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A Probabilistic Approach for Robustness Evaluation of Timber Structures

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Abstract
A probabilistic based robustness analysis has been performed for a glulam frame structure supporting the roof over the main court in a Norwegian sports centre. The robustness analysis is based on the framework for robustness analysis introduced in the Danish Code of Practice for the Safety of Structures and a probabilistic modelling of the timber material proposed in the Probabilistic Model Code (PMC) of the Joint Committee on Structural Safety (JCSS). Due to the framework in the Danish Code the timber structure has to be evaluated with respect to the following criteria where at least one shall be fulfilled: a) demonstrating that those parts of the structure essential for the safety only have little sensitivity with respect to unintentional loads and defects, or b) demonstrating a load case with 'removal of a limited part of the structure' in order to document that an extensive failure of the structure will not occur if a limited part of the structure fails, or c) demonstrating sufficient safety of key elements, such that the entire structure with one or more key elements has the same reliability as a structure where robustness is documented by b). Based on investigations with respect to criteria a) and b) the timber frame structure has one column with a reliability index a bit lower than an assumed target level. By removal three columns one by one no significant extensive failure of the entire structure or significant parts of it are obatined. Therefore the structure can be considered to behave robust according to the sued probabilistic approach. However, the present probabilistic approach for robustness evaluation has to be further developed for a general application to timber systems, and a simplified approache suitable for day-to-day engineering purposes must be identified.

Introduction
Robustness of strucutral systems has obtained a renewed interest due to a much more frequent use of advanced types of structures with limited redundancy and serious consequences in case of failure. The interst has also been facilitated due to recently severe structural failures such as that at Ronan Point in 1968 and the World Trade Centre towers in 2001. In order to minimise the likelihood of such disproportionated structural failures many modern building codes consider the need for robustness in structures and provides strategies and methods to obtain robustness, see e.g. [1, 2]. The requirement for robustness is specified in most buildings codes in a way like the general requirements in the two Eurocodes EN 1990 Eurocode0: Basis of Structural Design [3] and EN 1991-1-7 Eurocode 1: Part 1-7 Accidental Actions [4]. The first provides principles, e.g. it is stated that a structure shall be "designed in such a way that it will not be damaged by events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the original cause." The second provides strategies and methods to obtain robustness and the actions to consider, and consider design situations: 1) designing against identified accidental actions, and 2) designing unidentified actions (where designing against disproportionate collapse, or for robustness, is important). However, none specific criterion is delivered

which could be used to quantify the level of robustness of a structure which could have a benefit for design and analysis of structures. During the last decades a variety of research efforts have attempted to quantify aspects of robustness such as redundancy and identify design principles that can improve robustness. Several proposed methods for quantifying robustness are reviewed, and frameworks for robust design are proposed in [5, 6]. Several of the reviewed methods for quantifying robustness are based on a probabilistic framework, e.g. given as a redundancy index and a redundancy factor [7, 8]. Recently, a index of robustness has been proposed taking basis in decision analysis theory following [1] which states that a decision analysis theory framework can be used to assess robustness in a general manner. The index of rrobustness is assessed by computing both direct risk, which is associated with the direct consequences of potential damages to the system, and indirect risk, which corresponds to the increased risk of a damaged system. Indirect risk can be interpreted as risk from consequences disproportionate to the cause of the damage, and so the robustness of a system is indicated by the contribution of these indirect risks to total risk. In addition to quantifying the effect of the physical system's design, this approach can potentially account for the effect of inspection, maintenance and repair strategies as well as preparedness for accidental events, because those actions can reduce failure consequences and thus risk. The approaches mentioned above for defining structural robustness is in principle related to specific loads, accidental actions and damages which a structure should be designed for in any case. However, the requirements regarding structural robustness could also be to reduce the sensitivity of a structure with respect to unintentional loads and defects that are not included in the codes and design requirements. Such a robustness analysis framework is introduced in the Danish Code of Practice for the Safety of Structures [9, 10]. For the evaluation of robustness of timber construction, where size effects, moisture effects and creep, low strength perpendicular to grain and system effects are pronouced, the framework has a potential to outline the characteristics of timber systems regarding robustness. The approach will in the presented paper be considered for robustess evaluation of a timber structure: a Norwegian sports centre with a main structural system consisting of glulam frames. The robustness framework will shortly be presented in section 2 and the probabilistic modelling of the structure follows in section 3. Section 4 presents the results for the robustness evaluations.

Framework for Evaluation of Robustness of Structures

Robustness is introduced in the Danish Code of Practice for Safety of Structures [9, 10] as a general requirement to all structures in order to reduce the sensitivity of the structure with respect to unintentional loads and defects that are not included in the codes and design requirements. The background, the probabilistic model and an outline of implementation of robustness requirements are given in [11]. The Danish Code of Practice for the Safety of Structures [9, 10] defines a structure as robust

- when those parts of the structure essential for the safety only have little sensitivity with respect to **unintentional loads and defects**, or
- when **extensive failure of the structure will not occur** if a limited part of the structure fails.

This implies that a robust structure can be achieved by means of suitable choices of materials, general static layout and structural composition, and by suitable design of key elements. Robustness should be distinguished from accidental loads although some of the design procedures and measures are similar; structures should be robust regardless of the likelihood of accidental loads. A key element is defined as

- a limited part of the structure, which has an essential importance for the robustness of the structure such that any possible failure of the key element implies a failure of the entire structure or significant parts of it.

Examples of unintentional loads and defects are e.g. unforeseen load effects, geometrical imperfections, settlements and deterioration, unintentional deviations between the actual function of the structure and the applied computational models and between the executed project and the project material. The requirements to robustness of a structure should be related to the consequences of a failure of the structure. Therefore documentation of robustness is only required for structures in high safety class. For structures in high safety (consequence) class robustness shall be documented by preparation of a technical review where at least one of the following criteria shall be fulfilled:

a) by demonstrating that those parts of the structure essential for the safety only have little sensitivity with respect to unintentional loads and defects, or

b) by demonstrating a load case with 'removal of a limited part of the structure' in order to document that an extensive failure of the structure will not occur if a limited part of the structure fails, or

c) by demonstrating sufficient safety of key elements, such that the entire structure with one or more key elements has the same reliability as a structure where robustness is documented by b).

If robustness is verified using key elements c), then these can be designed based on by increasing the material partial safety factor by a factor 1.2. The design procedure to document sufficient robustness can be summarised in the following steps:

1. Review of loads and possible failure modes/scenarios and determination of acceptable collapse extent

2. Review of the structural systems and identification of key elements

3. Evaluation of the sensitivity of essential parts of the structure to unintentional loads and defects

4. Documentation of robustness by 'failure of key element' analysis

5. Documentation of robustness by increasing the strength of key elements if Step 4 is not possible.

The framework mentioned above considers the structural robustness at system-level and has the potential to take into account uncertatinties inherent in desciption of unintentional loads and defects, static layout and structural composition into account by working in a probabilistic format. Such a format can deal with e.g. that loads occur randomly in space and in time, and have uncertain magnitudes. Similarly, the variables describing the capacity of structural members and systems and other loads that act at the time the unintentional loads and defects events occur are also random. Consideration of system effects is particularly important when modelling robustness. In general design criteria stated in codes mainly consider individual elements or subsystems of a larger structural system. In principle such a frame work is sufficient as long as extensive failure of the structure can not occur if a limited part of the structure fails due to lack of robustness. A framework where robustness is related to an extensive failure of the structure due to unintentional loads and defects subjected to a limited part of the structure can be formulated in a probabilistic format [1, 11, 12]. Assume a structural damage D_j among j different types resulting from a number of exposures, i.e. unintentional loads and defects. If each of these i distinct exposures is represented by an event E_i then the total probability of structural collapse with the consequense C can be written as

$$P(C) = \sum_i \sum_j P(C|E_i \cap D_j) P(D_j|E_i) P(E_i) \qquad (1)$$

where the summations are over all exposures and damages. $P(D_j|E_i)$ is the probability of damage type j given exposure type i and $P(C|E_i \cap D_j)$ is the probability of collapse given exposure type i and damage type j . For damages related to key elements the probability of collapse is $P(C|E_i \cap D_j) \cong 1.$
From Equation (1) it can also be seen that the probability of collapse can be reduced (and robustness can be increased) by:

- Reducing one or more of the probabilities of exposures $P(E_1), P(E_2)$,…
- Reducing one or more of the probabilities of damages $P(D_1|E_1), P(D_2|E_2), \ldots$ or reducing the extent of the damages. Example: strengthen vital structural elements – key elements (for example: column): $P(D_j|E_i)$ is reduced
- Reducing one or more of the probabilities $P(C|E_1 \cap D_1), P(C|E_2 \cap D_2), \ldots$ Example: increase redundancy of structure.

Increasing the robustness at the design stage will in many cases only increase the cost of the structural system marginally – the key point is often to use a reasonable combination of a suitable structural system and materials with a ductile behaviour. In other cases increased robustness will influence the cost of the structural system. If more alternatives to increase the robustness are considered, then from a decision theoretical point of view, the optimal alternative is that which results in the smallest expected total costs

Probabilistic Model for the Norwegian Sports Centre

The Norwegian sports centre has a structural system consisting of 14 glulam frames supporting the roof over the main court, see Figure 1. Each frame consists of one 17.5 m long tapered main beam between two beams with approximately constant cross section. The beams are carried by 5 columns, see Figure 2. The frames are spaced 3 m apart and they support pulins which in turn support a wooden ceiling on which is placed insulation, tar paper, plastic, gravel and turf, see figure 2. The sports centre was erected in 1999 and had severe shear cracking in three of the 14 glulam frames in March 2003. An analysis of theses damages together with detailed data describing the structure are given in [13].

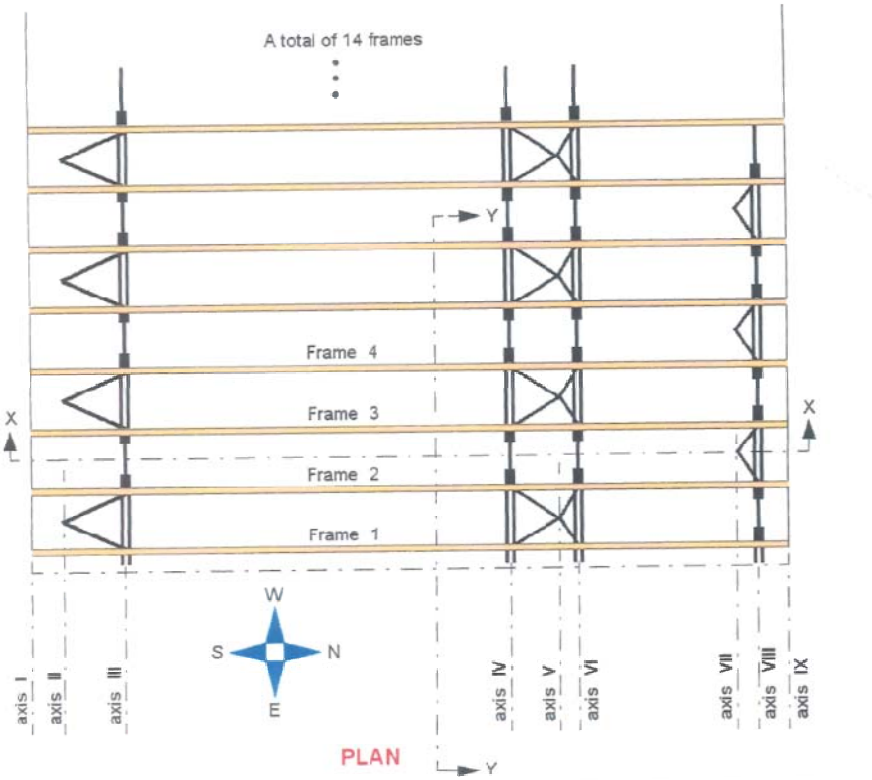


Figure 1: Plan of Norwegian Sports Centre.

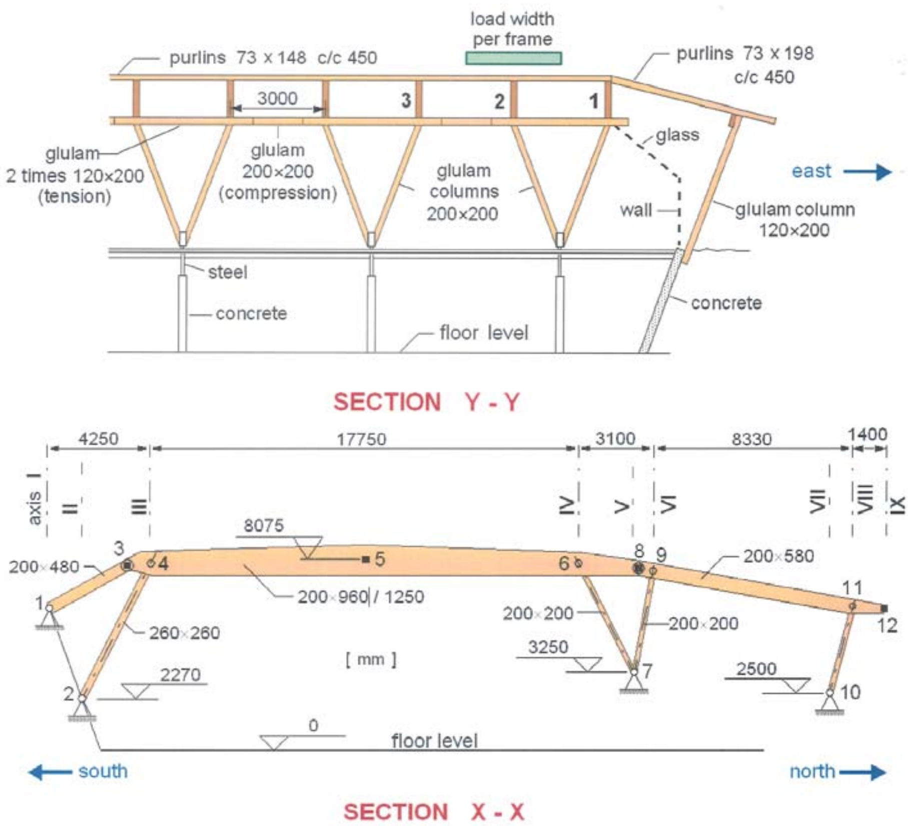
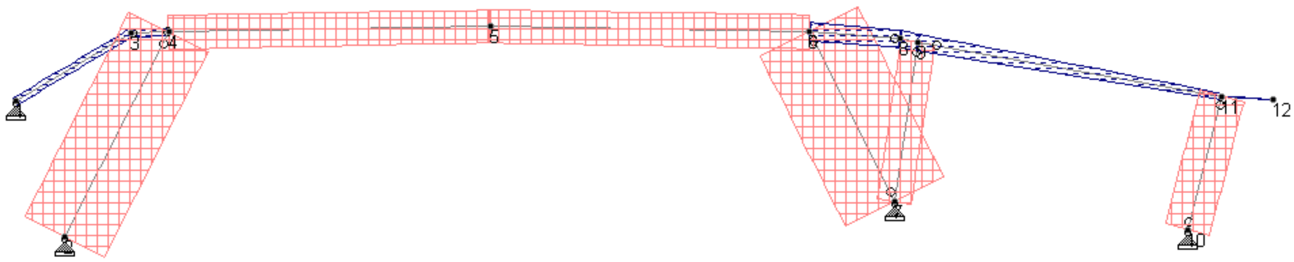


Figure 2: Sections of Norwegian Sports Centre.

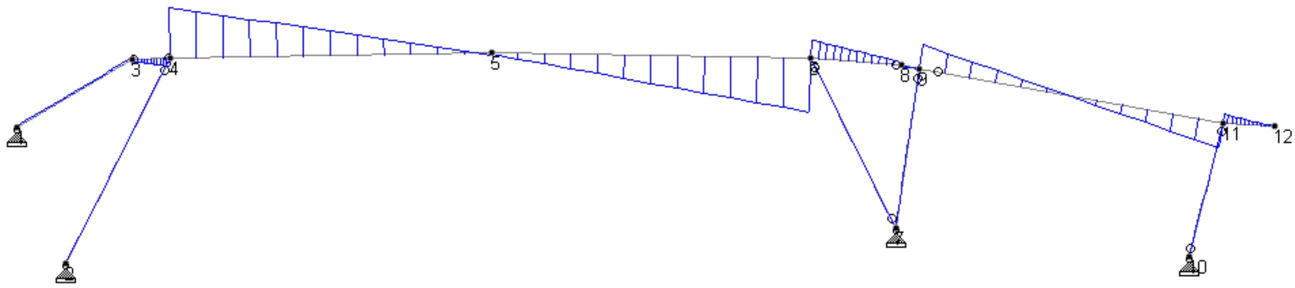
Failure Modes, Limit State Functions, Stochastic Model

The follwing sections outline the modelling used for the probabilistic calculations of the Norwegian sports centre by using First-Order Reliability Methods (FORM) where a reliability index β^F is estimated based on limit state function $g(\cdot)$ for each failure mode, see e.g. [14]. The probabilistic analysis will be performed with a stochastic model for the glulam frame number 3 with respect to the strength parameters for whole structural elements, and not to the strength for the single laminates and the glue [15]. Further, second order effects have be neglected for beams subjected to compresion and combined compression and bending, respectively. Buckling problems in the beams are assumed to be prevented by purlins and other secondary structural components attached to the main structural frame system. For the structural analysis a linear FEA has been performed where the glulam frame has be modelled by beam elements assuming hinges in joint 3 and 8, respectively. Figure 3 presents section forces for the glulam frame number 3 due to permanent load and a variable snow load. These loads will be described in next section. The magnitude of the section forces as well as the distribution corresponds to results presented in [13].

Axial Force



Shear Force



Bending

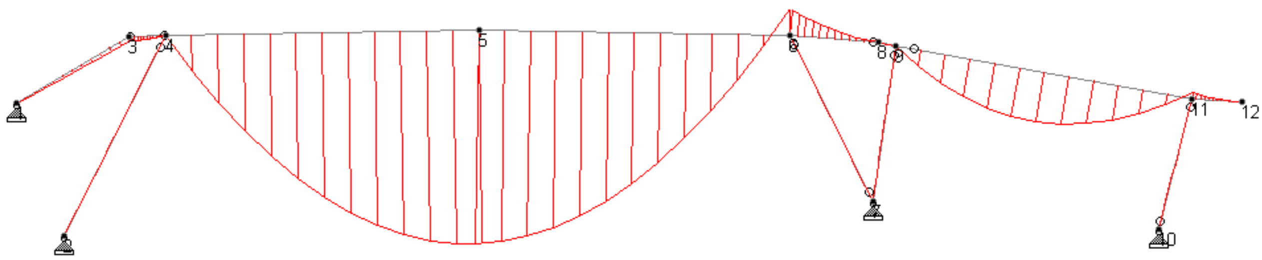


Figure 3: Section forces in glulam frame number 3 due to permanet load and snow load.

Failure modes

Related to ultimate limit state failure for the glulam frame 10 different failure modes are assumed, due to compression (N), tension (T), bending (M), combination of bending and compression (M+N), shear (V) and combination of tension perpendicular to the grain and shear. Also one service limit state failure mode is considered, i.e. deflection in the main beam. The 11 failure modes have been selected based on the section forces presented in figure 3 and the conclusions in the report [16] where a survey of a number of structural failures in large timber structures are given. The ultimate limt state failures are assumed to be brittle. This assumption and other failure modes which could be generated due to gross errors, e.g. failure in joints, will be discussed in section 4.

Following failure elements are considered for these failure modes

1. Failure in column 2-4 (N)
2. Failure in column 6-7 (N)
3. Failure in column 7-9 (N)
4. Failure in column 10-11 (N)
5. Failure in the main beam at point 5 (N+M)
6. Failure in the main beam at point 6 (N+M)
7. Failure in beam 9-11 (M+N)
8. Failure in the main beam at point 4 (V)
9. Failure in the main beam at point 6 (V)
10. Failure due to a combination of tension perpendicular to grain and shear at point 5
11. Failure in the main beam at point 5 due to deflection.

These 11 failure modes will be modelled according to the failure criteria stated in [17].

Limit state functions

The short-term ultimate limit state function is given for the failure elements 1-4

$$g_i = X_R - \frac{N_S}{N_R} = X_R - \frac{a_i G + b_i Q}{k_c A f_{c,0} k_{mod}} \qquad i = 1,..,4 \tag{3}$$

where A is the cross section area, $f_{c,0}$ the compressive strength along grain, X_R the model uncertainty , G the permanent load and Q the variable load. k_{mod} is a modification factor taking into account the effect of the duration of load and moisture content. k_c is a column instability factor. If the failure function is evaluated in section i, then the interal normal force N_S can be divided into a linear combination of the variable load Q, and the permanent load G. This gives $N = a_iG+b_iQ$ where a_i and b_i are constants depending on the geometry. These constants are obtained by a FE-analysis.

The failure elements 5-7 will be modelled with following short-term ultimate limit state function

$$g_i = X_R - \left(\frac{N_S}{N_R} + k_m \frac{M_S}{M_R}\right) = X_R - \left(\frac{a_i G + b_i Q}{k_c A f_{c,0} k_{mod}} + k_m \frac{c_i G + d_i Q}{W f_{m,\alpha} k_{mod} k_h}\right) \qquad i = 5,..,7 \tag{4}$$

where W is the section modolus. M_S and N_S are the internal bending moment and normal force, respectively given by linear combinations of the variable load Q and the permanent load G. M_R and N_R are the capacity values for bending moment and normal compressive force, respectively. $f_{m,\alpha}$ is the bending strength at an angle α to the grain, modelled $f_{m,\alpha} = k_{m,\alpha} f_m$ where $k_{m,\alpha}$ is a reduce factor due to the tapered beam shape. The factor k_m makes allowance for re-distribution of stresses and the effect of in homogeneities of the material in a cross-section and k_h is a size effect factor [17].

The shear failure elements 8-9 will be modelled with following short-term ultimate limit state function

$$g_i = X_R - \frac{V_S}{V_R} X_R = X_R - \frac{e_i G + f_i Q}{\frac{2}{3} A f_v k_{mod}} \qquad i = 8, \ldots, 9 \qquad (5)$$

where V_S is the internal shear force and V_R is the capacity value for shear force. V_S is given by a linear combination of the variable load Q and the permanent load,G. f_v is the shear strength.

The failure elements 10 for combined tension perpendicular to grain and shear has a short-term ultimate limit state function given by

$$g_{10} = X_R - \left(\frac{V_S}{V_R} + \frac{M_S}{M_{R,90}}\right) = X_R - \left(\frac{e_{10} G + f_{10} Q}{\frac{2}{3} A f_v k_{mod}} + \frac{(c_{10} G + d_{10} Q) k_p}{W k_{dis} k_{vol} f_{t,90} k_{mod} k_h}\right) \qquad (6)$$

where $M_{R,90}$ is the capacity value for bending moment related to tensile stress perpendicular til the grain. $f_{t,90}$ is tension strength perpendicular to the grain. k_{dis} is a factor which takes into account the effect of the stress distribution in the apex zone and k_{vol} is a volume factor, respectively [17]. The greatest tensile stress perpendicular to the grain is related to the bending moment by the factor k_p [17].

The deflection failure element 11 is given by the short-term serviceability limit state function

$$g_{11} = X_R - \frac{w_{net,fin}}{\delta_L} = X_R - \frac{e_{11} G (1 + k_{def}) + f_{11} Q}{\delta_L} \qquad (7)$$

where δ_L is an alloawable deflection limit given in [17] and $w_{net,fin}$ the net deflection given as a linear combination of permanent load G and variable load Q . The deflection contribution from permanent load is multiplied with the deformation factor $(1+ k_{def})$ where k_{def} is a factor for the evaluation of creep deformation due to permanet load.

Stochastic Model

The stochastic model is given in table 1 and is mainly based on information in [15], [13] and [18]. For the calculations permanent load G due to self weight and a variable snow load Q are taken into account. The permanent load of the roof structure, excluding the frame is Normal distributed with an expected value $\mu_G = 2.5$ kN/m^2 and a coefficient of variation (COV) $V_G = 0.1$, respectively. The load width per frame is 3 m. The self-weight of the frame is estimated during the FEA and added to the load from the roof structure and modelled by a Normal distribution with a COV at 10%.

For the region in Norway where the structure is located the annual maximum snow load at the groud Q_g is Gumbel distributed with a characteristic value Q_{g_k} = 6.5 kN/m^2 corresponding to a 98% quantile in an annual maximum distribution. The snow load at the roof Q is determined from

$$Q = CQ_g \qquad (8)$$

where C is a deterministic ground to roof snow load shape factor. Assuming the COV for ground snow load to be V_{Q_g} = 0.4 the expected value μ_{Q_g} is determined from the Gumbel cumulative distribution function $F_{Q_g}(\cdot)$

$$F_{Q_g}(Q_{gk}) = \exp\left(-\exp\left(-\alpha(Q_{gk}-\beta)\right)\right) \quad , \quad \mu_{Q_g} = \beta + \frac{0.577216}{\alpha}, \sigma_{Q_g} = \frac{\pi}{\alpha\sqrt{6}}, \quad V_{Q_g} = \frac{\sigma_{Q_g}}{\mu_{Q_g}} \qquad (9)$$

which gives $\mu_{Q_g} = 3.13\ \mathrm{kN/m^2}$. This value has to be multiplied by 3 m to determine the total expected ground snow load per frame.

The strength variables $f_{c,0}$, f_v, and $f_{t,90}$ are given as functions of the μ_{f_m} and V_{f_m} for the bending strength and the expected value for the density μ_ρ [15]. The initial (short term) bending strength is assumed to be Lognormal distributed with V_{f_m} = 0.15. Assuming a glulam material L40 with a charateristic value $f_{m,k} = 40$ MPa corresponding to a 5% quantile value the parameters $\mu_{\ln f_m}$ and $\sigma_{\ln f_m}$ of the Lognoral distribution can be determined from the equations

$$F_{f_m}(f_{m,k}) = \Phi\left(\frac{\ln f_{m,k} - \mu_{\ln f_m}}{\sigma_{\ln f_m}}\right), \quad \sigma_{\ln f_m} = \sqrt{\ln(V^2_{f_m}+1)}, \quad \mu_{\ln f_m} = \ln\mu_{f_m} - \frac{1}{2}\sigma^2_{\ln f_m}$$
(10)

where $\Phi(\cdot)$ is the cumulative standard normal distribution. Based on the parameters of the Lognormal distribution the expected value for the bending strength becomes $\mu_{f_m} = 49.9$ MPa. The density of the glulam is assumed to have an expected value μ_ρ = 490 $\mathrm{kg/m^3}$. The different strenght variables are mutually correlated as given in table 2.

Cross sections area A, cross section modulus W and the instability factor k_c are assumed normally distributed with a coefficient of variation of 1 %. All other parameters are assumed to be deterministc as presented in table 1.

Variable	Distribution	Expected value	COV	Designation
f_m	LN	49.9	0.15	Bending strength [13]
$f_{c,0}$	LN	$5\mu_{f_m}^{0.45}$	$0.8V_{f_m}$	Compression strength along grain [15]
f_v	LN	$0.2\mu_{f_m}^{0.8}$	V_{f_m}	Bending strength [15]
$f_{t,90}$	W	$0.0015\mu_\rho$	$2.5V_\rho$	Shear strength [15]
X_R	LN	1	0.05	Model uncertainty on short-term bearing capacity [15]
G	N	2.5 kN/m	0.1	Permanent load [13] (load width 3 m)
Q_g	G	3.13 kN/m	0.4	Variable load – snow [13] (load width 3 m)
A	N	1*	0.01	Area, *) multiplied with design value [18]
W	N	1*	0.01	Modulus,*) multiplied with design value [18]
k_c	N	1*	0.01	Instability,*) multiplied with design value [18]

C	D	0.8	-	Shape factor for snow [13]
k_h	D	1*	-	Size effect factor,*) multiplied with design value [17]
k_m	D	0.7	-	Re-distribution of stresses factor [17]
k_{dis}	D	1.4	-	Stress distribution factor in apex zone [17]
k_{vol}	D	1*	-	Volume factor in apex zone,*) multiplied with design value [17]
k_{mod}	D	0.9	-	Strength modification factor [15, 17]
k_{def}	D	1	-	Stiffness modification factor [15, 17]
k_p	D	0.007	-	Tensile stress perpendicular to the grain factor [17]
δ_L	D	0.089 mm	-	Deflection limit [17]

Table 1: Statistical characteristics (N:Normal, LN:Lognormal, G:Gumbel, W:2-pWeibull, D:Deterministic).

	f_m	$f_{c,0}$	f_v	$f_{t,90}$
f_m	1	0.8	0.4	0.4
$f_{c,0}$	0.8	1	0.4	0.2
f_v	0.4	0.4	1	0.6
$f_{t,90}$	0.4	0.2	0.6	1

Table 2: Correlation coeffficient matrix for strength parameters.

Robustness Evaluation of Glulam Frame

During the following sections the glulam frame will be analysed using the probabilistic model formulated in section 3. In section 4.1 a relability analysis of the undamaged glulam will be performed for the identification of those parts of the structure essential for the safety, and evaluate the sensitivity of structural safety with respect to the uncertainties included in the probabilistic model. Section 4.2 will present results with the load case 'removal of a limited part of the structure' in order to verify that an extensive failure of the structure will not occur if a limited part of the structure fails.

Reliability Analysis of the Glulam Frame - Identification of Key Elements

For each of the failure elements, formulated in section 3, element reliability β_i as well as system reliability β^s is estimated using first-order reliability methods (FORM) [14]. The reliability analysis is performed using the software PRADSS (Program for Reliability Analysis and Design of Structural Systems) [19].

The element reliability indices β_i given in Table 3 indicate that failure element 2 and 11 are the most significant failiure modes for the glulam frame. The relative ratio between the different reliability indices corresponds very well to the results from a deterministic analysis in [13] where cofficients of utilisation for each failure mode were estimated. E.g. the column 6-7 was found to be over-stressed with approximately 20 % compared with design criterion in the Norwegian building code while aother failiure modes had cofficients of utilisation in the region 75-90 %. The failure element corresponding to a deflection failiure mode has also a relatively low relability index. However, this estimate is strongly related to the choice of design criterion δ_L. In the following analysis only ultimate limit state failure modes will be considered.

1	2	3	4	5	6	7	8	9	10	11
5.58	3.40	6.55	5.76	6.58	5.37	6.05	4.96	4.81	6.31	3.18

Table 3: Element reliability indicesβ_i, reference period 1year.

The requirements to the safety of the glulam structure can be expressed in terms of an accepted minimum reliability index, i.e. a target reliability index. The Joint Committee on Structural Safety (JCSS) has proposed target reliability values for ultimat limit states for diffenent type of structures. The values presented in Table 4 [20] are obtained based on cost benefit analyses for the society at characteristic and representative but simple example structures and are compatible with calibration studies and statistical observations. The shadowed value in Table 4 should be considered as the most common design situation.

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences failure
Large (A)	$\beta = 3.1\ (P_f \approx 10^{-3})$	$\beta = 3.3\ (P_f \approx 5 \cdot 10^{-4})$	$\beta = 3.7\ (P_f \approx 10^{-4})$
Normal (B)	$\beta = 3.7\ (P_f \approx 10^{-4})$	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$
Small (C)	$\beta = 4.2\ (P_f \approx 10^{-5})$	$\beta = 4.4\ (P_f \approx 5 \cdot 10^{-6})$	$\beta = 4.7\ (P_f \approx 10^{-6})$

Table 4: Tentative target reliability indicies β (and associated target failure rates) related to one year reference period and ultimate limit state [20].

However, it should be noticed that the failure consequence also depend on the type of failure classfied as

a. Ductile failure with reserve strength capacity
b. Ductile failure with no reserve capacity
c. Brittle failiure.

Consequently, a structural element likely to result in an un-warned collapse should be designed for a higher reliability level than a structure with a more ductile like collapse scenario. Since the glulam frame is assumed to behave in a brittle mode one could argue for a higher reliability target level. Further, the relability indices in Table 4 are proposed for a structure with one dominant failure mode, i.e. for a structure with equally important failure modes a higher target relability level should be considered. The relability indices in Table 3 indicate only one signifacant failiure mode in ultimate limit state and therefore a target reliability index β^t = 4.2 is selected. Compared with a recommend target value the reliability analysis of the glulam frame indicates a structure with a bit too high probability of failure for the column 6-7.

So far only reliabilities of individual failure modes or limit states have been considered. Assuming the individual failure modes are combined in a series system of failure elements $y_i$the overall generalised system reliability for the glulam frame is given by

$$\beta^s \approx -\Phi^{-1}(P^s{}_f) \quad , \qquad P^s{}_f = \left(\bigcup_{i=1}^{10} g_i \leq 0 \right) \approx 1 - \Phi_{10}(\bar{\beta}, \bar{\rho})$$

The combination of failure elements in a series system can be understood as the glulam frame is non-redundant. In the present paper the generalised system reliability is estimated by the Hohenbichler approximation [14] after the element realiability indices have been organised in the vector $\bar{\beta}$ and the corresponding correlation between each failiure elemtent in the corellation matrix $\bar{\rho}$. For the 10 ultimate limit states the generalised system reliability $\beta^s = 3.32$ is estimated.

In order to analyse the sensitivity of the system reliability with respect to stochastic as well as deterministic parameters a sensitivity analysis has been performed. Figure 4 shows the sensitivity of the systems reliability index β^s to variations of the expected values μ of the stochastic variables $\frac{\partial \beta^s}{\partial \mu}\frac{\mu}{\beta^s}$ and standard deviations $\frac{\partial \beta^s}{\partial \sigma}\frac{\sigma}{\beta^s}$, respectively. This sensitivity measure, the *reliability elasticity coefficient*, gives the change in the reliability index in percentages due to 1 % changed of one of the parameters. The sensitivities of the system reliability to variations in deterministic parameters are estimated by modelling the deterministic parameters as fixed stochastic variables as presented in tabel 1. From figure 4 it is seen that the largest contribution to the overall uncertainty is due to the compression strength along grain $f_{c,0}$, column instability factor k_c and the cross section area A . Besides that, the model uncertainties turn out to be important. The systems reliability is also seen to be sensitive to variations of the snow load.

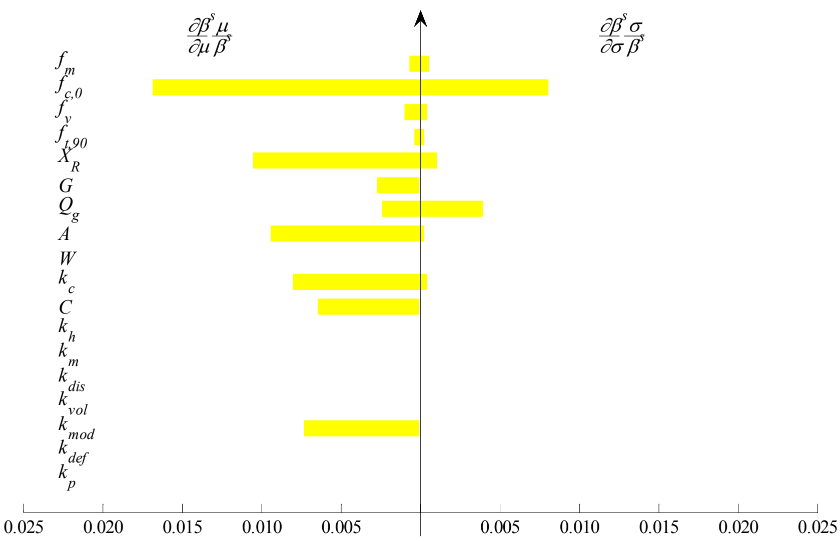


Figure 4: Sensitivity of the system reliability to variations of the parameters of the stochastic and deterministic variables.

Reliability Analysis of the Glulam Frame - Removal of Key Elements

In the following section 4 damage scenarios assuming columns with brittle failure modes are considered, see Figure 5. Only three failure modes will be considered due to a potentail significant failiure of the sports arena. Collapse of column 10-12 is assumed to give a minor significant failure. Horisontatal stability is assumed to be fulfilled by the primary structure during failure of one element. This means that following failure scenarios will be considered

1. Faliure of column 1-4
2. Faliure of column 6-7
3. Faliure of column 7-9

Each failure mode will be considered for the peremanet load G, permanent load and ekstreme snow G+Q and permament load combined with a daily snowload $G+Q_d$. The assumed daily snow load will be estimated using the Ferry Borges-Castanheta load model.

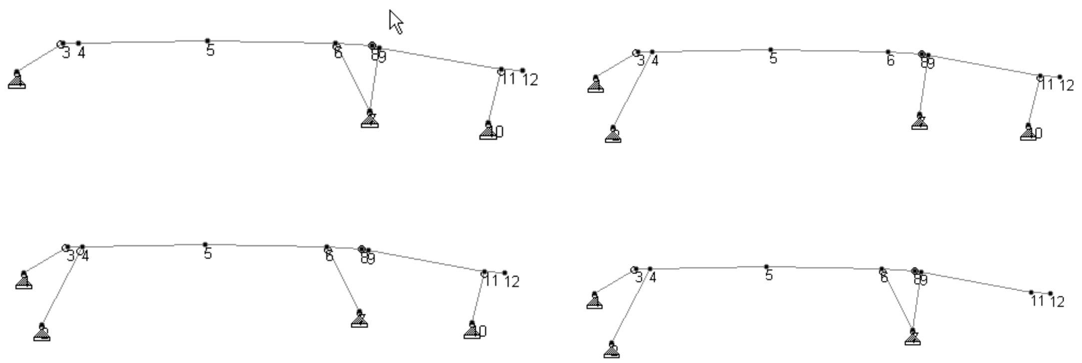


Figure 5: Four different failure scenarios.

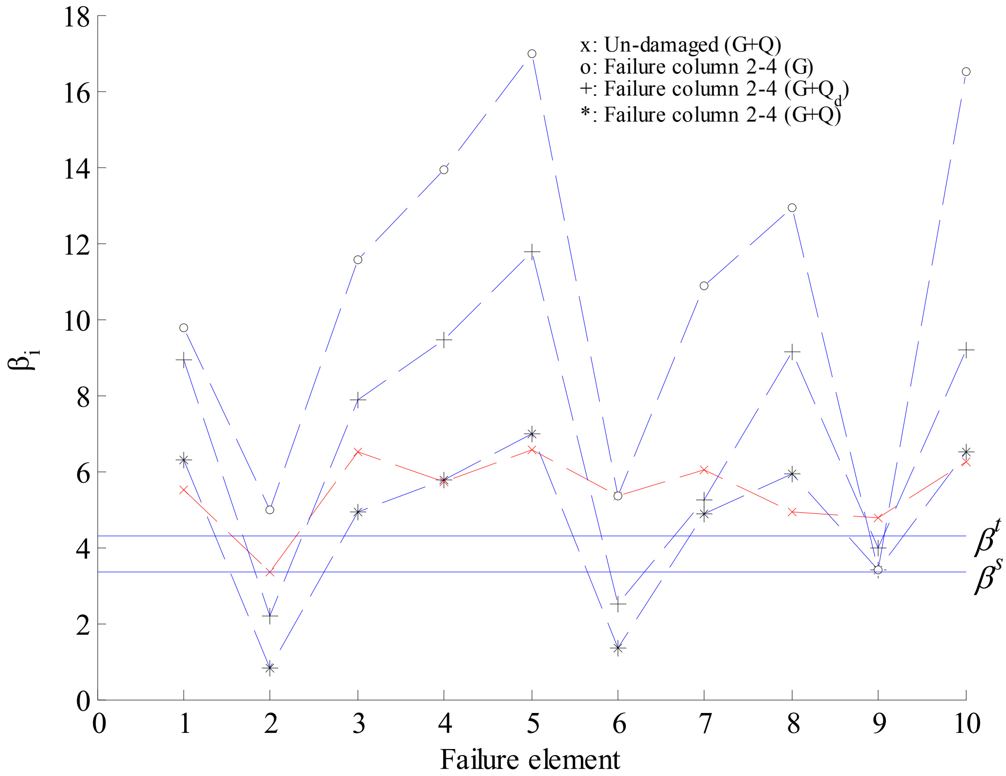


Figure 6: System reailibity indices for failure senario 1.

Figure 6,7 and 8 show the reliability indices for failure of columns 1-4, 6-7 and 7-9, respectively . Remark in figure 6 that failure element 1 is related to a failure mode with compression in colomn 1-3. This failure mode will only be considered related to failure scenario 1 where there will be compression in column 1-3, else it is a tension element.

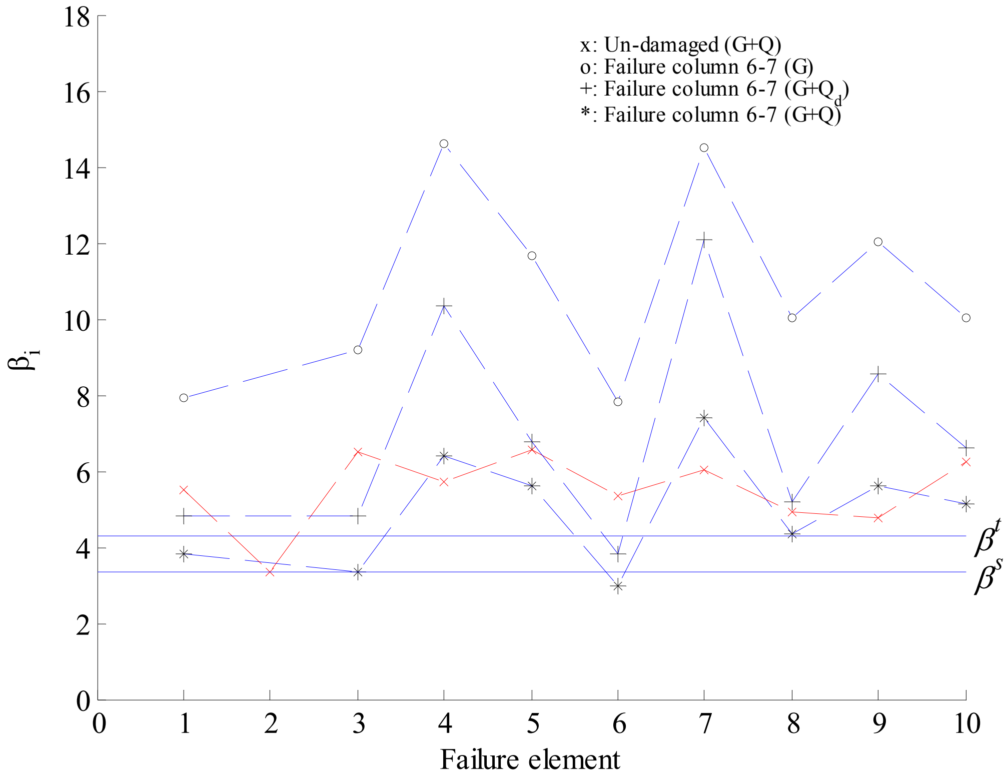


Figure 7: System reailibity indices for failure senario 2.

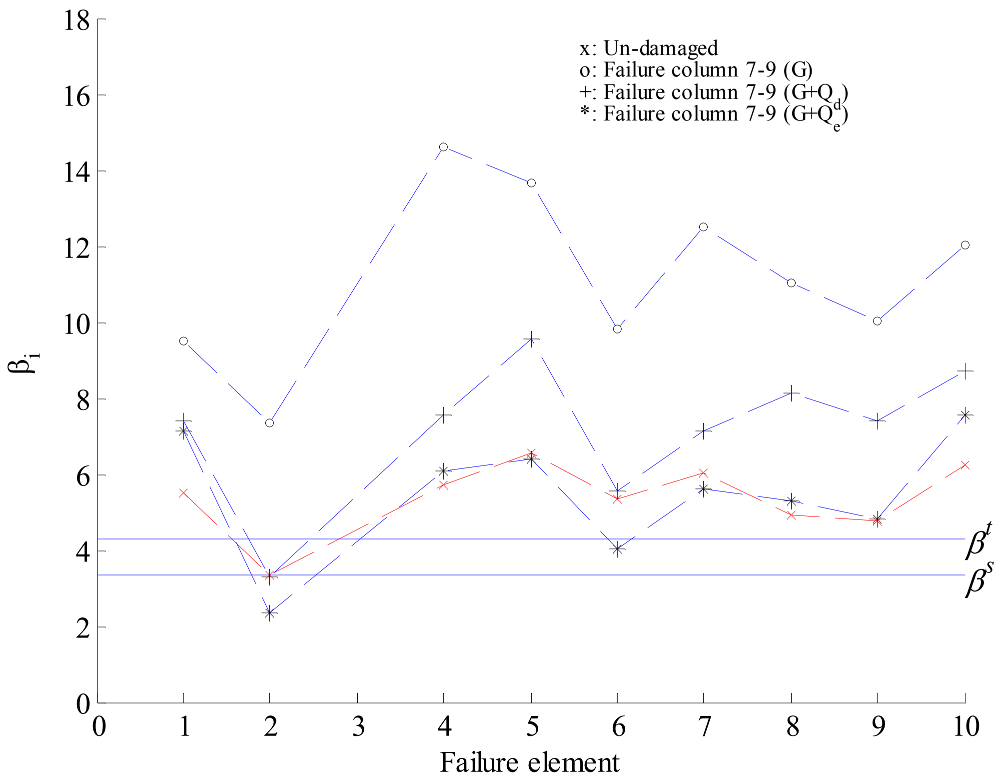


Figure 8: System reailibity indices for failure senario 3.

From the results in figure 6, 7 and 8 it is seen that the timber structure can be charaterised as robust with respect to the robustness framework used for the evaluation. By removal of three different columns one by one none significant extensive failure of the entire structure or significant parts of it is faciliated. However, this conclusion is strongly related to the choice of target reliability, modelling of the daily snow load and modelling of the joints.

Conclusion

The aim of the present paper was to investigate the robustness characteristics of timber structures. The robustness analysis is based on the framework for robustness analysis introduced in the Danish Code of Practice for the Safety of Structures and a probabilistic modelling of the timber material proposed in the Probabilistic Model Code (PMC) of the Joint Committee on Structural Safety (JCSS). The approach has been used for a case considering a glulam frame structure supporting the roof over the main court in a Norwegian sports centre. Compared with a recommend target value the reliability analysis of the glulam frame indicates a structure with a bit too high probability of failure for one out of 11 considered failure modes. Progressive collapse analyses are carried out by removing three columns one by one implying that the timber structure can be charaterised as robust with respect to the robustness framework used for the evaluation. However, the results are obtained based on a simplified modelling of the timber structure which does not consider a non-linear behaviour of the joints. Future invistigations should also consider redistribution of load effects, system effects and a modelling of possible gross erros, i.e. unintentional load and defects.

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