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Published in:
Nordic Concrete Research

Publication date:
1996

Document Version
Publisher's PDF, also known as Version of record

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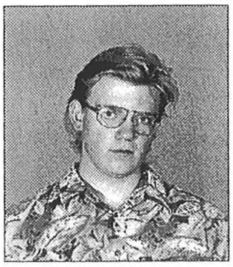

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ABSTRACT

An analytical model for load-displacement curves of lightly reinforced concrete beams is presented. The fracture is modelled by a fictitious crack in an elastic layer around the mid-section of the beam. The state of stress in the elastic layer is assumed to depend tri-linearly on the local elongation corresponding to a bi-linear softening relation for the fictitious crack.

Key Words: Concrete, reinforcement, fracture mechanics, fictitious crack.

The failure of a simply supported beam loaded in three-point bending is modelled by assuming development of a single fictitious crack of the mid-section of the beam. In the model material points on the crack extension path are assumed to be in one of four possible states: A) Linear elastic state, B^{I}) Softening behaviour of the concrete corresponding to the first part of the bilinear softening relation, B^{II}) Softening behaviour of the concrete corresponding to the second part of the bilinear softening relation and C) Development of real crack in the concrete.

The idea of modelling the bending failure of concrete beams by development of a fictitious crack in an elastic layer with a thickness proportional to the beam depth was introduced by Ulfkjær, Brincker and Krenk (1990) /1/. They have shown that results predicted by an analytical model for plain concrete beams with a linear softening relation are in good agreement with finite element analysis by assuming the thickness of the elastic layer to be equal to half the beam depth.

When the constitutive relations for the concrete and the steel are established, and assuming perfect bond between the concrete and steel, all the stresses can be determined by the strain field. By setting up equilibrium equations for each state, a relation between the beam deflection and the applied load can be established (see figure 1).

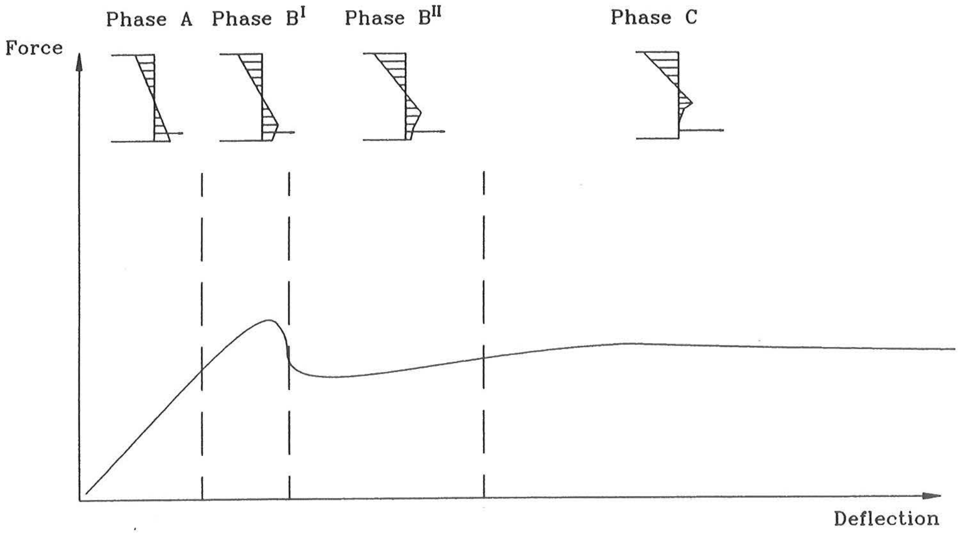

Figure 1. The result from the model is a relation between the beam deflection and the applied load.

The load displacement curves obtained from the analytical model have been compared with results obtained from laboratory tests. These tests were performed with 100x200x2400 mm and 200x400x4800 mm beams. Both with a reinforcement ratio equal to 0.14 %. Furthermore, the analytical model has been formulated with a power function proposed by Reinhardt (1990).

References

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/2/ Reinhardt, H. W. and Cornelis Van Der Veen.
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The article is part of the book "Applications of fracture mechanics to reinforced concrete" edited by Alberto Carpinteri, 1990.