Maximal Efficiencies in New Single $GaAs_{1-x}P_x$-Alloy Junction Solar Cells at 300 K

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Abstract:

In single $n^+(p^+) - p(n)$ $[X(x) \equiv GaAs_{1-x}P_x]$-alloy junction solar cells at 300 K, $0 \leq x \leq 1$, by basing on the same physical model and the same treatment method, as those used in our recent works (Van Cong, 2024), we will also investigate the highest (or maximal) efficiencies, $\eta_{Imax.(IImax.)}$ at the open circuit voltage $V_{oc}(= V_{ocI(ocII)})$, according to highest hot reservoir temperatures $T_H(K)$, obtained from the Carnot efficiency theorem, which was demonstrated by the use of the entropy law. Here, some concluding remarks are given in the following. (i)-First, with increasing x=(0, 0.5, 1), from Table 3, obtained for the single $n^+ - p$ $X(x)$-alloy junction solar cells, and for given $r_{Sn(Cd)}$-radius, for example, one obtains: $\eta_{Imax.}(\nearrow)=$ 31.18%, 33.495%, **35.99%**, according to $T_H(K) = 435.9, 451.1, 468.7$, at $V_{ocI}(V) = 1.07, 1.06, 1.05$, respectively. (ii)- Secondly, with increasing x=(0, 0.5, 1), from Table 5, obtained for the single $p^+ - n$ $X(x)$-alloy junction solar cells, and for given $r_{Cd(Sn)}$-radius, for example, one gets: $\eta_{IImax.}(\searrow)=$ **33.05%**, 31.95%, 31.37%, according to $T_H(K) = 448.0, 440.9, 437.1$, at $V_{ocII}(V)[> V_{ocI}(V)] = 1.20, 1.15, 1.12$, respectively, suggesting that such $\eta_{Imax.(IImax.)}$-and-T_H variations depend on $V_{ocII}(V)[> V_{ocI}(V)] - values$. Then, in particular, as given in Table 3, for $x = 0$ and $r_{d(a)} = r_{Te(Mg)}, one gets$: $\eta_I =$23.48 % and 29.76 % at $V_{oc} = 0.98\,V$ and 1.1272 V, respectively, which can be compared with the corresponding results obtained by Moon et al. (2016) and Green et al. (2022) for the single-junction GaAs thin-film solar cell, 22.08 % and 29.71 %, with relative deviations in absolute values, 6.34 % and 0.17 %. Finally, one notes that, in order to obtain the highest efficiencies, the single $GaAs_{1-x}P_x$-alloy junction solar cells could be chosen rather than the single crystalline GaAs-junction solar cell.

Keywords: *single $GaAs_{1-x}P_x$-alloy junction solar cell; single crystalline GaAs-junction solar cell; photovoltaic conversion factor; photovoltaic conversion efficiency.*

Introduction

In single $n^+(p^+) - p(n)$ $GaAs_{1-x}P_x$-alloy junction solar cells at 300 K, $0 \leq x \leq 1$, by basing on the same physical model and treatment method, as used in our two recent works (Van Cong, 2024), and also on other ones (Green et al., 2022; Kittel, 1976; Moon et al., 2016; Singh & Ravindra, 2012; Van Cong, 2024, 2016, 1999, 1995, 1991, 1975; Van Cong & Debiais, 1997, 1996, 1993; Van Cong & Doan Khanh, 1992; Van Cong et al., 1984), we will investigate the highest (or maximal) efficiencies, $\eta_{Imax.(IImax.)}$,

according to highest hot reservoir temperatures $T_H(K)$, obtained from the Carnot- efficiency theorem, being proved by the entropy law.

In the following, we will show that the energy-band-structure parameters, due to the effects of x-P concentration, size impurity, temperature T and heavy doping, affect strongly the dark (or total) minority-carrier saturation current density and the photovoltaic conversion effect.

Energy-Band-Structure Parameters and Dark Minority-Carrier Saturation Current Density, Due to the Effects of x- P Concentration, Impurity Size, and Heavy Doping

First of all, in the single $n^+(p^+) - p(n)$ $X(x)$- alloy junction solar cells, $X(\equiv GaAs_{1-x}P_x)$, we present the effects of x-concentration, donor (acceptor) [d(a)]-size, temperature T and heavy doping, affecting the energy-band-structure parameters, as those investigated in our recent works (Van Cong, 2024).

A. Effect of x-P Concentration

In the $n^+(p^+) - p(n)$ single $n^+(p^+) - p(n)$ $X(x)$- alloy junction at T=0 K, the energy-band-structure parameters (Kittel, 1976) are expressed as functions of x, are given in the following.

(i)-The unperturbed relative effective electron (hole) mass in conduction (valence) bands are given by:

$m_c(x)/m_o = 0.13 \times x + 0.066 \times (1 - x)$, and

$m_v(x)/m_o = 0.500 \times x + 0.291 \times (1 - x)$, (1)

so that one obtains: $m_c(x = 0)/m_o = m_{c(GaAs)}/m_o$=0.066, $m_v(x = 0)/m_o = m_{v(GaAs)}/m_o$=0.291, and $m_c(x = 1)/m_o = m_{c(GaP)}/m_o$=0.13, $m_v(x = 1)/m_o = m_{v(GaP)}/m_o$=0.5.

(ii)-The unperturbed relative static dielectric constant of the intrinsic of the single crystalline X- alloy is found to be defined by:

$\varepsilon_o(x) = 11.1 \times x + 13.13 \times (1 - x)$, (2)

which gives: $\varepsilon_o(x = 0) = \varepsilon_{GaAs}$=13.13 $\simeq$ 13.1, and $\varepsilon_o(x = 1) = \varepsilon_{GaP} = 11.1$.

(iii)-Finally, the unperturbed band gap at 0 K is found to be given by:

$E_{go}(x)$ in eV $= 1.796 \times x + 1.52 \times (1 - x)$, (3)

giving rise to: $E_{go}(x = 0) = E_{gGaAs} = 1.52$ eV, and $E_{go}(x = 1) = E_{gGaTe} = 1.796$ eV $\simeq 1.80$ eV.

Therefore, we can define the effective donor (acceptor)-ionization energy at $r_{d(a)} = r_{do(ao)}$ in absolute values as:

$$E_{do(ao)}(x) = \frac{13600 \times [m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2} \; meV, \qquad\qquad (4)$$

and then, the isothermal bulk modulus, by:

$$B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{(4\pi/3) \times (r_{do(ao)})^3}. \qquad\qquad (5)$$

B. Effects of Impurity-Size, with a Given x

Here, the effects of $r_{d(a)}$ and x-P concentration affect the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, in the following.

At $r_{d(a)} = r_{do(ao)} = r_{As(Ga)} = 0.118\,nm\,(0.126\,nm)$, respectively, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) σ, $\sigma_o = 0$. Further, the two important equations (Van Cong et al., 1984), used to determine the σ-variation, $\Delta\sigma \equiv \sigma - \sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}(\frac{d\sigma}{dV}) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\sigma(r_{d(a)}, x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \qquad \ln \qquad (\frac{V}{V_{do(ao)}}) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times$$

$$\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \qquad\qquad (6)$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and in the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained from the effective Bohr model, and then such the compression (dilatation) is represented respectively by: $\pm [\Delta\sigma(r_{d(a)}, x)]_{n(p)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] =$$

$$+ [\Delta\sigma(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] =$$

$$- [\Delta\sigma(r_{d(a)}, x)]_{n(p)}. \qquad\qquad (7)$$

Therefore, from Equations 6 and 7, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \dfrac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x),$

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times$$

$$ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \tag{8a}$$

according to the increase in both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \dfrac{\varepsilon_o(x)}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x),$ with a condition,

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1,$

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times$$

$$ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \tag{8.b}$$

corresponding to the decrease in both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, for a given x.

C. Effect of Temperature T, with Given x and $r_{d(a)}$

Here, the intrinsic band gap $E_{gin(gip)}(r_{d(a)}, x, T)$ at any T is given by (Van Cong, 2024):

$$E_{gin(gip)}(r_{d(a)}, x, T) \text{ in } eV = E_{gn(gp)}(r_{d(a)}, x) - 10^{-4} \times T^2 \times \left[\frac{3.065 \times x}{T + 94\,K} + \frac{5.405 \times (1-x)}{T + 204\,K}\right], \tag{9}$$

which gives: $E_{gin(gip)}(r_{do(ao)}, x = 0, T = 0K) = 1.52\ eV$ and $E_{gin(gip)}(r_{do(ao)}, x = 1, T = 0K) = 2.32\ eV$, and $E_{gin(gip)}(r_{do(ao)}, x = 0, T = 300\ K) \simeq 1.42\ eV$ and $E_{gin(gip)}(r_{do(ao)}, x = 1, T = 300\ K) \simeq 2.25\ eV$,

suggesting that, for given x and $r_{d(a)}$, $E_{gin(gip)}$ decreases with an increasing T.

Furthermore, in the n(p)-type $X(x)$-alloy, one can define the intrinsic carrier concentration $n_{in(ip)}$ by:

$$n_{i\,n(p)}^2(T, r_{d(a)}, x) \equiv N_c(T, x) \times N_v(T, x) \times exp\left(\frac{-E_{gin(p)}(T, r_{d(a)}, x)}{k_B T}\right), \tag{10}$$

where $N_{c(v)}(T, x)$ is the conduction (valence)-band density of states, being defined as:

$$N_{c(v)}(T, x) = 2 \times \left(\frac{m_{c(v)}(x) \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \ (cm^{-3}). \tag{11}$$

So, the numerical results of $E_{d(a)}(r_{d(a)}, x)$, $B_{do(ao)}(x)$, $\varepsilon(r_{d(a)}, x)$ and $E_{gin(gip)}(r_{d(a)}, x, T)$, calculated using Equations 4, 5, 8a (8b) and 9, are reported in Table 1.

Table 1 in Appendix 1.

D. Heavy Doping Effect, with given T, x and $r_{d(a)}$

Here, as given in our previous works (Van Cong, 2024; Van Cong & Debiais, 1993), the Fermi energy $E_{Fn}(-E_{Fp})$, band gap narrowing (BGN), and apparent band gap narrowing (ABGN), are reported in the following.

First, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and any d(a)-density, $N_{d(a)}$, being investigated in our previous paper (Van Cong & Debiais, 1993), with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\frac{E_{Fn}(u)}{k_B T}\left(\frac{-E_{Fp}(u)}{k_B T}\right) = \frac{G(u) + Au^B F(u)}{1 + Au^B}, A = 0.0005372 \text{ and } B = 4.82842262, \tag{12}$$

where u is the reduced electron density, $u(N_{d(a)}, T, x) \equiv \frac{N_{d(a)}}{N_{c(v)}(T,x)}$, $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[(3\sqrt{\pi}/4) \times u\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

Here, one notes that: (i) as $u \gg 1$, according to the HD [d(a)- $X(x)$- alloy] ER-case, or to the degenerate case, Eq. (12) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u \ll 1)}{k_B T}\left(\frac{-E_{Fp}(u \ll 1)}{k_B T}\right) \ll -1$, to the LD [a(d)- $X(x)$- alloy] BR-case, or to the non-degenerate case, Eq. (12) is reduced to the function G(u). Here, the notations: HD(LD) and ER(BR), mean: heavily doped (lightly doped)-cases and emitter (base)-regions, respectively.

Secondly, if denoting the effective Wigner-Seitz radius $r_{sn(sp)}$, characteristic of the interactions, by:

$$r_{sn(sp)}(N_{d(a)}, r_{d(a)}, x) = 1.1723 \times 10^8 \times \left(\frac{1}{N_{d(a)}}\right)^{1/3} \times \frac{m_{c(v)}(x)}{\varepsilon(r_{d(a)},x)}, \tag{13}$$

the correlation energy of an effective electron gas, $E_{cn(cp)}(N_{d(a)}, r_{d(a)}, x)$, is given as [4]:

$$E_{cn(cp)}(N_{d(a)}, r_{d(a)}, x) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - lnln(2)]}{\pi^2}\right) \times ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Now, taking into account various spin-polarized chemical potential-energy contributions such as (Van Cong, 2024): exchange energy of an effective electron (hole) gas, majority-carrier correlation energy of an effective electron (hole) gas, minority hole (electron) correlation energy, majority electron (hole)-ionized d(a) interaction screened Coulomb potential energy, and finally minority hole (electron)-ionized d(a) interaction screened Coulomb potential energy, the band gap narrowing (BGN) are given as follows.

Thus, in the n-type HD $X(x)$- alloy, the BGN is found to be given by (Van Cong, 2024):

$$\Delta E_{gn}(N_d, r_d, x) \simeq a_1 \times \frac{\varepsilon_o(x)}{\varepsilon(r_d,x)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_d,x)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_{cn}(r_{sn}) \times r_{sn}]\right) + a_3 \times$$

$$\left[\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}\right]^{5/4} \times \sqrt{\frac{m_v}{m_c}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}} \times N_r^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, \qquad N_r \equiv$$

$$\left(\frac{N_d}{9.999 \times 10^{17}\ cm^{-3}}\right), \tag{14n}$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type HD $X(x)$- alloy, as:

$$\Delta E_{gp}(N_a, r_a, x) \simeq a_1 \times \frac{\varepsilon_o(x)}{\varepsilon(r_a,x)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_a,x)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_{cp}(r_{sp}) \times r_{sp}]\right) + a_3 \times$$

$$\left[\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}\right]^{5/4} \times \sqrt{\frac{m_c}{m_v}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, \quad N_r \equiv \left(\frac{N_a}{9.999 \times 10^{17}\ cm^{-3}}\right),$$

$$\tag{14p}$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$ and

$a_5 = 9.80 \times 10^{-5}(eV)$.

Therefore, in the HD[d(a)- $X(x)$- alloy] ER, we can define the effective extrinsic carrier concentration, $n_{en(ep)}^*$, by :

$$n_{en(ep)}^*\left(N_{d(a)}, T, r_{d(a)}, x\right) \equiv \sqrt{N_{d(a)} \times p_o(n_o)} = n_{in(ip)}(T, r_{d(a)}, x) \times exp\left[\frac{\Delta E_{agn(agp)}}{2k_B T}\right], \quad (15)$$

where the apparent band gap narrowing (ABGN), $\Delta E_{agn(agp)}$, is defined by:

$$\Delta E_{agn}(N_d, T, r_d, x) \equiv \Delta E_{gn}(N_d, r_d, x) + k_B T \times ln\left(\frac{N_d}{N_c(T,x)}\right) - E_{Fn}(N_d, T, x), \tag{16n}$$

$$\Delta E_{agp}(N_a, T, r_a, x) \equiv \Delta E_{gp}(N_a, r_a, x) + k_B T \times ln\left(\frac{N_a}{N_v(T,x)}\right) + E_{Fp}(N_a, T, x)]. \tag{16p}$$

Total Minority-Carrier Saturation Current Density

In the two $n^+(p^+) - p(n)$ $X(x)$- alloy -junction solar cells, denoted respectively by I(II), the total carrier-minority saturation current density is defined by:

$$J_{oI(oII)} \equiv J_{Eno(Epo)} + J_{Bpo(Bno)} \tag{17}$$

where $J_{Bpo(Bno)}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-$X(x)$- alloy] BR, and $J_{Eno(Epo)}$ is the minority-hole (electron) saturation-current density injected into the HD[d(a)- $X(x)$- alloy] ER.

$J_{Bpo(Bno)}$ in the LD[a(d)- X(x)- alloy]BR

Here, $J_{Bpo(Bno)}$ is determined by (Van Cong, 2024):

$$J_{Bpo(Bno)}\left(N_{a(d)}, T, r_{a(d)}, x\right) = \frac{e \times n_{ip(in)}^2(T, r_{a(d)}, x) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, T, r_{a(d)}, x)}{\tau_{eB(hB)}(N_{a(d)})}}}{N_{a(d)}}, \tag{18}$$

where $n_{ip(in)}^2(T, r_{d(a)}, x)$ is determined Eq. (10), $D_{e(h)}(N_{a(d)}, T, r_{a(d)}, x)$ is the minority electron (minority hole) diffusion coefficient:

$$D_e(N_a, T, r_a, x) = \frac{k_B T}{e} \times \left[200 + \frac{8300}{1 + \left(\frac{N_a}{1.3 \times 10^{17} cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a, x)}{\varepsilon_o(x)}\right)^2 (cm^2 s^{-1}), \tag{19a}$$

$$D_h(N_d, T, r_d, x) = \frac{k_B T}{e} \times \left[130 + \frac{270}{1 + \left(\frac{N_d}{8 \times 10^{17} cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d, x)}{\varepsilon_o(x)}\right)^2 (cm^2 s^{-1}), \tag{19b}$$

and $\tau_{eB(hB)}(N_{a(d)})$ is the minority electron (minority hole) lifetime in the BR:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2, \tag{20a}$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76 \times 10^{-13} \times N_d + 2.78 \times 10^{-31} \times N_d^2. \tag{20b}$$

$J_{Eno(Epo)}$ in the HD[d(a)- X(x)- alloy]ER

In the non-uniformly and heavily doped emitter region of d(a)- X(x) devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in such the HD[d(a)- $X(x)$ alloy] ER-width W, as (Van Cong, 2024):

$$\rho_{d(a)}\left(y, N_{d(a)}, W\right) = N_{d(a)} \times exp\left\{-\left(\frac{y}{W}\right)^2 \times ln\left[\frac{N_{d(a)}}{N_{do(ao)}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{do(ao)}(W)}\right]^{-\left(\frac{y}{W}\right)^2}, \quad 0 \leq y \leq W,$$

$$N_{do(ao)}(W) \equiv 7.9 \times 10^{17}\,(2 \times 10^5) \times exp\left\{-\left(\frac{W}{184.2\,(1) \times 10^{-7}\,cm}\right)^{1.066\,(0.5)}\right\} (cm^{-3}), \quad (21)$$

where $\rho_{d(a)}(y = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(y = W) = N_{do(ao)}(W)$, which decreases with increasing W. Further, the "effective doping density" is defined by:

$$N_{d(a)}^*\left(y, N_{d(a)}, T, r_{d(a)}, x\right) \equiv \rho_{d(a)}(y)/exp\left[\frac{\Delta E_{agn(agp)}\left(\rho_{d(a)}, T, r_{d(a)}, x\right)}{k_B T}\right],$$

$$N_{d(a)}^*\left(y = 0, N_{d(a)}, T, r_{d(a)}, x\right) \equiv \frac{N_{d(a)}}{exp\left[\frac{\Delta E_{agn(agp)}\left(N_{d(a)}, T, r_{d(a)}, x\right)}{k_B T}\right]}, \text{ and}$$

$$N_{d(a)}^*\left(y = W, T, r_{d(a)}, x\right) \equiv \frac{N_{do(ao)}(W)}{exp\left[\frac{\Delta E_{agn(agp)}\left(N_{do(ao)}(W), T, r_{d(a)}, x\right)}{k_B T}\right]}, \quad (22)$$

where the apparent band gap narrowing $\Delta E_{agn(agp)}$ is determined in Eq. (16), replacing $N_{d(a)}$ by $\rho_{d(a)}\left(y, N_{d(a)}, W\right)$.

Now, we can define the minority hole (minority electron) transport parameter $F_{h(e)}$ as:

$$F_{h(e)}\left(y, N_{d(a)}, T, r_{d(a)}, x\right) \equiv \frac{n_{in(ip)}^2\left(T, r_{d(a)}\right)}{p_o(n_o) \times D_{h(e)}} = \frac{N_{d(a)}^*}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}} \times \left(\frac{n_{in(ip)}}{n_{in(ip)}^*}\right)^2 \equiv$$

$$\frac{N_{d(a)}}{D_{h(e)} \times exp\left[\frac{\Delta E_{agn(agp)}}{k_B T}\right]} (cm^{-5} \times s), \quad (23)$$

the minority hole (electron) diffusion length, $L_{h(e)}\left(y, N_{d(a)}, T, r_{d(a)}, x\right)$ by:

$$L_{h(e)}^{-2}\left(y, N_{d(a)}, T, r_{d(a)}, x\right) = \left[\tau_{hE(eE)} \times D_{h(e)}\right]^{-1} = \left(C \times F_{h(e)}\right)^2 = \left(C \times \frac{N_{d(a)}^*}{D_{h(e)}}\right)^2 = \left(C \times \frac{n_{in(ip)}^2\left(T, r_{d(a)}\right)}{p_o(n_o) \times D_{h(e)}}\right)^2,$$

where the constant C was chosen to be equal to: $2.0893 \times 10^{-30} \ (cm^4/s)$, and the minority hole (minority electron) lifetime $\tau_{hE(eE)}$ as:

$$\tau_{hE(eE)} \equiv \frac{1}{D_{h(e)} \times L_{h(e)}^{-2}} = \frac{1}{D_{h(e)} \times (C \times F_{e(h)})^2} \,. \tag{24}$$

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(y)[n_o(y)] \equiv \frac{n_{in(ip)}^2}{N_{d(a)}^*(y=W,T,r_{d(a)},x)}, \tag{25}$$

and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(y)[n(y)]$ and $p_o(y)[n_o(y)]$.

$$u(y) \equiv \frac{p(y)[n(y)] - p_o(y)[n_o(y)]}{p_o(y)[n_o(y)]}, \tag{26}$$

which must verify the two following boundary conditions as:

$$u(y=0) \equiv \frac{-J_h(y=0)[J_e(y=0)]}{eS \times p_o(y=0)[n_o(y=0)]},$$

$$u(y=W) = exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1.$$

Here, $n_{I(II)}(V)$ is the photovoltaic conversion factor, being determined later, $S\left(\frac{cm}{s}\right)$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(y, r_{d(a)}, x)$.

Further, from the Fick's law for minority hole (electron)-diffusion equations, one has (Van Cong, 2023):

$$J_{h(e)}(y, r_{d(a)}, x) = \frac{-e(+e) \times n_{in(ip)}^2}{F_{h(e)}(y)} \times \frac{du(y)}{dy} = \frac{-e(+e) n_{in(ip)}^2 D_{h(e)}(N_{d(a)}, r_{d(a)}, x)}{N_{d(a)}^*(y, r_{d(a)}, x)} \times \frac{du(y)}{dy}, \tag{27}$$

where $N_{d(a)}^*(y, r_{d(a)}, x)$ is given in Eq. (22), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (19) and (23), and from the minority-hole (electron) continuity equation as:

$$\frac{dJ_{h(e)}(y, r_{d(a)}, x)}{dy} = -e(+e) \times n_{i\ n(p)}^2 \times \frac{u(y)}{F_{h(e)}(y) \times L_{h(e)}^2(y)} = -e(+e) \times n_{i\ n(p)}^2 \times \frac{u(y)}{N_{d(a)}^*(y, r_{d(a)}, x) \times \tau_{hE(eE)}}, \tag{28}$$

Therefore, the following second-order differential equation is obtained:

$$\frac{d^2u(y)}{dy^2} - \frac{dF_{h(e)}(y)}{dy} \times \frac{du(y)}{dy} - \frac{u(y)}{L_{h(e)}^2(y)} = 0, \tag{29}$$

Then, taking into account the two above boundary conditions given in Eq. (22), one thus gets the general solution of this Eq. (29), as:

$$u(y) = \frac{sinh(P(y))+I(W,S)\times cosh(P(y))}{sinh(P(W))+I(W,S)\times cosh(P(W))} \times \left(exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \tag{30}$$

where the factor $I(W,S)$ is determined by: $\quad D_{h(e)}\big(N_d, T, r_{d(a)}, x\big)$

$$I(T, r_{d(a)}, x, W, S) = \frac{D_{h(e)}(y=W, N_{do(ao)}(W), T, r_{d(a)}, x)}{S\times L_{h(e)}(y=W, N_{do(ao)}(W), T, r_{d(a)}, x)}. \tag{31}$$

Further, since $\frac{dP(y)}{dy} \equiv C \times F_{h(e)}(y) = \frac{1}{L_{h(e)}(x)}$, $C = 2.0893 \times 10^{-30} \ (cm^4/s)$, for the X(x)-alloy, being an empirical parameter, chosen for each crystalline semiconductor, P(y) is thus found to be defined by:

$$P(y) \equiv \int_0^y \frac{dy}{L_{h(e)}(y)}, \quad 0 \leq y \leq W, \quad P(y=W) \equiv \left(\frac{1}{W} \times \int_0^W \frac{dy}{L_{h(e)}(y)}\right) \times W \equiv \frac{W}{L_{h(e)}^*(y)} = \frac{L_{h(e)}(y)}{L_{h(e)}^*(y)} \times$$
$$\frac{W}{L_{h(e)}(y)}, \tag{32}$$

where $L_{h(e)}^*(y)$ is the effective minority hole (minority electron) diffusion length. Further, the minority-hole (electron) current density injected into the HD[d(a)- X(x) alloy] ER is found to be given by:

$$J_{h(e)}\big(y, W, N_{d(a)}, T, r_{d(a)}, x, S, V\big) = -J_{Eno}(y, W, N_d, T, r_d, x, S) \ [J_{Epo}(y, W, N_a, T, r_a, x, S)] \times$$
$$\left(exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \tag{33}$$

where $J_{Eno(Epo)}$ is the saturation minority hole (minority electron) current density,

$$J_{Eno(Epo)}\big(y, W, N_{d(a)}, T, r_{d(a)}, x, S\big) = \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(y, N_{d(a)}, T, r_{d(a)}, x)\times L_{h(e)}} \times \frac{cosh(P(x))+I(W,S)\times sinh(P(x))}{sinh(P(W))+I(W,S)\times cosh(P(W))}. \tag{34}$$

In the following, we will denote P(W) and $I(W,S)$ by P and I, for a simplicity. So, Eq. (30) gives:

$$J_{Eno(Epo)}\left(y=0, W, N_{d(a)}, T, r_{d(a)}, x, S\right) = \frac{e n_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(y, N_{d(a)}, T, r_{d(a)}, x) \times L_{h(e)}} \times \frac{1}{\sinh(P) + I \times \cosh(P)},$$

$$(35)$$

$$J_{Eno(Epo)}\left(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S\right) = \frac{e n_{i\,n(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(y=W, N_{d(a)}, T, r_{d(a)}, x) \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)},$$

$$(36)$$

and then,

$$\frac{J_{h(e)}(y=0, W, N_{d(a)}, T, r_{d(a)}, x, S, V)}{J_{h(e)}(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S, V)} \equiv \frac{J_{Eno(Epo)}(y=0, W, N_{d(a)}, T, r_{d(a)}, x, S)}{J_{Eno(Epo)}(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S)} = \frac{1}{\cosh(P) + I \times \sinh(P)}.$$

$$(37)$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by:

$$Q_{h(e)}^*(y=W, N_{d(a)}, T, r_{d(a)}, x) \equiv \int_0^W \quad + e(-e) \times u(y) \times p_o(y)[n_o(y)] \times$$
$$\frac{\tau_{hE(eE)}(N_{d(a)}, T, r_{d(a)}, x)}{\tau_{hE(eE)}(\rho_{d(a)}(x), T, r_{d(a)})} dy,$$ and the effective minority hole (minority electron) transit time $[htt(ett)]$ by:
$$\tau_{htt(ett)}^*(y=W, W, N_{d(a)}, r_{d(a)}, x, S) \equiv Q_{h(e)}^*(y=W, N_{d(a)}, T, r_{d(a)}, x)/J_{Eno(Epo)}(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S),$$ and from Equations (24, 31), one obtains:

$$\frac{\tau_{htt(ett)}^*(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S)}{\tau_{hE(eE)}} \equiv 1 - \frac{J_{Eno(Epo)}(y=0, W, N_{d(a)}, T, r_{d(a)}, x, S)}{J_{Eno(Epo)}(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S)} = 1 - \frac{1}{\cosh(P) + I \times \sinh(P)}.$$

$$(38)$$

Now, some important results can be obtained and discussed below.

As $P \ll 1$ (or $W \ll L_{h(e)}$) and $S \to \infty$, $I \equiv I(W, S) = \frac{D_{h(e)}(N_{do(ao)}(W), T, r_{d(a)}, x)}{S \times L_{h(e)}(N_{do(ao)}(W), T, r_{d(a)}, x)} \to 0$, from Eq. (38),
one has: $\frac{\tau_{htt(ett)}^*(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S)}{\tau_{hE(eE)}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (36), one obtains:

$$J_{Eno(Epo)}\left(y=W, N_{d(a)}, T, r_{d(a)}, x, S \to \infty\right) \to \frac{e n_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(y=W, N_{d(a)}, T, r_{d(a)}, x) \times L_{h(e)}} \times \frac{1}{P(W)}.$$

$$(39)$$

Further, as $P \gg 1$ (or $W \gg L_{h(e)}$) and $S \to 0$, $I \equiv I(y=W, r_{d(a)}, x, S) = \frac{D_{h(e)}(N_{do(ao)}(W), T, r_{d(a)}, x)}{S \times L_{h(e)}(N_{do(ao)}(W), T, r_{d(a)}, x)} \to \infty$, and from Eq. (38) one has: $\frac{\tau_{htt(ett)}^*(y=W, W, N_{d(a)}, T, r_{d(a)}, x, S)}{\tau_{hE(eE)}} \to 1$,
suggesting a completely opaque emitter region (COER)-case, where, from Eq. (36), one gets:

$$J_{Eno(Epo)}\left(y=W, N_{d(a)}, T, r_{d(a)}, x, S \to 0\right) \to \frac{e n_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(y=W, N_{d(a)}, T, r_{d(a)}, x) \times L_{h(e)}} \times \tanh(P). \quad (40)$$

In summary, in the two $n^+(p^+) - p(n)$ X(x)-alloy junction solar cells, the dark carrier-minority saturation current density $J_{oI(oII)}$, defined in Eq. (17), is now rewritten as:

$$J_{oI(oII)}(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, r_{a(d)}, x) \equiv J_{Eno(Epo)}(W, N_{d(a)}, T, r_{d(a)}, x, S) + J_{Bpo(Bno)}(N_{a(d)}, T, r_{a(d)}, x),$$

(41)

where $J_{Eno(Epo)}$ and $J_{Bpo(Bno)}$ are determined respectively in Equations (36, 18).

Photovoltaic conversion effect at 300K

Here, in the $n^+(p^+) - p(n)$ X(x) -alloy junction solar cells at T=300 K, denoted respectively by I(II), and for physical conditions, respectively, as:

$$\mathbf{W = 15\ \mu m, N_{d(a)} = 10^{19} cm^{-3}(10^{20} cm^{-3}), r_{d(a)}, x, S = 100\ \left(\frac{cm}{s}\right); N_{a(d)} = 10^{17}\ cm^{-3},\ r_{a(d)}, x,}$$

(42)

we propose, at given open circuit voltages: $V_{ocI1(ocI2)}$ and $V_{ocII1(ocII2)}$, the corresponding data of the short circuit current density $J_{scI(II)}$, in order to formulate our following treatment method of two fix points (Green, 2022; Moon et al., 2016), as:

at $V_{ocI1(ocI2)}(V) = 0.980\ (1.1272),\ J_{scI1(scI2)}(mA/cm^2) = 27.06\ (29.76),$

at $V_{ocII1(ocII2)}(V) = 0.980\ (1.03), J_{scII1(scII2)}(mA/cm^2) = 24.2\ (29.84).$

(43)

Now, we define the net current density J at T=300 K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage V, flowing through the $n^+(p^+) - p(n)$ X(x)-alloy junction of solar cells, as:

$$J(V) \equiv J_{ph.}(V) - J_{oI(oII)} \times \left(e^{X_{I(II)}(V)} - 1\right),\ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V) \times V_T},\ V_T \equiv \frac{k_B T}{e} = 0.02585\ V,$$

(44)

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor (PVCF), noting that as $V = V_{oc}$, being the open circuit voltage, $J(V = V_{oc}) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(scII)}(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc})$, for $V_{oc} \geq V_{ocI1(ocII1)}$.

Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(scII)}\left(W, N_{d(a)}, T, r_{d(a)}x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right) \equiv$$
$$J_{oI(oII)}\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \qquad (45)$$

where $\quad n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, r_{a(d)}, x, V_{oc}\right)$, $\quad$ and $\qquad X_{I(II)}(V_{oc}) \equiv \dfrac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in the following calculation, (ii) the function $\left(e^{X_{I(II)}(V_{oc})} - 1\right)$ or the PVCF, $n_{I(II)}$, representing the photovoltaic conversion effect, converts the light, represented by $J_{scI(scII)}$, into the electricity, by $J_{oI(oII)}$, and finally, for given $\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right)$-values, $n_{I(II)}(V_{oc})$ is determined.

Now, for $V_{oc} \geq V_{ocI1(ocII1)}$, one can propose the general expressions for the PVCF, in order to get exactly the values of $n_{I1(II1)}\left(V_{ocI1(ocII1)}\right)$ and $n_{I2(II2)}\left(V_{ocI2(ocII2)}\right)$, as functions of V_{oc}, by:

$$n_{I(II)}\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right) = n_{I1(II1)}\left(V_{ocI1(ocII1)}\right) + n_{I2(II2)}\left(V_{ocI2(ocII2)}\right) \times$$
$$\left(\frac{V_{oc}}{V_{ocI1(ocII1)}} - 1\right)^{\alpha(\beta)}, \qquad (46)$$

where, for example, the values of $\alpha(\beta)$, obtained for $x = (0, 0.5$ and $1)$, will be reported in Tables 3 and 5, for these $[X(x) \equiv GaAs_{1-x}P_x]$-alloy junctions.

So, one can determine the general expressions for the fill factors, as:

$$F_{I(II)}\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right) = \frac{X_{I(II)}(V_{oc}) - ln[X_{I(II)}(V_{oc}) + 0.72]}{X_{I(II)}(V_{oc}) + 1}. \qquad (47)$$

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ X(x) alloy-junction solar cells, by:

$$\eta_{I(II)}\left(W, N_{d(a)}, T, r_{d(a)}, x, S; N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right) \equiv \frac{J_{scI(scII)} \times V_{oc} \times F_{I(II)}}{P_{in.}}, \qquad (48)$$

being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum $\left(P_{in.} = 0.100 \, \frac{W}{cm^2}\right)$.

It should be noted that the maximal values of $\eta_{I(II)}$, $\eta_{Imax.(IImax.)}$, are obtained at the corresponding ones of $\mathbf{V_{ocI(ocII)}}$, at which $\frac{\partial \eta_{I(II)}(W, N_{d(a)}, T, r_{d(a)}, S, N_{a(d)}, T, r_{a(d)}, V_{oc})}{\partial V_{oc}} = 0$, as those given in next Tables 3 and 5, being marked in bold. Further, from the well-known Carnot's theorem, being obtained by the second principle in thermodynamics, or by the entropy law, the maximum efficiency of a heat engine operating

between hot (H) and cold (C) reservoirs is the ratio of the temperature difference between the reservoirs, $T_H - T_C$, $T_C \equiv T = 300$ K, to the H-reservoir temperature, T_H, expressed as:

$$\eta_{I(II)}(T, V_{oc}) \leq \eta_{Imax.(IImax.)}\left(T, V_{oc} = V_{ocI(ocII)}\right) \equiv \eta_{Carnot} = \frac{T_H - T_C}{T_H}, \tag{49}$$

for a simplicity, noting that both $\eta_{Imax.(IImax.)}$ and T_H depend on $(W, N_{d(a)}, T, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{ocI(ocII)})$-parameters.

Numerical Results and Concluding Remarks

We will respectively consider the two following cases of $n^+(p^+) - p(n)$ -junctions such as:

$HD\ (Te\ ;\ Sb;\ Sn)\ X(x)\ alloy\ ER - LD\ (Mg\ ;\ In\ ;\ Cd)\ X(x) - alloy\ BR$ −case, according to: 3 (n^+p) − junctions denoted by: (Te^+Mg, Sb^+In, Sn^+Cd), and

$HD\ (Mg\ ;\ In\ ;\ Cd)\ X(x)\ alloy\ ER - LD\ (Te\ ;\ Sb;\ Sn)\ X(x) - alloy\ BR$ −case, according to: 3 (p^+n) − junctions denoted by: (Mg^+Te, In^+Sb, Cd^+Sn).

Now, by using the physical conditions, given in Eq. (42), we can determine various photovoltaic conversion coefficients as follows.

Firs case: $HD\ [\ Te;\ Sb;\ Sn]\ X(x) - Alloy\ ER - LD\ [\ Mg\ ;\ In\ ;\ Cd\ ]\ X(x) - \ Alloy\ BR$

Here, there are the 3 $(n^+p) - X(x)$ junctions, being denoted by: (Te^+Mg, Sb^+In, Sn^+Cd).

Then, the numerical results of $\frac{\tau_{htt}^*}{\tau_{hE}}, J_{Bpo}, J_{Eno}$ and J_{oI}, are calculated using Equations (38), (18), (36) and (41), respectively, and obtained, as those given in Table 2. Further, those of $n_I, J_{scI}, F_I, \eta_I$, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, and reported in the following Table 3.

Tables 2 and 3 in Appendix 1

Second case: $HD\ [\ Mg\ ;\ In\ ;\ Cd]\ X(x) - \ Alloy\ ER - LD\ [\ Te;\ Sb, Sn\ ]\ X(x) - \ Alloy\ BR$

Here, there are 3 $(p^+n) - X(x)$-junctions, being denoted by: (Mg^+Te, In^+Sb, Cd^+Sn).

Then, the numerical results of $\frac{\tau_{ett}^*}{\tau_{eE}}, J_{Bno}, J_{Epo}$ and J_{oII}, are calculated using Equations (38), (18), (36) and (41), respectively, and obtained, as those given in Table 4. Further, those of $n_{II}, J_{scII}, F_{II}, \eta_{II}$, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, and reported in the following Table 5.

Tables 4 and 5 in Appendix 1

Finally, some concluding remarks are obtained and discussed as follows.

(i)-First, with increasing x=(0, 0.5, 1), from Table 3, obtained for the single $n^+ - p\ X(x)$-alloy junction solar cells, and for given $r_{Sn(Cd)}$-radius, for example, one obtains: $\eta_{Imax.}(\nearrow)$= 31.18%, 33.495%, **35.99%**, according to $T_H(K) = 435.9, 451.1, 468.7$, at $V_{ocI}(V) = 1.07, 1.06, 1.05$, respectively.

(ii)- Secondly, with increasing x=(0, 0.5, 1), from Table 5, obtained for the single $p^+ - n\ X(x)$-alloy junction solar cells, and for given $r_{Cd(Sn)}$-radius, for example, one gets: $\eta_{IImax.}(\searrow)$= **33.05%**, 31.95%, 31.37%, according to $T_H(K) = 448.0, 440.9, 437.1$, at $V_{ocII}(V)[> V_{ocI}(V)] = 1.20, 1.15, 1.12$,

respectively, suggesting that such $\eta_{Imax.(IImax.)}$-and-T_H variations depend on $V_{ocII}(V)[> V_{ocI}(V)]-$ *values*.

Then, in particular, as given in Table 3, for $x = 0$ and $r_{d(a)} = r_{Te(Mg)}$, one $gets$: η_I =23.48 % and 29.76 % at $V_{oc} = 0.98$ V and 1.1272 V, respectively, which can also be compared with the corresponding results obtained for the single-junction GaAs thin-film solar cell, 22.08 % and 29.71 %, obtained with relative deviations in absolute values, 6.34 % and 0.17 %, and given respectively by Moon et al. (2016) and Green et al. (2022).

Finally, one notes that, in order to obtain the highest efficiencies, the single $GaAs_{1-x}P_x$-alloy junction solar cells could be chosen rather than the single crystalline GaAs-junction solar cell.

References

Green, M.A., Dunlop, E.D., Levi, D.H., Hohl-Ebinger, J., Yoshita, M., Kopidakis, N., … & Hao, X. (2022). Solar cell efficiency tables (version 60). *Progress in Photovoltaics Research & Applied*, 30, 687-701. https://doi.org/10.1002/pip.3595

Kittel, C. (1976). *Introduction to Solid State Physics.* New York: Wiley.

Moon, S., Kim, K., Kim, Y., Heo, J. & Lee, J. (2016). Highly efficient single GaAs thin-film solar cell on flexible substrate. *Scientific Report*, *6*, 30107. https://doi.org/10.1038/srep30107

Singh, P. & Ravindra, N. (2012). Temperature dependence of solar cell performance - An analysis. *Solar Energy Materials and Solar Cells, 101,* 36-45. http://doi.org/10.1016/j.solmat.2012.02.019

Van-Cong, H. (2024). Maximal Efficiencies in New Single GaAs1−xTex-Alloy Junction Solar Cells at 300 K. *European Journal of Theoretical and Applied Sciences*, *2*(1), 54-74. https://doi.org/10.59324/ejtas.2024.2(1).04

Van-Cong, H. (2024). Maximal Efficiencies in New Single GaAs 1−x Sb x-Alloy Junction Solar Cells at 300 K. *European Journal of Theoretical and Applied Sciences*, *2*(1), 75-95. https://doi.org/10.59324/ejtas.2024.2(1).05

Van Cong, H. (2016). Effects of impurity size and heavy doping on energy-band-structure parameters of various impurity-Si systems. *Physica B*, 487, 90-101. https://doi.org/10.1016/j.physb.2016.02.006

Van Cong, H. (1999). A new solution for minority-carrier injection into the heavily doped emitter of silicon devices. *Physica Status Solidi A*, 171, 631-664. https://doi.org/10.1002/(SICI)1521-396X(199902)171:2%3C631::AID-PSSA631%3E3.0.CO;2-%23

Van Cong, H. & Debiais, G. (1997). Energy band structure parameters and their data, derived from the measurements of minority carrier current density in heavily doped emitters of silicon devices. *Solar Ener. Mater. and Solar Cells*, 45, 385-399.

Van Cong, H. & Debiais, G. (1996). Apparent band-gap narrowing and its data derived from the measurements of minority-carrier current density in heavily doped emitters of silicon devices. *Physica Status Solidi A*, 155, 547-553. https://doi.org/10.1002/pssa.2211550229

Van Cong, H. (1995). A simple accurate solution to minority electron injection in the p-type heavily doped emitter region of silicon devices. *Physica Status Solidi A*, 149, 619-628. https://doi.org/10.1002/pssa.2211490212

Van Cong, H., & Debiais, G. (1993). A simple accurate expression of the reduced Fermi energy for any reduced carrier density. *Journal of Applied Physics*, 73, 1545-1546. https://doi.org/10.1063/1.353232

Van Cong, H. & Doan Khanh, B. (1992). Simple accurate general expression of the Fermi-Dirac integral $F_j(a)$ and for j> -1. *Solid-State Electron.*, 35, 949-951.

Van Cong, H. (1991). New series representation of Fermi-Dirac integral $F_j(-\infty < a < \infty)$ for arbitrary j> -1, and its effect on $F_j(a \geq 0_+)$ for integer j$\geq$ 0. *Solid-State Electron.*, 34, 489-492.

Van Cong, H., Brunet, S. & Martin, J. C. (1984). Size effect on different impurity levels in semiconductors. *Solid State Communications*, 49,

697-699.   https://doi.org/10.1016/0038-1098(84)90223-0

Van Cong, H. (1975). Fermi energy and band-tail parameters in heavily doped semiconductors. *J. Phys. Chem. Solids*, 36, 1237-1240. https://doi.org/10.1016/0022-3697(75)90197-3

Appendix 1

Table 1. From Equations (5, 8a, 8b, 9) and in the n(p)-type $GaAs_{1-x}P_x$-alloy, the numerical results of the energy-band-structure parameters, reported below, suggest that, with increasing x and $r_{d(a)}$, both $B_{do(ao)}(x)$ and $\varepsilon(r_{d(a)}, x)$ decrease, while the other ones increase

Donor		As	Te	Sb	Sn
r_d (nm)	↗	r_{do}=0.118	0.132	0.136	0.140
x	↗	0, 0.5, 1	0, 0.5, 1	0, 0.5, 1	0, 0.5, 1
$B_{do}(x)$ in 10^8 (N/m²)	↗	1.21, 2.11, 3.34			
$\varepsilon(r_d, x)$	↘	**13.1**, 12.1, **11.1**	12.3, 11.4, 10.4	11.8, 10.9, 10.0	11.3, 10.4, 9.57
$E_d(r_d, x)$ meV	↗	5.21, 9.08, 14.3	5.91, 10.3, 16.3	6.38, 11.1, 17.6	6.99, 12.2, 19.3
$E_{gn}(r_d, x)$ eV	↗	**1.52**, 1.92, **2.32**	1.52, 1.92, 2.32	1.52, 1.92, 2.32	1.52, 1.92, 2.32
$E_{gin}(T = 300K, r_d, x)$ eV	↗	**1.42**, 1.84, **2.25**	1.42, 1.84, 2.25	1.42, 1.84, 2.25	1.42, 1.84, 2.25
Acceptor		**Ga**	Mg	In	Cd
r_a (nm)	↗	r_{ao}=0.126	0.140	0.144	0.148
x	↗	0, 0.5, 1	0, 0.5, 1	0, 0.5, 1	0, 0.5, 1
$B_{ao}(x)$ in 10^8 (N/m²)	↗	4.39, 7.01, 10.5			
$\varepsilon(r_a, x)$	↘	**13.1**, 12.1, **11.1**	12.4, 11.5, 10.5	12.0, 11.1, 10.1	11.5, 10.6, 9.74
$E_a(r_a, x)$ meV	↗	23.0, 36.6, 55.2	25.7, 41.0, 61.7	27.5, 43.9, 66.1	29.8, 47.6, 71.7
$E_{gp}(r_a, x)$ eV	↗	**1.52**, 1.92, **2.32**	1.52, 1.92, 2.33	1.52, 1.93, 2.33	1.53, 1.93, 2.34
$E_{gip}(T = 300K, r_a, x)$ eV	↗	**1.42**, 1.84, **2.25**	1.43, 1.84, 2.26	1.43, 1.84, 2.26	1.43, 1.85, 2.27

Table 2. In the HD [(Te; Sb; Sn)- $GaAs_{1-x}P_x$-alloy] ER-LD[$(Mg; In; Cd)$-$GaAs_{1-x}P_x$-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of $\frac{\tau^*_{htt}}{\tau_{hE}}$, J_{Bpo}, J_{Eno} and J_{oI} , are computed, using Equations (38), (18), (36) and (41), respectively, noting that J_{oI} decreases slightly with increasing $r_{d(a)}$-radius for given x, but it increases strongly with increasing x for given $r_{d(a)}$-radius, being new results

n^+p		Te^+Mg	Sb^+In	Sn^+Cd
Here, **x=0,** and for the (Te^+Mg, Sb^+In, Sn^+Cd)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{htt}}{\tau_{hE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bpo} in 10^{-19} (A/cm^2)	↘	1.0315	0.9965	0.9565
J_{Eno} in 10^{-23} (A/cm^2)	↘	1.7830	1.7258	1.6624
J_{oI} in 10^{-19} (A/cm^2)	↘	1.0317	0.9967	0.9567
Here, **x=0.5,** and for the (Te^+Mg, Sb^+In, Sn^+Cd)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{htt}}{\tau_{hE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bpo} in 10^{-21} (A/cm^2)	↘	3.2121	3.1031	2.9786
J_{Eno} in 10^{-23} (A/cm^2)	↘	5.7338	5.4613	5.1539
J_{oI} in 10^{-21} (A/cm^2)	↘	3.2179	3.1086	2.9837
Here, **x=1,** and for the (Te^+Mg, Sb^+In, Sn^+Cd)junctions and from Eq. (34), one obtains: $\frac{\tau^*_{htt}}{\tau_{hE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bpo} in 10^{-21} (A/cm^2)	↘	7.4383	7.1859	6.8975
J_{Eno} in 10^{-23} (A/cm^2)	↘	3.8757	3.6185	3.3285
J_{oI} in 10^{-21} (A/cm^2)	↘	7.4771	7.2221	6.9308

Table 3. In the HD [(Te; Sb; Sn)- $GaAs_{1-x}P_x$-alloy] ER-LD[(Mg; In; Cd)-$GaAs_{1-x}P_x$-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of n_I, J_{scI}, F_I, η_I, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, noting that both $\eta_{Imax.}$ and T_H, marked in bold, increase with increasing x for given $r_{d(a)}$, being new results

$V_{oc}(V)$	n_I	$J_{scI}(\frac{mA}{cm^2})$	$F_I(\%)$	$\eta_I(\%)$
Here, **x=0**. For the (Te^+Mg, Sb^+In, Sn^+Cd) junctions, the value of α given in Eq. (46) is 1.0824.				
n^+p	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd
0.980	0.945; 0.944; 0.943	27.06; 27.06; 27.06	88.54; 88.55; 88.56	23.48; 23.48; 23.48
1.06	1.017; 1.016; 1.015	33.13; 33.14; 33.15	88.59; 88.60; 88.60	31.11; 31.12; 31.13
1.07	1.027; 1.026; 1.025	32.88; 32.88; 32.89	88.59; 88.59; 88.60	**31.16; 31.17; 31.18**
			$V_{ocI} = \mathbf{1.07\,V}$	**435.8; 435.8; 435.9**=$T_H(K)$
1.08	1.037; 1.036; 1.035	32.52; 32.52; 32.53	88.58; 88.59; 88.60	31.11; 31.12; 31.12
1.1272	1.084; 1.083; 1.082	29.81; 29.82; 29.82	88.56; 88.57; 88.58	29.76; 29.77; 29.77
3	3.318; 3.315; 3.311	0.159; 0.160; 0.158	87.28; 87.29; 87.30	0.418; 0.416; 0.414
Here, **x=0.5**. For the (Te^+Mg, Sb^+In, Sn^+Cd) junctions, the value of α given in Eq. (46) is 1.0796				
n^+p	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd
0.980	0.688; 0.688; 0.687	27.06; 27.06; 27.06	91.04; 91.05; 91.06	24.14; 24.14; 24.15
1.05	0.734; 0.733; 0.732	34.95; 34.95; 34.96	91.08; 91.08; 91.09	33.42; 33.43; 33.44
1.06	0.741; 0.740; 0.740	34.68; 34.68; 34.69	91.08; 91.08; 91.09	**33.48; 33.48; 33.49**
$T_H(K)$			$V_{ocI} = 1.06\,V$	**451.0; 451.0; 451.1**=$T_H(K)$
1.07	0.748; 0.747; 0.747	34.23; 34.24; 34.24	91.08; 91.08; 91.09	33.36; 33.37; 33.37
1.1272	0.790; 0.789; 0.789	29.63; 29.63; 29.64	91.06; 91.06; 91.07	30.42; 30.42; 30.42
3	2.413; 2.411; 2.410	0.025; 0.024; 0.024	90.04; 90.05; 90.05	0.066; 0.066; 0.066
Here, **x=1**. For the (Te^+Mg, Sb^+In, Sn^+Cd) junctions, the value of α given in Eq. (46) is 1.0785				
n^+p	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd	Te^+Mg; Sb^+In; Sn^+Cd
0.980	0.539; 0.538; 0.538	27.06; 27.06; 27.06	92.62; 92.63; 92.63	24.56; 24.56; 24.56
1.04	0.569; 0.569; 0.568	37.12; 37.12; 37.13	92.65; 92.65; 92.66	35.77; 35.77; 35.78
1.05	0.575; 0.574; 0.574	36.98; 36.98; 36.99	92.65; 92.65; 92.66	**35.97; 35.98; 35.99**
			$V_{ocI} = 1.05\,V$	**468.5; 468.6; 468.7**=$T_H(K)$
1.06	0.580; 0.579; 0.579	36.56; 36.57; 36.57	92.65; 92.65; 92.65	35.91; 35.91; 35.92
1.1272	0.619; 0.618; 0.618	29.72; 29.72; 29.72	92.63; 92.63; 92.64	31.03; 31.03; 31.03
3	1.889; 1.888; 1.887	0.003; 0.003; 0.003	91.78; 91.79; 91.79	0.010; 0.010; 0.010

Table 4. In the HD [$(Mg; In; Cd)$- $GaAs_{1-x}P_x$--alloy] ER-LD[(Te; Sb; Sn)-$GaAs_{1-x}P_x$-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of $\frac{\tau^*_{ett}}{\tau_{eE}}, J_{Bno}, J_{Epo}$, and J_{oII} are computed, using Equations (38), (18), (36) and (41), respectively, noting that J_{oII} decreases slightly with increasing $r_{a(d)}$-radius for given x, but it increases strongly with increasing x for given $r_{a(d)}$-radius, being new results

p^+n		Mg^+Te	In^+Sb	Cd^+Snvvv
Here, **x=0,** and for the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bno} in 10^{-20} (A/cm^2)	↘	3.0094	2.8418	2.6512
J_{Epo} in 10^{-23} (A/cm^2)	↘	1.1965	1.1298	1.0513
J_{oII} in 10^{-20} (A/cm^2)	↘	3.0106	2.8429	2.6522
Here, **x=0.5,** and for the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bno} in 10^{-21} (A/cm^2)	↘	9.6698	9.0066	8.2559
J_{Epo} in 10^{-23} (A/cm^2)	↘	2.7337	2.4692	2.1708
J_{oII} in 10^{-21} (A/cm^2)	↘	9.6971	9.0313	8.2776
Here, **x=1,** and for the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0,0)$ suggesting a completely transparent condition.				
J_{Bno} in 10^{-22} (A/cm^2)	↘	2.3368	2.1362	1.9118
J_{Epo} in 10^{-24} (A/cm^2)	↘	1.7488	1.4891	1.2139
J_{oII} in 10^{-22} (A/cm^2)	↘	2.3542	2.1511	1.9240

Table 5. In the HD [$(Mg; In; Cd)$- $GaAs_{1-x}P_x$-alloy] ER-LD[(Te; Sb; Sn)-$GaAs_{1-x}P_x$-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of n_{II}, J_{scII}, F_{II}, η_{II}, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, noting that both $\eta_{IImax.}$ and T_H, marked in bold, decrease with increasing x for given $r_{a(d)}$, being new results

$V_{oc}(V)$	n_{II}	$J_{scII}(\frac{mA}{cm^2})$	$F_{II}(\%)$	$\eta_{II}(\%)$
Here, **x=0**. For the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions, the value of β given in Eq. (46) is 1.052.				
p^+n	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn
0.980	0.919; 0.918; 0.917	24.20; 24.20; 24.20	88.78; 88.79; 88.81	21.05; 21.06; 21.06
1.03	0.961; 0.960; 0.958	29.84; 29.85; 29.86	88.83; 88.84; 88.85	27.30; 27.31; 27.33
1.19	1.110; 1.108; 1.106	31.21; 31.23; 31.24	88.84; 88.85; 88.86	33.00; 33.02; 33.03
1.20	1.119; 1.117; 1.116	30.97; 30.98; 30.99	88.83; 88.85; 88.86	**33.02; 33.03; 33.05**
			$V_{ocII} = 1.20\ V$	**447.8; 447.49; 448.0**=$T_H(K)$
1.21	1.129; 1.127; 1.125	30.70; 30.71; 30.72	88.83; 88.84; 88.86	33.01; 33.02; 33.04
3	2.977; 2.973; 2.968	2.547; 2.539; 2.528	88.29; 88.30; 88.32	6.746; 6.725; 6.699
Here, **x=0.5**. For the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions, the value of β given in Eq. (46) is 1.0422.				
p^+n	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn
0.980	0.675; 0.674; 0.673	24.20; 24.20; 24.20	91.18; 91.19; 91.20	21.62; 21.63; 21.63
1.03	0.706; 0.705; 0.704	29.83; 29.84; 29.85	91.21; 91.22; 91.23	28.03; 28.04; 28.05
1.14	0.782; 0.781; 0.779	30.68; 30.69; 30.70	91.21; 91.22; 91.23	31.91; 31.92; 31.93
1.15	0.789; 0.788; 0.786	30.44; 30.44; 30.45	91.21; 91.22; 91.23	**31.93; 31.94; 31.95**
			$V_{ocII} = 1.15\ V$	**440.7; 440.8; 440.9**=$T_H(K)$
1.16	0.796; 0.795; 0.793	30.16; 30.16; 30.17	91.21; 91.22; 91.23	31.91; 31.92; 31.93
3	2.176; 2.173; 2.170	1.390; 1.385; 1.379	90.81; 90.82; 90.83	3.788; 3.774; 3.757
Here, **x=1**. For the (Mg^+Te, In^+Sb, Cd^+Sn)-junctions, the value of β given in Eq. (46) is 1.0369.				
p^+n	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn	Mg^+Te; In^+Sb; Cd^+Sn
0.980	0.531; 0.530; 0.529	24.20; 24.20; 24.20	92.71; 92.72; 92.73	21.99; 21.99; 21.99
1.03	0.556; 0.555; 0.554	29.92; 29.93; 29.94	92.72; 92.73; 92.74	28.58; 28.58; 28.60
1.11	0.599; 0.598; 0.597	30.43; 30.44; 30.45	92.73; 92.74; 92.74	31.33; 31.34; 31.35
1.12	0.605; 0.604; 0.603	30.18; 30.19; 30.20	92.73; 92.73; 92.74	**31.34; 31.35; 31.37**
			$V_{ocII} = 1.12\ V$	**436.9; 437.0; 437.1**=$T_H(K)$
1.13	0.610; 0.609; 0.608	29.88; 29.89; 29.89	92.73; 92.73; 92.74	31.31; 31.32; 31.33
3	1.708; 1.706; 1.704	0.739; 0.735; 0.731	92.41; 92.42; 92.43	2.048; 2.039; 2.028