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Determination of the High Frequency Inductance Profile of Surface Mounted Permanent Magnet Synchronous Motors

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Abstract- **Accurate knowledge of the high frequency inductance profile plays an important role in many designs of sensorless controllers for Surface inductance. A special algorithm is used to decouple the cross-coupling effects between the d-axis and the q-axis, which allows Mounted Permanent Magnet (SMPM) synchronous motors. This paper presents an AC+DC measurement method for determination of the d-axis and q-axis high frequency inductance profiles of SMPM synchronous motors. This method uses DC currents to set a desired magnetic working point on the motor laminations, and then superimpose balanced small AC signals to measure the incremental a separate determination of the d, q inductance profiles as functions of the d, q currents. Experimental results on a commercial SMPM motor using the proposed method are presented in this paper.**

I. INTRODUCTION

High performance sensorless controllers for Surface Mounted Permanent Magnet (SMPM) synchronous motors have many attracting aspects for modern industry applications. Some sensorless control methods are based on the knowledge of the difference between the d- and q-axes high frequency inductances caused by magnetic saturation [1]~[3]. For a certain type of SMPM motors, the significance of the difference between d, q inductances plays an important role in a successful application of such sensorless control methods. This makes it necessary to have a reliable knowledge of the high frequency d, q inductances, which may be obtained by measurements.

The classical dq-model of a SMPM synchronous motor assumes that the d- and q-axes inductances have constant values. In reality, these two inductance values are functions of both the d- and q-axes currents. This is known as the cross-coupling effects. To be able to achieve an accurately model of the SMPM motor including cross-coupling effects, knowledge of the nonlinear inductance profiles as functions of d, q currents are necessary. Accurate knowledge of these nonlinear inductance profiles may only be obtained by measurements.

When the frequency of the injected signal for position estimation is far above the normal frequency range, according to [1], the inductance will behavior differently. In [1], it was illustrated that at low frequencies, the d-axis inductance is lower than the q-axis inductance due to the saturation caused by the rotor magnet fields. But at high frequencies, d-axis inductance becomes greater than the q-axis inductance, which is due to the change of the flux path from the main path to the leakage path [1]. More investigations on this phenomenon were reported in [4], [5].

Many reported measurement methods for determination of the nonlinear d, q inductances as functions of d, q currents are based on the steady-state motor equation with the back EMF pre-measured and assumed to be constant [6] ~ [8]. The steady-sate performance based measurement method requires a rotating test and mechanically loading the test motor to its rated operating point. Additional information, like speed, needs to be known. If inductance under high frequency excitation needs to be determined, this method is no longer valid.

The test may also be performed by locking the rotor shaft and supplying proper currents to the phases to create a desired d or q-axis field [6]. The flux linkage vs. current information is measured, e.g. using an inductance bridge [6]. This measurement method only requires a locked-rotor test, which is much simple to perform compared to the rotational test with a loading system. No speed measurement is needed for inductance determination in this method. But if high frequency inductance measurement is required, where high frequency AC signals are injected into the phases, due to the magnetic saturation caused by magnets, the d, q-axes inductances are not equal even for a SMPM motor, which results in unbalanced three phase high frequency currents. This brings difficulty in inductance determination using the motor equations, either in abc-reference frame or in dq-reference frame.

In this paper, a special algorithm is proposed for separate determination of the high frequency d- and q-axes inductances. This method uses DC currents to set a desired magnetic working point on the d- or q-axis, and then superimpose balanced small AC signals to measure the incremental inductance. A special algorithm is introduced for simplifying the motor equations, which allows a separate determination of the d, q-axes inductances.

II. FEM ANALYSIS OF THE HIGH FREQUENCY INDUCTANCE PROFILE OF A SMPM SYNCHRONOUS MOTOR

For a SMPM motor, ideally, the rotor exhibits no saliency and the d, q-axes inductances are equal. Though the d, q-axes are perpendicular to each other, which seems to offer a magnetic decoupling, they have common flux paths, e.g., in the stator core

back. If part of the stator core back region is saturated by the d-axis magnet and/or d-axis current, the flux produced by the q-axis current, which travels through this saturated region will be influenced, causing a reduction in the q-axis inductance. This phenomenon is illustrated in Fig. 1, where the regions indicated by circles are saturated regions caused by the d-axis flux, and where the q-axis flux will travel through.
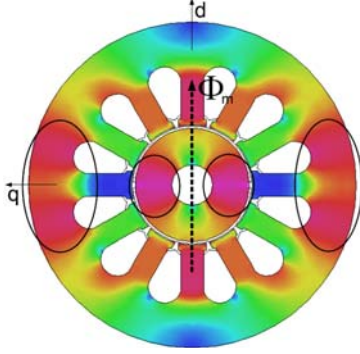

Fig. 1 Static 2D FE results showing the cross-coupling effects of a SMPM motor.

When high frequency signals are injected into the motor for position estimation, the flux path produced by high frequency currents will change, which is believed to be mainly due to the eddy current effects, as is illustrated in Fig. 2, where flux plot from AC transient analysis is shown.

(a) Very low frequency AC current excitation, at 0.1 Hz
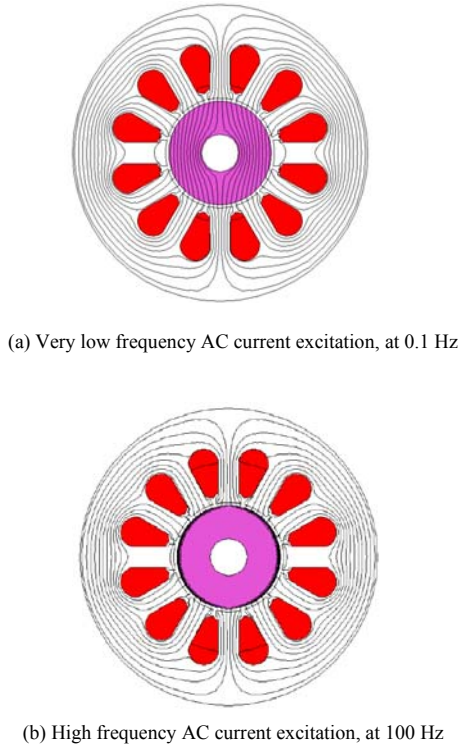

(b) High frequency AC current excitation, at 100 Hz

Fig. 2 Flux plot obtained from transient AC analysis showing the eddy current effects on flux path changing.

In this transient model, the rotor is assigned with a constant conductivity. It can be observed from Fig. 2 that at low frequency, the flux cross through the rotor region evenly. But when the frequency increases, eddy currents on the rotor side will be induced due to the alternating field created by stator currents. The field produced by the rotor eddy currents is always opposite to the change of the state flux. As a result, the rotor flux is forced to be squeezed in the area close to the rotor surface. This causes the rotor reluctance to increase (both due to decreased cross-section area and quick saturation in the flux path), and consequently, a decrease in the winding inductance.

A 2D model assumes the same geometry and material property on the direction perpendicular to the 2D plan. Such a model with conductivity assigned on the rotor side assumes the rotor is constructed by a bulk material. Many motors used in the industry often use laminations on the rotor side. So a 2D model may only give a rough estimation, especially eddy current effects are concerned. Accurate knowledge of how the inductance will behavior as a function of the d- and/or q-axis current and frequency, may be obtained by measurements. A reliable measurement method is required.

III. PROPOSED ALGORITHM FOR EXPERIMENTAL DETERMINATION OF HIGH FREQUENCY INDUCTANCE PROFILE

The proposed inductance measurement method uses AC+DC excitation. DC currents are used to set a desired magnetic working point on the d- or q-axis. Even in deep saturated regions, if the AC excitation is controlled to be small, the resultant AC current will still be sinusoidal, and the equivalent measurement circuit can be modeled by a simple R-L circuit. The corresponding incremental inductance can then be calculated based on the circuit impedance, which may be simply calculated from the measured terminal voltage, and line current. But the problem here is that due to the magnetic saturation caused by magnets and d, q currents, the d, q-axes inductances are not equal even for a SMPM motor. This causes the three phases self-induces differ from each other. Even balanced three-phase voltage is supplied on the phases, the resultant phase currents will differ in their magnitudes and their phase angles. The rotor is locked during the test. A constant angle value should be used in the Park transformation. Therefore the transformed d, q currents will be sinusoidal and also unbalanced. The zero-component will exist. If the d, q equivalent circuits are used for d, q inductances calculation, the instantaneous abc currents should be measured for calculation of the d, q currents. The measurement of the instantaneous current requires accurate data acquisition system, which is not convenient.

When the rotor is locked, the back EMF produced by the magnets is zero. The motor voltage equation becomes:

$$\underline{v}_{abc} = R\underline{i}_{abc} + \frac{d\underline{\lambda}_{abc}}{dt} \tag{1}$$

$$\underline{\lambda}_{abc} = \begin{bmatrix} \lambda_a \\ \lambda_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{ls} & 0 & 0 \\ 0 & L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_a \\ i_c \\ i_c \end{bmatrix} + L_{sm} \cdot \begin{bmatrix} i_a \\ i_c \\ i_c \end{bmatrix} \tag{2}$$

$$L_{sm} = L_{aad} \begin{bmatrix} \cos(\theta) \\ \cos\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} + L_{aaq} \begin{bmatrix} \sin(\theta) \\ \sin\left(\theta - \frac{2\pi}{3}\right) \\ \sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{3}$$

Where R is the stator phase resistance. L_{ls} is the phase leakage inductance, and L_{sm} is the main inductance matrix. The main inductance is defined as the inductance calculated from the flux travelling through the stator, the air-gap, and the rotor. It may be easily observed from (3) that inductance $L_{ls} + L_{aad}$ is the phase self-inductance when the rotor d-axis is aligned with this phase and the other two phases are open-circuited. Correspondingly, inductance $L_{ls} + L_{aaq}$ is the self-inductance when rotor q-axis is aligned with this phase. Angle θ is the rotor position angle, defined as the angle between the rotor d-axis and the stator phase-a axis. $\underline{v}_{abc}$, $\underline{i}_{abc}$, and $\underline{\lambda}_{abc}$ are column vectors of the three phase terminal voltage, line current, and flux linkage, respectively.

A special row vector is introduced as following:

$$k_q = \begin{bmatrix} \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{4}$$

It should be noticed that for each test where the rotor is locked, (4) is a constant row vector. Multiplying (4) to the both sides of (1) (and using (2) and (3)), and applying the following trigonometric relations [9]:

$$\frac{3}{2}\cos(x - y) = \sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) \tag{5}$$

$$\frac{3}{2}\sin(x - y) = \sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) \tag{6}$$

Equation (1) can be finally transformed to:

$$v_{eq} = Ri_{eq} + \frac{d}{dt}\left(L_{ls}i_{eq} + \frac{3}{2}L_{aaq}i_{eq}\right) \tag{7}$$

where

$$v_{eq} = \sin\theta \cdot v_a + \sin\left(\theta - \frac{2\pi}{3}\right) \cdot v_b + \sin\left(\theta + \frac{2\pi}{3}\right) \cdot v_c \tag{8}$$

$$i_{eq} = \sin\theta \cdot i_a + \sin\left(\theta - \frac{2\pi}{3}\right) \cdot i_b + \sin\left(\theta + \frac{2\pi}{3}\right) \cdot i_c \tag{9}$$

θ is the rotor position which is known.

Equation (7) forms a simple R-L circuit. The resistance is the phase resistance, which can be pre-measured using the DC method. The equivalent inductance in (7) is actually the q-axis inductance used in the dq-model. To calculate the inductance in (7), only the rms values of the equivalent voltage v_{eq}, and the equivalent current i_{eq} need to be known.

As has been discussed before, the resultant phase currents from a balanced three phase supply are not balanced. They differ in their magnitudes and phase angles. But they will still remain sinusoidal shapes. The coefficients evolved with θ in (8) and (9) are constants. The rms value of (9) may be calculated as:

$$i_{eq,rms} = \sqrt{i_{eq,rms,1}^2 + i_{eq,rms,2}^2} \tag{10}$$

$$i_{eq,rms,1} = \sin\theta \cdot i_{a,rms} + \sin\left(\theta - \frac{2\pi}{3}\right) \cdot i_{b,rms} \cdot \cos\theta_{ba} + \sin\left(\theta + \frac{2\pi}{3}\right) \cdot i_{c,rms} \cdot \cos\theta_{ca} \tag{11}$$

$$i_{eq,rms,2} = \sin\left(\theta - \frac{2\pi}{3}\right) \cdot i_{b,rms} \cdot \sin\theta_{ba} + \sin\left(\theta + \frac{2\pi}{3}\right) \cdot i_{c,rms} \cdot \sin\theta_{ca} \tag{12}$$

where θ_{ba} is the phase difference angle between phase a current and phase b current. θ_{ca} is the phase difference angle between phase a current and phase c current. The supplied voltage is balanced. Therefore the phase shift angle between phase voltages is known, which is $\frac{2\pi}{3}$. If the power factor angle, which tells the phase shift information between the current and the voltage, is measured, then the needed phase shift angles (θ_{ba}, θ_{ca} used in (11) and (12)) may be determined.

The rms value of the equivalent voltage (8) can be similarly calculated by replacing current with voltage in (10)~(12). For voltage calculation, $\theta_{ba} = \frac{2\pi}{3}$ and $\theta_{ca} = \frac{4\pi}{3}$.

For the d-axis inductance calculation, (4) should be substituted by:

$$k_d = \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{13}$$

Similar procedure may then be followed for d-axis inductance calculation.

As a short summary, for a measurement at a locked rotor position, first DC currents are supplied to the motor to set a desired magnetic working point on the d- and/or q-axis. Balanced three-phase AC voltages are then superimposed on motor terminals. The resultant AC current components and the power factor for each phase are then measured. For q-axis inductance determination, a special vector (4) is used to manipulate motor equations to form a equivalent R-L circuit as described in (7), where the rms equivalent current i_{eq} may be calculated by using (10)~(11). The rms equivalent voltage v_{eq} may be obtained similarly. For d-axis inductance determination, vector (13) instead of (4) should be used for motor equation manipulation, and similar procedure should be followed. By changing the DC currents and repeating the measurement, the d- and q-axis inductances as functions of the d- and q-axes currents may be obtained. The frequency of the AC signal may be changed to a desired value for a measurement of the inductances as functions of the frequency.

IV. EXPERIMENTAL RESULTS

The proposed measurement method has been applied to a 3-phase, 0.4 KW, commercial SMPM AC servomotor (manufacture: SEM, UK). The test bench is shown in Fig. 3.

The test motor is connected to a dividing head for locking the rotor at a desired position. A programmable power supply (Chroma) is used for supplying AC+DC voltages to the motor. The terminal voltage, line current and power factor are measured by three power analyzers.

The measured incremental inductances as functions of d, q currents are shown in Fig. 4 ~ Fig. 5. The results shown in Fig. 4~5 were obtained with a fixed AC excitation frequency. The AC current magnitudes were controlled to be the same for the measurements taken at different frequencies as shown in Fig. 6. All the measurements were performed at the position where rotor d-axis is aligned with stator phase a-axis.
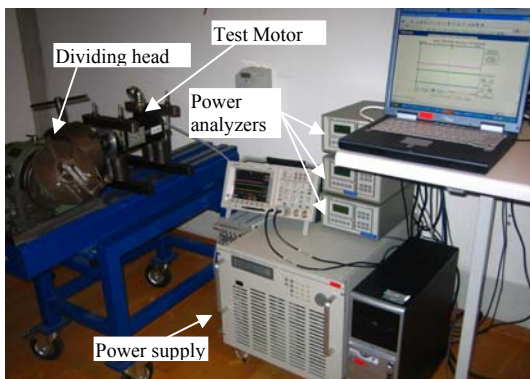

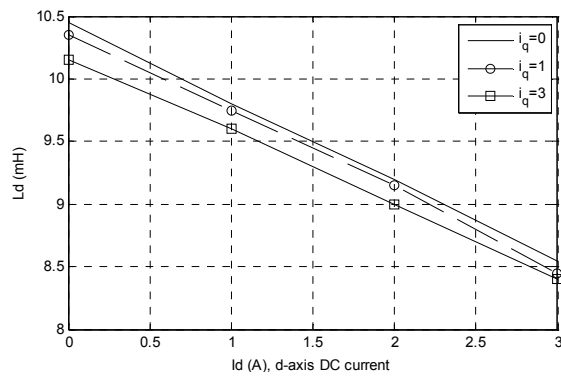Fig. 3 Test bench for inductance measurement

Fig. 4 Measured incremental d-axis inductance as a function of the d, q-axes currents, at the position where rotor d-axis is aligned with phase a-axis.
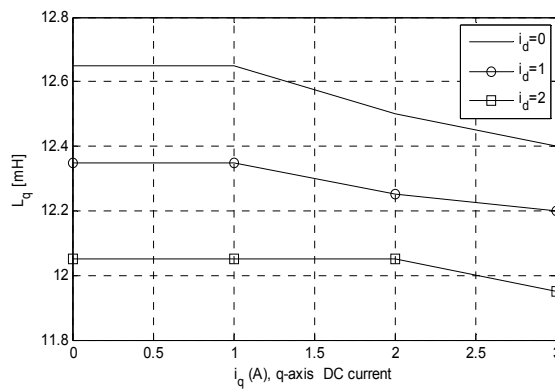

Fig. 5 Measured incremental q-axis inductance as a function of the d, q-axes currents, at the position where rotor d-axis is aligned with phase a-axis.
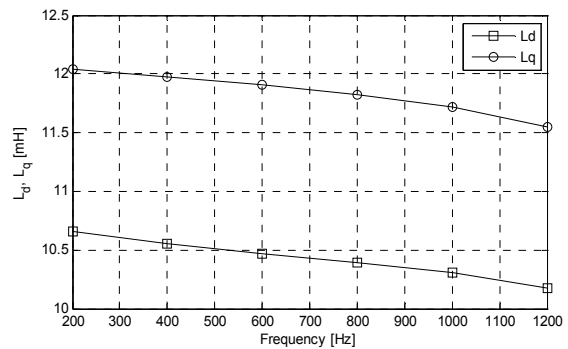

Fig. 6 Measured incremental d-axis and q-axis inductances as functions of the excitation AC frequency, at the position where rotor d-axis is aligned with phase a-axis.

It may be observed from Fig. 4~5 that both the d-axis and q-axis inductances are functions of the d, q-axes currents. As shown in Fig. 4, when the d-axis current increases, the d-axis inductance decreases, which is due to the increased saturation level on the d-axis path. When the q-axis current is increased, which causes saturation on the d-axis flux path, the d-axis inductance is future reduced. Similar effect of the d, q currents on the q-axis inductance may be observed from Fig. 5. When the AC excitation frequency increases, both the d-axis and q-axis inductances decrease, as shown in Fig. 6. This is mainly due to the eddy current effects which cause the change of the flux path, as briefly discussed in section II.

V. CONCLUSION

In this paper, a new d, q-axes incremental inductance measurement method was proposed. It is shown that by using two special row vectors, the original motor equations may be simplified and transformed into a simple equation that only involves the d-axis or the q-axis inductance. This allows the d, q-axes inductances to be separately determined, even with high nonlinearity effects present. Measurement results obtained using this method clearly show the cross-coupling effects of the d, q inductances and the frequency effects on the inductances.

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