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Frequency-Domain Generelaized Singular Peruturbation Method for Relative Error Model Order Reduction

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***Abstract*— A new mixed method for relative error model order reduction is proposed. In the proposed method the frequency domain balanced stochastic truncation method is improved by applying the generalized singular perturbation method to the frequency domain balanced system in the reduction procedure. Frequency domain balanced stochastic truncation method which is proposed in [14] and [16] by author is based on two recently developed methods, namely frequency domain balanced truncation within a desired frequency bound and inner-outer factorization techniques. The proposed method in this paper is a carry over of the frequency-domain balanced stochastic truncation and is of interest for practical model order reduction because in this context it shows to keep the accuracy of the approximation as high as possible without sacrificing the computational efficiency and important system properties. It is shown that some important properties of the frequency domain stochastic balanced reduction technique are extended to the proposed reduction method by using the concept and properties of the reciprocal systems. Numerical results show the accuracy, simplicity and flexibility enhancement of the method.**

I. INTRODUCTION

Over the past two decades, model reduction methods have become increasingly popular [1]-[3]. Such methods are designed to extract a reduced order state-space model that adequately describes the behavior of the system to study.

A low-order model for a large scale system brings us an easy implementation. As opposed to a high-order model that might require expensive or complicated hardware; the low-order model has less complicated and more easily available hardware. Furthermore in the high order systems the analysis problems can not be solved within a reasonable time and cost. It is advisable then to construct a reduced order model that approximates the physical behavior of the original system.

The reduction techniques are divided into two broad categories, namely SVD based methods and moment matching based techniques. The first category consists of the methods like balanced truncation that is stability preserving and has an upper bound for approximation error. Moment matching based methods like Krylov subspace method can be implemented iteratively, which leads to numerical efficient algorithms, but these do not automatically preserve stability and have no error bound[1], [3]. Some of the proposed reduction methods are trying to reduce the absolute error and some others are trying to reduce the relative error as a measure for the approximation accuracy. The balanced stochastic truncation (BST) approach belongs to the family of relative error methods. In contrast to absolute error methods like the balanced truncation (BT) or the singular perturbation approximation (SPA) method, the BST method has the main advantage in provision of a uniform approximation of the frequency response of the original system over the whole frequency-domain, and particularly, in preservation of phase information [4]. For example, for a minimum-phase original system, the BST-approximation is also minimum-phase. However this is not generally true for the absolute error methods. From a practical point of view a system is operating within a frequency bound and outside that the system shuts down. Because we do not have to keep the approximation good outside the operational bandwidth of the system, the accuracy can be increased if we confine the approximation within a frequency bound. Based on this idea, the frequency-domain balanced truncation within a frequency bound (FDBT) is proposed [5]-[15].

Frequency-domain balanced stochastic truncation (FBST) is a recently developed method for relative error model reduction which is based on BST and FDBT approaches[14][16]. In this paper we propose a new method in which FBST and generalized singular perturbation is mixed. The proposed method is more accurate and more flexible than previous methods in the context of relative error model reduction like BST or FBST. The paper is organized as follows. In Section II, we introduce some definitions, notations and concepts for BST. Section III consists of presenting the FDBT algorithm and its properties. In Section IV, the FBST method based on some of the numerical recent algorithms like inner-outer factorization is presented. Section V is the main part of this paper in which, by applying the generalized singular perturbation method to the system with frequency-domain stochastic balanced structure, a new mixed method is proposed. In Section VI, it is shown that by using the concept and properties of the reciprocal and σ-reciprocal systems, some important properties of FBST can be extended to the proposed reduction method. In Section VII, the proposed relative error model reduction method is applied to a practical CD player benchmark example and the results are shown. Finally in Section VIII the conclusion is presented.

II. Balanced Stochastic Truncation Model Reduction

Let $G(s)$ be a MIMO square transfer matrix with a minimal sate space realization $G := (A,B,C,D)$ and of order n. If D is nonsingular it is possible to compute the left spectral factor $\psi(s)$ of $G(s)G^T(-s)$ satisfying:

$$\psi^T(-s)\psi(s) = G(s)G^T(-s) \tag{1}$$

The state space realization of G is called a balanced stochastic realization if:

$$W_c^G = W_o^\psi = diag(\sigma_1,...,\sigma_n) \tag{2}$$

Where W_c^G is the controllability Gramian of $G(s)$, the matrix W_o^ψ is the observability Gramian of $\psi(s)$ and σ_i is the i^{th} Hankel singular value of the stable part of the so-called "phase matrix" $F(s) = (\psi^T(-s))^{-1}G(s)$. The singular values in (2) are ordered decreasingly [4],[17],[18].

We assume now that G is already stochastically balanced by an appropriate similarity transformation. The reduced model is obtained by eliminating the states related to the lowest set of singular values. The reduced model is stable and satisfies the relative error bound:

$$\left\|G^{-1}(G-G_r)\right\|_\infty \le \prod_{i=r+1}^{n}\frac{1+\sigma_i}{1-\sigma_i}-1 \tag{3}$$

where r is the order of reduced model.

This model reduction approach is called balanced stochastic truncation (BST) [18], [2].

III. Frequency-Domain Balanced Truncation within a Frequency Bound

Consider the following n^{th} order state-space model representation of an asymptotic stable LTI system:

$$\begin{pmatrix}\dot{x}\\ y\end{pmatrix} = \begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}x\\ u\end{pmatrix} \tag{4}$$

The problem is how to approximate the system with r^{th} order state-space model:

$$(A_r, B_r, C_r, D_r) \tag{5}$$

where $r < n$.

One commonly used and globally accurate approach is the so-called Balanced Model Reduction first introduced by Moore [19]. In this method, the system is transformed to a basis where the states which are difficult to reach are simultaneously difficult to observe. Then, the reduced model is obtained simply by truncating the states which have this property. Because of being operational the system within frequency bound, and outside that it is not important to have an accurate approximation the accuracy can be improved with applying balanced model reduction in the specified frequency band [5]-[15].

Controllability and observability Gramians in terms of w over a frequency band $[w_1, w_2]$ are defined by[5]-[15]:

$$\begin{aligned} W_{cf} &:= \frac{1}{2\pi}\int_{w_2}^{w_1}(Ijw-A)^{-1}BB^*(-Ijw-A^*)^{-1}dw \\ W_{of} &:= \frac{1}{2\pi}\int_{w_2}^{w_1}(-Ijw-A^*)^{-1}C^*C(Ijw-A)^{-1}dw \end{aligned} \tag{6}$$

Those are the solutions for the following Lyapunov equations:

$$\begin{aligned} AW_{cf} + W_{cf}A^* &= -(BB^*F^* + FBB^*) \\ A^*W_{of} + W_{of}A &= -(C^*CF + F^*C^*C) \end{aligned} \tag{7}$$

Where F is defined by:

$$F = \int_{w_1}^{w_2}(Ijw-A)^{-1}dw \tag{8}$$

With an appropriate similarity transformation T and change of the basis, the system realization in (1) can be transformed to a new balanced realization, so that the Gramians are equal and diagonal (in decreasing diagonal elements):

$$\overline{W}_{cf} = \overline{W}_{of} = diag(\sigma_1, \sigma_2, ..., \sigma_n) \qquad (9)$$

Here we have two important theorems that give us a physical interpretation for the reduction procedure [5]-[15]:

Theorem 1: The frequency-domain controllability Gramian represents the energy flow of the system through each state variable within the frequency range $[w_1, w_2]$.

It means that if the unit white Gaussian noise test input signal u(t) , and state vector x(t) of the system defined as follows:

$$u(jw)[u(jw)]^* = |u(jw)|^2 = \begin{cases} 1 & w_1 < |w| < w_2 \\ 0 & elsewhere \end{cases}$$

$$x(t) = e^{At}Bu(t) \overset{\mathbb{F}}{\Leftrightarrow} (jwI - A)^{-1}Bu(jw) = X(jw)$$

The energy of the system through controllability Gramian is as follows:

$$E_x = \int_0^\infty x(t)x^*(t)dt =$$
$$\frac{1}{2\pi}\int_{w_1}^{w_2} (jwI - A)^{-1}Bu(jw)u^*(jw)B^*(-jwI - A^*)^{-1}dw$$
$$= W_{cf}[w_1, w_2]$$

Theorem 2: The frequency-domain observability Gramian represents the energy flow of the system through each state variable within the frequency range $[w_1, w_2]$.

Consider a unit injected test signal x_0, where

$$x_0 x_0^* = \begin{cases} 1 & w_1 \prec w \prec w_2 \\ 0 & elsewhere \end{cases}$$

Define output

$$y(t) \triangleq Ce^{At}x_0 \overset{F}{\Leftrightarrow} C(jwI - A)^{-1}x_0 = Y(jw)$$

The energy E_y of the system through observability Gramian is obtained by:

$$E_y = \int_0^\infty y(t)y^*(t)dt =$$
$$x_0^*\left[\frac{1}{2\pi}\int_{w_1}^{w_2}(-jwI - A^*)^{-1}C^*C(jwI - A)^{-1}dw\right]x_0$$

Now, x_0 being a white noise test signal, the result follows:

$$E_y = W_{cf}[w_1, w_2]$$

From the above theorems and (9), it is understood that to have a good approximation, we should only truncate the states that are related to the lowest singular values in (9). This model reduction technique is called frequency-domain balanced truncation within a frequency bound (FDBT). This method is also stability preserving and provides an error bound for absolute error.

IV. FREQUENCY-DOMAIN STOCHASTIC BALANCED TRUNCATION

In this section a recently proposed method for large scale model reduction is surveyed. FBST keeps all of the advantages of BST and increases the accuracy of the approximation within a desired bounded frequency [14][16]. This model reduction method can easily be applied to models without solving Lyapunov equations by using the definition (6) and approximating the integral by summation for finding Gramians. Furthermore we can also use efficient available tools for solving Lyapunov equations and reach more efficiency. Numerical results in the next section show the accuracy enhancement of the proposed method.

In FBST algorithm like BST at first we should find the left spectral factor $\psi(s)$ of $G(s)G^T(-s)$ satisfying (1). In order to compute the left spectral function we apply inner-outer factorization of [4],[21] to factorize the state space realization $N = (A, W_c^G C^T + BD^T, -B^T(W_c^G)^{-1}, D^T)$ in the form $N_i(s)\psi(s)$ where $N_i(s)$ is the inner factor and $\psi(s)$ is the outer factor and the left spectral factor[4].

After the computation of the left spectral factor we change the state space representation by an appropriate similarity transform into stochastically balanced structure which we call "frequency-domain stochastic balanced realization". In the frequency-domain stochastic realization, the frequency-domain controllability Gramian of $G(s)$ and the frequency-domain observablity Gramian of the left spectral factor should be equal and diagonal and the diagonal elements should be in decreasing order:

$$W_{fc}^G = W_{fo}^\psi = diag(\sigma_1, ..., \sigma_n) \qquad (10)$$

The reduced model is obtained by eliminating the states related to the lowest set of singular values. The reduced model is also stable and satisfies the relative error bound similar to (3):

$$\left\|G^{-1}(jw)(G(jw) - G_r(jw))\right\|_\infty \le \prod_{i=r+1}^{n}\frac{1+\sigma_i}{1-\sigma_i} - 1 \qquad (11)$$

Figure (1) shows the overall algorithm of FBST method.

V. FREQUENCY-DOMIAN MIXED METHOD FOR RELATIVE ERROR MODEL REDUCTION

In FBST we eliminate the states related to the lowest set of the singular values. If we apply generalized singular perturbation approximation to frequency-domain stochastic balanced system, instead of elimination of the states, we obtain more accurate and more flexible method than FBST.

Inputs: system matrices (A,B,C,D) and the frequency ranges $[w_1, w_2]$

Outputs: reduced system matrices (A_r, B_r, C_r, D_r)

1- Form:
$N = (A, W_c^G C^T + BD^T, -B^T (W_c^G)^{-1}, D^T)$
2- Apply inner-outer factorization and find the left spectral factor $\psi(s)$
3- Compute the frequency domain controllability Gramian of the (A,B,C,D) system within a frequency bound $[w_1, w_2]$
4- Compute the frequency domain observability Gramian of the left spectral factor $\psi(s)$ within a frequency bound $[w_1, w_2]$
5- Find the similarity transformation T for stochastically balancing and balance the system stochastically.
6- Eliminate the states related to the lowest set of the singular values and find (A_r, B_r, C_r, D_r)

Figure 1. FBST model reduction algorithm

Given a state-space realization, a singular perturbation approximation is obtained by approximating some subset of the states by constants.
That is, if x denotes the state vector, we partition x as the slow and the fast modes:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and set } \dot{x}_2 = 0$$

To obtain a generalized singular perturbation approximation, we instead approximate x_2 by a pure exponential [22]:

$$\dot{x}_2 = \sigma x_2 \qquad (12)$$

In the frequency-domain mixed method for relative error model reduction we apply the generalized singular perturbation method to the frequency-domain stochastic balanced system which is obtained from step 5 in FBST algorithm of Figure (1). By partitioning the state vector in the frequency-domain stochastic balanced system into fast and slow modes we have:

$$\begin{pmatrix} \dot{\bar{x}}_1(t) \\ \dot{\bar{x}}_2(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{B}_1 \\ \bar{A}_{21} & \bar{A}_{22} & \bar{B}_2 \\ \bar{C}_1 & \bar{C}_2 & D \end{pmatrix}_f \begin{pmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \\ u(t) \end{pmatrix} \qquad (13)$$

and by using (12), the reduced system is obtained as:

$$\begin{pmatrix} \hat{A}_r \\ \hat{B}_r \\ \hat{C}_r \\ \hat{D}_r \end{pmatrix}_{mixed} = \begin{pmatrix} \bar{A}_{11} + \bar{A}_{12}(\sigma I_{n-r} - \bar{A}_{22})^{-1}\bar{A}_{21} \\ \bar{B}_1 + \bar{A}_{12}(\sigma I_{n-r} - \bar{A}_{22})^{-1}\bar{B}_2 \\ \bar{C}_1 + \bar{C}_2(\sigma I_{n-r} - \bar{A}_{22})^{-1}\bar{A}_{21} \\ D + \bar{C}_2(\sigma I_{n-r} - \bar{A}_{22})^{-1}\bar{B}_2 \end{pmatrix} \qquad (14)$$

where σ is a desired frequency in $[w_1, w_2]$.
In the frequency-domain mixed method for relative error model reduction like the method which is proposed in [19], we obtained:

$$G(\sigma) = G_r(\sigma) \qquad (15)$$

where $G(s)$ is the original and $G_r(s)$ is the reduced system by the frequency-domain stochastic balanced method within $[w_1, w_2]$ and for stable systems the reduced order system is also stable if we do not partition the system from equal Hankel singular values. These properties are shown similar to the proofs in [22] easily.

VI. ERROR BOUND AND SOME OTHER PROPERTIES

In this section, some properties of FBST reduction method are developed and related to the proposed mixed method by using the concept and properties of the reciprocal systems.

Definition 1: The reciprocal system $S(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ of an asymptotically stable system $S(A,B,C,D)$ is defined by[23]:

$$\begin{aligned} \tilde{A} &= A^{-1} \\ \tilde{B} &= -A^{-1}B \\ \tilde{C} &= CA^{-1} \\ \tilde{D} &= D - CA^{-1}B \end{aligned} \qquad (16)$$

The following propositions summarize some properties of a reciprocal system[23].

Proposition 1: Reciprocal mapping is a bijective mapping, i. e. the reciprocal of $S(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is $S(A,B,C,D)$.

Proposition 2: Let $G(s) = D + C(sI - A)^{-1}B$ and $\tilde{G}(s) = \tilde{D} + \tilde{C}(sI - \tilde{A})^{-1}\tilde{B}$. Then $G(s) = \tilde{G}(s^{-1})$.

Proposition 3: The controllability and observability Gramians of a system and its reciprocal are the same.

Proposition 4: If $S(A,B,C,D)$ is a minimal realization then $S(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is also minimal.

Proposition 5: The H_∞-norm is invariant under the reciprocal transformation.

$$\|G(s)\|_\infty = \|\tilde{G}(s)\|_\infty \quad (17)$$

Proposition 6: If $S(A,B,C,D)$ is a minimal realization of an asymptotically stable minimum phase system then $S(\tilde{A},\tilde{B},\tilde{C},\tilde{D})$ is also minimal, asymptotically stable and minimum phase.

Theorem 3: Consider a minimal realization of an n^{th} order asymptotically stable system $S(A,B,C,D)$ and let $S(\tilde{A},\tilde{B},\tilde{C},\tilde{D})$ be its corresponding reciprocal system. Let $S(A_r,B_r,C_r,D_r)$ be the r^{th} order reduced model of $S(A,B,C,D)$ by applying the proposed mixed method with $\sigma = 0$ and $S(\tilde{A}_r,\tilde{B}_r,\tilde{C}_r,\tilde{D}_r)$ be the r^{th} order reduced model of $S(\tilde{A},\tilde{B},\tilde{C},\tilde{D})$ by applying FBST. Then $S(A_r,B_r,C_r,D_r)$ is the reciprocal of $S(\tilde{A}_r,\tilde{B}_r,\tilde{C}_r,\tilde{D}_r)$.

The proof for Theorem 3 is similar to the same theorem in [23] which is proposed for BST. The only difference is we apply the frequency domain balancing instead of the ordinary balancing in our reduction procedure.
In view of this important theorem and the above propositions many results concerning the properties of reduced models obtained by using FBST can be extended to the mixed method reduced model for $\sigma = 0$.
In addition to having the best low frequency behavior $G(0) = G_r(0)$, the new mixed model reduction method $\sigma = 0$ is stability and passivity preserving and has the same error bound as what we have for FBST.
The following is a definition which enables us to study the properties of the proposed method for all σ.

Definition 2: The σ-reciprocal system $S(\tilde{A}_\sigma,\tilde{B}_\sigma,\tilde{C}_\sigma,\tilde{D}_\sigma)$ of an asymptotically stable system $S(A,B,C,D)$ is defined by [24]-[26]:

$$\begin{aligned}\tilde{A}_\sigma &= \sigma I_n + (A-\sigma I_n)^{-1} \\ \tilde{B} &= -(A-\sigma I_n)^{-1}B \\ \tilde{C} &= C(A-\sigma I_n)^{-1} \\ \tilde{D} &= D - C(A-\sigma I_n)^{-1}B\end{aligned} \quad (18)$$

Similar to Theorem 3, the following is a theorem which shows the connection between general proposed mixed method.
and FBST and it is obtained easily from the above definition and the σ-reciprocal properties .

Theorem 4: The reduced plant which is obtained by applying the general mixed model reduction method is the σ-reciprocal of the reduced plant which is obtained by applying FBST to σ-reciprocal of the full order system.

The proof of the above theorem is similar to the proof of the theorem in [23] which is proposed for the BST.
Based on Theorem 4 and some other properties of σ-reciprocal, many results concerning the properties of reduced models obtained by using FBST can be extended to the general mixed method reduced model.
For example based on Theorem 4 and the following proposition it can be easily shown that the proposed general mixed reduction method has the same error bound as the FBST[26].

Proposition 7: consider a realization of a system $S(A,B,C,D)$ with transfer matrix $G(s)$ and let $S(\tilde{A}_\sigma,\tilde{B}_\sigma,\tilde{C}_\sigma,\tilde{D}_\sigma)$ be its σ-reciprocal system with transfer matrix $\tilde{G}_\sigma(s)$. Then

$$G(s) = \tilde{G}_\sigma(\sigma + \frac{1}{s-\sigma}) \rightleftharpoons \tilde{G}_\sigma(\tilde{s}) = G(\sigma + \frac{1}{\tilde{s}-\sigma}) \quad (19)$$

VII. PRACTICAL CD PLAYER BENCHMARK EXAMPLE

In this section we applied the proposed method to a strictly proper SISO practical CD player model of order 120 and compare it with BST and FBST methods. This CD player model is a finite element model of the dynamics between the lenz actuator and radial arm position of a portable compact disc [4].The CD player model is reduced to 4^{th} order model by the frequency-domain mixed method which is more accurate than 4^{th} order reduced models by BST and FBST. Figure (2) shows the Hankel singular values for FBST method related to the reduction frequency bound in Figure (3).

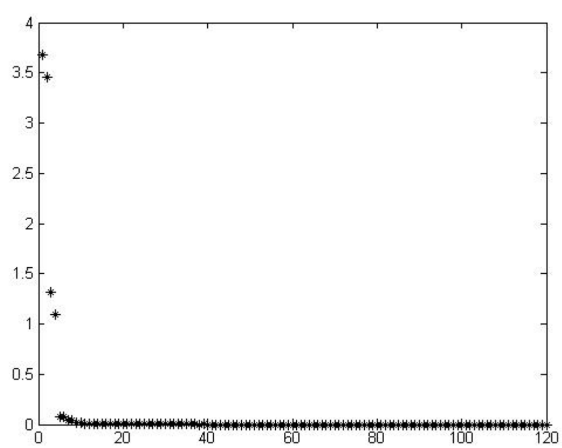
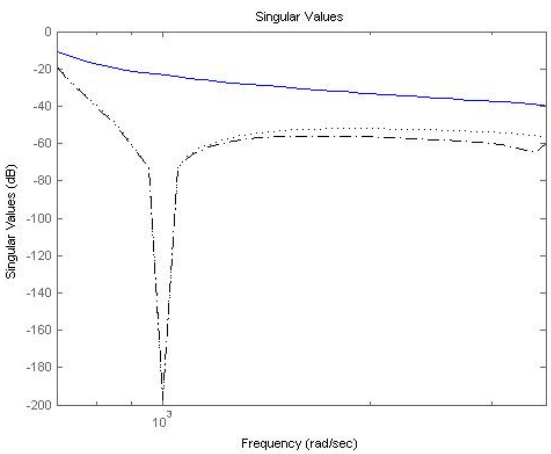


Figure. 2 .FBST Hankel singular values.

The infinity norm errors of BST, FBST and the proposed mixed method are shown in Figure (3) and (4) in different frequency bounds. As expected, the FBST method is more accurate than BST technique and because of the Gramian approximations it is more efficient. The mixed method is more accurate than BST and FBST. The selection of σ value within the frequency bandwidth enables us to have the best approximation at a desired frequency σ within our desired frequency domain and it causes more flexibility. In Figure (3) we selected $\sigma = w_2 = 250$ and in Figure (4) we selected $\sigma = 1000$.

In Figure (5) the step responses of original system and the reduced model by BST and FBST are shown.



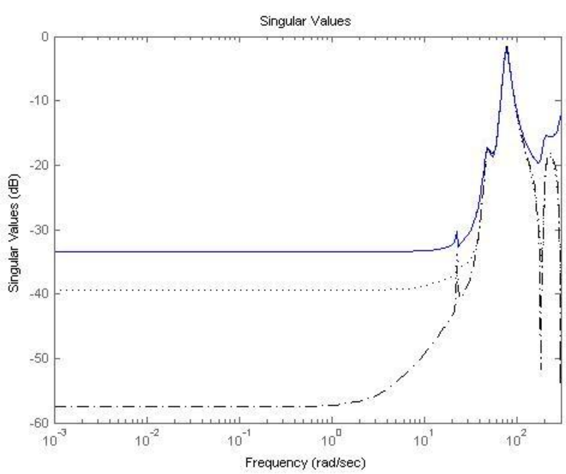


Figure. 3 . The proposed mixed method error (dash dotted), FBST model reduction error (dotted) and BST model order reduction error (solid lines).

Figure. 4 . The proposed mixed method error (dash dotted), FBST model reduction error (dotted) and BST model order reduction error (solid lines).

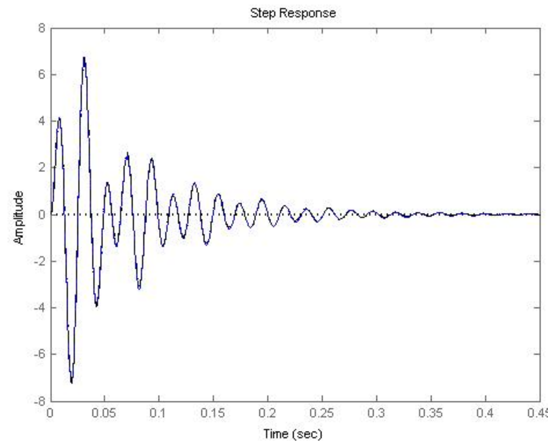


Figure. 5 . The proposed mixed method step response (dash dotted), FBST model reduction step response (dotted) and BST model order reduction error (solid lines).

VIII. CONCLUSION

In this paper, we have proposed a new relative error model reduction method. The reduction method is based on FBST method and the generalized singular perturbation method. FBST is a recently developed method which is based on stochastic balancing of a system within a frequency bound. Inner-Outer factorization is used in the numerical algorithm of the method as an accurate and an efficient numerical approach.

High accuracy, more flexibility and some important properties preservation make the proposed method suitable for the practical relative error model reduction. Furthermore computation of the Grammians in this method can be easily done without involving the problem of solving Lyapunov equation.

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