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Abstract – **Knowledge of actual flux linkage versus current profiles plays an important role in design verification and performance prediction for switched reluctance motors (SRM's) and permanent magnet motors (PMM's). Various measurement methods have been proposed and discussed so far but each method has its own limitations and constraints. In this paper, first an investigation and comparison of existing measurement methods is reported. Next, a simple pure AC flux linkage measurement method is discussed and evaluated, which does not require a search coil. Experimental results are given, obtained using the described AC method on an SRM and on a PM motor. For these two motors, the measured flux-linkage-current curves are compared to those measured using other methods. The comparison results show good effectiveness of the proposed AC method for both the SRM and the PM motor.**

Keywords **-** *flux linkage profile, measurement and instrumentation, switched reluctance motors, permanent magnet motors.*

I. INTRODUCTION

Switched Reluctance Motors (SRM's) have very nonlinear magnetization (flux-linkage-current) curves at positions near the aligned position. Saturation in permanent magnet motors (PMM's) often occurs due to the rotor magnet flux, which causes the magnetization curves to saturate near the aligned position. Experimental determination of nonlinear magnetization curves is important in design verification and where there is a need for accurate performance prediction, especially when using the flux-MMF diagram technique [1].

Various magnetization curve measurement methods have been reported and discussed in the literature, especially for SR motors. For SR motors, and PM motors without starting cages, changing stator current, induces no rotor current. This implies that most measurement methods for SR motors may be applied to PM motors without rotor cages. Eddy current effects in the magnets may normally be neglected.

A clear overview of the existing measurement methods and an understanding of their advantages and limitations are important when selecting a suitable measurement method for a particular application. This may also indicate how the measurement may be performed simply while maintaining a satisfactory accuracy.

This paper presents an overview of previous reported measurement methods with an attempt to demonstrate their limitations and constraints, and an AC excitation based flux linkage measurement method, which has not been properly reported. This method uses an equivalent core loss resistance in parallel with the winding inductance to determine the instantaneous flux linkage profile. Comparison with the static torque method, and AC+DC method shows that the proposed AC method is simple and yields accurate results.

A review of the previous reported methods for measuring the magnetization curve is given in section II. Section III presents the proposed AC measurement method, for linear and nonlinear cases with analytical evidence. In section IV, experimental results are presented using the proposed AC method on an SR motor and on a PM motor with a pure ferrite magnet rotor. Comparison is made with results obtained using other methods. The sensitivity and accuracy of the proposed method is discussed in section V. Section VI concludes this paper.

II. METHODS FOR FLUX LINKAGE CURVE MEASUREMENT

The measurement of flux linkage curves may be carried out in many different ways:

1) From static interaction torque measurement and a known flux linkage value at an arbitrary position (normally this is the unaligned position for SR motors and the q-axis position for PM motors, where the magnetization curve is linear over a wide range of current) [2],[3].
2) Using a DC power supply and measuring the transient current response during the energizing or the de-energizing period [4] ~ [11].
3) Without a search coil fitted, applying an AC power supply, and measuring the RMS values of the voltage and current [18].
4) Supplying an AC voltage superimposed on a DC voltage to the winding and measuring the RMS values of the AC voltage and AC current.
5) Supplying AC power to the winding with a search coil fitted, measuring the instantaneous current in the excitation coil and the voltage induced in the search coil [12] ~ [14].
6) Using an AC power supply and no search coil, measuring the instantaneous current and voltage in the motor phase winding. This is a simple method but so far is not fully discussed. This method will be studied in detail in this paper.

Determination of the flux linkage profile from static torque measurement is recognized as the indirect method in [2], and was recently reported again in [3]. The theory behind this method is well-known: the static torque is equal to the

differential of the co-energy with respect to rotor position while the armature current is held constant. The co-energy may be calculated from the flux linkage vs. current profile by integration. Thus it is possible to reconstruct the flux linkage profile as a function of the current and rotor position by measuring the static torque. This is described in detail in [3]. The advantage of this method is that only DC current is supplied to the phase winding and measurement is carried out at steady state. There are no specific requirements for the DC power supply, and no extra losses incurred (e.g. core loss and eddy current loss) that would affect the measurement. The torque transducer can be a simple strain-gauge torque transducer on the shaft because continuous rotation is not required. The test is easily performed and automation of the test procedure is simple. A major problem with this method is that the information in the torque measurement itself is not sufficient to reconstruct the flux linkage profile. This is because the torque is determined by differentiating of the co-energy profile. Co-energy profiles that differ from each other by a constant will therefore result in the same torque. Measurement of the inductance at a known position using other methods is required to eliminate this uncertainty [2], [3]. A second problem with this method is that frictional torque always exists and it is difficult to align the shafts of the motor and the torque transducer perfectly, which may introduce a static torque offset. This problem becomes extremely severe if the torque to be measured is small and comparable to the torque offset caused by friction and mechanical misalignment. A large relative error may be expected and this method is therefore unsuitable for motors with small torque.

The DC and AC flux linkage measurement methods are based on the fact that the flux linkage is equal to the integral of the terminal voltage minus the stator resistive voltage drop:

$$\lambda = \int_{o}^{t} [u(t) - R \cdot i(t)] dt + \lambda(0) \qquad (1)$$

where $\lambda(0)$ is the value of the flux linkage at time $t = 0$. R is the armature resistance. $u(t)$ and $i(t)$ are the measured terminal voltage and line current. The DC method is to supply the phase winding with DC voltage and measure the transient current response during the energising or de-energising periods [4]~[11]. The AC method is to measure the voltage and current at steady state, alternatively the instantaneous voltage and current waveforms [12],[14], or the rms values could be measured [18].

Often in the DC and AC methods, the measured signal must be integrated. In the early stages, elaborate hardware was applied to perform the integration [4]~[6]. There are many problems associated with hardware integration as discussed in [7]. The hardware is affected by temperature, causing drift, and the integration time constant must be properly adjusted to ensure that the integration period is greater than the transient current period [7]. Latterly, instead of using elaborate hardware, the trend is to use data acquisition systems to capture the transient voltage and current, as reported in [7]~[14]. It was also asserted that numerical integration can yield good results provided that the data sampling frequency is high compared to the frequency of the current to be measured [4], [14]. This requirement is easy to achieve with modern data acquisition systems.

A problem with the DC method is that if the inductance is determined by measuring the time constant of the transient current response to a step change of the input voltage (switching-on or switching-off the DC power supply), capacitor-free DC power supplies are needed [7]. This is because a capacitor embedded in the DC power supply will cause oscillation of the measured current. This effect is evident from the results presented in [8], where clear oscillations may be observed. In [7], a lead acid battery was used as the DC power supply. However, using a battery as the DC supply presents a difficulty in adjusting the DC voltage to a desired value.

A second problem with the DC method is that it requires extra care for synchronization and control of the captured data length. If the energizing transient current is measured, the data acquisition should start before switching on the DC power supply. If the current is measured during the de-energizing period, a constant DC current is first established and data acquisition should then start shortly before switching off the DC power supply. If the time constant of the motor winding is unknown, before useful measurements can be recorded, it may be necessary to repeat the measurement several times to find a suitable combination of the sampling frequency of the data acquisition system and desired data storage length.

A third problem with the DC method is that it requires extra power electronics circuits to help switch on or switch off the DC power supply [8]~[11]. For different test motors with different power ratings, the power electronics circuits may need to be modified. If the measurement is carried out in the de-energizing period, a mechanical switch may replace the power electronics circuit. However, a freewheeling circuit is required for energy dissipation purposes [2], [7].

It should be noted that the inductance bridge method used for PM motors introduced and discussed in [15]~[17] may be treated as a special DC method. Using an inductance bridge, during the de-energizing period after switching off the power supply, the energy can be dissipated in the resistances, and no extra freewheeling circuits are needed. If the lower branch resistances are equal the bridge may be balanced by varying the resistance in series with the phase winding. Knowledge of the value of any resistor is not needed for the flux linkage calculation [17]. Disadvantage of this method are that theoretically it needs three non-inductive resistors, and, because the transient current is measured, extra care is needed for synchronization and control of the captured data length in the data acquisition procedure.

The simplest AC methods is to measure the rms values of terminal voltage and line current. The inductance is then calculated based on the equivalent circuit of the motor at steady state [18]. A disadvantage of this method is the difficulty of making accurate measurements of rms values of current waveforms, distorted due to saturation effects [14].

The AC+DC method is a more accurate way to determine the inductance or flux linkages than the previous AC method. In this method, the working point of the magnetic circuit of a motor is set by the DC current. An AC voltage is then imposed on the DC voltage. The amplitude of this AC voltage should be controlled to be small so that the resultant AC current will be sinusoidal even when operating in saturated regions (the current in the AC circuit is governed by the incremental inductance). Using the measured AC voltage and AC current components, a steady state motor equivalent circuit may be constructed and the incremental inductance calculated as described in [18]. The measured incremental inductance vs. DC current curve must be integrated to yield the desired flux linkage vs. current curve. The main disadvantage of this method is that it needs a special power supply which has an AC+DC output.

Using a search coil in the AC method is likely to give accurate measurement results [12]~[14]. The idea is that the flux linkages are equal to the integral of the back EMF induced in the search coil. The search coil is open-circuited so in (1), $i = 0$. The voltage to be integrated is equal to the terminal voltage. There is no need to measure the resistance. The instantaneous current in the excitation coil (phase winding) is measured, and its amplitude is varied. Different flux linkages vs. excitation current loops can then be formed. For each measured loop, the vertex represents a point on the flux linkages vs. current curve to be determined [12]. In [14], a bifilar winding is used as the search coil, which functions similarly. In this method, the need for a search coil is inconvenient, and is the main problem associated with this method. The eddy current effect is neglected. Note that it is the flux linkages (not the flux as mentioned in [12]) that are measured. Leakage flux, which links only part of the turns of the winding, will be present. The relationship between flux and flux linkages cannot be taken as simply the number of turns of the windings, as presented in [12].

There are many advantages in the AC methods. There is no need for extra external power electronics circuits or freewheeling circuits. The AC method uses measurements taken at steady state. Compared to the DC methods, there are fewer requirements for synchronization of the start of the measurement and the commencement of data acquisition. The frequency of the excitation voltage and the sampling frequency of the data acquisition system may be pre-determined. So the desired data storage length and measurement duration time may be pre-specified. If a data acquisition card is used, the need for real-time integration, which normally requires elaborate excessive hardware, may be avoided. Based on the foregoing considerations, it is easier to construct a fully automatic measurement system for the AC method. The AC method can yield acceptable accuracy and, 'as far as test conditions are concerned, the requirements of the AC excitation method can be more easily satisfied' [12].

III. A SIMPLE AC MEASUREMENT METHOD

If the search coil can be avoided, the AC method is a good method which is simple, accurate, and easy to perform.

Without the search coil, the direct measurable signals are terminal voltages and line currents at steady state. The following part examines if the desired flux-linkage-current curve can be derived from the measured winding voltages and currents.

In this section, a description of the AC method is given first. Analysis of this method is first carried out for linear cases, and then extended to nonlinear cases. A discussion of this method is given at the end of this section.

A. Description of the AC measurement method

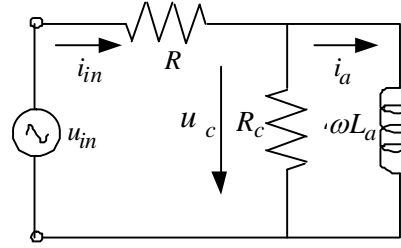At steady state, with AC excitation, the equivalent measurement circuit can be shown in Fig. 1.

Fig.1 The equivalent circuit for measurements at steady state with AC excitation

Where R is the winding resistance, R_c is the equivalent core loss resistance including eddy current loss. ω is the angular frequency of the terminal voltage u_{in}, and L_a is the motor winding inductance, which is a function of the current when saturation effects are taken into account.

The measurement procedure using an AC excitation and without using a search coil is described as follows:
- The resistance of the phase winding R is first measured.
- Select an AC excitation frequency and a sampling frequency to achieve the desired data storage length.
- Supply the motor phase winding with the required AC voltage. Record the instantaneous terminal voltage and line current. The voltage and current sensors may be hall-effect sensors.

  The equivalent core loss resistance may be calculated from the measured (or calculated) input power, measured terminal voltage and line current. This is carried out by:

$$R_c = \frac{U_{c,rms}^2}{P_{in} - RI_{in,rms}^2} \qquad (2)$$

  where P_{in} is the input power. $U_{c,rms}$ and $I_{in,rms}$ are the rms values of the winding voltage u_c and input line current i_{in}.

- The flux linkage can be calculated by integrating the winding voltage u_c.

- The current i_a, which flows only through the inductance, can be calculated if R_c is known.
- The curve of winding current i_a vs. the integral of the winding voltage u_c with respect to time is the flux-linkage-current curve required to be measured. For each rotor position, only one measurement is needed at the maximum required current.

This method requires measurement of the phase resistance, which is temperature dependent. In the test bench used to carry out the test in the laboratory, the power supply and the data acquisition card are controlled by computer. For each measurement, only two periods of the supply need to be measured and the duration of current flow in the winding is very short. The time between two successive measurements, which is known as the idle time, may be suitably selected, to avoid significant temperature rise.

Since instantaneous waveforms are recorded for complete periods, the rms values may be calculated accurately even though the current is not sinusoidal. In this AC method, the rms values are used only for equivalent core loss resistance determination.

B. Analysis for linear cases

If there are assumed to be no core loss in the machine, e.g. R_c disappears in Fig.1, it is apparent that the AC method described above enables accurately determination of the instantaneous flux-linkage-current curve. In reality, the core loss effects must be considered and here, core loss is represented by an equivalent core loss resistance in parallel with the armature winding.

The introduction of the core loss resistance to the equivalent circuit causes one problem in the determination of the flux linkages. It is the instantaneous flux-linkage-current information that is needed. However, the core loss resistance is a representation of the rms core loss effect. In linear situations, when the supply voltage is sinusoidal, the resultant current is also sinusoidal. It can be proven mathematically that, with sinusoidal terminal voltage and line current waveforms and using this equivalent rms core loss resistance, the trajectory formed by the instantaneous winding current i_a and the integral of the winding voltage u_c (which gives the instantaneous flux linkage) is a straight line. Its slope is equal to the inductance that can be calculated directly from the equivalent circuit (Fig. 1) using rms voltage and current values. This validates the use of equivalent rms core loss resistance for instantaneous flux-linkage-current curve determination. Mathematical proofs are given in the appendix.

C. Analysis for nonlinear cases

With saturation effects accounted for, the current waveform is no longer sinusoidal, even with a sinusoidal voltage supply. It is not difficult to maintain a sinusoidal shape of the supply voltage with modern power supply equipment. Therefore, in saturated situations, the terminal voltage and line current may be expressed by:

$$u_{in} = U_m \cos \omega t \tag{3}$$

$$i_{in} = I_{m1} \cos(\omega t - \varphi_1) + \sum_{k=2}^{\infty} I_{mk} \cos(k\omega t - \varphi_k) \tag{4}$$

where U_m is the amplitude of the rms terminal voltage and I_{mk} is the amplitude of the kth rms current harmonic. ω is the electrical angular frequency. φ_k represents the phase shift of the current with respect to the voltage. It should be noted that there is no DC component in the non-sinusoidal line current.

The winding voltage u_c now becomes:

$$u_c = u_{in} - R i_{in} = u_{c1} - R \sum_{k=2}^{\infty} I_{mk} \cos(k\omega t - \varphi_k) \tag{5}$$

Where u_{c1} is the fundamental component of the winding voltage u_c. This is determined by:

$$u_{c1} = U_m \cos(\omega t) - R I_{m1} \cos(\omega t - \varphi_1) \tag{6}$$

The equivalent core loss resistance may be calculated by (2). According to [19], it is not difficult to find that now the rms winding voltage and line current have the following expression:

$$U_{c,rms}^2 = \frac{1}{T} \int_0^T (u_{in} - R i_{in})^2 dt = \frac{1}{2} U_{cm1}^2 + \frac{R^2}{2} \sum_{k=2}^{\infty} I_{mk}^2 \tag{7}$$

$$I_{in,rms}^2 = \frac{1}{T} \int_0^T i_{in}^2 dt = \frac{1}{2} \sum_{k=1}^{\infty} I_{mk}^2 \tag{8}$$

Where U_{cm1} is the amplitude of the fundamental component of winding voltage u_{c1}. T is the fundamental period.

The flux linkage λ is obtained by integrating the instantaneous winding voltage u_c. If the instantaneous flux linkage λ vs. winding current i_a should form a flux-linkage-current curve, then the area enclosed by the λ vs. i_a trajectory should be zero. Based on [20], it is not difficult to derive that the area under the trajectory formed by arbitrary periodic waveforms of λ vs. i_a may be calculated by:

$$\text{Trajectory area} = \frac{1}{T} \int_0^T i_a d\lambda \tag{9}$$

Noting that $d\lambda = u_c dt$, $i_a = i_{in} - \dfrac{u_c}{R_c}$, and the relationship between u_c and u_{in}, i_{in} as indicated in (5), yields:

$$\text{Trajectory area} = \frac{1}{T} \int_0^T u_{in} i_{in} dt - R \frac{1}{T} \int_0^T i_{in}^2 dt - \frac{1}{R_c} \frac{1}{T} \int_0^T u_c^2 dt$$
$$= P_{in} - R I_{in,rms}^2 - \frac{U_{c,rms}^2}{R_c} \tag{10}$$

Substituting R_c with (2), gives that the enclosed trajectory area is actually zero. This suggests that for a non-sinusoidal current waveform as would occur in nonlinear situations, and using an equivalent core loss resistance represented by (2), a flux-linkage-current curve will be obtained by using the integral of the winding voltage u_c vs. winding current i_a.

From electromagnetic theory, it is clear that (9) yields the average energy stored in the magnetic field. As the total input power is now completely balanced by the power losses on the stator resistance and on the equivalent core loss resistance, which is evident from (2), there should be no extra energy taken by the winding and thus (9) will be zero.

D. Discussion

As previously discussed, if there is no core loss in the machine (e.g., $R_c = 0$), it appears that the above described AC method accurately determines the instantaneous flux-linkage-current curve. Note that it is the changing magnetic field that causes core loss. The two end points of the flux-linkage-current curve obtained represent maximum and minimum flux linkage values. At these two points, the rate of change of the flux linkage with respect to time ($d\lambda/dt$) is zero, and therefore the core loss is zero. This suggests that in the proposed equivalent measurement circuit, the equivalent core loss resistance should be in parallel with the armature winding. Thus at the moments when $d\lambda/dt$ is zero, the instantaneous winding voltage u_c is zero and therefore the core loss u_c^2/R_c is zero. If the core loss resistance is put in series with the stator winding resistance, when the winding voltage u_c is zero, there will still be some current flowing through the core loss resistance, resulting in finite core loss, and a smaller flux linkage value will be calculated by integrating the winding voltage u_c [3].

Using the advantage gained by neglecting the effects of core loss on the determination of the flux-linkage-current curve at the maximum and minimum flux linkage points, the vertices of flux linkage measured using a search coil vs. current loops, were used as points on the flux-linkage-current curve to be determined in [12]. The amplitude of the current in the phase winding should be varied to obtain sufficient points to form the desired flux-linkage-current curve in [12]. In the proposed method, one measurement only, is needed at the maximum current. To validate this method, the curve measured at the maximum current value was compared with the curve formed by the vertices of flux-linkage-current loops measured at different current amplitudes. Results presented in the next section show a good agreement of these two curves even in deeply saturated regions.

Integrating the winding voltage does not determine the flux linkage uniquely, because differentiation with respect to time of different flux linkage profiles that differ from each other by a constant will result in the same voltage waveform. In this method, a complete period of the voltage is used for integration. The fact that the mean value of the flux linkage

waveform for one complete period should always be zero, makes it very easy to find the actual location of the flux linkage waveform obtained from voltage integration on the flux-linkage axis. It is easy to realize automation of the test using this method and within the limits of the power supply and the limits of voltage and current sensors, this test system may be applied directly to motors with different power ratings without modification.

IV. EXPERIMENTAL RESULTS USING THE AC MEASUREMENT METHOD

The test bench used for the proposed flux linkage curve measurement is shown in Fig. 2. It can also be used to perform the static torque measurements.
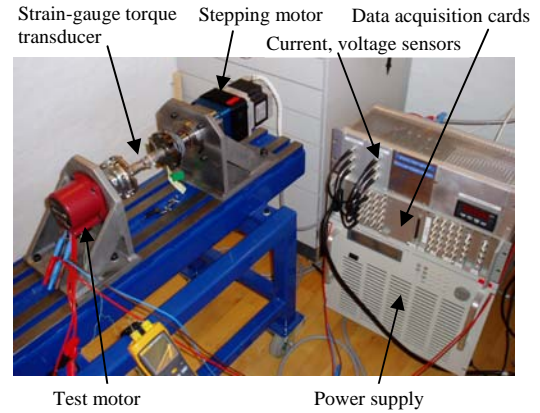

Fig. 2 Test bench for flux linkage curve measurement

The stepping motor rotates the test motor to a desired position and locks it. The programmable power supply (Chroma) can supply AC, DC and AC+DC voltages to the test motor. The static torque is measured using the strain-gauge torque transducer. Voltage and current sensors use the Hall-effect modules. The data acquisition card stores the measured voltage and current and transfers the data to the PC. The stepping motor and power supply are controlled by the PC using serial ports.

The proposed AC measurement method discussed in the previous section was first applied to an 800 W, 3-phase SR motor for flux-linkage-current profile measurement. The measured flux linkage profiles are shown in Fig. 3. The rated torque of this motor is 12 Nm. The effects of friction and shaft misalignment may be neglected in static torque measurement for this motor, and the flux linkage curves can be obtained from the torque measurement as described in [3]. Measurement results using torque measurement are presented in Fig. 3 for comparison to the proposed AC method. The reason for using the torque measurement method here for comparison is that if the torque drift caused by friction and mechanical misalignment can be ignored, this method is considered to be the most accurate because it is measured at DC steady state and there are no core loss or eddy current effects. It may be observed from Fig. 3 that the measured

curves using the AC method agree very well with the curves obtained indirectly from the torque measurement.

During the measurement using the proposed AC method, different excitation frequencies were used to illustrate the insensitivity of this method to the excitation frequency. For the curves taken close to the unaligned position, the excitation frequency was 150 Hz. For the curves near the aligned position, the excitation frequency was set to be 60 Hz. It can be observed from Fig. 3 that the accuracy of this AC method is not sensitive to the excitation frequency. Similar results may be found in [18], where the frequency was changed from 10 to 55 Hz and the measured inductance profiles were almost unchanged.
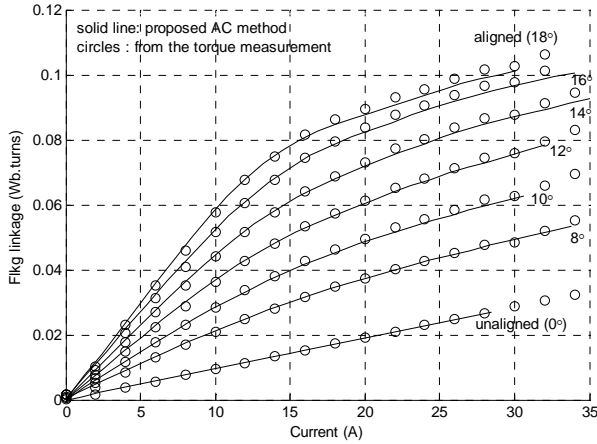

Fig. 3 Comparison of the flux linkage vs. current curves measured using the AC method to the curves obtained indirectly from the static torque measurement.

In the proposed AC method, for each position, only one measurement is needed at the desired maximum current. In [12], where a search coil was used with AC excitation, the vertices of the measured flux linkage vs. current loops were used as points on the flux-linkage-current curve to be determined, for different current amplitudes. This measurement was repeated until a suitable number of measured points are obtained, coving the required current range. As discussed, the reason for doing this is that at the vertices, the measurement results are the most accurate, due to zero instantaneous core loss. Based on this consideration another test was carried out, using the proposed AC method but with different current amplitudes. The vertices of the flux linkage vs. current trajectories obtained for different current amplitudes were recorded and used to construct the flux-linkage-current curve to be measured. The results are shown in Fig. 4.

It can be observed that the results obtained using the vertices fit very well with the flux linkage curves obtained from a single test at the maximum current for different positions. This further validates the accuracy of this AC method.
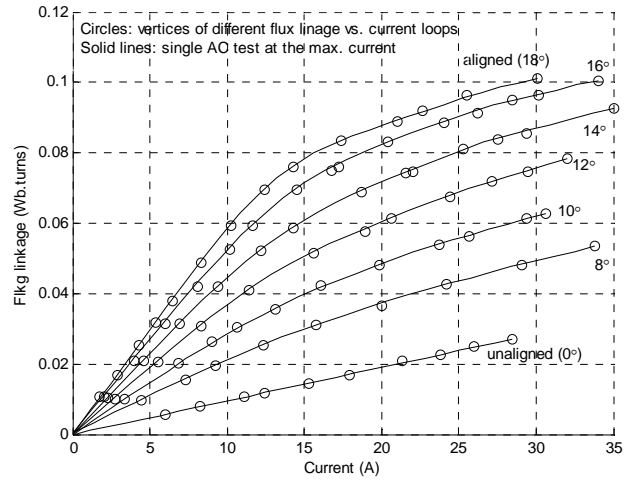

Fig. 4 Comparison of the flux linkage vs. current curves measured from a single AC test at the maximum current amplitude to the curves obtained from the vertices of the flux linkage vs. current loops for different current amplitudes.

The second test motor was a 30 watts, 2-phase, PM motor with a pure ferrite magnet rotor. Most of the measurement methods, including the method proposed in this paper, require a locked-rotor test. It is the rate of change of the flux linkage that will induce a voltage. At locked rotor, the PM flux linkage will not induce any voltage. This means that the PM flux linkage is not directly measurable from a locked-rotor test. The actual PM flux linkage profile may be obtained using the superposition principle. First, a back EMF test is performed, with all phases open-circuited. The PM flux linkage, which is a function of the rotor position, may be calculated by integrating the back EMF voltage. Then the flux-linkage-current curve is measured in the same manner as for an SRM. The measured curve will cross the origin of the flux linkage vs. current coordinate system. The measured curve is then shifted along the flux linkage axis to ensure that when the current is zero, the flux linkage is equal to the measured PM flux linkage value at this position. To validate this superposition method, another test was carried out to compare the directly measured static torque with the torque calculated using the co-energy method and the measured flux-linkage-current curves for different positions. The results agreed closely with each other.

The flux-linkage-current curve measured at the aligned position, for one phase of this PM motor is shown in Fig. 5. At the same position, the flux linkage curve was measured using the AC+DC method, and is also shown in Fig. 5 for comparison. These two curves agree with each other very well. The test was repeated for different supply frequencies (20Hz, 40Hz, 60Hz, 80Hz, 100Hz). All the measured flux linkage curves obtained at different frequencies fit very well with each other. The absolute maximum relative flux linkage error observed, referred to the measurements taken at 20 Hz, is 0.3%.
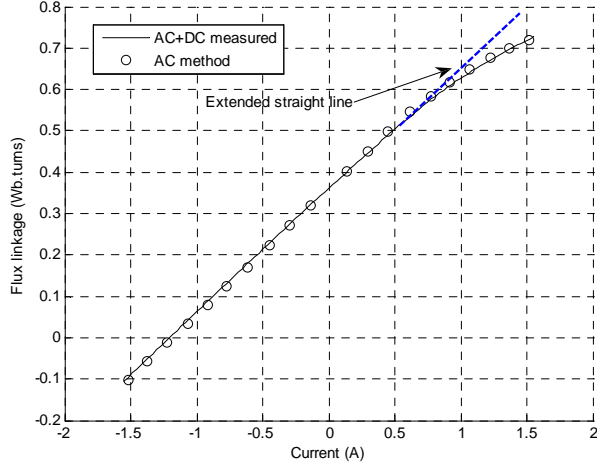
Fig. 5 Comparison of the flux linkage vs. current curves measured using the AC method and the AC+DC method.

V. SENSITIVITY AND ACCURACY

In this proposed test system, the programmable power supply (Chroma) has a maximum THD (Total Harmonic Distortion) of 0.3% at 50 Hz and 1.5% between 15~1.2K Hz. The current and voltage sensors are hall-effect sensors having a typical accuracy of 0.5% and 0.9% respectively, which are considered to be the main error sources that affect the measurement accuracy. The data acquisition system employs a 14-bit A/D converter, with a full scale range of $V_{fs} = \pm 10$ V. It samples the inputs in a sequential manner at a frequency of $f_s = 50$ KHz for each channel. If the flux linkage measurement is considered, the maximum theoretical measurement sensitivity may be similarly calculated as [12], [21]:

$$S = \frac{V_{fs}}{2^{14} f_s} = 0.24 \cdot 10^{-7} \text{ (Wb)} \qquad (11)$$

As was discussed in [12], although it is theoretically possible to determine the measurement accuracy from the errors of e.g., current voltage sensors, A/D converter, and discrete integration algorithm, the result is not reliable, due to the uncertainty of individual error determination and environmental influences. In [12] and [21], a practical accuracy evaluation was carried out by testing a standard mutual inductor. An impression of the accuracy of the proposed measurement method may be obtained by comparing with results obtained using other measurement methods, e.g. the static torque measurement based method, which is more reliable for machines with negligible influence of drifting torque caused by friction and mechanical misalignment. The comparison of results shows that the absolute relative flux linkage error, referred to the static torque measurement based method, is between 0.2%~1.8%, for all 7 different positions that were measured for the SRM described in section IV.

VI. CONCLUSION

A review was presented, of the existing methods for experimental determination of the flux linkage vs. current profiles, which can help to clarify their limitations and constraints. It was shown that the AC method has many advantages compared to the other methods.

A simple method of AC measurement, which allows easy and accurate determination of the flux linkage profiles, was proposed and demonstrated. The arrangement of the test bench is simple and may easily be automated. There is no need for search coils to be added to the test machine, or extra power switching circuits. The proposed test system is flexible, allowing different types of motors, or motors with different power ratings to be tested. Experimental results were presented, for an SR motor and a PM motor obtained using the proposed method and these compare favourably with results obtained using previous proposed measurement methods.

APPENDIX

In this appendix, evidence is provided for using the equivalent rms core loss resistance in the determination of the instantaneous flux-linkage-current curve for linear conditions.

Assuming the terminal voltage and current are sinusoidal:

$$u_{in} = U_m \cos \omega t \qquad (A.1)$$

$$i_{in} = I_m \cos(\omega t - \varphi) \qquad (A.2)$$

where U_m and I_m are the amplitudes of the terminal voltage and current. φ is the power factor angle. It is easy to show that the voltage across the core loss resistance is:

$$u_c = U_{cm} \cos(\omega t + \alpha) \qquad (A.3)$$

where

$$U_{cm} = \sqrt{U_m^2 - 2RU_m I_m \cos \varphi + I_m^2 R^2} \qquad (A.4)$$

$$\tan \alpha = \frac{I_m R \sin \varphi}{U_m - I_m R \cos \varphi} \qquad (A.5)$$

The instantaneous winding current i_a (in Fig. 1) may be calculated by:

$$i_a = i_{in} - \frac{u_c}{R_c} \qquad (A.6)$$

For a sinusoidal waveform, its relationship between the amplitude and the rms value is given by $\sqrt{2}$. Substituting the rms values in (2) with amplitudes, and then substituting (2) into (A.6), after several steps of manipulation, it may be shown that:

$$i_a = \frac{U_m I_m \sin \varphi}{U_{cm}} \sin(\omega t + \alpha) \qquad (A.7)$$

The flux linkage may be calculated from the integral of u_c to be:

$$\lambda = \frac{1}{\omega} U_{cm} \sin(\omega t + \alpha) + C \qquad (A.8)$$

Where C represents the residual flux linkage for SR motors and the PM flux linkage for PM motors. From (A.7) and (A.8), the instantaneous relationship between λ and i_a is time independent and is characterized by:

$$\lambda = \left(\frac{1}{\omega} \frac{U_{cm}^2}{U_m I_m \sin\varphi}\right) \cdot i_a + C = L_{linear} \cdot i_a + C \quad (A.9)$$

where the inductance is found to be

$$L_{linear} = \left(\frac{1}{\omega} \frac{U_{cm}^2}{U_m I_m \sin\varphi}\right) \qquad (A.10)$$

From the equivalent circuit presented in Fig. 1, the inductance can be calculated from the rms values of the terminal voltage and line current. The equation is

$$\frac{U_{c,rms}}{I_{in,rms}} = \frac{1}{\sqrt{\frac{1}{(\omega L_{linear})^2} + \frac{1}{R_c^2}}} \qquad (A.11)$$

Substituting R_c in (A.11) with (2), and using the relationship obtained from the voltage, and current phasor diagram, it is simple to find that the inductance in (A.11) may be expressed as (A.10). This suggests that even though the equivalent core loss resistance is calculated from the rms core loss (not the instantaneous core loss), the instantaneous flux-linkage-current curve obtained will be the desired curve.

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