

VACUUM RADIATION IN CONFORMALLY INVARIANT QUANTUM FIELD THEORY

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Although the whole conformal group $SO(4,2)$ can be considered as a symmetry in a classical massless field theory, the subgroup of special conformal transformations (SCT), usually related to transitions to uniformly accelerated frames, causes vacuum radiation in the corresponding quantum field theory, in analogy to the Fulling-Unruh effect. The spectrum of the outgoing particles can be calculated exactly and proves to be a generalization of the Planckian one.

The conformal group $SO(4,2)$ has ever been recognized as a symmetry of the Maxwell equations for *classical* electro-dynamics¹, and more recently considered as an invariance of general, non-abelian, massless gauge field theories at the classical level. However, the *quantum* theory raises, in general, serious problems in the implementation of conformal symmetry, and much work has been devoted to the study of the physical reasons for that (see e.g. Ref. ²). Basically, the main trouble associated with this quantum symmetry (at the second quantization level) lies in the difficulty of finding a vacuum, which is *stable* under special conformal transformations acting on the Minkowski space in the form:

$$x^\mu \rightarrow x'^\mu = \frac{x^\mu + c^\mu x^2}{\sigma(x, c)}, \quad \sigma(x, c) = 1 + 2cx + c^2 x^2. \quad (1)$$

These transformations, which can be interpreted as transitions to systems of relativistic, uniformly accelerated observers (see e.g. Ref. ³), cause *vacuum radiation*, a phenomenon analogous to the Fulling-Unruh effect^{4,5} in a non-inertial reference frame. To be more precise, if $a(k), a^+(k)$ are the Fourier components of a scalar massless field $\phi(x)$, satisfying the equation

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi(x) = 0, \quad (2)$$

then the Fourier components $a'(k), a'^+(k)$ of the transformed field $\phi'(x') = \sigma^{-l}(x, c)\phi(x)$ by (1) (l being the conformal dimension) are expressed in terms of both $a(k), a^+(k)$ through a Bogolyubov transformation

$$a'(\lambda) = \int dk [A_\lambda(k)a(k) + B_\lambda(k)a^+(k)]. \quad (3)$$

In the second quantized theory, the vacuum states defined by the conditions $a(k)|0\rangle = 0$ and $a'(\lambda)|0'\rangle = 0$, are not identical if the coefficients $B_\lambda(k)$ in (3) are not zero. In this case the new vacuum has a non-trivial content of untransformed particle states.

This situation is always present when quantizing field theories in curved space as well as in flat space, whenever some kind of global mutilation of the space is involved. This is the case of the natural quantization in Rindler coordinates⁴, which leads to a quantization inequivalent to the normal Minkowski quantization, or that of a quantum field in a box, where a dilatation produces a rearrangement of the vacuum⁴. Nevertheless, it must be stressed that the situation for SCT is more peculiar. The rearrangement of the vacuum in a massless QFT due to SCT, even though they are a symmetry of the classical system, behaves as if the conformal group were *spontaneously broken*, and this fact can be interpreted as a kind of topological *anomaly*.

Thinking of the underlying reasons for this anomaly, we are tempted to make the singular action of the transformations (1) in Minkowski space responsible for it, as has been in fact pointed out in⁶. However, a deeper analysis of the interconnection between symmetry and quantization reveals a more profound obstruction to the possibility of implementing unitarily STC in a generalized Minkowski space (homogeneous space of the conformal group), free from singularities. This obstruction is traced back to the impossibility of representing the entire $SO(4, 2)$ group unitarily and irreducibly on a space of functions depending arbitrarily on \vec{x} (see e.g. Ref.²), so that a Cauchy surface determines the evolution in time. Natural representations, however, can be constructed by means of wave functions having support on the hole space-time and evolving in some kind of *proper time*. It is proved (see⁷) that unitary irreducible representations of the conformal group require the generator P_0 of time translations to have dynamical character (i.e., it has a canonically conjugated pair), as it happens with the spatial component P_1 , due to the appearance of a central term $\hat{1}$ in the quantum commutators

$$[P_\mu, K_\nu] = -\eta_{\mu\nu}(2D + 4N\hat{1}) \quad (4)$$

(K_ν and D denote the generators of SCT and dilatations, respectively) proportional to a parameter N , which characterizes the unitary irreducible representations of the conformal group. So, conformal wave functions $\psi^{(N)}$ have support on the whole space-time. If we forced the functions $\psi^{(N)}$ to evolve in time according to the Klein-Gordon-like equation

$$Q\psi^{(N)} = P_\mu P^\mu \psi^{(N)} = 0, \quad (5)$$

(“null square mass condition”, i.e., by selecting those functions nullified by the

Casimir operator Q of the Poincaré subgroup of $SO(4, 2)$) we would find that the appearance of *quantum* terms proportional to N at the right-hand side of the quantum commutators

$$[K_\mu, Q] = f_\mu(x, t)Q + 8NP_\mu \quad (6)$$

(where $f_\mu(x, t)$ are some functions on the generalized Minkowski space), terms which do not appear at the classical level ($N = 0$), prevent the whole conformal group to be an exact symmetry of the massless quantum field. This way, the quantum time evolution itself destroys the conformal symmetry, leading to some sort of *dynamical symmetry breaking* which preserves the Weyl subgroup (Poincaré + dilatations). The SCT do not leave the Eq. (5) invariant, and this fact manifests, at the second quantization level, through a *radiation* of the vacuum of the massless quantum field (“Weyl vacuum”) under the action of SCT, i.e., from the point of view of an uniformly accelerated observer. The spectrum of the outgoing particles can be calculated exactly⁷ and proves to be a generalization of the Planckian one, this recovered in the limit $N \rightarrow 0$. The temperature of this thermal bath is linear in the acceleration parameter, as in the reference⁵.

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