# A Novel Technique for the Numerical Evaluation of the Green's Functions Associated to Cavity Backed Antennas in Circular Waveguides

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### Abstract

In this contribution a simple and effective technique for the numerical calculation of the Green's Functions in Cylindrical shaped enclosures is developed. The technique is based on the numerical imposition of the boundary conditions for the fields at the cylindrical walls, using the theory of images. Numerical results for the Green's functions inside a cylindrical cavity are presented, including convergence test of the algorithm. Results show that numerical convergence is attained fast, therefore demonstrating the usefulness of the developed algorithm.

## 1 Introduction

The analysis of shielded circuits and cavity backed antennas is a subject that has attracted recently the attention of many investigations [1]. For the analysis of shielded circuits and cavity backed antennas, the integral equation technique has grown in popularity due to its efficiency, and to the capability to push to a maximum the analytical treatment of the problem.

For the calculation of the relevant Green's functions, only the rectangular enclosure has been extensively treated in the past [2]. In the case of circular waveguides, the Green's functions are formulated by using the corresponding vector modal series based on the Bessel functions [3]. However, this approach shows to be critical from the numerical point of view, since the higher order Bessel functions are not easily computed with high accuracy. On the other hand, spatial domain formulations have not been applied to the computation of the Green's functions in circular waveguide geometries. This is mainly because an analytical solution for the spatial images of a point source in the presence of cylindrical metallic structures does not exist.

This contribution presents a numerical technique that can be used for the computation of the Green's functions in circular cavities. The technique is formulated for the first time in the spatial domain, and it uses the theory of images to enforce the proper boundary conditions for the fields. Results of convergence

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show that the derived technique is indeed efficient, and can be used for the numerical calculation of the relevant Green's functions avoiding slow convergence series of previous formulations [4].

## 2 Theory

The geometry for the calculation of the mixed potential Green's functions is presented in Fig. 1. As shown, a unit dipole is placed inside a circular metallic cavity. For the electric scalar potential Green's function we should impose null potential on the cavity wall. If we impose this condition at only one point of the wall, then a proper choice will be to place an infinite plane tangent to the cylindrical wall at the point of interest, and then by image theory take a negative charge at the mirror position with respect to the plane.

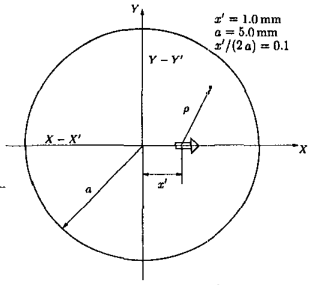


**Figure 1:** Unitary dipole inside a cylindrical cavity studied in this paper.

It is desirable, however, to be able to impose the boundary conditions at more than one point of the cylindrical wall. If a similar procedure is repeated in order to impose the boundary conditions for the potential at $N$ distinct points of the cylindrical wall, the following system of linear equations is obtained

$$\sum_{i=1}^{N} q_i \, G_V(\bar{r_j}, \bar{r_i}') = -G_V(\bar{r_j}, \bar{r_0}'); \qquad j = 1, 2, \cdots, N \tag{1}$$

where $G_{V_{\mathrm{cyl}}}(\bar{r})$ is the scalar potential Green's function inside the cylindrical cavity, and $G_V(\bar{r}, \bar{r}')$ is the potential Green's function of a unit point charge. The solution of above system gives the value of the $N$ image charges $q_i$ needed to satisfy the boundary conditions for the potential at $N$ distinct points of the cylindrical wall. The final scalar potential Green's function inside the circular cavity is simply evaluated by reusing the already computed charge amplitudes, namely

$$G_{V_{\mathrm{cyl}}}(\bar{r}) = G_V(\bar{r}, \bar{r_0}') + \sum_{i=1}^{N} q_i \, G_V(\bar{r}, \bar{r_i}') \tag{2}$$

For the evaluation of the magnetic vector potential dyadic Green's function, a similar procedure is followed, but taking into account the vector nature of the quantity to be computed.

# 3 Results and Conclusions

The procedure described in the previous section has been implemented for the numerical calculation of the Green's functions inside the structure shown in Fig. 1. First, the algorithm has been tested in the evaluation of the static electric scalar potential in this structure. Fig. 2 shows a comparison between the computed scalar potential (along the X-X' cut) and the potential inside an sphere of equal radius, with known analytical solution [5]. It can be seen that both solutions satisfy the boundary conditions at the metallic wall. The figure also shows the convergence behavior of the algorithm, by presenting the results obtained when the boundary conditions are enforced in 2 and 20 points of the cavity wall. It can be seen that the convergence is attained fast since the numerical results obtained in both cases are very similar.

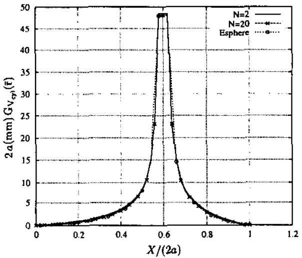
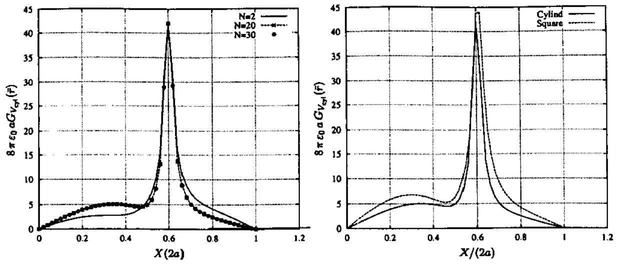


**Figure 2:** Comparison for the static electric scalar potential between a cylindrical cavity and a sphere cavity.

In order to check the numerical behavior of the technique when frequency increases, we present in Fig. 3(a) the electric scalar potential Green's function along the X-X' cut, at 30 GHz. The figure shows the results obtained when 2, 20 and 30 points are used to enforce the boundary conditions. It can be observed that the results with 20 and 30 points are very similar, showing that convergence has been reached. Finally, in Fig. 3(b) the same results are compared against the Green's function obtained inside a square cavity of length side equal to the diameter of the cylinder. The electric scalar potential inside the square cavity is evaluated following the technique described in [6], while inside the cylinder we use the technique described in this paper. It can be seen that the behavior of the potential is very similar in both cases.

As a final result we have checked the stability of the algorithm when frequency increases. In Fig. 4 we present a 3D plot of the electric scalar potential inside the same structure as before but at 70 GHz. It can be observed similar results as before, but now the standing wave created by the presence of the cavity wall is more strongly excited, as characterized by the increasing number of *peaks* in the response. A part from this fact, the convergence is reached again with 20 points in the calculations, and as can be observed in the 3D plot of Fig. 4, the boundary conditions are satisfied through the whole cylindrical wall contour.



(a) Convergence results. (b) Comparison with square cavity.

**Figure 3:** Electric scalar potential Green's function inside the cylindrical cavity along the X-X' cut of Fig. 1 (frequency is 30 GHz).

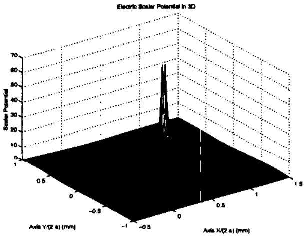


**Figure 4:** Electric scalar potential Green's function in three dimensions inside the cylindrical cavity of Fig. 1 (frequency is 70 GHz).

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