Global restrictions to the Mixing Angle θ_W

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1 Introduction

In spite of its firm hold in particle physics, the gauge group $SU(2)\otimes U(1)_Y$ proves not to be the most appropriate to describe the unification of weak and electromagnetic interactions. In fact, a look at the structure of $SU(2)\otimes U(1)_Y$ tells us that both, the electric charge generator Q and its corresponding electromagnetic gauge field $A_\mu(x)$, are not basic constituents of this gauge group. Rather, the "weak isospin" T_1, T_2, T_3 (with its corresponding gauge fields $A^{(1)}_\mu, A^{(2)}_\mu, A^{(3)}_\mu$) and the hyphercharge $T_4 \equiv Y$ (which provides another gauge field $A^{(4)}_\mu$) are its basic generators. Nevertheless, there is a possibility of defining a proper electric charge through the Gell'Mann-Nishijima relationship

$$Q \equiv \tilde{T}_4 = T_3 + \frac{1}{2}T_4 \tag{1}$$

and the associated electromagnetic field through the mixture

$$A_\mu \equiv \tilde{A}^{(4)}_\mu = R^4_3 A^{(3)}_\mu + R^4_4 A^{(4)}_\mu \,. \tag{2}$$

It can be seen that *canonical independence* requirements in the Poisson brackets of the gauge field theory (see e.g. []) force us to define the counterpart mixture, that which leads to the neutral vectorial boson

$$Z^{(0)}_\mu \equiv \tilde{A}^{(3)}_\mu = R^3_3 A^{(3)}_\mu + R^3_4 A^{(4)}_\mu \,, \tag{3}$$

and force the transformation in (2) and (3) to be ortogonal, i.e.,

$$\begin{array}{c}\text{canonical}\\ \text{independence}\\ \text{conditions}\end{array} \Rightarrow \begin{pmatrix} R^3_3 & R^3_4 \\ R^4_3 & R^4_4 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} . \tag{4}$$

Thus, the neutral weak charge would be given by a mixture

$$T_0 \equiv \tilde{T}_3 = C^3_3 T_3 + C^4_3 T_4 \,, \tag{5}$$

where the coefficients C_3^3, C_3^4 depend on the mixing angle θ_W. To be more precise, the invariance of the connection 1-form $\Lambda = (g)^a_b T_a A_\mu^{(b)} dx^\mu$ under a general Lie algebra transformation $\tilde{T}_a = C_a^b T_b$, induces a transformation in the gauge fields of the form

$$\tilde{A}_\mu^{(a)} = (\tilde{g}^{-1})^a_b (C^{-1})^b_c (g)^c_d A_\mu^{(d)} \equiv R^a_d A_\mu^{(d)} \qquad (6)$$

where $(g) = \mathrm{diag}(r, r, r, r')$ and $(\tilde{g}) = \mathrm{diag}(\tilde{r}, \tilde{r}, \tilde{r}, \tilde{r}')$ are the initial and final (bare) coupling-constant matrices. For the specific Lie algebra transformation given in (1,5), the identification of R^a_d in (6) with the rotation (4) gives the value of $\sin^2\theta_W = \frac{\tilde{r}'^2}{4r^2} (\equiv \frac{e^2}{g^2}$ in the standard notation) in terms of the quotient of final and initial coupling constants. This characterization of the mixing angle differs from the more conventional expression of $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ in terms of the masses of the vectorial bosons $Z_\mu^{(0)}$ and $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^{(1)} \pm i A_\mu^{(2)})$. This other characterization of θ_W is strongly related to the Symmetry Breaking mechanism and arises as an orthogonal transformation which diagonalizes a symmetric mass matrix. Even though these two definitions of θ_W are usually identified, there are conceptual differences, as in fact pointed out in []. We shall go further and show that, whereas the quantity $1 - M_W^2/M_Z^2$ is a free parameter which has to be fixed by experiments, there are strong mathematical restrictions to the value of e^2/g^2 so as to fix it to $1/2$ ($\theta_W = \pi/4$).

2 Global restrictions to θ_W

If we disregard the global structure of the gauge group, then any one-to-one Lie algebra transformation $\tilde{T}_a = C_a^b T_b$ (as that given in (1,5)) would clearly be allowed for arbitrary coefficients C_a^b. However, the increasing importance in gauge theories of some basic topological issues (for example, the existence or not of monopoles and solitons, topological properties of the Symmetry Breaking, the Bohm-Aharonov effect itself, etc.) demands a revision of some local (Lie algebra level) transformations. In fact, since the group $SU(2) \otimes U(1)_Y$ is non-simply connected, there are strong restrictions to the number of globally exponentiable Lie algebra transformations []; in other words, not all the Lie algebra isomorphisms can be realized as the derivative of global homomorphisms of the corresponding gauge group. Without going into details (see [] for more information), we can say that the mere embedding of the electromagnetic subgroup $U(1)_Q$ in the torus $T^2 = U(1)_{T_3} \otimes U(1)_Y$, as suggested by (1,5), imposes non-trivial restrictions (rational values) to the tangent of the integral curves (*closed* geodesics) associated with its generator. That is, the coefficients C_3^3, C_4^3 in (5) have to be rational numbers, this fact leading to fractional values for $\tan^2\theta_W = n/m$, according to (6). Then, the additional requirement that $U(1)_{T_3}$ be a subgroup of $SU(2)$ imposes further, severe restrictions so as to fix $\tan^2\theta_W = 1$. The corresponding group homomorphism proves to be

$$SU(2) \otimes U(1)_Y \stackrel{\theta_W = \pi/4}{\longrightarrow} \left(SU(2) \otimes U(1)_Y\right)/Z_2 \simeq U(2)\,. \qquad (7)$$

3 Conclusions

In summary, the only global homomorphism from $SU(2) \otimes U(1)_Y$ to a locally isomorphic group defining a proper rotation on the gauge fields compatible with the Gell'Mann-Nishijima relationship (thus providing an electric charge) is the homomorphism (7), which leads to the value $\sin^2\theta_W = 1/2$ for the mixing angle. This means that there is only one final coupling constant –essentially the electric charge, i.e., $e \equiv \tilde{r}' = \sqrt{2}r (\equiv g/\sqrt{2}$ in the standard notation)–, even though the gauge group ($U(2)$) is not a simple group. According to general settings, however, the theory must contain a coupling constant for each simple or abelian term in the Lie algebra decomposition of the gauge group. An immediate conclusion is that the *assignment of coupling constants* should be done according to factors in the direct product decomposition of the *group*, rather than the algebra.

With regard to the structure of the currents, this particular value of θ_W means that the neutral weak current is pure V−A for the neutrino and pure V+A for the lepton.

In the light of the difference between the value of $e^2/g^2 = 1/2$ (previous to any mechanism intended to supply masses) and the experimental value of $1 - M_W^2/M_Z^2 \approx 0.23$, only the hope remains that our characterization of θ_W really corresponds to an *asymptotic limit* (high energies), or to that state of the Universe in which the electroweak interaction was not yet "spontaneously broken", i.e. the masses of the vector bosons are zero and therefore the quantity $1 - M_W^2/M_Z^2$ makes no physical sense. In any case, our results provide strong support to the idea that those two quantities cannot be directly identified, as it was in fact pointed out in [].

Acknowledgments

This work was partially supported by the DGICYT. M. Calixto thanks the Spanish MEC for a FPI grant.

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