Stretched exponential relaxation for growing interfaces in quenched disordered media

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(February 1, 2008)

We study the relaxation for growing interfaces in quenched disordered media. We use a directed percolation depinning model introduced by Tang and Leschhorn for $1+1$-dimensions. We define the two-time autocorrelation function of the interface height $C(t',t)$ and its Fourier transform. These functions depend on the difference of times $t-t'$ for long enough times, this is the steady-state regime. We find a two-step relaxation decay in this regime. The long time tail can be fitted by a stretched exponential relaxation function. The relaxation time τ_α is proportional to the characteristic distance of the clusters of pinning cells in the direction parallel to the interface and it diverges as a power law. The two-step relaxation is lost at a given wave length of the Fourier transform, which is proportional to the characteristic distance of the clusters of pinning cells in the direction perpendicular to the interface. The stretched exponential relaxation is caused by the existence of clusters of pinning cells and it is a direct consequence of the quenched noise.

I. INTRODUCTION

For decades the investigation of growing surfaces and interfaces has attracted much attention due to its importance in many fields, such as motion of liquids in porous media, growth of bacterial colonies, crystal growth, fronts of fire, etc. In these problems we have a nonequilibrium interface. The d-dimensional interface described by a single valued-function $h(\mathbf{x},t)$ evolves in a $d+1$-dimensional medium. The disorder affects the motion of the interface and leads to its roughness. A phenomenological non-linear Langevin equation, the Kardar-Parisi-Zhang equation (KPZ) [1], and the directed percolation depinning (DPD) models [2,3] have been used in order to study growing interfaces. Two main kinds of disorder have been proposed in these models: the annealed noise that depends only on time and the quenched disorder due to the inhomogeneity of the media, which does not depend on time. In the DPD models the disorder is quenched and they describe very well some experiments such as the growth of bacterial colonies and the motion of liquids in porous media. These models were proposed simultaneously by Tang and Leschhorn [2] and Buldyrev *et al.* [3].

In many glassy systems a non-exponential relaxation is found when they are close to some temperature above to the static transition. As an example, in structural glasses a two-step relaxation decay is found near the so called "ideal glass transition" [4]. The long relaxation step has the stretched exponential form

$$f(t) = f_0 \exp\left[-(t/\tau_\alpha)^\beta\right] \ , \qquad (1)$$

where $0 < \beta < 1$ does not depend on the temperature. There are two mechanisms driving non-exponential relaxation. In disordered systems such as spin glasses that behavior is caused by the existence of non-frustrated ferromagnetic-type clusters of interactions [5] which is a direct consequence of the quenched disorder [6]. Another mechanism in frustrated systems is based on the percolation transition of the Kasteleyn-Fortuin and Coniglio-Klein cluster [7], here disorder is not needed to obtain non-exponential relaxation [8]. Recently, Colaiori and Moore [9] have found a stretched exponential relaxation for the KPZ equation with annealed noise.

In this paper, we use the DPD model proposed by Tang and Leschhorn (TL) [2] in order to investigate the relaxation of the two-time autocorrelation functions in quenched disordered media at the steady-state regime. We relate the relaxation properties to the clusters of pinning cells. The paper is organized as follows. In Sec. II we present the model and some properties of the clusters of pinning cells. In Sec. III the steady-state relaxation is studied. Finally, in Sec IV we present some conclusions.

II. THE MODEL

In the TL model for $1+1$-dimensions [2], the advance of the fluid through the media is modeled by a driving force p, while the disorder of the media, that brakes this advance, is represented by a quenched noise in the substratum. The interface grows in a square lattice of edge L with periodic boundary conditions. We assign a random pinning force $g(\mathbf{r})$ uniformly distributed in the interval $[0,1]$ to every cell of the square lattice. For a given applied driving force $p>0$, we can divide the cells into two groups: those with $g(\mathbf{r}) \le p$ (free cells), and those with $g(\mathbf{r}) > p$ (pinning cells). Denoting by q the density of pinning cells on the lattice, we have $q = 1-p$ for $0 < p < 1$ and $q = 0$ for $p \ge 1$. The interface is specified completely by a set of integer column heights h_i ($i = 1, \ldots, L$). At $t = 0$ all columns are assumed to have the same height, which is zero. During growth, a column is selected at random, say column i, and its height is compared with those of the neighbor columns $(i-1)$ and $(i+1)$. The growth event is defined as follows. If h_i is greater than either h_{i-1} or h_{i+1} by two or more units, the height of the lower of the two columns $(i-1)$ and $(i+1)$ is incremented in one (in case of the two being equal, one of them is chosen with equal probability). In the opposite case, $h_i < \min(h_{i-1}, h_{i+1}) + 2$, the column i advances by one unit provided that the cell to be occupied is a free cell. Otherwise no growth takes place. In this model, the time unit is defined as one growth attempt. In numerical simulations at each growth attempt the time t is increased by δt, where $\delta t = 1/L$. Thus, after L growth attempts the time is increased in one unit. In our simulations we use $L = 10000$ and take the averages over 100 different realizations of quenched noise.

A. Clusters of pinning cells

As it has been shown in Ref. [2], this model has a depinning transition at a driving force $p_c = 0.461$. For driving forces below the critical one p_c, the advance of the interface is halted (pinning phase), while above this driving force the interface moves without stopping (moving phase). At the transition, the characteristic length ξ of the pinned regions diverges. A directed percolation cluster of pinning cells which extends over the whole system appears in the pinning phase. In the moving phase, a typical connected cluster of pinned cells extends over a distance of the order of $\xi_\parallel$ in the direction parallel to the interface and a distance of the order of $\xi_\perp$ in the direction perpendicular to the interface. On both sides of the percolation transition, the two lengths have a power-law behavior $\xi_\parallel \sim |p - p_c|^{-\nu_\parallel}$ and $\xi_\perp \sim |p - p_c|^{-\nu_\perp}$, with $\nu_\parallel = 1.733 \pm 0.001$ and $\nu_\perp = 1.097 \pm 0.001$. $\xi_\perp$ sets a characteristic scale for the height while $\xi_\parallel$ sets characteristic scales for both the distance parallel to the interface and the time. For the mean interface height the scaling form $H(t) \approx \xi_\perp \Phi(t/\xi_\parallel)$ is obtained, denoting Φ a scaling function which is different for the two phases. In the moving phase there is a crossover from a power-law growth $H(t) \sim t^{\nu_\perp/\nu_\parallel}$ at $t \ll \xi_\parallel$ to a linear behavior $H(t) = vt$ at $t \gg \xi_\parallel$. The steady-state velocity can be expressed as $v(p) \sim (p - p_c)^{\nu_\parallel - \nu_\perp}$.

III. STEADY-STATE RELAXATION

We define the two-time autocorrelation function of the surface height as

$$C(t', t) = \frac{1}{L} \sum_j \delta\left[h_j(t') - h_j(t)\right] , \quad (2)$$

where $\delta[x]$ is the delta function. Its Fourier transform is

$$C_k(t', t) = \frac{1}{L} \sum_j e^{-i\left[h_j(t') - h_j(t)\right]k} , \quad (3)$$

where k is the wave number.

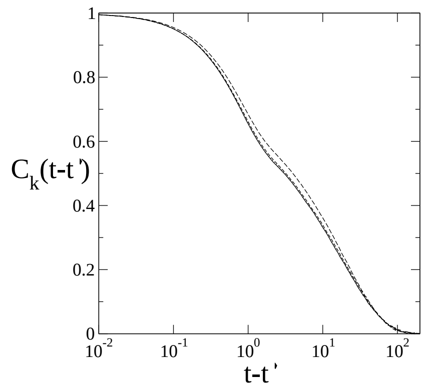

FIG. 1. $C_k(t - t')$ for $p = 0.5$ and $k = \pi$ and for different initial times $t' = 10^2$ (long dashed line), 10^3 (dashed line), 10^4 (dotted line), and 10^5 (solid line). Dotted and solid lines overlap for any $t - t'$.

For long enough times, $t' \gg \xi_\parallel$, these functions depend only on the difference of times $t - t'$, this is the steady-state regime where $H(t) = vt$. This regime is reached at longer times when we approach to the critical driving force p_c. Fig. 1 shows $C_k(t - t')$ for $p = 0.5$ and $k = \pi$ and for different initial times $t' = 10^2$, 10^3, 10^4, and 10^5. As we see $C_k(t - t')$ is independent of t' when $t' \geq 10^4$ for this value of p.

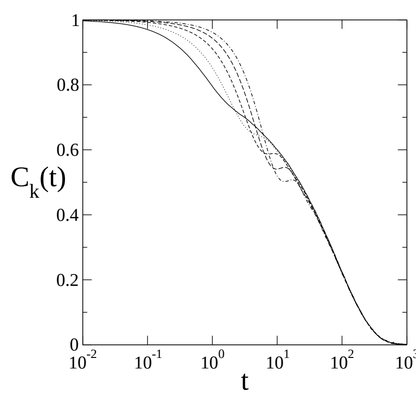

FIG. 2. $C_k(t)$ in the steady-state regime for $p = 0.5$ and $k = \pi$ (solid line), $\pi/2$ (dotted line), $\pi/3$ (dashed line), $\pi/4$ (long dashed line), and $\pi/5$ (dot-dashed line).

We find a two-step relaxation decay in the steady-state regime. We can see in Fig. 2 that the time interval of the first and second relaxation step depends on the wave number k. Nevertheless, the form of the second relaxation step does not depend on k. For small enough k we only have one step relaxation process. So, there is a wave number k_e where the two-step relaxation decay is lost for $k < k_e$. Fig. 3 shows $C_k(t)$ for $k = \pi$ and different values of the driving force p. The two-step relaxation decay is observed from the highest value of $p = 0.95$, but the time interval of the second step increases when p is decreased, i.e. when the system approaches to the criticality. This behavior is also found in other glassy systems, where the time interval of the second relaxation step increases when the systems approach to the critical temperature. As we can see in Fig. 4, the second relaxation step can be fitted by a stretched exponential relaxation function Eq. (1) with the exponent $\beta = 0.805 \pm 0.05$. This exponent is in practice independent of p and it brings to the master equation

$$C_k(t) = \widetilde{C_k}(t/\tau_\alpha) \tag{4}$$

for $t > \tau_\alpha$, where $\widetilde{C_k}(t/\tau_\alpha)$ does not depend on p. In glassy systems Eq. (4) is also called time-temperature superposition principle [4], because the temperature plays the role of the driving force in that systems. In the inset of Fig. 4 we show an equivalent time-driving force superposition principle for our system. This stretched exponential relaxation means that in the system there is a broad distribution of relaxation times [10].

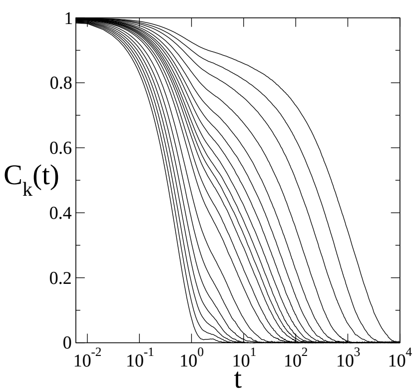

FIG. 3. $C_k(t)$ in the steady-state regime for $k = \pi$ and $p = 0.95$, 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.68, 0.56, 0.55, 0.54, 0.53, 0.52, 0.51, 0.5, 0.49, 0.48, 0.475, and 0.47 (from left to right).

The relaxation time τ_α can be obtained from the fit of $C_k(t)$ with a stretched exponential function, it is shown in Fig. 5. We see that it is very well fitted by a power law $\tau_\alpha \propto (p - p_c)^{-\nu_\parallel}$ where $p_c = 0.462 \pm 0.001$ and $\nu_\parallel = 1.733 \pm 0.001$. This means that τ_α is proportional to the characteristic distance of the clusters of pinning cells in the direction parallel to the interface $\xi_\parallel$, which also diverges as $\tau_\alpha \sim \xi_\parallel \sim (p - p_c)^{-\nu_\parallel}$.

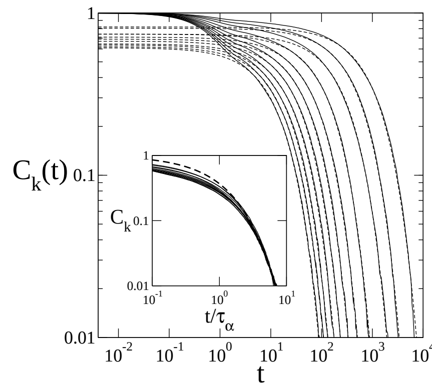

FIG. 4. Log-log plot of $C_k(t)$ where $k = \pi$ and $p = 0.56$, 0.55, 0.54, 0.53, 0.52, 0.51, 0.5, 0.49, 0.48, 0.475, and 0.47 (from left to right). Dashed curves are fitting functions corresponding to the stretched exponential functions. Inset: time-driving force superposition principle. The dashed curve is a stretched exponential function with $\beta = 0.8$.

We can obtain a wave length $\lambda_e = \pi/k_e$ in the direction perpendicular to the interface from the wave number k_e where the two-step relaxation is lost. In the inset of Fig. 5 we show λ_e as a function of $(p - p_c)$. This length diverges as a power law $\lambda_e \propto (p - p_c)^{-\nu_\perp}$ with $p_c = 0.46 \pm 0.01$ and $\nu_\perp = 1.1 \pm 0.01$. So that, λ_e is proportional to the characteristic distance of the clusters of pinning cells in the direction perpendicular to the interface, $\lambda_e \sim \xi_\perp \sim (p - p_c)^{-\nu_\perp}$.

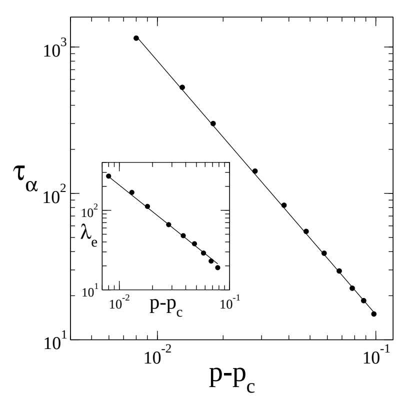

FIG. 5. Log-log plot of the relaxation time τ_α, obtained by the stretched exponential fit in Fig. 4, as a function of $p - p_c$, for $p = 0.56$, 0.55, 0.54, 0.53, 0.52, 0.51, 0.5, 0.49, 0.48, 0.475, and 0.47. The solid curve is a power law function $\tau_\alpha = 0.275(p - 0.462)^{-1.733}$. Inset: log-log plot of λ_e as a function of $p - p_c$, for $p = 0.54$, 0.53, 0.52, 0.51, 0.5, 0.49, 0.48, 0.475, and 0.47. The solid curve is a power law function $\lambda_e = 1.25(p - 0.46)^{-1.1}$.

We see that the characteristic length of the clusters of pinning cells in the direction parallel to the interface $\xi_\parallel$ sets the time scale of the stretched relaxation function $\widetilde{C_k}(t/\xi_\parallel)$. On the other hand, the stretched relaxation step is lost for $\lambda \geq \lambda_e$, that is for $\lambda \gtrsim \xi_\perp$. So, the stretched exponential relaxation is caused by the clusters of pinning cells.

IV. CONCLUSIONS

We have studied relaxation properties for growing interfaces in quenched disordered media. We have used the TL model in which properties of clusters of pinning cells are known. We have studied the relaxation properties of the Fourier transform of the autocorrelation of the surface height and found a two-step relaxation process in which the second step is well fitted by a stretched relaxation function with $\beta = 0.805 \pm 005$. The relaxation time diverges as a power law and it is proportional to the characteristic distance of the clusters of pinning cells in the direction parallel to the interface. The form of the second step relaxation does not depend on the wave number of the Fourier transform. This step is lost for a given wave length λ_e which is proportional to the characteristic distance of the clusters of pinning cells in the direction perpendicular to the interface. From these results, we can say that the stretched exponential relaxation behavior is caused by the clusters of pinning cells, which is a direct consequence of the quenched noise as it happens in other glassy systems [6].

ACKNOWLEDGMENTS

This work was supported in part by the project No. PI-60/00858/FS/01 from the Fundación Séneca, Región de Murcia.

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