

# Non-Data Aided Carrier Offset Compensation for SDR Implementation

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**Abstract** - This paper deals with the design and implementation of an algorithm for non-data aided carrier offset compensation in a QPSK communication system. The work concentrates on frequency synchronization and we propose a new modulation removal algorithm where the carrier offset is corrected in two steps, reducing the computational complexity. Moreover, a FPGA-accelerated implementation is introduced. Initial tests show that the implemented algorithm works as expected and that the Simulink simulation time is reduced by half when accelerated by the FPGA.

## I. INTRODUCTION

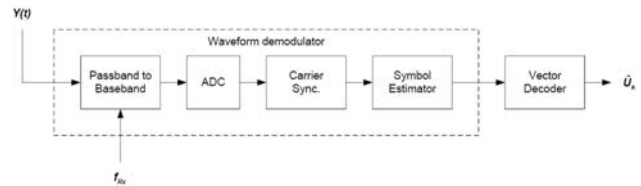
Carrier synchronization of transmitters and receivers is one of challenges in modern communication systems. When transmitting data through a channel it is modulated on a predefined carrier frequency. The transmitter modulates the transmission burst using a local oscillator to generate the carrier frequency and the receiver demodulates the signal using another local oscillator. As ideal oscillators do not exist, a frequency offset between the transmitter and the receiver oscillators is inevitable. This offset increases the Bit-Error Rate (BER), which calls for compensation techniques. The offset estimation can be aided by transmitting training data, which is known as data aided estimation but has the major drawback of occupying channel bandwidth. To save bandwidth, non-data aided estimation is preferred since the offset is estimated from the actual transmitted data.

The contribution presented in this paper is the design and implementation of a new Modulation Removal (MR) algorithm for carrier offset compensation which can cope with higher sampling rates than existing algorithms and has a limited computational complexity and hardware usage.

## II. CARRIER SYNCHRONISATION SYSTEM

Fig. 1 shows the receiver model used in this work. The received signal is pass-banded to baseband in the analog domain and then converted to a digital sequence where the carrier synchronisation is performed.

Finally, the symbols are estimated and decoded. If the frequency in the transmitter,  $f_{Tx}$ , is not equal to the frequency in the receiver,  $f_{Rx}$ , then the BER is increased. To accommodate this error a method for frequency offset compensation is introduced. This is done by the carrier synchronization block placed between the Analog to Digital Converter (ADC) and the symbol estimator.



**Fig. 1.** The receiver model including carrier synchronization where  $Y(t)$  is the received signal,  $f_{Rx}$  the receiver frequency and  $\hat{U}_k$  the estimated signal.

The signal specifications in this work are as follows:

- Symbol rate: 8400 symbols/s,
- Channel bandwidth: 10.4 kHz,
- Modulation scheme: QPSK,
- Symbols encoding: Gray,
- Carrier offset frequency interval: +/- 40 kHz,
- Burst duration: 80 ms.

Several assumptions have been made:

- It is only the desired channel that is located in the interval Frequency Carrier,  $f_c$ , +/- 40 kHz. If more than one channel is located in the interval  $f_c$  +/- 40 kHz it is impossible for a non-data aided communication system to detect the correct channel.
- In the MR algorithm, a FFT is performed on the received transmission burst. Since the MR algorithm estimates one frequency set per burst the frequency offset must be constant for the entire burst.
- The phase offset of the received transmission bursts are zero. This means that is not necessary to correct the phase before the frequency offset is estimated and the signal corrected.

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- The transmission channel is assumed to be an AWGN channel. Distortion of the signal like echoes and Doppler effects are neglected.

### III. INITIAL MR ALGORITHM

Our initial MR algorithm is inspired by [1], which proposes a modulation removal algorithm used to estimate and correct carrier frequency offset. To remove the frequency offset of the received baseband signal, the offset has to be estimated. An intuitive approach is to make a FFT of the baseband signal and choose the frequency bin with the most energy. When noise is applied to the signal the power spectrum is added with noise. When the frequencies in the neighborhood of the offset frequency almost have the same power, it is difficult to detect the correct offset frequency. If the modulation is removed from the received signal, the bandwidth is ideally reduced to 0 Hz. The power of the received signal is then centered at a single frequency, which makes the offset easier to detect when noise is present. Such a system is illustrated in fig. 2.

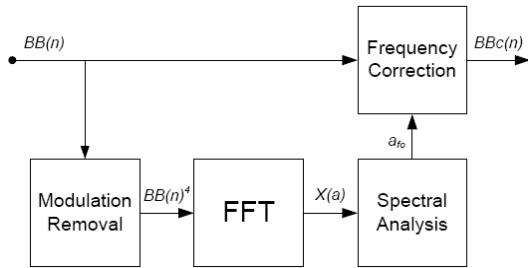


Fig. 2. The MR algorithm proposed by [1].

The authors of [1] assume that the baseband signal is sampled with one sample per symbol. In this work it is necessary to apply a higher sampling rate. In this section we propose a modified version of the algorithm found in [1] to support several samples per symbol and in section 4 we propose a 2-step approach to reduce the computational complexity.

In the initial algorithm proposal the baseband signal is sampled with one sample per symbol. When the modulation is removed, the offset carrier frequency is located at four times the original frequency. In our case the symbol rate of 8400 symbols/s makes that the initial algorithm is only able to correct the carrier frequency of  $\pm 1050$  Hz when fulfilling the Nyquist criterion.

Our system specifications tell that the offset carrier frequency  $f_o$  can have values in the area from -40 kHz to +40 kHz. This requires a sample rate such as  $\text{samplerate} > 2 \cdot 4 \cdot 40 = 320$  kHz. The number of samples per symbol,  $k$ , should then be:  $k > \text{samplerate}/\text{symbolrate} = 320/8.40 = 38.1$ , i.e.,  $k \geq 39$ .

The high number of samples per symbol increases the computational complexity of the algorithm components with a factor  $k$ . Unfortunately, the higher values of  $k$  decreases the accuracy of the FFT. If  $k$  is set to 40, there will be a total of  $k \cdot \text{symbolrate} \cdot \text{burstlength} =$

$40 \cdot 8400 \text{ Hz} \cdot 80 \text{ ms} = 26880$  samples per transmission burst. This supports a radix-2 FFT of length 16384.

An approach to reduce the computation complexity of the carrier synchronization is presented in section 4.

### IV. PROPOSED 2-STEP MR ALGORITHM

The key ideas behind making the carrier correction in two steps is: 1) Make a computational inexpensive low precision estimation of the carrier offset frequency and correct the frequency, 2) Lowpass filter and downsample the baseband signal and 3) Make a high precision estimation of the carrier offset frequency and correct the frequency. This approach is illustrated in fig. 3.

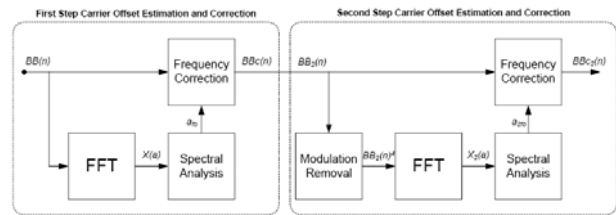


Fig. 3. The proposed 2-step carrier correction algorithm.

The argument for making this 2-step carrier correction is to reduce the carrier frequency error to a maximum of half the width of the SQPSK main lobe after the first frequency offset correction step. This low precision correction will make it possible to reduce the number of sample per symbol needed in the second frequency offset correction step, without losing signal information. By doing this, the computational complexity of the high precision estimation and carrier correction is reduced.

#### A. First step of the carrier correction algorithm

The low precision estimation is performed without the use of modulation removal. There are three reasons for this: 1) lower sample rate: the MR algorithm needs four times higher sample rate as the algorithm multiplies the carrier frequency with four. This is not the case without the MR algorithm where the sample rate only has to be one fourth compared to the MR algorithm, 2) better SNR: as only a low precision estimate is needed the length of the FFT can be reduced, the estimation without MR algorithm gives a better SNR at the offset frequency than the MR algorithm for short FFT lengths, and 3) less complex: there is no need to raise to the power of four.

Selecting the FFT length,  $N_1$ , is equal to selecting the FFT precision,  $\Delta f_1$ . There are no high demands to  $\Delta f_1$  except that it has to be smaller than half the main lobe width of 8484 Hz. With  $\Delta f_1$  set to be less than 100 Hz, the length of  $N_1$  is  $> 840$ . The FFT algorithm has the best performance using a length that is a radix-2 number. Therefore  $N_1$  is set to 1024. This gives a  $\Delta f_1$  of 82 Hz and a total correction error of maximum  $\pm 8566$  Hz.

#### B. Second step of the carrier correction algorithm

To obtain a high precision estimation the MR algorithm is used in the second correction step. From the first correction step the maximum frequency error is  $\pm 8566$  Hz. When the signal is raised to the power of four in the

MR algorithm, the carrier frequency is multiplied by four which results in a maximum frequency deviation of  $\pm 34.3$  kHz. Using this information, the number of samples per symbol in the second correction step,  $k_2$ , is calculated using the Nyquist theorem:  $k_2 > (2 \cdot 34.3 \text{ kHz}) / 8400 \text{ Hz}$ , i.e.,  $k_2 \geq 9$ .

### C. Test of the Initial MR Algorithm versus the 2-Step Carrier Correction Algorithm

Each algorithm has processed 1000 different transmission bursts. The Matlab profiler is used to evaluate the execution time of each algorithm. In the performance test for low SNR, each algorithm has processed 1000 different transmission bursts at each simulated SNR value. The known frequency offset for each burst is then compared to the estimated frequency offset. If the difference is larger than 10 Hz the estimation is rated as a failure. The specifications for the algorithms are listed in table 1.

TABLE 1 ALGORITHM SPECIFICATIONS

	Initial MR algorithm	2-step algorithm	
		Step 1	Step 2
FFT length	16384	1024	4096
Samples/symbol	40	10	10

Each individual burst has a symbol rate of 8400 Hz and a length of 80ms which corresponds to 672 symbols. The carrier frequency offset is randomly chosen in the interval from -40 kHz to 40 kHz. The execution times for 1000 random generated bursts are listed in table 2.

TABLE 2 1000 BURST EXECUTION TIMES (S)

	Initial MR algorithm	2-step algorithm	
		Step 1	Step 2
Modulation removal	0.844	-	0.197
Spectral analysis	0.537	0.114	0.179
FFT	0.471	0.040	0.121
Carrier Correction	1.524	0.673	0.639
Execution time	3.376	0.827	1.136
		1.963	

Table 2 shows that the proposed algorithm is 38 % faster than the initial MR algorithm. The speed gain is obtained via the sampling rate which is lowered by 1/4. Fig. 4 illustrates the error ratio for both algorithms at different SNR values.

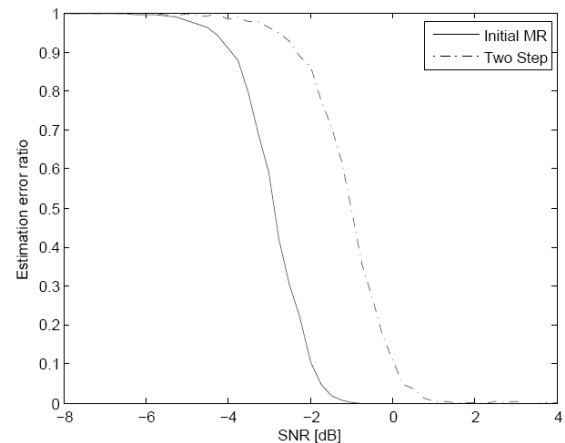


Fig. 4. Comparison of the SNR estimation errors.

Fig. 4 shows that the proposed 2-step algorithm has a good performance above 4 dB SNR whereas the initial MR algorithm performs well as low as 2 dB SNR. This is normal since a SNR at 4 times the offset frequency increases with the length of the FFT. The second step in the proposed algorithm is identical to the initial MR algorithm except that the FFT length is 1/4 of the one in the initial MR algorithm. So the 2-step carrier correction algorithm is faster at the cost of low-SNR performance. However, performance can be significantly improved, as discussed later on in section 6.

## V. FPGA ACCELERATED SIMULATION

The FPGA used in this work is an Altera Cyclone II [2] containing 35,000 logic elements and connected to a 50 MHz clock. The method used in this work is called Hardware In the Loop (HIL), where the FPGA executes part(s) of the algorithm and a PC sends/receives data to/from the FPGA. This can potentially accelerate the simulation if the algorithm exhibits inherent parallelism. The MR algorithm is described using Altera's DSP Builder [3] which contains Simulink blocks that can be executed on the FPGA after a synthesis process.

### A. Modulation Removal

When the modulation removal is performed, the input signal is raised to the power of four. This is done by squaring the signal twice. The input signal to the square block is a signed 16 bit Q14 number, therefore the output of the multiplications in this block is a signed 32 bit Q28 number. This number is truncated such that the output of the block is a signed 16 bit Q14 number.

### B. FFT

The FFT block is implemented using the Altera MegaCore FFT block. The essential settings in this work are: 1) data precision: 16 bit, 2) data flow: streaming allowing the FFT length to change dynamically, which is convenient for the two step carrier correction algorithm, 3) FFT length: 64, and 4) implementation of multipliers: logic elements and DSP blocks.

### C. Spectral Analysis

The input to this block is a complex number from the FFT analysis; therefore the magnitude of this number is

computed. Next the 64 first magnitudes are stored, as they represent the FFT analysis of the input signal. This is done using a state machine, with the four states: init, write, read and wait controlling a FIFO memory block for storing the 64 magnitudes.

#### D. Frequency Correction

The frequency correction consists of an exponential function and a sine/cosine generator. The sine and cosine generator is created from a MegaCore block called NCO. The output from the NCO is a sine and a cosine that is used for the exponential function for the frequency carrier correction.

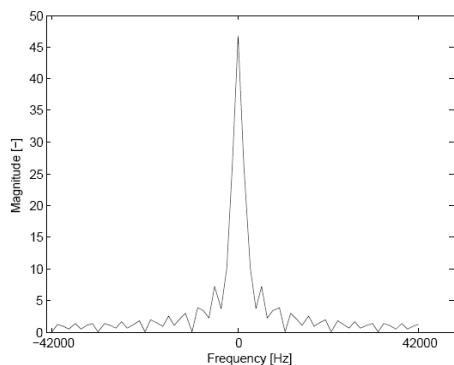
#### E. Test of the FPGA implementation MR Algorithm

The MR algorithm is tested without noise, as the hardware-implemented MR algorithm only works when the input SNR is approximately 30 dB, which practically is a noise free input. The FFT analysis with a length of 64 is covering a spectra of +/-42 kHz, meaning that the precision of the FFT is = 1.3125 kHz.

The FPGA implementation (HIL) has been tested for two scenarios, i.e., when the frequency offset is a multiple of the FFT precision or not, both on an Intel Centrino 1.7 GHz PC with 1 GB of RAM. The implementation uses 95 % of the multipliers and 51 % of the logic elements. After the data is processed by the FPGA, the results are saved in the Matlab workspace, where they are compared with the results from the non-accelerated Simulink tests.

##### E.1 Frequency Offset is a Multiple of the FFT Precision

Here the frequency offset is chosen to -13.125 kHz. The QPSK signal is then frequency-shifted according to the frequency offset.



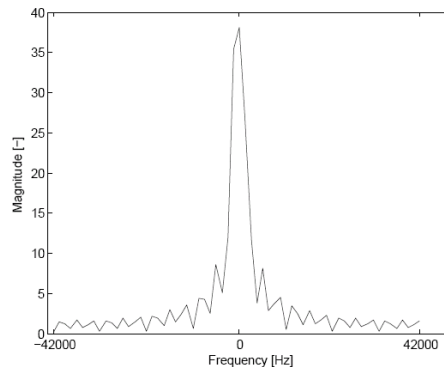
**Fig. 5.** Frequency corrected output when the offset is a multiple of the FFT Precision.

Fig. 5 shows that there is no frequency offset in the signal, and that the FFT analysis is symmetric. The output from the test of the MR algorithm is compared with the non frequency-shifted QPSK signal, and the bit error rate is 0, i.e., the algorithm works properly.

##### E.2 Frequency Offset is not a Multiple of the FFT Precision

In this second scenario the frequency offset is now +12.5 kHz. Fig. 6 illustrates the frequency corrected output: it is seen that the frequency offset is removed, but the FFT

analysis is not symmetric. This results in a bit error rate of 0.5 which was expected as a consequence of the low FFT precision.

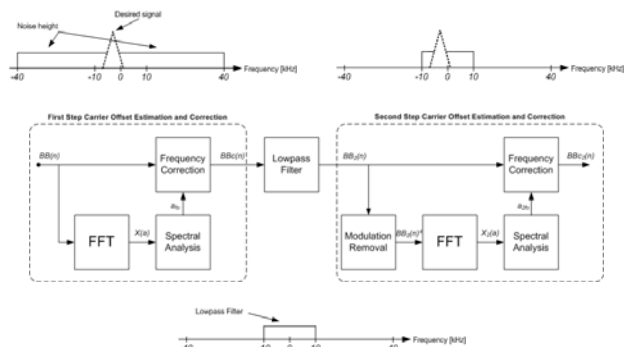


**Fig. 6.** Frequency corrected output when the offset is not a multiple of the FFT Precision.

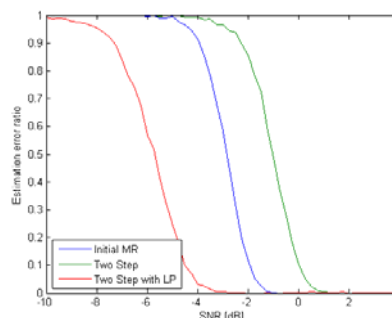
SNR-wise the FPGA-accelerated and non-accelerated results are identical. However, the execution times are significantly different: 2.5 minutes for the non-accelerated version vs. 1.2 minute for the FPGA-accelerated one, which about twice as fast.

## VI. IMPROVED 2-STEP ALGORITHM

Inserting a Low-pass Filter (LP) between the 2 steps, cf. fig. 7, significantly improves the performance of the proposed solution. Fig. 8 shows that the LP 2-step version outperforms not only the non-LP version but also the initial one.



**Fig.7** LP-version of the 2-step algorithm.



**Fig.8** SNR estimation errors. Center: initial, right: 2-step, left: 2-step with LP.

## VII. CONCLUSION

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The design and implementation of an algorithm for non-data aided carrier offset compensation in a QPSK communication system has been presented. A new 2-step modulation removal algorithm has been proposed; it supports high sampling-rates and minimizes the computational complexity. This is achieved, in the non-LP version, at the cost of a slightly degraded performance for SNRs below 4dB. A FPGA-accelerated implementation of the non-LP version performs as expected and the simulation time is reduced by half. Finally, inserting a LP filter between the 2 steps significantly increases the performance. Future work will investigate the FPGA implementation of that version.

#### REFERENCES

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