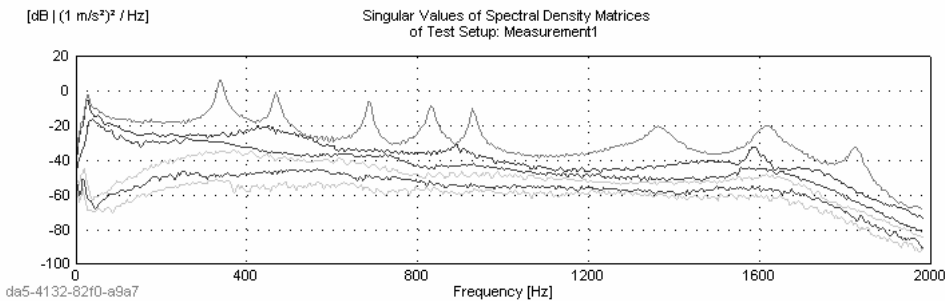


Figure 3.2: 6 projection channels were chosen resulting in only 6 singular values per frequency. Only the lowest singular values is significantly flat (horizontal) this indicates that 6 measurement channels are sufficient to contain all information about the system dynamics.

### 3.2 Reducing the Number of Frequencies Needed Check

In figure 3.3 the singular values of the spectral densities of a system excited with a broad-banded excitation as well as a sinusoidal excitation. The natural frequency first mode appear at 354 Hz, and only the first singular value is significant larger that the rest at this frequency. This indicates that this particular mode is dominating at this particular frequency. On the other hand, at 374 Hz and even more clear at 748 Hz a distinct narrow peak appear in several of the singular values. The peak at 374 Hz indicates the rotational speed of the harmonic excitation and the peak at 748 Hz is the first over-harmonic originating from the same sinusoidal excitation source. That more than one singular value has a peak at these two frequencies indicates that several modes have been significantly excited at these frequencies compare to the surrounding frequencies.

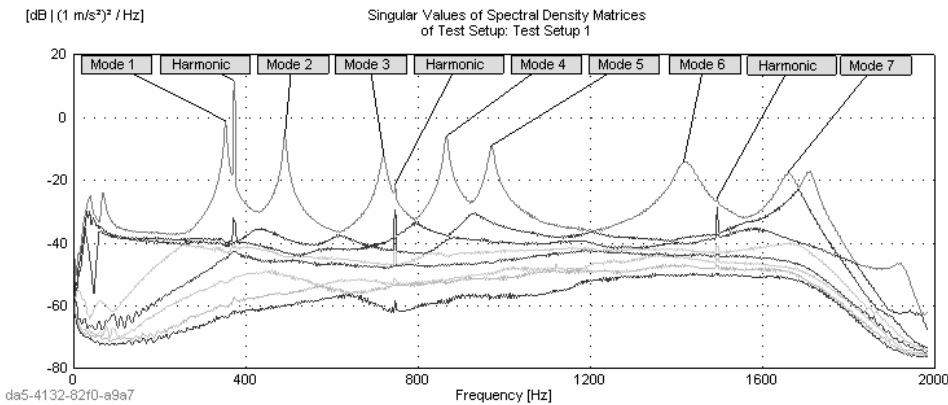


Figure 3.3: Similar structure as in figure 3.3, but now also with harmonic excitation. The excitation is a single point shaker with a sinusoidal force at 374 Hz. Number of discrete frequencies between DC and the Nyquist is 1024. First, second and fourth harmonic are clearly shown at 374 Hz, 748 Hz and 1496 Hz.

We will make use of this phenomenon that always happens in case of sinusoidal excitation, when the ambient excitation acting on a structure is much weaker. If the sinusoidal excitation is much weaker than the broad-banded ambient excitation, the sinusoidal excitation becomes negligible and does not affect the modal analysis algorithms. If an abrupt change happens at the same frequency at least in two singular value lines, then we have detected a potential harmonics. This potential harmonics should then be tested using the basic algorithm described in section 2.3. In this way it is possible to limit the number of times that the Kurtosis needs to be checked. In the example in figure 3.3 it reduces the number of times from 1024 to 7 times. There are several ways to test if there is an abrupt change on a curve, see e.g. Basseville et al. [4]. Here we apply a simple approach based on a median calculation. Given a sequence of positive and non-zero singular values  $S_{i,j}$  of length  $n_s$ , where index  $i$  is the singular value number and  $j$  the discrete frequency index, we construct the following normalized sequence  $X_{i,j}$

$$X_{i,j} = \begin{cases} \log_{10} \left( \frac{S_{i,j}}{\text{median}(S_{i,j-k}, \dots, S_{i,j+k})} \right) & , j > k \wedge j < n_s - k \\ 0 & , j \leq k \wedge j \geq n_s - k \end{cases} \quad (3)$$

where  $k$  is a small number, say 2-5. If the median of the values  $S_{i,j-k}$  to  $S_{i,j+k}$  is equal to the value  $S_{i,j}$  then  $X_{i,j}$  is 0 and otherwise  $X_{i,j}$  will be a non-zero value. Since the sequence is normalized using the median that is robust towards outliers, the result is that  $X_{i,j}$  will have significant values at the locations where the singular values have significant but narrow peaks.

The algorithm used here contains the following steps:

1. For each singular value  $S_{i,j}$  calculate the sequences  $X_{i,j}$  for  $i = 1$  to  $n_p$  and  $j = 1$  to  $n_s$ .
2. Calculate the sampled standard deviations of the sequences  $X_{i,j}$ .
3. Check if some of the values of  $X_{i,j}$  exceeds a threshold of say 2-3 times the standard deviation of  $X_{i,j}$ .
4. If, for a certain index  $j$ , more than one of the  $n_{sB}$  sequences  $X_{i,j}$  exceeds the threshold, then a potential harmonic has been detected at position  $j$ .
5. Apply Kurtosis check described in section 2.3 at position  $j$ .

In figure 3.4 the sequences  $X_{i,j}$  are shown for 3 singular values corresponding to the example shown in figure 3.3. The number of potential harmonics to check has been decreased to only 7. Once the harmonics have been detected they can easily be removed from any frequency domain based modal parameter estimation algorithm, see e.g. Jacobsen et al. [3],[4], Brincker et al. [6] and Andersen et al. [7].

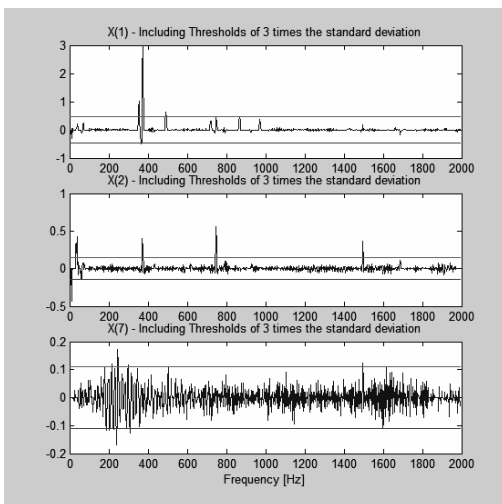


Figure 3.4: Sequences  $X_{i,j}$  shown for 3 singular values (1,2 and 7) of the example shown in figure 3.3. Thresholds are exceeded in more than one singular value at frequencies 28 Hz, 372 Hz, 374 Hz, 376 Hz, 746 Hz, 748 Hz and 1496 Hz.

#### 4 Example

In the following the complete harmonic detection algorithm is tested on measurements of a Canadian gravity dam. In figure 4.1 two pictures display the dam from both the low and high water level sides. The dam is 130 m long and 58 m high, and built in 1930. An ambient vibration test was conducted using 20 setups of 8 channels. In setup 8, channel 8 was dead and was disabled from the analysis. A 3D accelerometer served as reference station mounted on the dam itself. Some part of the rock at both side of the dam was also measured.

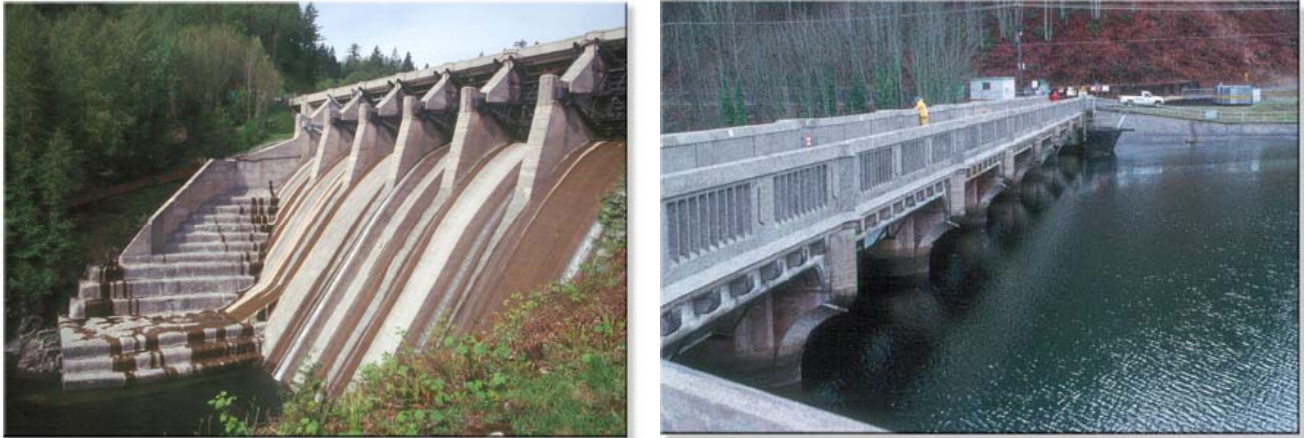


Figure 4.1: The dam seen from low and high water level.

In figure 4.2 all measured degrees of freedom are presented on the test geometry used by the operational modal analysis software ARTEMIS Extractor. The measurements were conducted using an 8-channel measurement system for 819 seconds. The sensors were Kinemetrics Episensors accelerometers of the forced balanced type. Due to the turbines running the measurements are affected by harmonics at every 2 Hz.

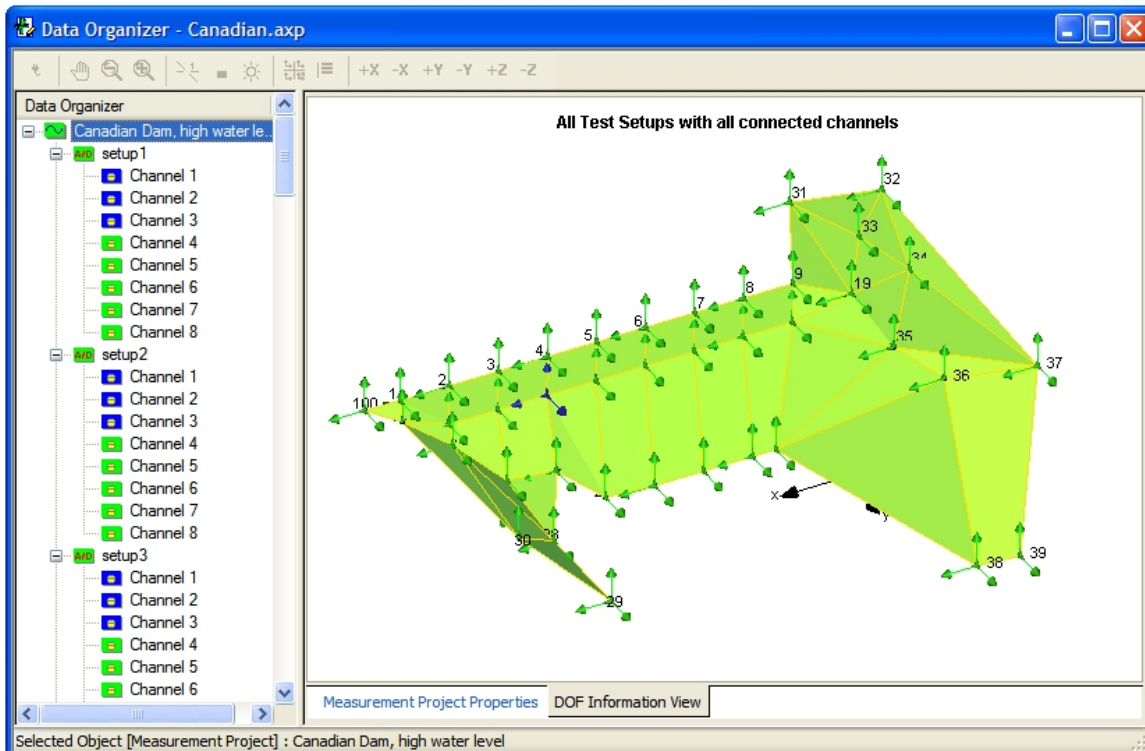


Figure 4.2: All measured degrees of freedom of all 20 setups on the dam as well as the surrounding rock on both sides. The three dark arrows below point 4 is the reference station.

Since setup 8 only have 7 active channels the number of projection channels used in this analysis is 7. The 7 largest singular values of the spectral densities of all 20 setups were then averaged, and the harmonic detection described in section 3.2 was applied using  $k = 2$  in eq. (3). In step 3 of the algorithm, the threshold was set to 2 times the standard deviation of the sequence  $X_{i,j}$ . In figure 4.3 the results of the harmonic detection analysis are shown. All harmonics at 2 Hz intervals have been detected. The algorithm has mistakenly detected two harmonics at 19.5 Hz and 39 Hz. Taking the scatter of the SVD spectrum from the poor signal to noise ratio into account, it is a quite satisfactory results.

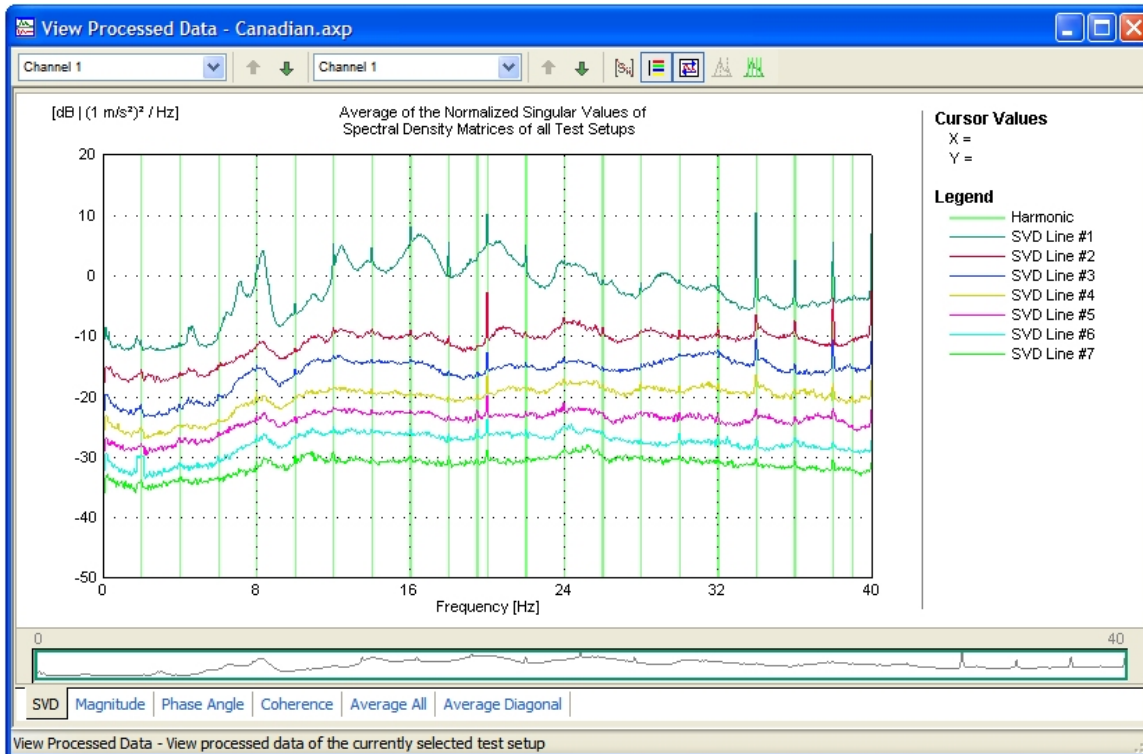


Figure 4.3: Result of the harmonic detection analysis. All harmonics at 2 Hz intervals have been detected. The algorithm has mistakenly detected harmonics at 19.5 Hz and 39 Hz. The harmonics are indicated with vertical lines at the harmonic frequencies.

## 5 Conclusions

In this paper, we have presented a fast algorithm for detection of harmonics from sinusoidal excitation. A statistical assessment algorithm of potential harmonics, based on evaluation of the Kurtosis of band pass filtered measurement, has been introduced. The speed of this algorithm has been optimized by applying a search algorithm that looks for abrupt changes in more than one singular value at a certain frequency, since this is a typical phenomenon in case of harmonics. This search algorithm typically increases the performance of the statistical assessment algorithm significantly. The harmonic detection approach has been demonstrated on measurements from a gravity dam.



## 6 Reference List

- [1] P. H. Wirsching, T.L. Paez and K. Ortiz: *Random Vibrations, Theory and Practice*. Wiley, ISBN 0-471-58579-3, 1995.
- [2] A. Papoulis: *Probability and Statistics*. Prentice-Hall, ISBN 0-13-711730-2, 1990.
- [3] N.J. Jacobsen, P. Andersen, R. Brincker: *Using Enhanced Frequency Domain Decomposition as a Robust Technique to Harmonic Excitation in Operational Modal Analysis*. Proceedings of ISMA2006, Leuven, Belgium, 2006.
- [4] N.J. Jacobsen, P. Andersen, R. Brincker: *Eliminating the Influence of Harmonic Components in Operational Modal Analysis*. Proceedings of The 25th International Modal Analysis Conference (IMAC), Orlando, Florida, 2007.
- [5] M. Basseville and I. Nikiforov: *Detection of Abrupt Changes - Theory and Applications*. Prentice Hall, ISBN 0-13-126780-9, 1993.
- [6] R. Brincker, P. Andersen, N.J. Jacobsen: *Automated Frequency Domain Decomposition for Operational Modal Analysis*. In Proceedings of The 25<sup>th</sup> International Modal Analysis Conference (IMAC), Orlando, Florida, 2007.
- [7] P. Andersen, R. Brincker, M. Goursat, L. Mevel: *Automated Modal Parameter Estimation of Civil Engineering Structures*. In Proceedings of The Evaces Conference, Porto, Portugal, 2007.