Detecting LFM Parameters in
Joint Communications and Radar Frequency Bands

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Introduction

As one of the most common radar signals, **linear frequency modulation (LFM)** is extensively applied to sense the targets in both military and civilian applications. In the meantime, **orthogonal frequency division multiplexing (OFDM)** signals are common waveforms generally used in both 4G and 5G communication systems. Thus, to research the scenario when the radar signals and the communication waveforms are coexisting simultaneously has gradually attracted the attention of researchers. In this poster, we consider the scenario when the representative radar signal, the LFM signal, the communication OFDM waveform, and additive white Gaussian noise (AWGN) are present in the same frequency band. Under this scenario, this poster presents a **discrete chirp Fourier transform (DCFT)** method to detect LFM parameters with a high accuracy and proposes to apply **maximum mutual information (MMI)** algorithm to apply **compressive sensing (CS)** to the received dataset from the combined OFDM and LFM signals for future signal processing.

System Model

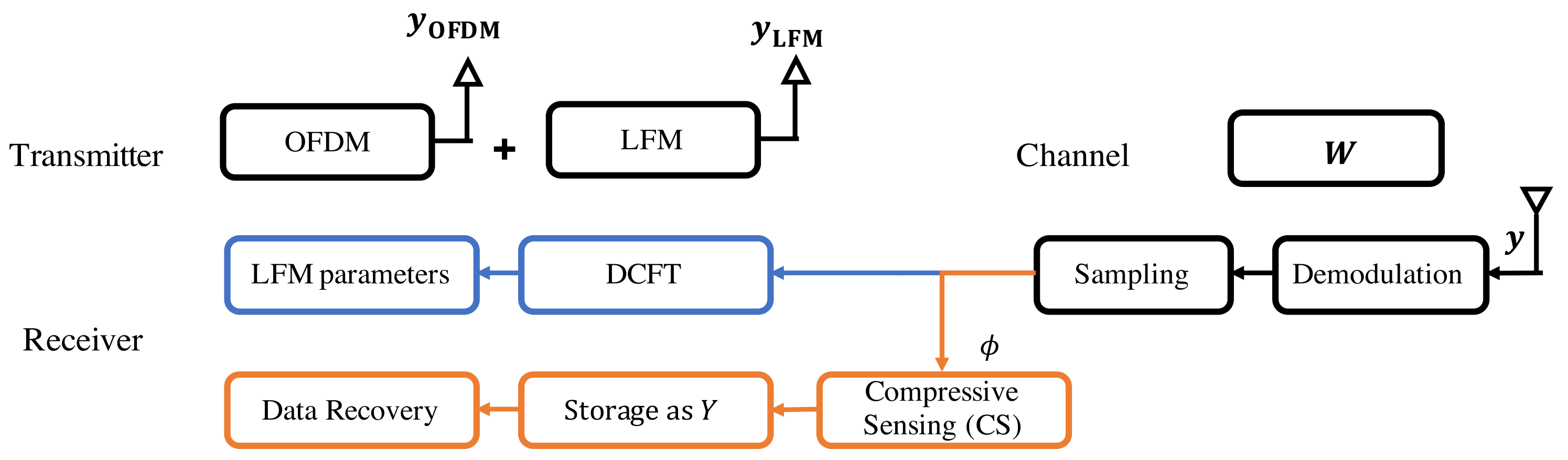

Figure 1: Model of the Transmitters and Receivers

At transmitter part,

- OFDM waveform $\boldsymbol{y}_{\mathbf{OFDM}}$
- LFM waveform $\boldsymbol{y}_{\mathbf{LFM}}$

At receiver part,

- Received waveform $\boldsymbol{y} = \boldsymbol{y}_{\mathbf{OFDM}} + \boldsymbol{y}_{\mathbf{LFM}} + \boldsymbol{W}$.

Blue part is the DCFT estimation part

Orange part is the current CS research area.

Discrete Chirp Fourier Transform

DCFT [1] is one technique similar to the discrete Fourier transform and is applied to calculate the value of the quadratic frequency components.

- When $W_N = \exp(-2\pi j/N)$, N-point DCFT is

$$X[l,k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] W_N^{ln^2+kn}, \qquad l,k = 0,1,\cdots,N-1.$$

- For sampling rate $t = n/N^M, M \in (0,1]$, the LFM waveform is

$$f[n,l,k] = W_N^{-(ln^2+kn)}.$$

- After estimating the DCFT parameters $\tilde{l}$ and $\tilde{k}$, the estimated chirp frequency $\tilde{f_l}$ and the estimated start frequency $\tilde{f_c}$ of the LFM signal can be calculated as:

$$\tilde{f_l} = 2N^{2M-1}\tilde{l}, \qquad \tilde{f_c} = N^{M-1}\tilde{k}.$$

Simulation

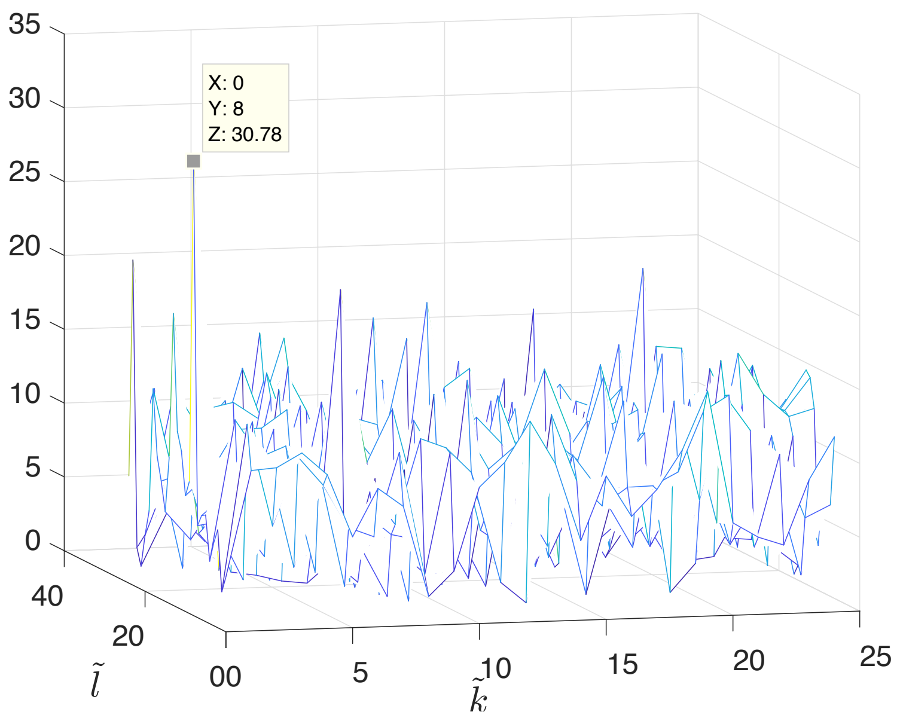

Figure 2: Simulation of the DCFT

Simulation Parameters

- $N = 25$
- $M = 1$
- LFM $f_l = 400\text{Hz}$ and $f_c = 0\text{Hz}$
- SNR = 0dB

Simulation Result

- Highest value is at $(\tilde{k}, \tilde{l}) = (0,8)$
- LFM parameters estimated
  $\tilde{f_l} = 400\text{Hz}$ $\tilde{f_c} = 0\text{Hz}$
- LFM signal correctly detected

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Maximum Mutual Information Algorithm

Reference [2] presents a basic approach to CS measurement design that utilises an information-theoretic metric (ITM) [3] with Shannon mutual information (MI) and shows that such an approach obtains a higher level of performance than other similar metrics designed by non-Shannon MI.

Based on the research result above, reference [4] proposed an advanced model as follows with two independent inputs, non-Gaussian inputs, $\boldsymbol{X}$ and $\boldsymbol{P}$, and the additional noise component $\boldsymbol{Q}$. Then, the authors applied the MMI algorithm to this new model to compress the dataset and still achieve a high detection or reconstruction performance.

- In the advanced model $\boldsymbol{Y} = \boldsymbol{\Phi}(\boldsymbol{X} + \boldsymbol{P}) + \boldsymbol{Q}$, the MMI algorithm is

$$\max_{\boldsymbol{\Phi}} F(\boldsymbol{\Phi}, \beta) \coloneqq \max_{\boldsymbol{\Phi}} \{ I(\boldsymbol{X}; \boldsymbol{Y}) - \beta I(\boldsymbol{P}; \boldsymbol{Y}) \}$$

where $\beta \in R$ controls the relative importance of the two mutual-information metrics. To advance the work of [4], this project will investigate the application of MMI when $\boldsymbol{X}$ and $\boldsymbol{P}$ are radar and communication signals, respectively. Initially, the following presents some simulations to discuss MMI algorithm.

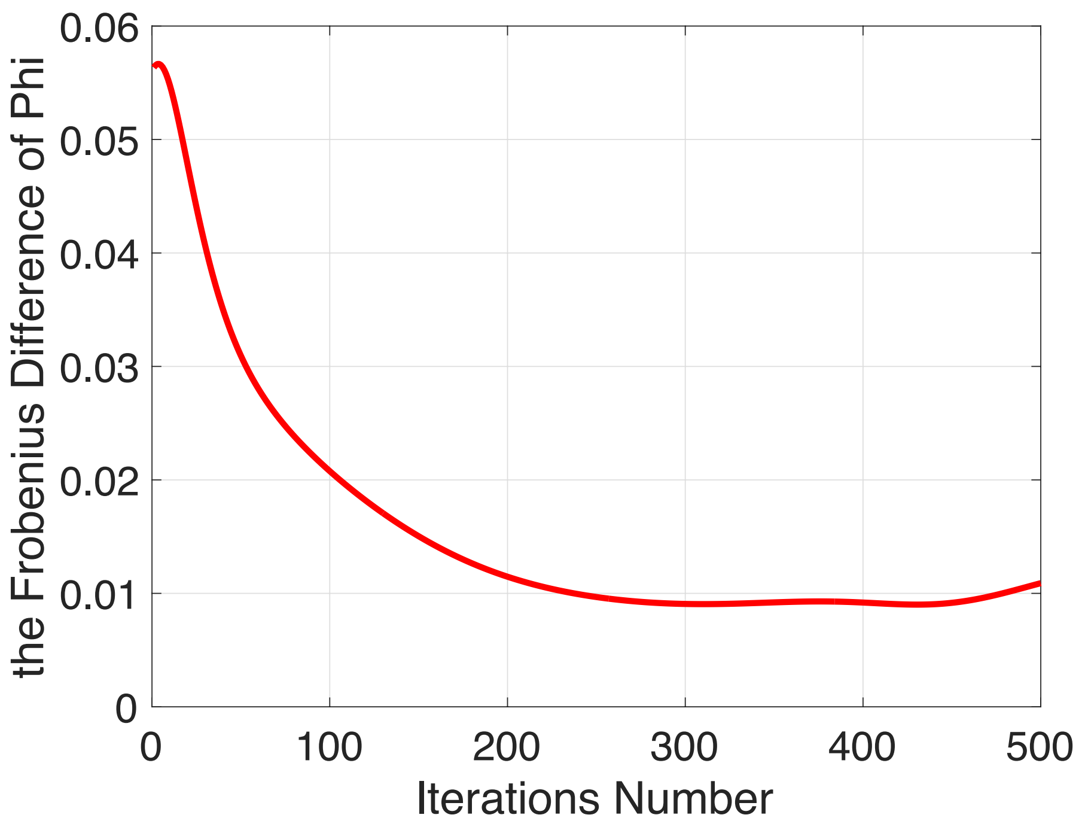

Figure 3: Simulation of the MMI method

Simulation Settings

- $\boldsymbol{Y} = \boldsymbol{\Phi}\boldsymbol{X} + \boldsymbol{Q}$
- $\boldsymbol{Q}$ is the Gaussian Noise
- step size $= 10^{-3}$
- $\boldsymbol{X}$ is a 50×1 vector
- $\boldsymbol{Y}$ is a 10×1 vector
- $\boldsymbol{\Phi}$ is a 10×50 matrix
- Number of samples = 100

In Figure 3, the Frobenius norm of the difference in $\boldsymbol{\Phi}$ gradually decreases until it reaches a value of 0.01; this is expected, as it shows that our method is able to achieve a good solution to compress the radar waveform. After convergence, the matrix $\boldsymbol{\Phi}$ should will remain constant as the number of iterations is increasing. At this point, the MMI method can be stopped and the final matrix $\boldsymbol{\Phi}$ can be used to detect the radar signal.

Discussion and Future Work

- This poster mainly discusses the available methods to estimate LFM parameters and the potential CS technique at the receiver with the following results:
  1. The DCFT method is able to estimate LFM parameters correctly for the coexisting the communication and radar signals scenario.
  2. The MMI technique via [4] for the joint communication and radar signals to compress the received signal is the next step work.
- In future research, we will keep improving the blue part and investigating the implementation of the orange blocks as shown in system model from:
  1. improving the DCFT method into general DCFT without the influence of parameters setting;
  2. discovering the alternative method, such as Hough transform [5] to estimate LFM parameters;
  3. applying general compressive sensing techniques to this communication and radar spectrum sharing application;
  4. exploiting the input-output mutual information compressive sensing technique for two independent sources, which could be radar and communication data, respectively.

References

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