Investigation of the Relationship between Sensitivity and Stored Energy

Monica Martinez-Mendoza #1, Christoph Ernst #2, Alejandro Alvarez Melcon *3

#TEC-ETM ESA/ESTEC. Keplerlaan 1 - 2202 AZ Noordwijk ZH - The Netherlands
1shanaz00@hotmail.com

*Universidad Politecnica de Cartagena. Campus Muralla del Mar-Antiguones Cartagena-Spain

***Abstract*— This paper is focused on the theoretical investigation of the relationship between the time average stored energy (t.a.s.e) in conventional lumped-element Chebychev low-pass prototype filters, and the sensitivity of the return loss with respect to properly tuned resonators. It is well known that the resonant frequencies are the most sensitive parameters in a filter to achieve the required return loss and often some form of manual adjustment is needed. Different degree transfer functions, and also different topologies, are investigated throughout the paper. It is shown that both, the t.a.s.e and the sensitivity behaviors are strongly related. This work provides some new insight in fundamental properties of passive filters and can be useful for the design of filters with reduced sensitivity for high power applications and tuningless filters, for example.**

***Index Terms*— Transversal filters, Ladder filters, Sensitivity.**

I. Introduction and Motivation

The improvement of power handling capability in microwave filters, represents an important issue for current space and terrestrial communication systems [1]. Several contributions have been published in this sense [2], [3]. An interesting approach based on the selection of the best topology to minimize the maximum stored energy in the resonators can be found in [4], [5].

On the other hand, it is also important to consider the sensitivity of the different networks that can be used to synthesize a specific filtering response [6]. This will allow to find the least sensitive coupling scheme for each specific case. A wide range of studies have been carried out about this topic [7], [8], [9]. These studies can be important for the understanding of the behaviour of the filter to temperature variations and to uncertainties in manufacturing tolerances.

The investigation carried out in this paper arise from the observation of the distribution of energy in different filter topologies. As highlighted in [4], in a transversal network, the stored energy in each resonator contributes to the total stored energy mainly in one specific part of the passband, whereas it is almost null in the rest. On the contrary, the total stored energy in an inline network is distributed among the different resonators in a much more uniform way. Intuitively, it is expected that manufacturing errors in resonators with larger values of stored energy provoke larger variations of the reflection parameter of the filter than the errors which affect to the resonators storing a small amount of energy.

This paper investigates the relationship between the t.a.s.e and the sensitivity of the S_{11} parameter when the diagonal elements in the coupling matrix of the prototype are varied (variation in resonant frequencies). The investigation starts from a single resonator (Fig. 1), and it is extended to second and third order filter topologies. The best of all topologies that leads to the lowest t.a.s.e and lowest sensitivity for the transfer functions treated is identified. It is shown that indeed there is a strong relationship between sensitivity and stored energy, specially in transversal networks.

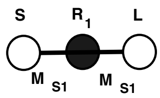

Fig. 1. Single resonator.

II. Theoretical Background

The sensitivity of a specific filter topology can be evaluated by studying the components of the gradient of the reflection and transmission coefficients with respect to the entries of the coupling matrix as detailed in [8]. In that contribution different topologies were compared taking into account all the possible errors, in order to consider the worst case scenario.

In this paper the individual components of the gradients of the reflection coefficient with respect to the diagonal entries of the coupling matrix are evaluated, in order to investigate its relationship with the stored energy in the individual resonators. In this way, the effects due to errors in the resonant frequencies of the individual resonators for the topologies under study will be assessed.

The K_{ii} parameter has been defined here, following [8]:

$$K_{ii} = \frac{\partial |S_{11}|}{\partial M_{ii}} \tag{1}$$

The K_{ii} parameter represents the variations of S_{11} with respect to the variation of the diagonal coupling term corresponding to the resonator i. The sensitivity of the reflection coefficient S_{11} was selected as it is the most sensitive parameter for the in-band performance of a filter. Besides, only the diagonal elements are studied because the resonators are much more sensitive than the couplings elements themselves.

On the other hand, the stored energy of a low-pass prototype filter can be obtained by adding the stored energy in the individual elements of the prototype [4]. Thereby, the total stored energy W of a specific topology can directly be obtained from the value of the capacitors and the voltages that form each resonator.

$$W(\omega) = \Sigma W_{Ci} \tag{2}$$

$$W_{Ci} = \frac{1}{4} \cdot |V(\omega)|^2 \cdot C_i \tag{3}$$

III. Single Resonator Case

Since a first order network only has one resonator, the total stored energy of the network will be the same as the energy stored by the resonator.This stored energy can be calculated as explained in [4], [10]:

$$W = \frac{1}{M_{S1}^2} \cdot \frac{2}{4 + \frac{\omega^2}{M_{S1}^8}} \tag{4}$$

where M_{S1} is the coupling value shown in Fig. 1, obtained from the $N+2$ coupling matrix for this network. In a similar way, the sensitivity can be analytically obtained, starting from equation (1b) given in [8]:

$$K_{11} = \frac{\partial |S_{11}|}{\partial M_{11}} = -2 \cdot j \cdot [A^{-1}]_{12} \cdot [A^{-1}]_{21} \tag{5}$$

After some lengthy algebraic manipulation, the sensitivity results to be:

$$K_{11} = \frac{4 \cdot M_{S1}^4 \cdot \sqrt{\omega^2 + 4 \cdot M_{S1}^4}}{(\omega^2 + 4 \cdot M_{S1}^4)^2} \tag{6}$$

In Fig. 2, the sensitivity K_{11} parameter together with the t.a.s.e for a first order network with a coupling to the source of $M_{s1} = 2.9795$ is shown. Note that the dimensions have been omitted. The sensitivity is given in $|S_{11}|$ dimension change over change in the M_{ii} coupling, whereas the t.a.s.e is given in Joule.

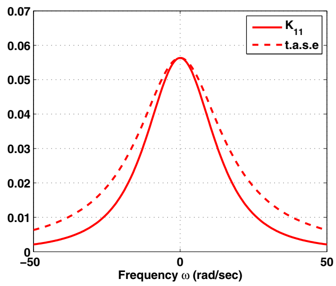

Fig. 2. Sensitivity and stored energy comparison for a single resonator implementing a Chebychev response with 25 dB return loss.

It is interesting to observe that the maximum value of the stored energy and the sensitivity are the same, and both of them occur at the centre frequency of the response. This can be also checked by substituting $\omega = 0$ in (4) and (6):

$$K_{11,max} = W_{max} = \frac{1}{2 \cdot M_{S1}^2} \tag{7}$$

IV. Comparison: Inline and Transversal Topologies.

In higher degree networks there is more than one topology with the same power transfer function. The topologies that can be used to synthesize a specific transfer function, depend on the number and position of the transmission zeros that the transfer function exhibits. In the following, all pole filter transfer functions are considered, and transversal networks are compared to inline topologies.

First, the second degree all-pole transfer function, shown in Fig. 3, will be studied. In Fig. 4, the transversal and the inline second order topologies are shown.

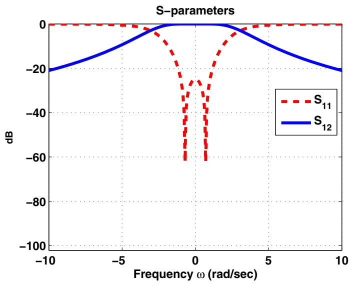

Fig. 3. Second order Chebychev response, with 25 dB return loss.

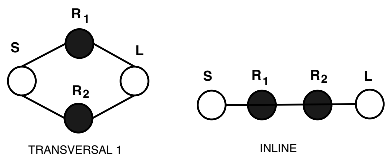

Fig. 4. Transversal and Inline second order Topologies.

In the transversal network the t.a.s.e of each resonator is asymmetric with respect to the center frequency. This can be observed in Fig. 5. It is clearly noticed that each resonator contributes to the total t.a.s.e mainly in one side of the passband. On the contrary, in the inline network, the t.a.s.e

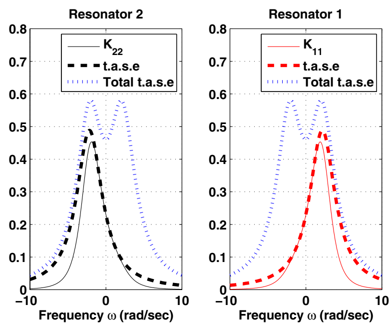

Fig. 5. Sensitivity and stored energy comparison for the transfer function of Fig. 3. Transversal topology (Fig. 4).

of each resonator is symmetric with respect to the center frequency. This is shown in Fig. 6, where it is also observed that the two resonators store an equal amount of energy around the upper and lower band edges where the total t.a.s.e rises considerably. Besides, K_{11} and K_{22} in the inline topology, result to be equal for both resonators.

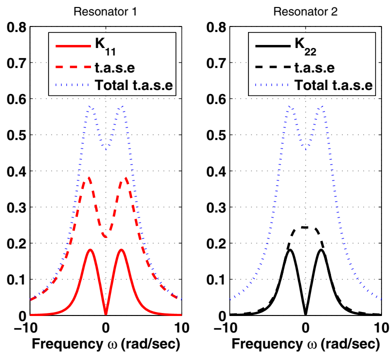

Fig. 6. Sensitivity and stored energy comparison for the transfer function of Fig. 3. Inline topology (Fig. 4).

Comparing the t.a.s.e and the $K_{11} = K_{22}$ parameters (sensitivity), shown in Fig. 6 and Fig. 5, a strong connection can be noticed. Similar to the stored energy, the sensitivity of the resonators in the inline network has two similar maximum values around the center frequency, whereas in the transversal network each resonator is responsible for the sensitivity at each side of the passband. The best topology in terms of minimum t.a.s.e and minimum sensitivity for the all pole transfer function results to be the ladder network, whereas the transversal topology is the one with maximum sensitivity and maximum t.a.s.e.

Note that it seems that the connection between the t.a.s.e and the sensitivity is stronger in the transversal topology. This could be explained because in the transversal topology each resonator contributes to the response more independently in a certain part of the bandwidth.

This observation has also been verified in higher order filters. For example, considering a 3rd order transfer function, the comparison between the sensitivity and the stored energy in the resonators of the transversal network is shown in Fig. 7. Again, this is an asynchronous network with different resonant frequencies in the resonators. The plots in Fig. 7 correspond to the three resonators sorted by its resonant frequency, in such a way that the one in the left has the lowest resonant frequency, and the one in the right has the highest resonant frequency. As in the second order case, the t.a.s.e of each resonator in the transversal network contributes to the total t.a.s.e in only narrow parts of the passband, being larger in those with a resonant frequency closer to the band edges. The sensibility of each resonator is surprisingly similar to the t.a.s.e. In fact, they coincide in the interval $\omega = [-1, 1]$ (inside the passband).

For the inline network, the comparison between the sensitivity and the stored energy in the resonators is shown in Fig. 8. It can be observed that the t.a.s.e of each resonator is symmetrical with respect to the center frequency, and also that the contribution to the total stored energy is spread among the three resonators. Note that a similar behavior is observed for the sensitivities, where the effect of an error in any of the resonator is not concentrated in just a small portion of the passband, but in different parts of it.

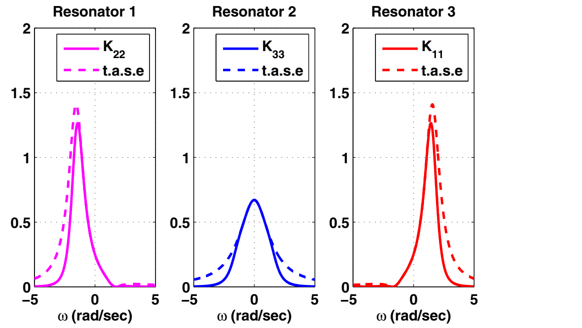

Fig. 7. Sensitivity and stored energy comparison for a third order transfer function, with 25 dB return loss. Transversal topology.

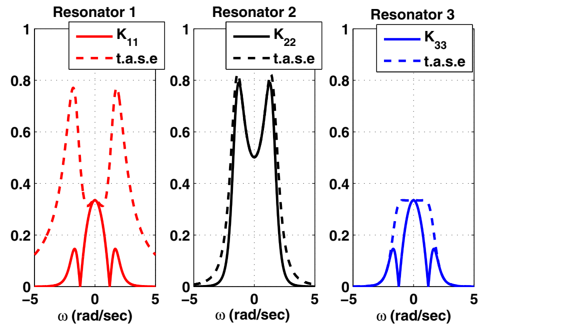

Fig. 8. Sensitivity and stored energy comparison for a third order transfer function, with 25 dB return loss. Inline topology.

In Table I and Table II, the peak value of the sensitivity as well as the peak value of the stored energy for each resonator are listed, for networks up to 6th order. Table I shows the results for transversal networks, where resonators 1 and 6 are the resonators tuned at the lowest and the highest frequency respectively. Table II shows the results for inline networks, where the resonators are ordered by proximity to the source, in such a way that resonator 1 is the closest to the source, and resonator 6 is the closest to the load.

Several facts can be noticed from these tables. Looking at Table I, corresponding to the results of transversal networks, it is observed that the maximum values of the sensitivity and the stored energy for a given order, are always in the resonators tuned closer to the band edges, whereas the minimum values are in the resonators tuned in the proximity of the center frequency of the filter. Besides, symmetry with respect to the resonator/s tuned in the center of the passband is observed, both in peak stored energy and peak sensitivity. Finally, note that the value of the peak stored energy and the maximum sensitivity in the central resonator when the degree of the filter is odd, are always the same.

On the other hand, in Table II where the results of inline networks are listed, it is observed that the sensitivity is symmetric with respect to the resonator/s located in the center of the network. Besides, the minimum value of the sensitivity occurs always in the resonators located closer to the source and the load, whereas the resonators located in the center of

TABLE I

PEAK T.A.S.E VS MAXIMUM VALUE OF K_{ii} IN EACH RESONATOR. CHEBYCHEV NETWORK, RL=25 DB. TRANSVERSAL TOPOLOGY

	R1	R2	R3	R4	R5	R6
Order	K_{11} t.a.s.e	K_{22} t.a.s.e	K_{33} t.a.s.e	K_{44} t.a.s.e	K_{55} t.a.s.e	K_{66} t.a.s.e
1	0.0563 0.0563	–	–	–	–	–
2	0.4531 0.4882	same as R_1	–	–	–	–
3	1.2650 1.4098	0.6704 0.6704	same as R_1	–	–	–
4	2.4080 2.7463	1.1684 1.1691	same as R_2	same as R_1	–	–
5	3.7984 4.4148	1.7985 1.8007	1.6064 1.6064	same as R_2	same as R_1	–
6	5.3474 6.3242	2.5398 2.5437	2.1552 2.1562	same as R_3	same as R_2	same as R_1

TABLE II

PEAK T.A.S.E VS MAXIMUM VALUE OF K_{ii} IN EACH RESONATOR. CHEBYCHEV NETWORK, RL=25 DB. INLINE TOPOLOGY.

	R1	R2	R3	R4	R5	R6
Order	K_{11} t.a.s.e	K_{22} t.a.s.e	K_{33} t.a.s.e	K_{44} t.a.s.e	K_{55} t.a.s.e	K_{66} t.a.s.e
1	0.0563 0.0563	–	–	–	–	–
2	0.1816 0.3831	0.1816 0.2441	–	–	–	–
3	0.3352 0.7709	0.7913 0.8245	0.3352 0.3352	–	–	–
4	0.3755 1.0906	0.9707 1.4605	0.9707 1.0101	0.3755 0.3766	–	–
5	0.3980 1.3222	0.8778 2.0862	1.6159 1.7037	0.8778 1.1051	0.3980 0.3980	–
6	0.4094 1.4792	0.8890 2.6613	1.8187 2.4058	1.8187 1.8251	0.8890 1.1612	0.4094 0.4102

the network show the maximum values for the peak sensitivity. Note that the maximum value of the sensitivity is always lower than the peak value of the stored energy.

It is also important to highlight that comparing Table I and Table II, it is noticed that the maximum value of the sensitivity, as well as the peak value of the stored energy among all the resonators of a specific filter, is always lower employing the inline topology than the transversal.

More complex transfer functions implemented in other topologies can also be of interest. A simple study was already carried out for second order networks when transmission zeros are present in the transfer function [11]. Similar relations as described in this paper were also found for the topologies shown in Fig. 9.

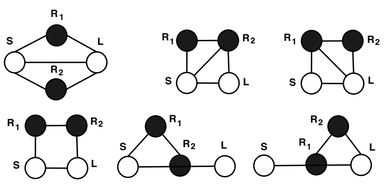

Fig. 9. Second order topologies.

V. CONCLUSIONS

In this work the relationship between the stored energy in a resonator, and the sensitivity of the return losses to resonator resonance frequency variation have been investigated. Chebychev filters of degree up to 6 have been considered and the sensitivities and stored energies in in-line and transversal topologies have been compared to each other. A close relationship between the t.a.s.e and the sensitivity has been observed. Resonators with larger stored energies, present larger sensitivities (Tables I, II). Besides, it is curious that the value of the peak stored energy and the maximum sensitivity in the central resonator of odd degree filters is identical. Finally, the inline topology has been found to be the best topology in terms of minimum peak t.a.se and minimum sensitivity when compared with the transversal.

VI. ACKNOWLEDGMENTS

This work has been conducted under ESA contract 22736, Ministerio de Educacion y Ciencia Grant TEC2007-67630-C03-02 and Fundacion Seneca Project Ref. 08833/PI/08.

REFERENCES

[1] M. Yu, “Power-handling capability for rf filters” *IEEE Microwave Magazine*, pp. 889774, October 2007.

[2] C. Ernst and V. Postoyalko, “Comparison of the stored energy distribution in a qc-type and a tc-type prototype with the same power transfer function” in *IEEE International Microwave Symposium Digest, IEEEIMS*, April 1999.

[3] B. Senior, I. Hunter, V. Postoyalko, and R. Parry, “Optimum network topologies for high power microwave filters” in *6th High Frequency Postgraduate Student Colloquium, (Cardiff, UK)*, pp. 5358, IEEE, September 2001.

[4] C. Ernst, Energy Storage in Microwave Cavity Filter Networks. *Leeds, UK: PhD Thesis*, 2000.

[5] C. Ernst, V. Postoyalko, and N. Khan, “Relationship between group delay and stored energy in microwave filters” *IEEE Transactions on Microwave Theory and Techniques*, vol. 49, pp. 192196, January 2001.

[6] R. Cameron, J. C. Faugere, and F. Seyfert, “Coupling matrix synthesis for a new class of microwave filters” in *IEEE International Microwave Symposium Digest, IEEE-IMS*, 2005.

[7] S. Amari, “Sensitivity of coupled resonator filters” *IEEE Circuits Syst. II*, vol. 47, pp. 10171022, October 2000.

[8] S. Amari and U. Rosenberg, “On the sensitivity of coupled resonator filters without some direct couplings” *IEEE Transactions on Microwave Theory and Techniques*, vol. 51, pp. 17671773, June 2003.

[9] F. Seyfert, S. Bila, and P. Lenoir, “Dedale-hf a new tool for the design and tuning of microwave filters” in *International Workshop on Microwave Filters*, (Toulouse, France), ESA/ESTEC, 16-18, Oct 2006.

[10] R. E. Collin, Foundation for Microwave Engineering. New York. St Louis: McGraw-Hill.

[11] M. M. Mendoza, A. A. Melcon, and C. Ernst, “Sensitivity and stored energy investigation” in *International Workshop on Microwave Filters*, (Toulouse, France), ESA/ESTEC, 16-18, November 2009.