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Published in:
Proceedings of the 9th International Conference on Experimental Mechanics

Publication date:
1990

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Brincker, R., Jensen, J. L., & Krenk, S. (1990). Spectral Estimation by the Random Dec Technique. In Proceedings of the 9th International Conference on Experimental Mechanics: 20-24 August 1990, Copenhagen Denmark (pp. 2049-2058). Technical University of Denmark (DTU).

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SPECTRAL ESTIMATION BY THE RANDOM DEC TECHNIQE

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SUMMARY

This paper contains an empirical study of the accuracy of the Random Dec (RDD) technique. Realizations of the response from a single-degree-of-freedom system loaded by white noise are simulated using an ARMA model. The Autocorrelation function is estimated using the RDD technique and the estimated functions are compared to an analytical solution. The RDD and FFT techniques are compared, and the influence of trig level, damping ratio, type of trig window, and selection of trig points is investigated. The RRD technique appears to be accurate and fast compared to the FFT technique, and the technique is accurate also at small damping ratios and when the estimated functions contain only few points. It is illustrated how bias might be introduced or removed by the choice of trig window and the selection of trig points.

1. INTRODUCTION

The Random Dec Technique was developed at NASA in the late sixties and early seventies by Henry Cole and co-workers [1-4]. The purpose was to develop a simple data analysis algorithm for the characterization of stochasic response of space stuctures and aeroelastic systems and to identify damage in such systems by identifying system changes.

The technique has been used for system identification of large structures, Ibrahim [5], for structural damage detection and determination of fluid damping, Yang, [6-8], and for vehicle system identification and damping measurements of soil [9-10].

The basic idea of the technique is to estimate a so-called Random Dec (RDD) signature which can be used to characterize stochastic time series. If the time series $x(t)$ is given, then the Random Dec signature $\hat{D}(\tau)$ is formed by averaging N segments of the time series

$$\hat{D}(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(\tau - t_i) \qquad (1)$$

where the time series at the times t_i satisfies a certain condition, the so-called trig condition. The condition might be that $x(t_i) = a$, or that $x(t_i) = 0 \wedge \dot{x}(t_i) > 0$. One of the advantages of the RDD technique is the simplicity of the algorithm given by eq. (1) which means that the technique establishes a basis for simple and fast on-line system identification.

However, it is a problem that only little experience with the technique has been gained so far, and therefore there is a need for systematic investigations of factors that might be important in practical applications. Only few investigations have been caried out. Cole [3] and Chang [4] performed simulation studies to investigate the technique, and later, Nasir and Sunder [12] performed a simulation study of the application of the RDD technique for determination of damping ratios.

This paper describes an impirical investigation of the sources of error when using the RDD technique for estimation of autocorrelation functions for systems with one degree of freedom. Three damping ratios are considered: 1%, 5% and 25%. In this investigation only the level crossing trig condition and horizontal trig windows (defined in next section) are used. The accuracy and the speed of the RDD technique are compared to the FFT technique, and the influence of the trig level is investigated. Finally, it is shown how the estimates might be biased by improper choice of trig windows or by improper selection of trig points.

2. THE RANDOM DEC (RDD) TECHNIQUE

One of the advantages of the RDD technique is that the algorithm is extremely simple as it appears from eq. (1). However, before the technique can be used succesfully in practical applications the meaning of the estimated function $\hat{D}(\tau)$ must be known, and the problems of implementing the algorithm must be solved.

The investigations in this paper will be restricted to a specific trig condition, namely the so-called level trig condition. For this condition the exact mathematical definition of the RDD signature reads

$$D_{XX}(\tau) \;=\; E[X(t+\tau)|\; X(t)=a] \qquad (2)$$

where $X(t)$ is a stationary stochastic process and a is the trig level. Vandiver et al. [11] have shoved that if $X(t)$ is a stationary Gaussian process, then the RDD signature $D_{XX}(\tau)$ and the autocorrelation function $R_{XX}(\tau)$ are related by

$$D_{XX}(\tau) \;=\; \frac{R_{XX}(\tau)}{\sigma_X^2}a \qquad (3)$$

where σ_X^2 is the variance of the process $X(t)$. In this case, therefore, the function given by eq. (1) is simply an estimate of the autocorrelation function $R_{XX}(\tau)$. This means that accurate estimates of spectral densities might be determined by forming an RDD signature by averaging segments of the time series according to eq. (1) and then tranforming to the frequency domain by applying the FFT only once. However, in practical applications where digital equipment is used, eq. (1) has to be modified, since one has to sample the realizations of the stochastic proccesses, i.e. to work in a discrete time space. The signature will therefore only be known in a finite number of equally spaced time points

$$\hat{D}(m\Delta t) \;=\; \frac{1}{N}\sum_{i=1}^{N} x(m\Delta t - t_i)\;; \;\; -M \le m \le M \qquad (4)$$

where Δt is the time between the sample points. It is not usual to store the signature for both negative and positive time increments. However, in order to extract maximum statistical information from the basic data, both negative and positive time increments are used in the following investigations. Furthermore, to remove and illustrate the bias error on the RDD signature it proves convenient to introduce the even and the odd part of the signature

$$\begin{aligned}\hat{D}_{even}(m\Delta t) &= \frac{1}{2}(\hat{D}(m\Delta t)+\hat{D}(-m\Delta t))\\ \hat{D}_{odd}(m\Delta t) &= \frac{1}{2}(\hat{D}(m\Delta t)-\hat{D}(-m\Delta t))\;;\;\; 0\le m\le M\end{aligned} \qquad (5)$$

However, one major problem arises because of the discrete time space. For a sampled finite length time series the event $x(t_i) = a$ has the probability zero, which means that the level crossing condition cannot be used without modification. One possibility is to replace a with an interval which will introduce a finite height vertical window. However, in this investigation only horizontal windows have been used. A horizontal window with trig point t_i is introduced by the condition

$$(x(t_i) < a \wedge x(t_i+\Delta t) > a) \;\vee\; (x(t_i) > a \wedge x(t_i+\Delta t) < a) \qquad (6)$$

This is a horizontal trig window with the length Δt. It is seen that the trig point is placed at the left edge of the window, and that the first part of the condition detects the upcrossings and the last part detects the downcrossings. In the RDD technique the trig windows are very important. A proper choice of trig window is absolutely essential for the accuracy of the correlation function estimates. These problems are illustrated in section 6.

3. SIMULATION BY AN (2,1) ARMA MODEL

In this investigation autocorrelation functions are estimated for the response of a system with one degree of freedom loaded by stationary Gaussian white noise. The response $X(t)$ is the solution to the second order differential equation

$$\ddot{X} + 2\zeta\omega_0\dot{X} + \omega_0^2 X \;=\; Q(t) \qquad (7)$$

where ω_0 is the undamped natural angular frequency, ζ is the damping ratio and $Q(t)$ is stationary zero mean Gaussian white noise. For this case the normalized (corresponding to variance one) autocorrelation function is given by, Crandall [13]

$$R_{XX}(\tau) \;=\; \exp(-\zeta\omega_0\tau)\,(\cos(\omega_d\tau) + \frac{\zeta\omega_0}{\omega_d}\sin(\omega_d\tau))\;;\;\; \tau\ge 0 \qquad (8)$$

where ω_d is the damped natural frequency $\omega_d = \omega_0\sqrt{1-\zeta^2}$. The most accurate way to perform simulations of a system formulated in continuous time, is to transform the system model to the discrete time space. This can be done by using an ARMA model. It can be shown, Pandit [14], that a second order system formulated in continuous time may be represented in the discrete time space by a (2,1) ARMA model given by

$$x_m \;=\; \Phi_1 x_{m-1} + \Phi_2 x_{m-2} + a_m - \Theta a_{m-1} \qquad (9)$$

where m is the discrete time ($t_m = m\Delta t$), Φ_1, Φ_2 is the Auto Regressive (AR) parameters, Θ is the Moving Average (MA) parameter and a_m is a time series of independent Gaussian distributed numbers. The model is denoted (2, 1) since it has 2 AR parameters and 1 MA parameter. If the ARMA parameters are chosen as

$$\Phi_1 = 2\exp(-\zeta\omega_0\Delta t)\ \cos(\omega_d\Delta t) \qquad (10.a)$$

$$\Phi_2 = \exp(-2\zeta\omega_0\Delta t) \qquad (10.b)$$

$$\Theta = -P \pm \sqrt{P^2-1}\ ;\ \ |\Theta| < 1 \qquad (10.c)$$

where

$$P = \frac{\omega_d \sinh(2\zeta\omega_0\Delta t) - \zeta\omega_0 \sin(2\omega_d\Delta t)}{2\zeta\omega_0 \sin(\omega_d\Delta t)\cosh(\zeta\omega_0\Delta t) - 2\omega_d \sinh(\zeta\omega_0\Delta t)\cos(\omega_d\Delta t)}$$

then the ARMA model given by eq. (9) is the representation of the continuous system given in eq. (7) in the discrete time space. It can be shown, Pandit [14], that the discrete autocorrelation function of the time series x_m given by eq. (9) is equal to the sampled autocorrelation function of the continuous process $X(t)$.

All the simulations were performed modelling a system with a period of 0.5 seconds corresponding to $\omega_0 = 12.57$ rad/s, the time spacing between sample points was 0.051 seconds and the length of all the time series was 4000 points. The simulations were performed using the PC version of the MATLAB software package, [15], except the algorithm for estimation of the RDD signature which was programmed in the C programming language and linked to the MATLAB software by the MATLAB user function interface.

4. THE RDD TECHNIQUE COMPARED TO FFT

In this section the accuracy and speed of the RDD technique are compared to ordinary Fast Fourier Transformation (FFT), Brigham [16]. An FFT based estimate of the autocorrelation function is obtained in the following way. First, the 4000 points are divided into segments of $2M$ points each. Then the segments are FFT'ed, the results are averaged, multiplied by its complex conjugate and the resulting power spectrum is then tranformed to the time domain by inverse FFT. A radix-2 FFT algorithm is used in all cases. No spectral windows have been used.

The RDD technique was used with a trig level $a = 1.5\sigma_X$ and using the symmetrical trig window recommended in section 6. A few examples of estimates using RDD and FFT technique are shown in figure 1.

To investigate the influence of number of points and the damping on the accuracy and the speed, the number of points were varied from $M = 16$ to $M = 512$ points, and the damping ratio was varied using the values $\zeta = 0.01$, $\zeta = 0.05$ and $\zeta = 0.25$. The error ϵ is defined as

$$\epsilon = \frac{1}{M\sigma_X^2}\sum_{m=0}^{M}(R_{XX}(m\Delta t) - \hat{R}_{XX}(m\Delta t))^2 \qquad (11)$$

where $\hat{R}_{XX}(m\Delta t)$ is the autocorrelation function estimated by FFT or RDD. The results are shown in figure 2. Each point in the figure is the average of the result from 4 time series (realizations). From these results it is seen that the errors on the RDD estimates are generally

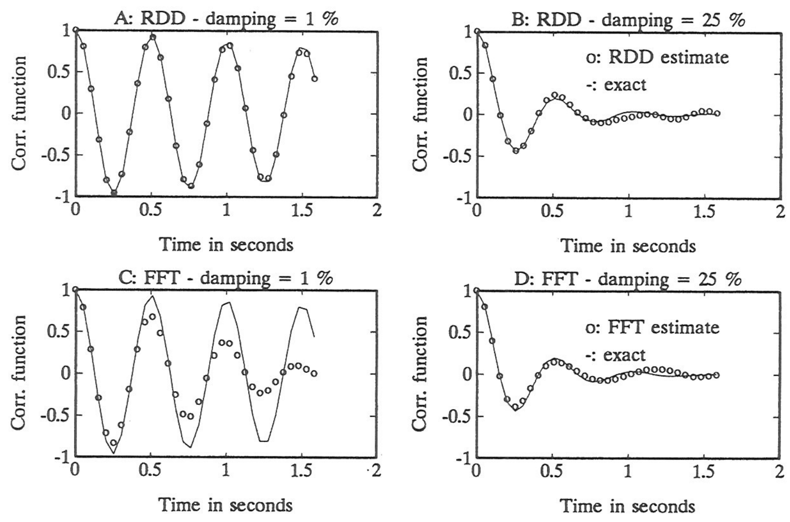

Figure 1. Autocorelation functions estimated by FFT and RDD technique for $M = 32$.

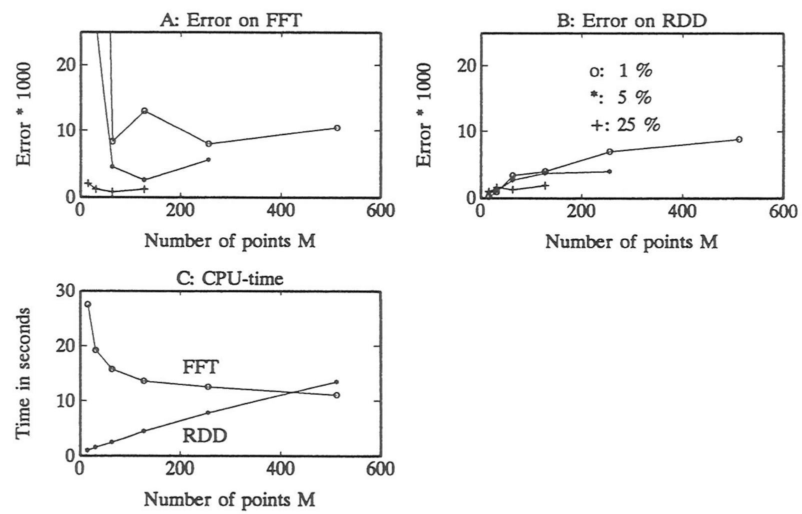

Figure 2. Errors and CPU-times when estimating autocorrelation functions by FFT and RDD.

smaller than the errors on the FFT estimates. Especially in the case were M is small the RDD estimates are substantially better than the FFT estimates. The accuracy of the FFT might be improved by introduction of a spectral window. However, that will be at the expense of the speed.

The speed of the RDD technique is also high compared to the FFT technique; again the great advantage of the RDD technique is when M is small - for the FFT technique the CPU-time increases as M approches zero, whereas for the RDD technique the CPU-time is approximately proportional to M. The RDD algotithm was implemented as a floating point implementation. However, because of the simplicity of the RDD algorithm it is possible to implement the RDD algorithm as an integer implementation making the algorithm much faster. Therefore, the absolute speed results only show that it is not difficult to make the RDD algorithm faster than an FFT algorithm.

5. INFLUENCE OF THE TRIG LEVEL

One of the important differences between RDD and FFT estimation is, that the user has to choose a trig level when using the RDD technique. From a user point of wiev it is important to know how to choose a proper trig level and to know whether or not the method is sensitive to the choice of trig level. This issue is investigated in this section.

For each of the damping ratios $\zeta = 0.01$, $\zeta = 0.05$ and $\zeta = 0.25$ the trig level has been varied from $a = 0.25\sigma_X$ to $a = 2\sigma_X$ and the error ϵ has been calculated according to eq. (11). In all cases the symmetrical window recommended in section 6 was used. The results are shown in figure 3. Each point in the figures corresponds to the average of the results from 4 realizations. As it appears from the results, the method is not very sensitive to the choice of trig level - if the estimate consists of only a few cycles ($M = 32$) then the error is not changing significantly in the range from $a = 0.5\sigma_X$ to $a = \sigma_X$.

When the trig level a is large, only few trig points are detected, and therefore the variance on the RDD estimate is increased because only a small number of segments are averaged. On the other hand, if the trig level a is chosen too small, the number of trig points approaches an asymptotic value equal to the number of zero crossings in the time series, the information is not increased much by the increased number of averaged segments since they becomes highly

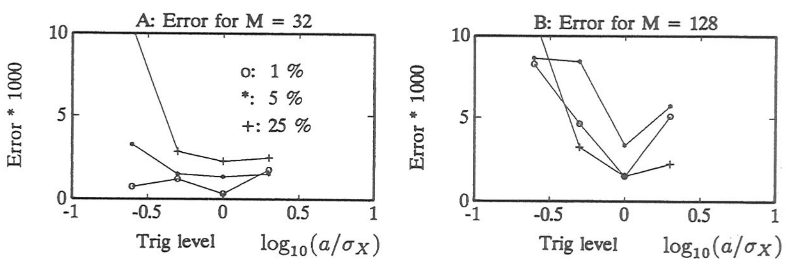

Figure 3. Influence of trig level on the error of the RDD estimate.

correlated as the number of trig points increases, but accuracy is lost due to averaging segment with a large quotient between variance and trig level. An optimal choice therefore exists.

The optimal choice of trig level is a trade off between accuracy and speed. If the speed is not important the optimal trig level seems to be about $a = \sigma_X$. However, if the speed is important, the trig level might be chosen in the interval from $a = \sigma_X$ to $a = 2\sigma_X$ without any significant loss of accuracy.

6. INTRODUCTION OF BIAS ON THE RDD ESTIMATE

It is well known that FFT estimates suffer from strong bias errors if the effective bandwidth is too large compared to the width of the spectral peaks, i.e. bias is introduced if the damping is small and the length of the time series is not properly adjusted. Therefore, in practice it might be difficult to establish a reliable identification of small system damping on the basis of FFT estimates, since very long time series have to be used in order to reduce the bias error as well as the random error.

Using the RDD technique a similar problem does not seem to be present. The estimates seem to be unbiased - or the bias is very small - independently of the damping ratio as well as the number of points in the autocorrelation function estimate. The average error increases for increasing number of points in the estimate as it appears from figure 2, but this is due to random errors - the random error increases with the distance from the origin. This is only natural since the correlation between the trig value and neighbour values decrease with the distance from the trig point - furthermore, this effect has been predicted by Vandiver et al [11].

However, the quality of the preceeding results has been obtained by a careful choice of trig window and a proper selection of trig points. It is absolutely important how the algorithm is implemented - a small change that might seem of no importance might turn out to be essential for the accuracy of the technique. This is believed to be a new observation.

In the following the problems of choice of trig windows and selection of trig points will be illustrated.

As mentioned in section 2, when signals are sampled, a finite length window must be applied. It turns out that if a window like the one proposed in eq. (6) is applied, the RDD estimate of the auto correlation function will be biased. The trig window given in eq. (6) has the trig point at the left edge (point no. i is taken as the origin of the data segment), however, if point no. $i+1$ is chosen as trig point then the trig point is moved to the right edge, and if both points are used as trig point, the resulting trig point will be in the midle of the window, and a symmetrical window is defined. RDD estimates of the autocorrelation function using these three windows are shown in figure 4. The figure shows the even and the odd part of $\hat{R}_{XX}(\tau)$ given by eq. (5). As it appears from the figure the unsymmetrical windows introduce a biased estimate (the odd part of $\hat{R}_{XX}(\tau)$ deviates systematically from zero). This is only natural, since the unsymmetrical windows will shift the unbiased estimate of the autocorrelation function the time $\Delta t/2$ along the time axis. As it appears from the results, when a symmetrical window is used, the bias disappers. Therefore, when simultaneous sampled time series are used, a symmetrical window should be applied. The symmetrical window has been used in all the preceeding cases. Furthermore, it is interesting to note that time shift bias can be removed by averaging the positive and the negative part of the RDD estimate, i.e. the even part of the estimate will be unbiased as indicated by the results given in figure 4.

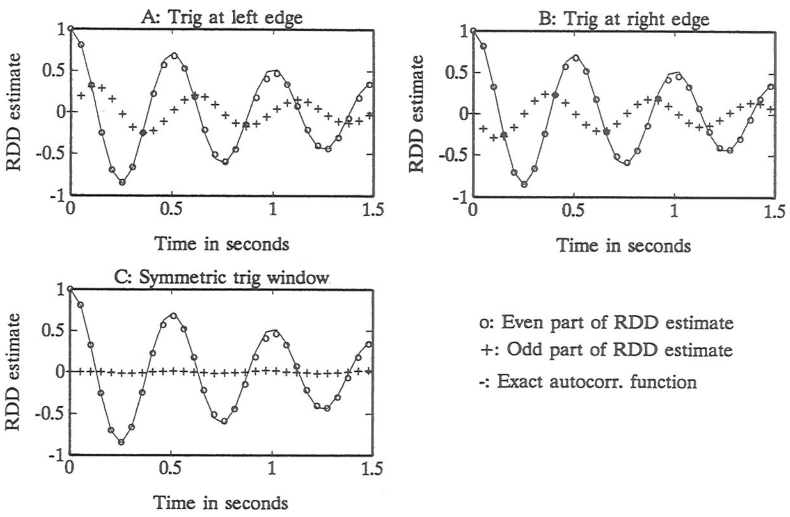

Figure 4. Bias introduced by the trig window.

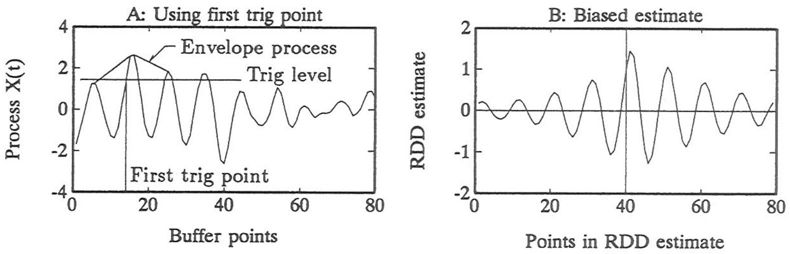

Figure 5. Bias introduced by selection of trig points.

The selection of trig points might also introduce bias. In all the preceeding investigations a time series large enough to contain many trig points has been applied, and all detected trig points has been used, i.e. the corresponding data segments have been used in the averaging. In this case, the selection of trig points does not introduce bias, because the information is representative for the process and all the information is used. However, if the number of trig points becomes too small or if not all trig points are used, the estimate might be biased. In

fact it is very difficult to exclude some of the trig points without introducing bias. This is illustrated in the following example. A time series of the usual length of 4000 points was divided into smaller series of 40 points each. Then the RDD estimate was found in the usual way (the symmetric trig window was used) except that trig points were searched in the small time series, and only the first trig point in each of the small time series was used. As it appears from the result shown in figure 5 the estimate is heavily biased.

In this case there are two resons for the bias. First, if the trig level is large (here equal to $1.5\sigma_X$) the first trig point will typically be an upcrossing (since the probability that $X > a$ at the first point in the time series is small). This will shift the estimate to the right along the time axis. Second, the first crossing of the envelope process will allso typically be an upcrossing (for the same reason), and therefore the system energy will be inreasing, causing the right part of the RDD estimate to get too large amplitudes (damping is underestimated) and the left part of the estimate to get too small amplitudes (the damping is overestimated). Both effects are significant in the example shown in figure 5.

In order avoid this problem time series should be large enough to hold a number of trig points that is large compared to the typical number of trig points between an upcrossing and a downcrossing of the envelope process, and all the detected trig points should be used.

7. CONCLUSIONS

At the present theoretical basis the RDD technique is a technique for estimation of autocorrelation functions. When a reliable estimate has been obtained by averaging, the estimate might be Fast Fourier Transformed to obtain the corresponding spectral density. The advantages of the RDD technique might be summarized as :

- the algorithm is very simple, so simple that the algorithm in every programming language can be programmed in only a few lines.

- the algorithm is fast compared to FFT, especially if the estimate contains only few points.

- the RDD technique is more accurate than the FFT, especially if the estimate contains only a few cycles and if the damping is small.

- the quality of the RDD estimates is not very sensitive to the choice of trig level. A trig level in the range from $a = \sigma_X$ to $a = 2\sigma_X$ is recommended.

- the choice of trig window is very important for the quality of the RDD estimates. If an improper window is used, the RDD estimate becomes biased. A so-called symmetrical trig window is recommended.

- the selection of trig points might bias the RDD estimate. If the trig points are not representative or if some of the trig points are not used, bias is introduced. It is recommended that time series used are large enough to contain many trig points, and that all trig points are used.

ACKKNOWLEDGEMENTS

Financial support from the Danish Technical Research Council is gratefully acknowledged.

REFERENCES

[1] Cole, A. Henry: "On-The-Line Analysis of Random vibrations". AIAA/ASME 9th Structures, Structural and Materials Conference, Palm Springs, California, April 1-3, 1968.

[2] Cole, A. Henry: "Failure Detection of a Space Shuttle Wing Flutter by Random Decrement". NASA, TMX-62,041, May 1971.

[3] Cole, A. Henry: "On-Line Failure Detection and Damping Measurement of Space Structures by Random Decrement Signatures". NASA, CR-2205, March 1973.

[4] Chang, C.S.: "Study of Dynamic Characteristics of Aerodynamic Systems Utilizing Randomdec Signatures". NASA, CR-132563, Febr. 1975.

[5] Ibrahim, S.R.: "Random Decrement Technique for Modal Identification of Structures". J. Spacecraft, Vol. 14, No. 11, Nov. 1977.

[6] Yang, J.C.S., J. Chen and N.G. Dagalakis: "Damage Detection in Offshore Structures by the Random Decrement Technique". ASME, Vol. 106, March 1984.

[7] Yang, J.C.S., N. Dagalakis and M. Hirt: "Application of the Random Decrement Technique in the Detection of a Induced Crack on an Offshore Platform Model". Comput. Methods for Offshore Struct. Winter Annu. Meet. of ASME, Nov. 16-21, 1980.

[8] Yang, J.C.S., C.H. Marks, J. Jiang, D. Chen, A. Elahi and W-H Tsai: "Determination of Fluid Damping Using Random Excitation". ASME, Vol. 107, June 1985.

[9] Siviter, R. and M.G. Pollard: "Measurement of Railway Vehicle Kinetic Behaviour Using the Random Decrement Technique". Vehicle System Dynamics, Vol. 14, No 1-3, June 1985.

[10] Al-Sanad, A.A., M.S. Aggour and M.I. Amer: "Use of Random Loading in Soil Testing. Indian Geotechnical Journal, Vol. 16, No.2, 1986.

[11] Vandiver, J.K., A.B. Dunwoody, R.B. Campbell and M.F. Cook: "A Mathematical Basis for the Random Decrement Vibration Signature Analysis Technique". J. Mechanical Design, Vol. 104, April 1982.

[12] Nasir, J and S.S. Sunder: "An Evaluation of the Random Decrement Technique of Vibration Signature Analysis for Monitoring Offshore Platforms", Research Report, R82-52, Sept. 1982, Dept. of Civil Eng., Massachusetts Inst. of Technology, 1982.

[13] Crandall, S and W.D. Mark: "Random Vibration in Mechanical Systems". Academic Press, 1973.

[14] Pandit, S.M. and S. Wu: "Time Series and System Analysis with Applications", John Wiley and Sons, 1983.

[15] PC-MATLAB for MS-DOS Personal Computers, The Math Works, Inc., 1989.

[16] Brigham, E.O.: "The Fast Fourier Transform". Prentice-Hall, Inc., 1974.