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APPROXIMATE SOLUTION OF FRACTIONAL ORDER MATHEMATICAL MODEL ON THE CO-TRANSMISSION OF ZIKA AND CHIKUNGUNYA VIRUS USING LAPLACE ADOMIAN DECOMPOSITION METHOD

By

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ABSTRACT

Gaining insight into the transmission dynamics of the Zika and Chikungunya viruses, as well as their co-infection, is essential for implementing efficient public health interventions. This paper presents a comprehensive fractional order mathematical model consisting of thirteen non-linear compartments to accurately represent the intricate interactions between humans and infected mosquito populations, as well as the challenges associated with their identification. In order to solve this model, we utilize the Laplace Adomians Decomposition Method (LADM), which is a very effective analytical technique for solving nonlinear differential equations. By utilizing LADM, we obtained infinite series solutions for the previously given model that ultimately converged to its precise solutions. The numerical simulations of the model demonstrate the transmission patterns of Zika virus, Chikungunya virus, and their co-infections for different values of . We utilized the fmicon algorithm, a MATLAB optimization tool, to accurately fit into the model, real-life data from Espirito Santos State in Brazil, where two viruses are concurrently spreading. The simulation deduce that, reducing mosquito biting rates and promoting compliance with treated bed net usage can substantially mitigate Zika-Chikungunya co-infection dynamics.

KEYWORDS

Laplace Adomians Decomposition Method (LADM) , Zika virus , Chikungunya virus, co-infections, series solutions.

Introduction

Zika virus disease, usually known as Zika, is an infectious disease transmitted by mosquitoes, stemming from the Zika virus initially detected in rhesus monkeys in Uganda's bush in 1947 [1]. Subsequently, it was isolated from humans in Uganda and Tanzania in 1952 [2]. For decades, only occasional instances were documented in Africa and Southeast Asia [3], until 2007 when Zika arose on Yap Island in Micronesia, located in the western Pacific Ocean [4]. In early 2015, researchers found Zika virus infection in Brazil [5], leading to its rapid spread to Northern Europe, Australia, the United States, Canada [6, 7, 8, 9], and later to Japan, China, India, and other nations [10, 11, 12], posing substantial hazards to human health. Presently, Zika exists at a low level across Central and South America. From January 1 to April 30, 2022, Brazil reported a total of 6171 suspected Zika cases, with 541 cases confirmed [13]. Zika virus is a huge global public health challenge due to its quick and broad transmission, facilitated by many pathways. The virus primarily infects humans through the bites of infected Aedes aegypti and Aedes albopictus mosquitoes [14]. Additionally, it can spread among humans through heterosexual or homosexual sexual contact [15, 16], vertically from infected mother mosquitoes to their offspring [17], and from contaminated water to mosquitoes in their aquatic stages [18]. The incubation time for the virus in the human body normally ranges from 3 to 14 days [14]. Most infected persons display no symptoms, while around a quarter have moderate symptoms such as fever, rash, conjunctivitis, and joint discomfort, with just a few confirmed fatalities [14]. Despite its low death rate, Zika infection during pregnancy is related to microcephaly and other congenital abnormalities in fetuses and infants [19]. Moreover, Zika infection can produce Guillain-Barre syndrome, myelitis, and neuropathy, particularly in adults and older children [20]. Unfortunately, there are still no licensed vaccinations or antiviral medications for the Zika virus.

Chikungunya, like Zika, is a mosquito-borne illness carried largely by female Aedes mosquitoes, comparable to dengue fever. Chikungunya virus was first diagnosed in 1952 on the Makonde Plateau in Africa [21]. Chikungunya experienced outbreaks between 1960 and 1980, impacting various countries in Asia and Africa. Unlike dengue, which can be life-threatening, Chikungunya normally results in few documented fatalities. Given that both diseases are transmitted by the same mosquito species, there's a potential for simultaneous infection in both humans and mosquitoes, as indicated by reports in California [22], Africa [23], and Colombia [24].

Atokolo et. al. [25] presented a fractional order sterile insect technology (SIT) model to prevent Zika virus transmission, applying the Laplace-Adomian decomposition technique (LADM) to derive an analytical solution. They proved that the fractional model offers additional flexibility, allowing for diverse responses by modifying the fractional order. Their work contributes to the literature by exhibiting the usefulness of LADM in addressing SIT models, a novel method in the field.

Acheneje et al. [26] proposed a fractional order compartmental model to study the transmission dynamics of COVID-19 and Monkeypox. Their model utilized the Laplace-Adomian Decomposition Method to obtain approximate solutions in the form of infinite series, complemented by MATLAB fmincon techniques for optimization. Through data fitting analysis with real-life data, their study demonstrated the potential to significantly reduce the incidence of diseases within the human population by improving treatment effectiveness and capacity. This approach highlights the importance of mathematical modeling and data-driven strategies in informing public health interventions and disease control efforts.

Adejoh and Mbah [27] presented the application of fractional differential equations to develop approximate and numerical solutions for a cancer disease model containing control measures. Similarly, Fazal et al. [28] applied the Laplace–Adomian decomposition method to obtain a numerical solution for a fractional order epidemic model of a childhood disease. Unlike other numerical approaches, the Laplace–Adomian decomposition method does not require discretization or linearization, resulting in more effective and realistic results. The models described in [25–28] serve as important tools for studying the applicability of this method in solving both linear and nonlinear differential equations.
Fractional order models offer a more thorough description of system behavior compared to integer order models, which only capture local aspects. They are particularly effective for systems demonstrating memory effects [29–32]. In biological contexts, the Caputo derivative and the Riemann–Liouville derivative are regarded singular kernels of fractional derivatives. Other nonsingular operators, such as the Mittag-Leffler and Atangana–Baleanu operators, also play key roles in simulating biological events [31, 33]. These findings emphasize the versatility and relevance of fractional calculus in solving difficult real-world challenges in epidemiology and disease dynamics. Jan et al. [34] investigated the transmission dynamics of rift valley fever with vaccination policy in fractional framework. They proved that the results of fractional derivative are more accurate and flexible than the classical derivative. Jan et al. also presented an epidemic model for HIV [35] through fractional derivative with real data. The authors showed that the fractional models provide more accurate results due to an extra parameter in the system. The incorporation of fractional-order systems in mathematical modeling of vector-borne infection can effectively capture and represent these types of phenomena. In this study, our main goal is to explore the approximate series solutions of the transmission dynamics of Zika and Chikungunya virus co-infection. To achieve this, we employ fractional differential equations, which are commonly used in modeling such infectious diseases

2.0 Model description:

The proposed deterministic compartmental model focuses on the transmission dynamics of the co-infection of Zika and Chikungunya viruses. It consists of two main populations: humans and mosquitoes. The human population is further subdivided into eight compartments, namely: susceptible human population (S_H), Exposed humans to Zika only (E_Z), Exposed humans to Chikungunya virus only (E_C), Exposed humans to both Zika and Chikungunya virus (E_{ZC}), individuals infected with Zika only (I_Z), individuals infected with Chikungunya virus only (I_C), individuals co-infected with Zika and Chikungunya virus (I_{ZC}) and the recovery class (R). Similarly the mosquito compartment is sub-divided into five (5) compartments namely; the susceptible Mosquito (S_M), Exposed mosquito to Zika virus (E_{MZ}), Exposed mosquito to Chikungunya virus (E_{MC}), mosquito infected with Zika virus (I_{MZ}) and mosquito infected with Chikungunya virus (I_{MC}). The recruitment rate of humans (Mosquitoes) into the susceptible class is denoted ($\Lambda_H(\Lambda_M)$), β_{HZ} and β_C is the effective contact rate with the probability of susceptible humans been infected with Zika virus and Chikungunya virus per contact with the infected mosquitoes with Zika virus (I_{MZ}) and infected mosquitoes with Chikungunya virus (I_{MC}) respectively. β_Z is the effective contact rate with the probability of susceptible humans been infected with Zika virus per sexual contact with humans infected with Zika virus (I_Z), the effective probability of the contact rates of susceptible humans infected with both Zika and Chikungunya virus per capital contact with humans infected with both Zika and Chikungunya virus (I_{ZC}) is denoted by (β_{ZC}). β_{MZ} and β_{MC} are the effective contact rates of the susceptible mosquitoes been infected with Zika and Chikungunya virus after a successful contact with humans infected with Zika (I_{MZ}) and humans infected with Chikungunya virus (I_{MC}) respectively. Similarly, β_{MZC} is the effective probability of the contact rates of susceptible mosquitoes been infected with both Zika and Chikungunya virus per capital contact with humans infected with both Zika and Chikungunya virus (I_{ZC}). σ_1, σ_3 and σ_5 are the progression rates of exposed individuals to Zika, Chikungunya virus and both Zika and Chikungunya virus to an infected humans with Zika, Chikungunya and both Zika and Chikungunya co-infection class respectively. Similarly, α_1, α_3 and ψ_2 are the recovery parameters of the infected class of humans with Zika , Chikungunya and co-infection of both Zika and Chikungunya virus respectively. θ_1 is the progression rate from the infected humans with Zika virus (I_Z) to the co-infected class of Zika and Chikungunya virus (I_{ZC}), η_1 is the progression rate from the infected humans with Chikungunya virus (I_C) to the co-infected class of

Zika and Chikungunya virus (I_{ZC}) and the natural death for human (mosquitoes) is $\mu_H(\mu_M)$. $\delta_Z(\delta_C)$ are the respective disease induced death for Zika and Chikungunya virus.

2.1 Model Assumptions:

(i) Mosquitoes can transmit only one disease after each interaction with humans who are susceptible to it [37].

(ii) The model exclusively accounts for Zika virus transmission through vectors and sexual contact, excluding vertical transmission [37, 38].

(iii) Individuals who have fully recovered from both Zika and Chikungunya viruses are believed to have lifelong immunity [39].

(iv) The model incorporates control options for co-infections of Zika and Chikungunya viruses [37].

(v) It is not possible for individuals to be simultaneously infected with both Zika and Chikungunya viruses [37].

(vi) Individuals have the ability to fully recover from either the Zika or Chikungunya virus [39].

2.2 Model equations

$$\frac{dS_H}{dt} = \Lambda_H - \left[\frac{m\beta_{HZ}I_{MZ} + \beta_Z I_Z}{N_H}\right]S_H - \left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H - \mu_H S_H,$$

$$\frac{dE_Z}{dt} = \left[\frac{m\beta_{HZ}I_{MZ} + \beta_Z I_Z}{N_H}\right]S_H - \left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z - (\sigma_1 + \mu_H)E_Z,$$

$$\frac{dE_C}{dt} = \left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z - \left[\frac{m\beta_{HZ}I_{MZ} + \beta_Z I_Z}{N_H}\right]S_H - (\sigma_3 + \mu_H)E_C,$$

$$\frac{dE_{ZC}}{dt} = \left[\frac{m\beta_{HZ}I_{MZ} + \beta_Z I_Z}{N_H}\right]S_H + \left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z - (\sigma_5 + \mu_H)E_{ZC},$$

$$\frac{dI_Z}{dt} = \sigma_1 E_Z - (\alpha_1 + \eta_1 + \delta_Z + \mu_H)I_Z,$$

$$\frac{dI_C}{dt} = \sigma_3 E_C - (\alpha_3 + \theta_1 + \delta_C + \mu_H)I_C,$$

$$\frac{d_{ZC}}{dt} = \sigma_5 E_{ZC} - (\psi_1 + \delta_{ZC} + \mu_H)I_{ZC}, \qquad 1$$

$$\frac{dR}{dt} = \alpha_1 I_Z + \alpha_3 I_C + \psi_1 I_{ZC} - \mu_H R,$$

$$\frac{dS_M}{dt} = \Lambda_M - \left[\frac{m\beta_{HZ} I_Z}{N_H}\right] S_M - \left[\frac{m\beta_{MC} I_C}{N_H}\right] S_M - \left[\frac{m\beta_{MZC} I_{ZC}}{N_H}\right] S_M - \mu_M S_M,$$

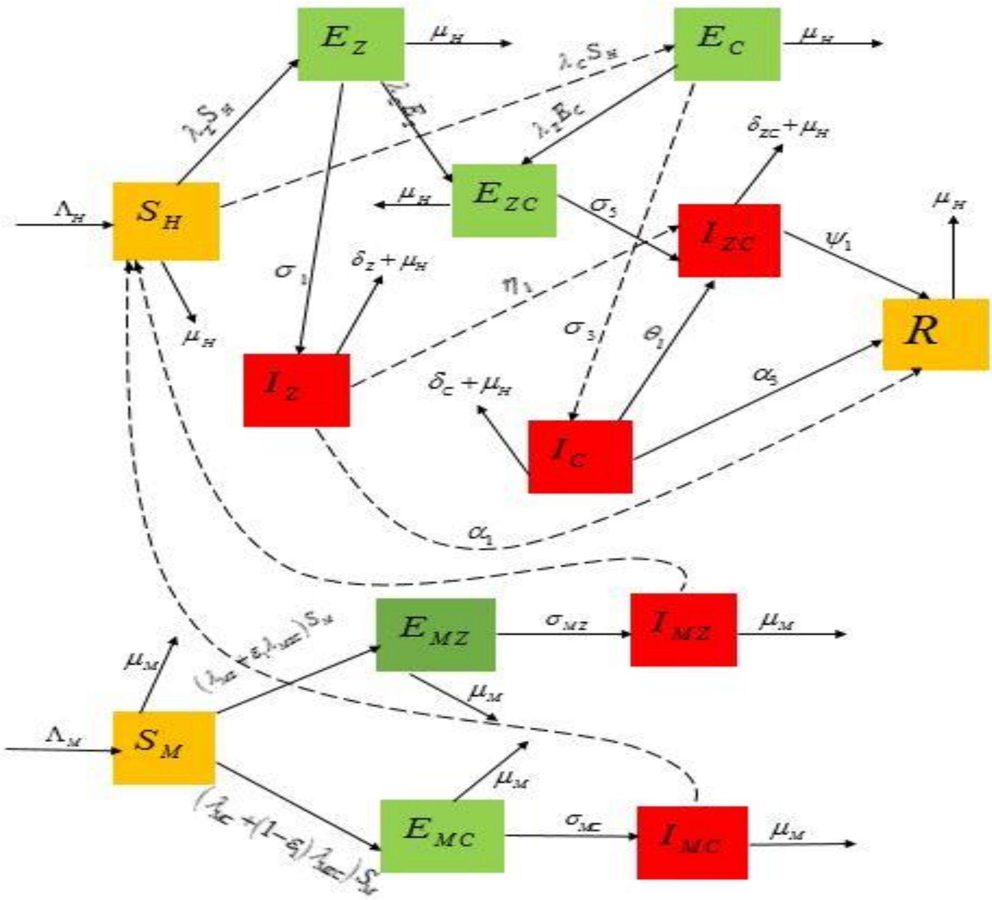
$$\frac{dE_{MZ}}{dt} = \left[\frac{m\beta_{HZ} I_Z}{N_H}\right] S_M - \left[\frac{\varepsilon_1 m\beta_{MZC} I_{ZC}}{N_H}\right] S_M - (\sigma_{MZ} + \mu_M) E_{MZ},$$

$$\frac{dE_{MC}}{dt} = \left[\frac{m\beta_{MC} I_C}{N_H}\right] S_M - \left[\frac{(1-\varepsilon_1) m\beta_{MZC} I_{ZC}}{N_H}\right] S_M - (\sigma_{MC} + \mu_M) E_{MC},$$

$$\frac{dI_{MZ}}{dt} = \sigma_{MZ} - \mu_M I_{MZ},$$

$$\frac{dI_{MC}}{dt} = \sigma_{MC} - \mu_M I_{MC},$$

2.3 Model flow diagram



Variables	***Descriptions***

N_H	Total human population
$S_H(S_M)$	Susceptible humans (Mosquito) to both diseases
$E_Z(E_C)$	Humans exposed to both Zika and Chikungunya virus
E_{ZC}	Exposed Human to both ZIKV and CHIKV respectively
$I_Z(I_C)$	HumanInfected with ZIKV and CHIKV
I_{ZC}	Infectious Human with ZIKV and CHIKV respectively.
R	Recovery class of humans
$E_{MZ}(E_{MC})$	Exposed mosquito to Zika virus and Chikungunya virus
$I_{MZ}(I_{MC})$	Infectious Mosquito with Zika virus (Chikungunya virus
Parameters	***Interpretation***
$\Lambda_H(\Lambda_M)$	Recruitment rate for Human(Mosquito) population
m	Mosquito biting rate
$\beta_{MZ}(\beta_{MC})$	Transmission probability from I_{MZ} and I_{MC} to S_H respectively
β_{MZC}	Transmission probability from $I_{MZC,}$ and I_{MDC} to S_H respectively
$\mu_H(\mu_M)$	Natural death rate of Human (Mosquito)
ε_1	Proportion of Mosquito to E_{mz}
$\sigma_{MZ}(\sigma_{MC})$	Progression rate from E_{MZ} and E_{MC} class to I_{MZ} and I_{MC} respectively.
$\sigma_1(\sigma_3)$	Progression rate from Exposed human with ZIKV and CHIKV to Infected Human with ZIKV respectively.
η_1	progression rate from Infected Human with ZIKV to Infected Human with both ZIKV and DF
θ_1	progression rate from Infected Human with CHIKV to Infected Human with both ZIKV and CHIKV
$\delta_H(\delta_Z,\delta_C)$	Disease induced death rate for humans with ZIKV (CHIKV)

δ_{ZC}	Disease induced death rate for humans with ZIKV and CHIKV
ψ_1	Recovery rate of infectious Human with I_{zc}
$\alpha_1(\alpha_3)$	Recovery rate of infectious Human with ZIKV and CHIKV respectively
σ_5	Transmission rate from exposed Human with both ZIKV and CHIKV to infected Human with both ZIKV and CHIKV

Table 1: Variables and Parameters descriptions

3.0 FRACTIONAL ORDER ZIKA-CHIKUNGUNYA MODEL

The Caputo derivative is considered a differential operator in this model. In the Caputo fractional initial value problem, the initial condition can be described with an initial integer order, whose physical interpretation is easy for real-life applications; consequently, it is ideal for the Zika-Chikungunya virus co-infection model. We now offer in this part some of the key definitions that we will be using throughout this work.

3.1 Definitions of basic terms

Definition1. The Caputo fractional order derivative of a function (f) on the interval $[0,T]$ is defined as follows:

$$\left[{}^{c}D_0^{\gamma} f(t)\right] = \frac{1}{\Gamma(n-\gamma)} \int_0^t (t-s)^{n-\gamma-1} f^{(n)}(s)ds, \tag{2}$$

Where $n=[\gamma]+1$ and $[\gamma]$ represent the integer parts of γ. Particularly, $0<\gamma<1$, the Caputo derivatives becomes as follows:

$$\left[{}^{c}D_0^{\gamma} f(t)\right] = \frac{1}{\Gamma(n-\gamma)} \int_0^t \frac{f(s)}{(t-s)^{\gamma}} ds, \tag{3}$$

Definition2: Laplace transform of Caputo derivatives is defined as follows:

$$\mathrm{L}\left[{}^{c}D^{\gamma} p(\mathrm{t})\right] = S^{\gamma} h(\mathrm{s}) - \sum_{k=0}^{n} S^{\gamma-i-1} y^{k}(0),\ n-1<\gamma<n,\ \forall n \in \mathrm{N}, \tag{4}$$

For arbitrary $c_i \in R$, $i = 0,1,2,\dots,n-1$, where $n=[\gamma]+1$ and $[\gamma]$ represent the non-integer parts of γ.

Lemma1. The following results holds for fractional differential equations:

$$I^{\gamma}\left[{}^{c}D^{\gamma}h\right](t)=h(t)+\sum_{k=0}^{n-1}\frac{h^{(!)}(0)}{i!}t^{!}, \quad (5)$$

For arbitrary $c_i \in R$, $i=0,1,2,...,n-1$, where $n=[\gamma]+1$ and $[\gamma]$ represent the non-integer parts of γ.Introducing the fractional order into the system (1), we now present a new system described by the following sets of fractional equations of order (γ), for $0<\gamma<1$.

$$D^{\gamma}\left(S_H\right)=\Lambda_H-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H-\mu_H S_H,$$

$$D^{\gamma}\left(E_Z\right)=\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_1+\mu_H\right)E_Z,$$

$$D^{\gamma}\left(E_C\right)=\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left(\sigma_3+\mu_H\right)E_C,$$

$$D^{\gamma}\left(E_{ZC}\right)=\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H+\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_5+\mu_H\right)E_{ZC},$$

$$D^{\gamma}\left(I_Z\right)=\sigma_1 E_Z-\left(\alpha_1+\eta_1+\delta_Z+\mu_H\right)I_Z,$$

$$D^{\gamma}\left(I_C\right)=\sigma_3 E_C-\left(\alpha_3+\theta_1+\delta_C+\mu_H\right)I_C,$$

$$D^{\gamma}\left(I_{ZC}\right)=\sigma_5 E_{ZC}-\left(\psi_1+\delta_{ZC}+\mu_H\right)I_{ZC},$$

$$D^{\gamma}\left(R\right)=\alpha_1 I_Z+\alpha_3 I_C+\psi_1 I_{ZC}-\mu_H R,$$

$$D^{\gamma}\left(S_M\right)=\Lambda_M-\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M-\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M-\mu_M S_M,$$

$$D^{\gamma}\left(E_{MZ}\right)=\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M+\left[\frac{m\varepsilon_1\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ},$$

$$D^{\gamma}\left(E_{MC}\right)=\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M+\left[\frac{m\left(1-\varepsilon_1\right)\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ},$$

$$D^{\gamma}\left(I_{MZ}\right)=\sigma_{MZ}-\mu_M I_{MZ},$$

$$D^{\gamma}\left(I_{MC}\right)=\sigma_{MC}-\mu_M I_{MC}, \quad (6)$$

3.2 Application of Laplace Adomian Decomposition Method (LADM)

This section discusses the procedures for LADM with the given initial conditions. Applying Laplace transform to both sides of system (6), we obtain the following system of differential equations:

$$S^{\gamma}\mathrm{L}\left(S_H\right)-S^{\gamma-1}S_H(0)=\mathrm{L}\left[\Lambda_H-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H-\mu_H S_H\right],$$

$$S^{\gamma}\mathrm{L}\left(E_Z\right)-S^{\gamma-1}E_Z(0)=\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_1+\mu_H\right)E_Z\right],$$

$$S^{\gamma}\mathrm{L}\left(E_C\right)-S^{\gamma-1}E_C(0)=\mathrm{L}\left[\left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]E_C-\left(\sigma_3+\mu_H\right)E_C\right],$$

$$S^{\gamma}\mathrm{L}\left(E_{ZC}\right)-S^{\gamma-1}E_{ZC}(0)=\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]E_C+\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_5+\mu_H\right)E_{ZC}\right],$$

$$S^{\gamma}\mathrm{L}\left(I_Z\right)-S^{\gamma-1}I_Z(0)=\mathrm{L}\left[\sigma_1 E_Z-\left(\alpha_1+\eta_1+\delta_Z+\mu_H\right)I_Z\right],$$

$$S^{\gamma}\mathrm{L}\left(I_C\right)-S^{\gamma-1}I_C(0)=\mathrm{L}\left[\sigma_3 E_C-\left(\alpha_3+\theta_1+\delta_C+\mu_H\right)I_C\right],$$

$$S^{\gamma}\mathrm{L}\left(I_{ZC}\right)-S^{\gamma-1}\,\mathrm{I}_{ZC}(0)=\mathrm{L}\left[\sigma_5 E_{ZC}-\left(\psi_1+\delta_{ZC}+\mu_H\right)I_{ZC}\right],$$

$$S^{\gamma}\mathrm{L}\left(R\right)-S^{\gamma-1}\,\mathrm{R}(0)=\mathrm{L}\left[\alpha_1 I_Z+\alpha_3 I_C+\psi_1 I_{ZC}-\mu_H R\right],$$

$$S^{\gamma}\mathrm{L}\left(S_M\right)-S^{\gamma-1}S_M(0)=\mathrm{L}\left[\Lambda_M-\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M-\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M-\mu_M S_M\right],$$

$$S^{\gamma}\mathrm{L}\left(E_{MZ}\right)-S^{\gamma-1}E_{MZ}(0)=\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M+\left[\frac{m\varepsilon_1\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ}\right],$$

$$S^{\gamma}\mathrm{L}\left(E_{MC}\right)-S^{\gamma-1}E_{MC}(0)=\mathrm{L}\left[\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M+\left[\frac{m\left(1-\varepsilon_1\right)\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ}\right],$$

$$S^{\gamma}\mathrm{L}\left(I_{MZ}\right)-S^{\gamma-1}\,\mathrm{I}_{MZ}(0)=\mathrm{L}\left[\sigma_{MZ}-\mu_M I_{MZ}\right],$$

$$S^{\gamma}\mathrm{L}\left(I_{MC}\right)-S^{\gamma-1}I_{MC}(0)=\mathrm{L}\left[\sigma_{MC}-\mu_M I_{MC}\right], \quad (7)$$

With initial conditions

$S_H(0)=n_1,\ I_{MZ}(0)=\mathrm{n}_{12}$,

$E_Z(0)=n_2,\ I_{MC}(0)=\mathrm{n}_{13}$

$E_C(0)=n_3,$

$E_{ZC}(0)=\mathrm{n}_4,$

$I_Z(0)=\mathrm{n}_5,$

$I_C(0)=\mathrm{n}_6,$

$I_{ZC}(0)=\mathrm{n}_7,$

$R(0)=\mathrm{n}_8,$

$S_M(0)=\mathrm{n}_9,$

$E_{MZ}(0)=\mathrm{n}_{10},$

$E_{MC}(0)=\mathrm{n}_{11},,$ (8)

Dividing (7) by S^γ gives

$$\mathrm{L}\left(S_H\right)(t)=\frac{n_1}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\Lambda_H-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H-\mu_H S_H\right],$$

$$\mathrm{L}\left(E_Z\right)(t)=\frac{n_2}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]S_H-\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_1+\mu_H\right)E_Z\right],$$

$$\mathrm{L}\left(E_C\right)(t)=\frac{n_3}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\left[\frac{m\beta_C I_{MC}}{N_H}\right]S_H-\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]E_C-\left(\sigma_3+\mu_H\right)E_C\right],$$

$$\mathrm{L}\left(E_{ZC}\right)(t)=\frac{n_4}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_{MZ}+\beta_Z I_Z}{N_H}\right]E_C+\left[\frac{m\beta_C I_{MC}}{N_H}\right]E_Z-\left(\sigma_5+\mu_H\right)E_{ZC}\right],$$

$$\mathrm{L}\left(I_Z\right)(t)=\frac{n_5}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\sigma_1 E_Z-\left(\alpha_1+\eta_1+\delta_Z+\mu_H\right)I_Z\right],$$

$$\mathrm{L}\left(I_C\right)(t)=\frac{n_6}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\sigma_3 E_C-\left(\alpha_3+\theta_1+\delta_C+\mu_H\right)I_C\right],$$

$$\mathrm{L}\left(I_{ZC}\right)(t)=\frac{n_7}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\sigma_5 E_{ZC}-\left(\psi_1+\delta_{ZC}+\mu_H\right)I_{ZC}\right],$$

$$\mathrm{L}\left(R\right)(t)=\frac{n_8}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\alpha_1 I_Z+\alpha_3 I_C+\psi_1 I_{ZC}-\mu_H R\right],$$

$$\mathrm{L}\left(S_M\right)(t)=\frac{n_9}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\Lambda_M-\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M-\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M-\mu_M S_M\right],$$

$$\mathrm{L}\left(E_{MZ}\right)(t)=\frac{n_{10}}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\left[\frac{m\beta_{HZ}I_Z}{N_H}\right]S_M+\left[\frac{m\varepsilon_1\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ}\right],$$

$$\mathrm{L}\left(E_{MC}\right)(t)=\frac{n_{11}}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\left[\frac{m\beta_{MC}I_C}{N_H}\right]S_M+\left[\frac{m\left(1-\varepsilon_1\right)\beta_{MZC}I_{ZC}}{N_H}\right]S_M-\left(\sigma_{MZ}+\mu_M\right)E_{MZ}\right],$$

$$\mathrm{L}\left(I_{MZ}\right)(t)=\frac{n_{12}}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\sigma_{MZ}-\mu_M I_{MZ}\right],$$

$$\mathrm{L}\left(I_{MC}\right)(t)=\frac{n_{13}}{S}+\frac{1}{S^\gamma}\mathrm{L}\left[\sigma_{MC}-\mu_M I_{MC}\right], \tag{9}$$

We decompose the non linear terms of system (5), we assume an infinite series solution of $S_H(\mathrm{t}), E_Z(\mathrm{t}), E_C(\mathrm{t}), E_{ZC}(\mathrm{t}), I_Z(\mathrm{t}), I_C(\mathrm{t}), I_{ZC}(\mathrm{t}), R(\mathrm{t}), S_M(\mathrm{t}), E_{MZ}(\mathrm{t}), E_{MC}(\mathrm{t}), I_{MZ}(\mathrm{t})$ and $I_{MC}(\mathrm{t})$ in form:

$$S_H(\mathrm{t})=\sum_{n=0}^{\infty}S_H(\mathrm{n}),\ I_{MZ}(\mathrm{t})=\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n}),$$

$$E_Z(\mathrm{t})=\sum_{n=0}^{\infty}E_Z(\mathrm{n}),\ I_{MC}(\mathrm{t})=\sum_{n=0}^{\infty}I_{MC}(\mathrm{n}),$$

$$E_C(\mathrm{t})=\sum_{n=0}^{\infty}E_C(\mathrm{n}),$$

$$E_{ZC}(\mathrm{t})=\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n}),$$

$$I_Z(\mathrm{t})=\sum_{n=0}^{\infty}I_Z(\mathrm{n}),$$

$$I_C(\mathrm{t})=\sum_{n=0}^{\infty}I_C(\mathrm{n}),$$

$$I_{ZC}(\mathrm{t})=\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n}),$$

$$R(\mathrm{t})=\sum_{n=0}^{\infty}R(\mathrm{n}),$$

$$S_M(\mathrm{t})=\sum_{n=0}^{\infty}S_M(\mathrm{n}),$$

$$E_{MZ}(\mathrm{t})=\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n}),$$

$$E_{MC}(\mathrm{t})=\sum_{n=0}^{\infty}E_{MC}(\mathrm{n}), \tag{10}$$

Thus, the nonlinear terms in system (1) are :

$I_{MZ}S_H(\mathrm{t})$, $I_ZS_H(\mathrm{t})$, $I_{MC}S_H(\mathrm{t})$, $I_CS_M(\mathrm{t})$, $I_ZS_M(\mathrm{t})$ and $I_{MZC}S_M(\mathrm{t})$

(11)

The nonlinear terms in (11) are decomposed by the Adomian polynomial in the following form

$$I_{MZ}S_H(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{P}(\mathrm{n}),$$

$$I_ZS_H(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{Q}(\mathrm{n}),$$

$$I_{MC}S_H(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{R}(\mathrm{n}),$$

$$I_CS_M(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{T}(\mathrm{n}),$$

$$I_ZS_M(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{U}(\mathrm{n}),$$

$$I_{MZC}S_M(\mathrm{t})=\sum_{n=0}^{\infty}\mathrm{V}(\mathrm{n}), \tag{12}$$

Where $M(\mathrm{n})$, $\mathrm{N}(\mathrm{n})$, $\mathrm{O}(\mathrm{n})$, $\mathrm{P}(\mathrm{n})$, $\mathrm{Q}(\mathrm{n})$ and $M(\mathrm{n})$ are the Adomian polynomial defined as follows:

$$P(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_{MZ}(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_H(\mathrm{k})\right\}_{|\lambda=0},$$

$$Q(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_Z(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_H(\mathrm{k})\right\}_{|\lambda=0},$$

$$R(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_{MC}(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_H(\mathrm{k})\right\}_{|\lambda=0},$$

$$\mathrm{T}(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_{Z}(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_M(\mathrm{k})\right\}_{|\lambda=0},$$

$$U(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_{C}(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_M(\mathrm{k})\right\}_{|\lambda=0},$$

$$V(\mathrm{n})=\frac{1}{\Gamma(\mathrm{n}+1)}\frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n}\lambda^k I_{Z}(\mathrm{k})\sum_{k=0}^{n}\lambda^k S_M(\mathrm{k})\right\}_{|\lambda=0}, \tag{13}$$

We substitute (9) $n=0$ into (10) and (12) to obtain the following:

$$\mathrm{L}S_H(0)=\frac{n_1}{S}, \qquad \mathrm{L}R(0)=\frac{\mathrm{n}_8}{S},$$

$$\mathrm{L}E_Z(0)=\frac{n_2}{S}, \qquad \mathrm{L}S_M(0)=\frac{\mathrm{n}_9}{S},$$

$$\mathrm{L}E_C(0)=\frac{n_3}{S}, \qquad \mathrm{L}E_{MZ}(0)=\frac{\mathrm{n}_{10}}{S},$$

$$\mathrm{L}E_{ZC}(0)=\frac{\mathrm{n}_4}{S}, \qquad \mathrm{L}E_{MC}(0)=\frac{\mathrm{n}_{11}}{S},$$

$$\mathrm{L}I_Z(0)=\frac{\mathrm{n}_5}{S}, \qquad \mathrm{L}I_{MZ}(0)=\frac{\mathrm{n}_{12}}{S},$$

$$\mathrm{L}I_C(0)=\frac{\mathrm{n}_6}{S}, \qquad \mathrm{L}I_{MC}(0)=\frac{\mathrm{n}_{13}}{S},$$

$$\mathrm{L}I_{ZC}(0)=\frac{\mathrm{n}_7}{S}, \tag{14}$$

Similarly for $n=1$ and $n=n+1$ we have the following solutions

$$P(0)=\mathrm{I}_{MZ}(0)\mathrm{S}_H(0),$$

$$\mathrm{P}(1)=\mathrm{I}_{MZ}(0)\mathrm{S}_H(1)+\mathrm{I}_{MZ}(1)\mathrm{S}_H(0),$$

$$\mathrm{P}(2)=\mathrm{I}_{MZ}(0)\mathrm{S}_H(0)+\mathrm{I}_{MZ}(1)\mathrm{S}_H(1)+\mathrm{I}_{MZ}(2)\mathrm{S}_H(0),$$

$$\mathrm{Q}(0)=\mathrm{I}_Z(0)\mathrm{S}_H(0),$$

$\mathrm{Q}(1) = \mathrm{I}_Z(0)\mathrm{S}_H(1) + \mathrm{I}_Z(1)\mathrm{S}_H(0)$,

$\mathrm{Q}(2) = \mathrm{I}_Z(0)\mathrm{S}_H(0) + \mathrm{I}_Z(1)\mathrm{S}_H(1) + \mathrm{I}_Z(2)\mathrm{S}_H(0)$,

$\mathrm{R}(0) = \mathrm{I}_{MC}(0)\mathrm{S}_H(0)$,

$\mathrm{R}(1) = \mathrm{I}_{MC}(0)\mathrm{S}_H(1) + \mathrm{I}_{MC}(1)\mathrm{S}_H(0)$,

$\mathrm{R}(2) = \mathrm{I}_{MC}(0)\mathrm{S}_H(0) + \mathrm{I}_{MC}(1)\mathrm{S}_H(1) + \mathrm{I}_{MC}(2)\mathrm{S}_H(0)$,

$\mathrm{T}(0) = \mathrm{I}_Z(0)\mathrm{S}_M(0)$,

$\mathrm{T}(1) = \mathrm{I}_Z(0)\mathrm{S}_M(1) + \mathrm{I}_Z(1)\mathrm{S}_M(0)$,

$\mathrm{T}(2) = \mathrm{I}_Z(0)\mathrm{S}_M(0) + \mathrm{I}_Z(1)\mathrm{S}_M(1) + \mathrm{I}_Z(2)\mathrm{S}_M(0)$,

$\mathrm{U}(0) = \mathrm{I}_C(0)\mathrm{S}_M(0)$,

$\mathrm{U}(1) = \mathrm{I}_C(0)\mathrm{S}_M(1) + \mathrm{I}_C(1)\mathrm{S}_M(0)$,

$\mathrm{U}(2) = \mathrm{I}_C(0)\mathrm{S}_M(0) + \mathrm{I}_C(1)\mathrm{S}_M(1) + \mathrm{I}_M(2)\mathrm{S}_M(0)$,

$\mathrm{V}(0) = \mathrm{I}_{ZC}(0)\mathrm{S}_M(0)$,

$\mathrm{V}(1) = \mathrm{I}_{ZC}(0)\mathrm{S}_M(1) + \mathrm{I}_{ZC}(1)\mathrm{S}_M(0)$, $\quad \mathrm{V}(2) = \mathrm{I}_{ZC}(0)\mathrm{S}_M(0) + \mathrm{I}_{ZC}(1)\mathrm{S}_M(1) + \mathrm{I}_{ZC}(2)\mathrm{S}_M(0)$

, (15)

Substituting (10) and (12) into (9)

$$\mathrm{L}\left(\sum_{n=0}^{\infty} S_H(\mathrm{n})\right) = \frac{n_1}{S} + \frac{1}{S^{\gamma}}\mathrm{L}\left[\Lambda_H - \left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} P(\mathrm{n})\right) + \beta_Z\left(\sum_{n=0}^{\infty} Q(\mathrm{n})\right)}{N_H}\right] - \left[\frac{m\beta_C \sum_{n=0}^{\infty} R(\mathrm{n})}{N_H}\right] - \mu_H \sum_{n=0}^{\infty} S_H(\mathrm{n})\right]$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_Z(\mathrm{n})\right) = \frac{n_2}{S} + \frac{1}{S^{\gamma}}\mathrm{L}\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} P(\mathrm{n})\right) + \beta_Z\left(\sum_{n=0}^{\infty} Q(\mathrm{n})\right)}{N_H}\right] - \left[\frac{m\beta_C \sum_{n=0}^{\infty} I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty} E_Z(\mathrm{n}) - \left(\sigma_1 + \mu_H\right)E_Z\right]$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty}E_C(\mathrm{n})\right)=\frac{n_3}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\left[\left[\frac{m\beta_C\sum_{n=0}^{\infty}R(\mathrm{n})}{N_H}\right]-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}I_Z(\mathrm{n})\right)}{N_H}\right]\sum_{n=0}^{\infty}E_C(\mathrm{n})-\left(\sigma_3+\mu_H\right)\sum_{n=0}^{\infty}E_Z(\mathrm{n})\right]$$

$$\left(\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n})\right)=\frac{n_4}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}I_Z(\mathrm{n})\right)}{N_H}\right]\sum_{n=0}^{\infty}E_C(\mathrm{n})+\left[\frac{m\beta_C\sum_{n=0}^{\infty}I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty}E_Z(\mathrm{n})-\left(\sigma_5+\mu_H\right)\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n})\right]$$

$$\left(\sum_{n=0}^{\infty}I_Z(\mathrm{n})\right)=\frac{n_5}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\sigma_1\sum_{n=0}^{\infty}E_Z(\mathrm{n})-\left(\alpha_1+\eta_1+\delta_Z+\mu_H\right)\sum_{n=0}^{\infty}I_Z(\mathrm{n})\right],$$

$$\left(\sum_{n=0}^{\infty}I_C(\mathrm{n})\right)=\frac{n_6}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\sigma_3\sum_{n=0}^{\infty}E_C(\mathrm{n})-\left(\alpha_3+\theta_1+\delta_C+\mu_H\right)\sum_{n=0}^{\infty}I_C(\mathrm{n})\right],$$

$$\left(\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n})\right)=\frac{n_7}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\sigma_5\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n})-\left(\psi_1+\delta_{ZC}+\mu_H\right)\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n})\right],$$

$$\left(\sum_{n=0}^{\infty}R(\mathrm{n})\right)=\frac{n_8}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\alpha_1\sum_{n=0}^{\infty}I_Z(\mathrm{n})+\alpha_3\sum_{n=0}^{\infty}I_C(\mathrm{n})+\psi_1\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n})-\mu_H\left(\sum_{n=0}^{\infty}R(\mathrm{n})\right)\right],$$

$$\left(\sum_{n=0}^{\infty}S_M(\mathrm{n})\right)=\frac{n_9}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\Lambda_M-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}T(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty}U(\mathrm{n})\right)}{N_H}\right]-\mu_M\sum_{n=0}^{\infty}S_M(\mathrm{n})\right],$$

$$\left(\sum_{n=0}^{\infty}E_{MZ}(\mathrm{n})\right)=\frac{n_{10}}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}T(\mathrm{n})\right)}{N_H}\right]+\left[\frac{m\varepsilon_1\beta_{MZC}\left(\sum_{n=0}^{\infty}V(\mathrm{n})\right)}{N_H}\right]-\left(\sigma_{MZ}+\mu_M\right)\sum_{n=0}^{\infty}E_{MZ}(\mathrm{n})\right]$$

,

$$\left(\sum_{n=0}^{\infty}E_{MC}(\mathrm{n})\right)=\frac{n_{11}}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\left[\left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty}U(\mathrm{n})\right)}{N_H}\right]+\left[\frac{m\left(1-\varepsilon_1\right)\beta_{MZC}\left(\sum_{n=0}^{\infty}V(\mathrm{n})\right)}{N_H}\right]-\left(\sigma_{MZ}+\mu_M\right)\sum_{n=0}^{\infty}E_{MC}(\mathrm{n})\right]$$

,

$$\left(\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n})\right)=\frac{n_{12}}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\sigma_{MZ}-\mu_M\sum_{n=0}^{\infty}I_{MZ}(\mathrm{n})\right],$$

$$\left(\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})\right)=\frac{n_{13}}{S}+\frac{1}{S^{\gamma}}\mathrm{L}\left[\sigma_{MC}-\mu_M\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})\right], \tag{16}$$

Evaluating the Laplace transform of the second terms on the RHS of equation (16)

$$\mathrm{L}\left(\sum_{n=0}^{\infty} S_H(\mathrm{n})\right)=\frac{n_1}{S}+\left[\Lambda_H-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}Q(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_C\sum_{n=0}^{\infty}R(\mathrm{n})}{N_H}\right]-\mu_H\sum_{n=0}^{\infty}S_H(\mathrm{n})\right]\frac{1}{S^{\gamma+1}}$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_Z(\mathrm{n})\right)=\frac{n_2}{S}+\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}Q(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_C\sum_{n=0}^{\infty}I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty}E_Z(\mathrm{n})-(\sigma_1+\mu_H)E_Z\right]\frac{1}{S^{\gamma+1}}$$

,

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_C(\mathrm{n})\right)=\frac{n_3}{S}+\left[\left[\frac{m\beta_C\sum_{n=0}^{\infty}I_{MC}(\mathrm{n})\sum_{n=0}^{\infty}E_Z(\mathrm{n})}{N_H}\right]-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}Q(\mathrm{n})\right)}{N_H}\right]-(\sigma_3+\mu_H)E_C\right]\frac{1}{S^{\gamma+1}}$$

,

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_{ZC}(\mathrm{n})\right)=\frac{n_4}{S}+\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty}Q(\mathrm{n})\right)}{N_H}\right]+\left[\frac{m\beta_C\sum_{n=0}^{\infty}I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty}E_Z(\mathrm{n})-(\sigma_5+\mu_H)\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}}$$

,

$$\mathrm{L}\left(\sum_{n=0}^{\infty} I_Z(\mathrm{n})\right)=\frac{n_5}{S}+\left[\sigma_1\sum_{n=0}^{\infty}E_Z(\mathrm{n})-(\alpha_1+\eta_1+\delta_Z+\mu_H)\sum_{n=0}^{\infty}I_Z(\mathrm{n})\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} I_C(\mathrm{n})\right)=\frac{n_6}{S}+\left[\sigma_3\sum_{n=0}^{\infty}E_C(\mathrm{n})-(\alpha_3+\theta_1+\delta_C+\mu_H)\sum_{n=0}^{\infty}I_C(\mathrm{n})\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} I_{ZC}(\mathrm{n})\right)=\frac{n_7}{S}+\left[\sigma_5\sum_{n=0}^{\infty}E_{ZC}(\mathrm{n})-(\psi_1+\delta_{ZC}+\mu_H)\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} R(\mathrm{n})\right)=\frac{n_8}{S}+\left[\alpha_1\sum_{n=0}^{\infty}I_Z(\mathrm{n})+\alpha_3\sum_{n=0}^{\infty}I_C(\mathrm{n})+\psi_1\sum_{n=0}^{\infty}I_{ZC}(\mathrm{n})-\mu_H\left(\sum_{n=0}^{\infty}R(\mathrm{n})\right)\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} S_M(\mathrm{n})\right)=\frac{n_9}{S}+\left[\Lambda_M-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty}T(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty}U(\mathrm{n})\right)}{N_H}\right]-\mu_M\sum_{n=0}^{\infty}S_M(\mathrm{n})\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_{MZ}(\mathrm{n})\right)=\frac{n_{10}}{S}+\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} T(\mathrm{n})\right)}{N_H}\right]+\left[\frac{m\varepsilon_1\beta_{MZC}\left(\sum_{n=0}^{\infty} V(\mathrm{n})\right)}{N_H}\right]-\left(\sigma_{MZ}+\mu_M\right)\sum_{n=0}^{\infty} E_{MZ}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}}$$

,

$$\mathrm{L}\left(\sum_{n=0}^{\infty} E_{MC}(\mathrm{n})\right)=\frac{n_{11}}{S}+\left[\left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty} U(\mathrm{n})\right)}{N_H}\right]+\left[\frac{m\left(1-\varepsilon_1\right)\beta_{MZC}\left(\sum_{n=0}^{\infty} V(\mathrm{n})\right)}{N_H}\right]-\left(\sigma_{MZ}+\mu_M\right)\sum_{n=0}^{\infty} E_{MC}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}}$$

,

$$\mathrm{L}\left(\sum_{n=0}^{\infty} I_{MZ}(\mathrm{n})\right)=\frac{n_{12}}{S}+\left[\sigma_{MZ}-\mu_M\sum_{n=0}^{\infty} I_{MZ}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}},$$

$$\mathrm{L}\left(\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})\right)=\frac{n_{13}}{S}+\left[\sigma_{MC}-\mu_M\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})\right]\frac{1}{S^{\gamma+1}}, \tag{17}$$

Taking the inverse Laplace transform of both sides of (17)

$$\sum_{n=0}^{\infty} S_H(\mathrm{n})=n_1+\left[\Lambda_H-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty} Q(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_C\sum_{n=0}^{\infty} R(\mathrm{n})}{N_H}\right]-\mu_H\sum_{n=0}^{\infty} S_H(\mathrm{n})\right]\frac{t^{\gamma}}{\Gamma(\gamma+1)}$$

$$\sum_{n=0}^{\infty} E_Z(\mathrm{n})=n_2+\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} P(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty} Q(\mathrm{n})\right)}{N_H}\right]-\left[\frac{m\beta_C\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty} E_Z(\mathrm{n})-\left(\sigma_1+\mu_H\right)\sum_{n=0}^{\infty} E_Z(\mathrm{n})\right]\frac{t^{\gamma}}{\Gamma(\gamma+1)}$$

$$\sum_{n=0}^{\infty} E_C(\mathrm{n})=n_3+\left[\left[\frac{m\beta_C\sum_{n=0}^{\infty} R(\mathrm{n})}{N_H}\right]-\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} I_{MZ}(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty} I_Z(\mathrm{n})\right)}{N_H}\right]\sum_{n=0}^{\infty} E_C(\mathrm{n})-\left(\sigma_3+\mu_H\right)\sum_{n=0}^{\infty} E_C(\mathrm{n})\right]\frac{t^{\gamma}}{\Gamma(\gamma+1)}$$

$$\sum_{n=0}^{\infty} E_{ZC}(\mathrm{n})=n_4+\left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} I_{MZ}(\mathrm{n})\right)+\beta_Z\left(\sum_{n=0}^{\infty} I_Z(\mathrm{n})\right)}{N_H}\right]\sum_{n=0}^{\infty} E_C(\mathrm{n})+\left[\frac{m\beta_C\sum_{n=0}^{\infty} I_{MC}(\mathrm{n})}{N_H}\right]\sum_{n=0}^{\infty} E_Z(\mathrm{n})-\left(\sigma_5+\mu_H\right)\sum_{n=0}^{\infty} E_{ZC}(\mathrm{n})\right]\frac{t^{\gamma}}{\Gamma(\gamma+1)}$$

$$\sum_{n=0}^{\infty} I_Z(\mathrm{n}) = n_5 + \left[\sigma_1 \sum_{n=0}^{\infty} E_Z(\mathrm{n}) - (\alpha_1 + \eta_1 + \delta_Z + \mu_H) \sum_{n=0}^{\infty} I_Z(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$\sum_{n=0}^{\infty} I_C(\mathrm{n}) = n_6 + \left[\sigma_3 \sum_{n=0}^{\infty} E_C(\mathrm{n}) - (\alpha_3 + \theta_1 + \delta_C + \mu_H) \sum_{n=0}^{\infty} I_C(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$\sum_{n=0}^{\infty} I_{ZC}(\mathrm{n}) = n_7 + \left[\sigma_5 \sum_{n=0}^{\infty} E_{ZC}(\mathrm{n}) - (\psi_1 + \delta_{ZC} + \mu_H) \sum_{n=0}^{\infty} I_{ZC}(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$\sum_{n=0}^{\infty} R(\mathrm{n}) = n_8 + \left[\alpha_1 \sum_{n=0}^{\infty} I_Z(\mathrm{n}) + \alpha_3 \sum_{n=0}^{\infty} I_C(\mathrm{n}) + \psi_1 \sum_{n=0}^{\infty} I_{ZC}(\mathrm{n}) - \mu_H \left(\sum_{n=0}^{\infty} R(\mathrm{n})\right)\right] \frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$\sum_{n=0}^{\infty} S_M(\mathrm{n}) = n_9 + \left[\Lambda_M - \left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} T(\mathrm{n})\right)}{N_H}\right] - \left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty} U(\mathrm{n})\right)}{N_H}\right] - \left[\frac{m\beta_{MZC}\left(\sum_{n=0}^{\infty} V(\mathrm{n})\right)}{N_H}\right] - \mu_M \sum_{n=0}^{\infty} S_M(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)}$$

,

$$\sum_{n=0}^{\infty} E_{MZ}(\mathrm{n}) = n_{10} + \left[\left[\frac{m\beta_{HZ}\left(\sum_{n=0}^{\infty} T(\mathrm{n})\right)}{N_H}\right] + \left[\frac{m\varepsilon_1\beta_{MZC}\left(\sum_{n=0}^{\infty} V(\mathrm{n})\right)}{N_H}\right] - (\sigma_{MZ} + \mu_M) \sum_{n=0}^{\infty} E_{MZ}(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)}$$

,

$$\sum_{n=0}^{\infty} E_{MC}(\mathrm{n}) = n_{11} + \left[\left[\frac{m\beta_{MC}\left(\sum_{n=0}^{\infty} U(\mathrm{n})\right)}{N_H}\right] + \left[\frac{m(1-\varepsilon_1)\beta_{MZC}\left(\sum_{n=0}^{\infty} V(\mathrm{n})\right)}{N_H}\right] - (\sigma_{MZ} + \mu_M) \sum_{n=0}^{\infty} E_{MC}(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)}$$

,

$$\sum_{n=0}^{\infty} I_{MZ}(\mathrm{n}) = n_{12} + \left[\sigma_{MZ} - \mu_M \sum_{n=0}^{\infty} I_{MZ}(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$\sum_{n=0}^{\infty} I_{MC}(\mathrm{n}) = n_{13} + \left[\sigma_{MC} - \mu_M \sum_{n=0}^{\infty} I_{MC}(\mathrm{n})\right] \frac{t^\gamma}{\Gamma(\gamma+1)}, \tag{18}$$

Setting $n = 0$ and equating the corresponding terms in (18) yields

$$S_H(0) = \mathrm{n}_1,\ E_Z(0) = \mathrm{n}_2,\ E_C(0) = \mathrm{n}_3,\ E_{ZC}(0) = \mathrm{n}_4,\ I_Z(0) = \mathrm{n}_5,\ I_C(0) = \mathrm{n}_6,\ I_{ZC}(0) = \mathrm{n}_7,$$

$$R(0) = \mathrm{n}_8,\ S_M(0) = \mathrm{n}_9,\ E_{MZ}(0) = \mathrm{n}_{10}, E_{MC}(0) = \mathrm{n}_{11}, I_{MZ}(0) = \mathrm{n}_{12}, I_{MC}(0) = \mathrm{n}_{13}$$

For $n = 1$,

$$S_H(1)=\left[\Lambda_H-\left[\frac{m\beta_{HZ}P(0)+\beta_Z Q(0)+m\beta_C R(0)}{N_H}\right]-\mu_H S_H(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_Z(1)=\left[\frac{m\beta_{HZ}P(0)+\beta_Z Q(0)}{N_H}-\frac{m\beta_C\,\mathrm{I}_{MZ}(0)\,\mathrm{E}_Z(0)}{N_H}-(\sigma_1+\mu_H)E_Z(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_C(1)=\left[\frac{m\beta_C R(0)}{N_H}-\left[\frac{m\beta_{HZ}I_{MZ}(0)+\beta_Z I_Z(0)}{N_H}\right]E_C(0)-(\sigma_3+\mu_H)E_C(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{ZC}(1)=\left[\left[\frac{m\beta_{HZ}I_{MZ}(0)+\beta_Z I_Z(0)}{N_H}\right]E_C(0)+\frac{m\beta_C\,\mathrm{I}_{MZ}(0)\,\mathrm{E}_Z(0)}{N_H}-(\sigma_5+\mu_H)E_{ZC}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)}$$

,

$$I_Z(1)=\left[\sigma_1 E_Z(0)-(\alpha_1+\eta_1+\delta_Z+\mu_H)I_Z(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_C(1)=\left[\sigma_3 E_C(0)-(\alpha_3+\theta_1+\delta_C+\mu_H)I_C(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{ZC}(1)=\left[\sigma_5 E_{ZC}(0)+\eta_1 I_Z(0)+\theta_1 I_C(0)-(\psi_1+\delta_{ZC}+\mu_H)I_{ZC}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$R(1)=\left[\alpha_1 I_Z(0)+\alpha_3 I_C(0)+\psi_1 I_{ZC}(0)-\mu_H R(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$S_M(1)=\left[\Lambda_M-\left[\frac{m\beta_{MZ}\,\mathrm{T}(0)+m\beta_{MC}\,\mathrm{U}(0)+m\beta_{MZC}\,\mathrm{V}(0)}{N_H}\right]-\mu_M S_M(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{MZ}(1)=\left[\frac{m\beta_{MZ}T(0)}{N_H}+\frac{\varepsilon_1 m\beta_{MZC}\,\mathrm{V}(0)}{N_H}-(\sigma_{MZ}+\mu_M)E_{MZ}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{MC}(1)=\left[\frac{m\beta_{MC}\,\mathrm{U}(0)}{N_H}+\frac{(1-\varepsilon_1)m\beta_{MZC}\,\mathrm{V}(0)}{N_H}-(\sigma_{MC}+\mu_M)E_{MC}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{MZ}(1)=\left[\sigma_{MZ}E_{MZ}(0)-\mu_M I_{MZ}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{MC}(1)=\left[\sigma_{MC}E_{MC}(0)-\mu_M I_{MC}(0)\right]\frac{t^\gamma}{\Gamma(\gamma+1)}, \tag{19}$$

For $n=2$, we have,

$$S_H(2)=\left[\Lambda_H-\left[\frac{m\beta_{HZ}P(1)+\beta_Z Q(1)+m\beta_C R(1)}{N_H}\right]-\mu_H S_H\left(1\right)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$E_Z(2)=\left[\frac{m\beta_{HZ}P(1)+\beta_Z Q(1)}{N_H}-\frac{m\beta_C\,\mathrm{I}_{MZ}(1)\,\mathrm{E}_Z(1)}{N_H}-\left(\sigma_1+\mu_H\right)E_Z(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$E_C(2)=\left[\frac{m\beta_C R(1)}{N_H}-\left[\frac{m\beta_{HZ}I_{MZ}(1)+\beta_Z I_Z(1)}{N_H}\right]E_C(1)-\left(\sigma_3+\mu_H\right)E_C(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$E_{ZC}(2)=\left[\left[\frac{m\beta_{HZ}I_{MZ}(1)+\beta_Z I_Z(1)}{N_H}\right]E_C(1)+\frac{m\beta_C\,\mathrm{I}_{MZ}(1)\,\mathrm{E}_Z(1)}{N_H}-\left(\sigma_5+\mu_H\right)E_{ZC}(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)}$$

,

$$I_Z(2)=\left[\sigma_1 E_Z(1)-\left(\alpha_1+\eta_1+\delta_Z+\mu_H\right)I_Z(2)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$I_C(2)=\left[\sigma_3 E_C(1)-\left(\alpha_3+\theta_1+\delta_C+\mu_H\right)I_C(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$I_{ZC}(2)=\left[\sigma_5 E_{ZC}(1)+\eta_1 I_Z(1)+\theta_1 I_C(1)-\left(\psi_1+\delta_{ZC}+\mu_H\right)I_C(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$R(2)=\left[\alpha_1 I_Z(1)+\alpha_3 I_C(1)+\psi_1 I_{ZC}(1)-\mu_H R(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$S_M(2)=\left[\Lambda_M-\left[\frac{m\beta_{MZ}\,\mathrm{T}(1)+\beta_{MC}\,\mathrm{U}(1)+m\beta_{MZC}\,\mathrm{V}(1)}{N_H}\right]-\mu_M S_M\left(1\right)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$E_{MZ}(2)=\left[\frac{m\beta_{MZ}T(1)}{N_H}+\frac{\varepsilon_1 m\beta_{MZC}\,\mathrm{V}(1)}{N_H}-\left(\sigma_{MZ}+\mu_M\right)E_{MZ}(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$E_{MC}(2)=\left[\frac{m\beta_{MC}\,\mathrm{U}(1)}{N_H}+\frac{\left(1-\varepsilon_1\right)m\beta_{MZC}\,\mathrm{V}(1)}{N_H}-\left(\sigma_{MC}+\mu_M\right)E_{MC}(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$I_{MZ}(2)=\left[\sigma_{MZ}E_{MZ}(1)-\mu_M I_{MZ}(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)},$$

$$I_{MC}(2)=\left[\sigma_{MC}E_{MC}(1)-\mu_M I_{MC}(1)\right]\frac{t^{\gamma}}{\Gamma\left(\gamma+1\right)}, \qquad (20)$$

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$$S_H(\mathrm{n+1}) = \left[\Lambda_H - \left[\frac{m\beta_{HZ}P(\mathrm{n}) + \beta_Z Q(\mathrm{n}) + m\beta_C R(\mathrm{n})}{N_H}\right] - \mu_H S_H(n)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_Z(\mathrm{n+1}) = \left[\frac{m\beta_{HZ}P(\mathrm{n}) + \beta_Z Q(\mathrm{n})}{N_H} - \frac{m\beta_C\,\mathrm{I}_{MZ}(\mathrm{n})\,\mathrm{E}_Z(\mathrm{n})}{N_H} - (\sigma_1 + \mu_H)E_Z(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_C(\mathrm{n+1}) = \left[\frac{m\beta_C R(\mathrm{n})}{N_H} - \left[\frac{m\beta_{HZ}I_{MZ}(\mathrm{n}) + \beta_Z I_Z(\mathrm{n})}{N_H}\right]E_C(\mathrm{n}) - (\sigma_3 + \mu_H)E_C(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{ZC}(\mathrm{n+1}) = \left[\left[\frac{m\beta_{HZ}I_{MZ}(\mathrm{n}) + \beta_Z I_Z(\mathrm{n})}{N_H}\right]E_C(\mathrm{n}) + \frac{m\beta_C\,\mathrm{I}_{MZ}(\mathrm{n})\,\mathrm{E}_Z(\mathrm{n})}{N_H} - (\sigma_5 + \mu_H)E_{ZC}(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)}$$

,

$$I_Z(\mathrm{n+1}) = \left[\sigma_1 E_Z(\mathrm{n}) - (\alpha_1 + \eta_1 + \delta_Z + \mu_H)I_Z(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_C(\mathrm{n+1}) = \left[\sigma_3 E_C(\mathrm{n}) - (\alpha_3 + \theta_1 + \delta_C + \mu_H)I_C(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{ZC}(\mathrm{n+1}) = \left[\sigma_5 E_{ZC}(\mathrm{n}) + \eta_1 I_Z(\mathrm{n}) + \theta_1 I_C(\mathrm{n}) - (\psi_1 + \delta_{ZC} + \mu_H)I_C(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$R(\mathrm{n+1}) = \left[\alpha_1 I_Z(\mathrm{n}) + \alpha_3 I_C(\mathrm{n}) + \psi_1 I_{ZC}(\mathrm{n}) - \mu_H R(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$S_M(\mathrm{n+1}) = \left[\Lambda_M - \left[\frac{m\beta_{MZ}\,\mathrm{T}(\mathrm{n}) + \beta_{MC}\,\mathrm{U}(\mathrm{n}) + m\beta_{MZC}\,\mathrm{V}(\mathrm{n})}{N_H}\right] - \mu_M S_M(n)\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{MZ}(\mathrm{n+1}) = \left[\frac{m\beta_{MZ}T(1)}{N_H} + \frac{\varepsilon_1 m\beta_{MZC}\,\mathrm{V}(\mathrm{n})}{N_H} - (\sigma_{MZ} + \mu_M)E_{MZ}(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$E_{MC}(\mathrm{n+1}) = \left[\frac{m\beta_{MC}\,\mathrm{U}(\mathrm{n})}{N_H} + \frac{(1-\varepsilon_1)m\beta_{MZC}\,\mathrm{V}(\mathrm{n})}{N_H} - (\sigma_{MC} + \mu_M)E_{MC}(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{MZ}(\mathrm{n+1}) = \left[\sigma_{MZ}E_{MZ}(\mathrm{n}) - \mu_M I_{MZ}(\mathrm{n})\right]\frac{t^\gamma}{\Gamma(\gamma+1)},$$

$$I_{MC}(\mathrm{n+1}) = \left[\sigma_{MC}E_{MC}(\mathrm{n}) - \mu_M I_{MC}(\mathrm{n})\right]\frac{t^{\gamma}}{\Gamma(\gamma+1)}, \tag{21}$$

Hence, the series solution of each of the epidemiological classes is given below:

$$S_H(\mathrm{t}) = \mathrm{S}_H(0) + \mathrm{S}_H(1) + \mathrm{S}_H(2) + \ldots$$

$$\mathrm{E}_Z(\mathrm{t}) = \mathrm{E}_Z(0) + \mathrm{E}_Z(1) + \mathrm{E}_Z(2) + \ldots$$

$$\mathrm{E}_Z(\mathrm{t}) = \mathrm{E}_Z(0) + \mathrm{E}_Z(1) + \mathrm{E}_Z(2) + \ldots$$

$$\mathrm{E}_{ZC}(\mathrm{t}) = \mathrm{E}_{ZC}(0) + \mathrm{E}_{ZC}(1) + \mathrm{E}_{ZC}(2) + \ldots$$

$$I_Z(\mathrm{t}) = I_Z(0) + I_Z(1) + I_Z(2) + \ldots$$

$$I_C(\mathrm{t}) = I_C(0) + I_C(0) + I_C(2) + \ldots$$

$$I_{ZC}(\mathrm{t}) = I_{ZC}(0) + I_{ZC}(1) + I_{ZC}(2) + \ldots$$

$$\mathrm{R(t)} = R(0) + \mathrm{R}(1) + \mathrm{R}(2) + \ldots$$

$$\mathrm{S}_M(\mathrm{t}) = \mathrm{S}_M(0) + \mathrm{S}_M(1) + \mathrm{S}_M(2) + \ldots$$

$$\mathrm{E}_{MZ}(\mathrm{t}) = \mathrm{E}_{MZ}(0) + \mathrm{E}_{MZ}(1) + \mathrm{E}_{MZ}(2) + \ldots$$

$$\mathrm{E}_{MC}(\mathrm{t}) = E_{MC}(0) + \mathrm{E}_{MC}(0) + \mathrm{E}_{MC}(2) + \ldots$$

$$I_{MZ}(\mathrm{t}) = I_{MZ}(0) + I_{MZ}(1) + I_{MZ}(2) + \ldots$$

$$I_{MC}(\mathrm{t}) = I_{MC}(0) + I_{MC}(0) + I_{MC}(2) + \ldots \tag{22}$$

3.3 Numerical Solutions of Fractional order Derivatives of Zika-Chikungunya virus co-infection using Laplace Adomian Decomposition Method (LADM)

In this section, we discuss the numerical solution of system (1), considering the initial conditions, we obtain an infinite series approximate solutions using Laplace Adomian Decomposition Method (LADM) as shown below:

$$S_H(\mathrm{t}) = -1667508.099\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 13335.12218\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{E}_Z(\mathrm{t}) = -15.13300550\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 0.2757449485\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{E}_{C}(\mathrm{t}) = -346.2375190\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 178.3698430\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{E}_{ZC}(\mathrm{t}) = -11.51110796\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 2.650112855\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$I_{Z}(\mathrm{t}) = -72.5500\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 121.9636744\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$I_{C}(\mathrm{t}) = 324.725\frac{t^{\gamma}}{\Gamma(\gamma+1)} - 239.4722593\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$I_{ZC}(\mathrm{t}) = 57.86111111\frac{t^{\gamma}}{\Gamma(\gamma+1)} - 95.03049621\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{R}(\mathrm{t}) = 35.820\frac{t^{\gamma}}{\Gamma(\gamma+1)} - 17.20911556\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{S}_{M}(\mathrm{t}) = -8.0953115\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 6.575796849\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{E}_{MZ}(\mathrm{t}) = -25.43549931\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 4.313070900\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$\mathrm{E}_{MC}(\mathrm{t}) = -62.57139594\frac{t^{\gamma}}{\Gamma(\gamma+1)} + 21.75121294\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$I_{MZ}(\mathrm{t}) = 18.10220673\frac{t^{\gamma}}{\Gamma(\gamma+1)} - 3.963900003\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots$$

$$I_{MC}(\mathrm{t}) = 53.0\frac{t^{\gamma}}{\Gamma(\gamma+1)} - 21.29522830\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \ldots \tag{23}$$

The series solution for $\gamma = 1$ are as follows:

$$S_{H}(\mathrm{t}) = -1667508.099t + 13335.12218t^{2} + \ldots$$

$$\mathrm{E}_{Z}(\mathrm{t}) = -15.13300550t + 0.2757449485t^{2} + \ldots$$

$$\mathrm{E}_{C}(\mathrm{t}) = -346.2375190t + 178.3698430t^{2} + \ldots$$

$$\mathrm{E}_{ZC}(\mathrm{t}) = -11.51110796t + 2.650112855t^{2} + \ldots$$

$$I_Z(\mathrm{t}) = -72.5500t + 121.9636744t^2 \ldots$$

$$I_C(\mathrm{t}) = 324.725t - 239.4722593t^2 + \ldots$$

$$I_{ZC}(\mathrm{t}) = 57.86111111t - 95.03049621t^2 + \ldots$$

$$\mathrm{R(t)} = 35.820t - 17.20911556t^2 + \ldots$$

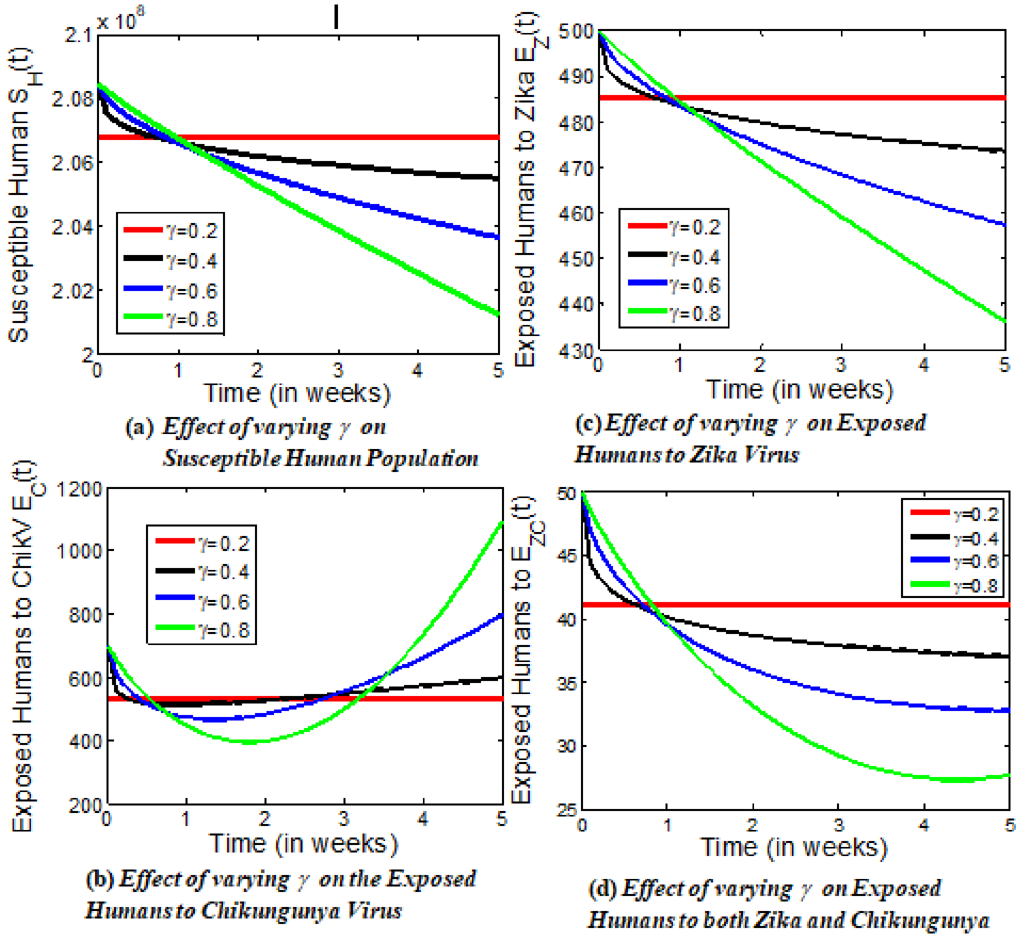
$$\mathrm{S}_M(\mathrm{t}) = -8.0953115t + 6.575796849t^2 + \ldots$$

$$\mathrm{E}_{MZ}(\mathrm{t}) = -25.43549931t + 4.313070900t^2 + \ldots$$

$$\mathrm{E}_{MC}(\mathrm{t}) = -62.57139594t + 21.75121294t^2 + \ldots$$

$$I_{MZ}(\mathrm{t}) = 18.10220673t - 3.963900003t^2 + \ldots$$

$$I_{MC}(\mathrm{t}) = 53.0t - 21.29522830t^2 + \ldots \tag{24}$$



(a) *Effect of varying γ on Susceptible Human Population*

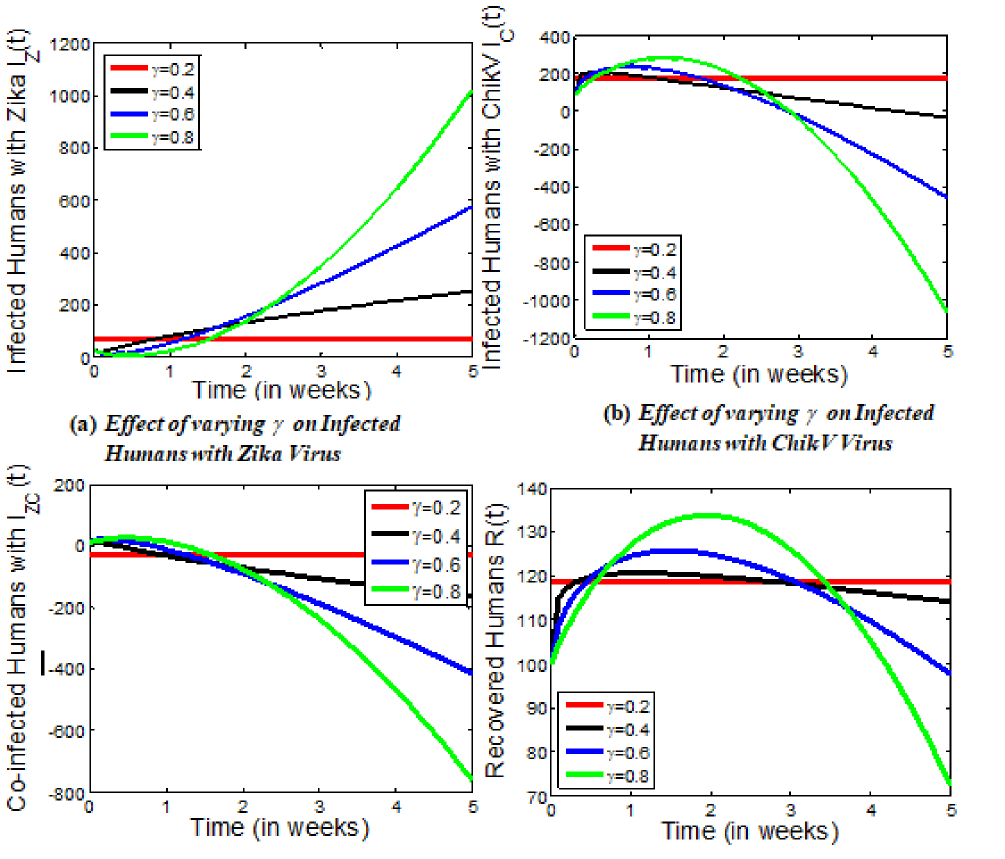
(c) *Effect of varying γ on Exposed Humans to Zika Virus*

(b) *Effect of varying γ on the Exposed Humans to Chikungunya Virus*

(d) *Effect of varying γ on Exposed Humans to both Zika and Chikungunya*

Figure 1

The plot in figure 1a shows a drastic decrease in the human population as a result of a bite from infectious female Aedes egypti mosquitoes and sexual interaction between susceptible humans and those already infected with Zika, thereby causing steady increases in the exposed classes of Zika virus, Chikungunya virus, and the co-infection of Zika and Chikungunya virus, as shown in figures 1b, 1c, and 1d.



(a) ***Effect of varying*** γ ***on Infected Humans with Zika Virus***

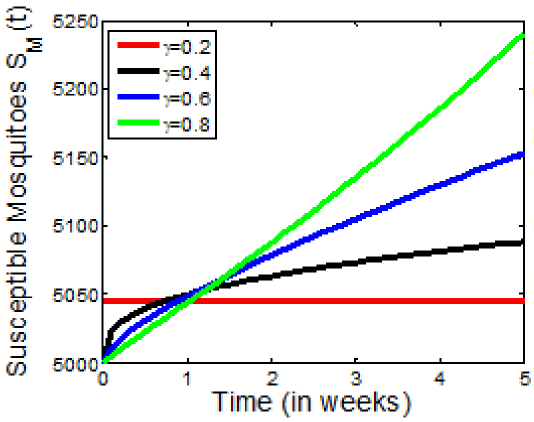
(b) ***Effect of varying*** γ ***on Infected Humans with ChikV Virus***

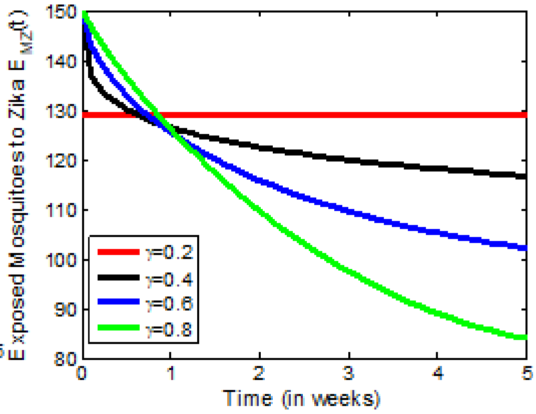
(c) ***Effect of varying*** γ ***on co-Infected Humans with both Zika and ChikV***

(d) ***Effect of varying*** γ ***on Infected Humans with Zika Virus***

Figure 2

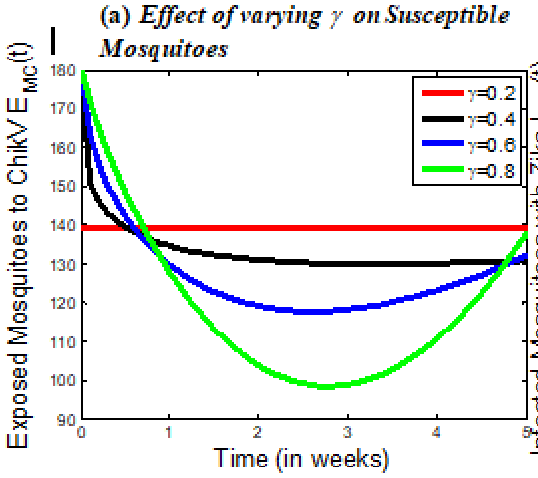
Figures 2a and 2b exhibit the effect of variation in the infected class of Zika and Chikungunya viruses, respectively. Plot 2a clearly reproduced the growth rates at which exposed humans to Zika virus increased the infected class of humans with Zika virus as a result of the unavailability of adequate vaccines for controlling Zika virus and non-compliance with the use of male and female condoms. Similarly, plot 2b increases as exposed persons to the Chikungunya virus falls in population due to their fast development into the infected class.





(a) ***Effect of varying γ on Susceptible Mosquitoes***

(b) ***Effect of varying γ on Exposed Mosquitoes with Zika Virus***



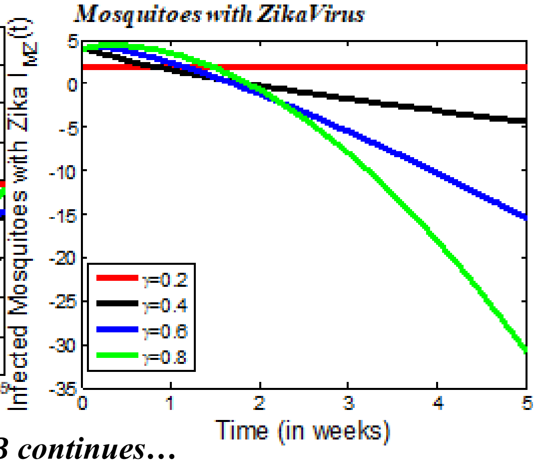


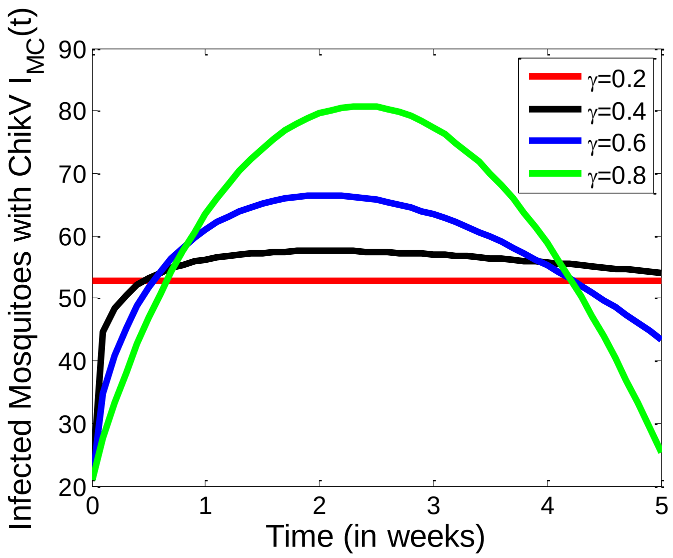
Figure 3 continues…

(c) ***Effect of varying γ on Exposed Mosquitoes with ChikV Virus***

(d) ***Effect of varying γ on Infected Mosquitoes with Zika Virus***

Figure 3

Figure 2c depicts a gradual increase in the co-infection rate due to the lack of treatment for individuals infected with either Zika virus, Chikungunya virus, or both. However, significant compliance among adults and children with the use of treated bed nets and condoms has effectively reduced the spread of Zika virus, thereby alleviating the burden and leading to the recovery of some co-infected individuals, as illustrated in Figure 2d.



(e) ***Effect of varying γ on Infected Mosquitoes with Chikungunya Virus***

Figure 3 continues…

Figure 3a demonstrates a decline in the susceptible mosquito population due to interactions with humans infected with either Zika virus or Chikungunya virus, but not both simultaneously. In Figures 3b and 3c, there is an initial steady increase followed by a peak, reflecting uncontrolled interactions between infectious humans and susceptible mosquitoes. In contrast, Figures 3d and 3e depict the dynamics of mosquitoes infected with Zika virus and Chikungunya virus, respectively.

.4. Analysis of Convergence of the Laplace Adomian Decomposition Method (LADM)

The solution to system (1), expressed as an infinite series, converges uniformly to its exact solution. To verify the convergence of the series in equation (23), we employ the method described in [25].

For a sufficient condition ensuring the convergence of the Laplace Adomian Decomposition Method, we refer to the following theorem:

Theorem 3

Let y be a Banach and $T: y \to y$ be a constructive nonlinear operator such that for $y, y \in Y$, $\|Ty - Ty'\|, 0 < k < 1$. Then T has a unique point y such that $Ty = y$, where $y = S_H, E_Z, E_C, E_{ZC}, I_Z, I_C, I_{ZC}, R, S_M, E_{MZ}, E_{MC}, I_{MZ}, I_{MC}$. The series in (23) can be written by applying the Laplace Adomian Decomposition Method as follows $y_n = Ty_{n-1},\ y_{n-1}$

$$\sum_{i=1}^{n-1} y_1,\ n = 1, 2, 3, 4, \ldots$$

And we assume that $y_0 \in B, y$, where $B, y = y \in Y : \|\tilde{y} - y\| < r$: then we have the following

(i) $y_n \in B_r; y$

(ii) $\lim_{n \to \infty} y_n = y_n$

Proof

For condition (i), employing mathematical induction:

For $n = 1$,

We have the following results

$$\|y_o - y\| \le k^{q-1} \|y_0 - y\|,$$

This gives

$$\|y_q - y\| \le \|Ty_{q-1} - Ty\| \le k\|y_{n-1} - y\| \le k^0 \|y_0 - y\|$$

Therefore; $\|y_q - y\| \leq k^0 \|y_0 - y\| \leq k^n{}_r < r$, this directly implies that $y_n \in B_r\ y$

Also for (ii), we have that since $\|y_q - y\| < k^n \|y_0 - y\|$ and $\lim_{n \to \infty} k^n = 0$, we can rewrite

$$\lim_{n \to \infty} y_n = y.$$

PARAMETERS	Values	References	Parameters	Values	Reference
Λ_H	100	[36]	θ_1	0.42	Assumed
Λ_M	100	[39]	β_C	0.0423	Fitted
σ_1	0.0235	Fitted	β_{MC}	0.25	Fitted
σ_3	0.049	Fitted			
σ_5	$1/4.5$	Assumed			
α_1	$0.09-0.15$	[37]			
α_3	$0.09-0.15$	[37]			
m	0.45				
β_Z	0.01	Fitted			
β_{HZ}	0.065	Fitted			
β_{MZ}	0.0441	Fitted			
δ_{ZC}	0.089	Assumed			
μ_H	$1/(365\times 67)$	[39]			
μ_M	$1/14$	[39]			
σ_{MZ}	$1/8.2$	[40]			
σ_{MC}	0.3	Assumed			
δ_Z	0.0098	Assumed			
δ_C	0.05	Assumed			
η_1	0.5	Assumed			
ψ_1	0.2	Assume			

Table 2: Table of parameters values

5.0 Data fitting for Zika and Chikungunya Virus

In this section, we present how we carried out data estimates for some of the major parameters in our models. The data fitting is performed to Zika, and Chikungunya virus model using the fmincon algorithm which is a component of optimization toolbox of MATLAB software. Weekly epidemiological data was collected from Espirito Santos state Brazil where the two vector borne diseases co-circulated. The number of confirmed cases of Zika and Chikungunya viruses are as presented in table below.

Table 2: Real life data as obtained for Zika and Chikungunya virus

Week	Period	ZIka	Chikungunya
1	03/01/2021–09/01/2021	24	85
2	10/01/2021–16/01/2021	31	95
3	17/01/2021–23/01/2021	28	105
4	24/01/2021–30/01/2021	32	102
5	31/01/2021–06/02/2021	22	72
6	07/02/2021–13/02/2021	23	63
7	14/02/2021–20/02/2021	31	63
8	21/02/2021–27/02/2021	19	58
9	28/02/2021–06/03/2021	28	101
10	07/03/2021–13/03/2021	25	97
11	14/03/2021–20/03/2021	26	100
12	21/03/2021–27/03/2021	25	99
13	28/03/2021–03/04/2021	31	99
14	04/04/2021–10/04/2021	31	93
15	11/04/2021–17/04/2021	31	99

Numbers of cases of Zika, and Chikungunya per epidemiological week in Espirito Santos State in Brazil from 3rd January, 2021 to 17th April, 2021 as extracted from Omame et al.,[37].

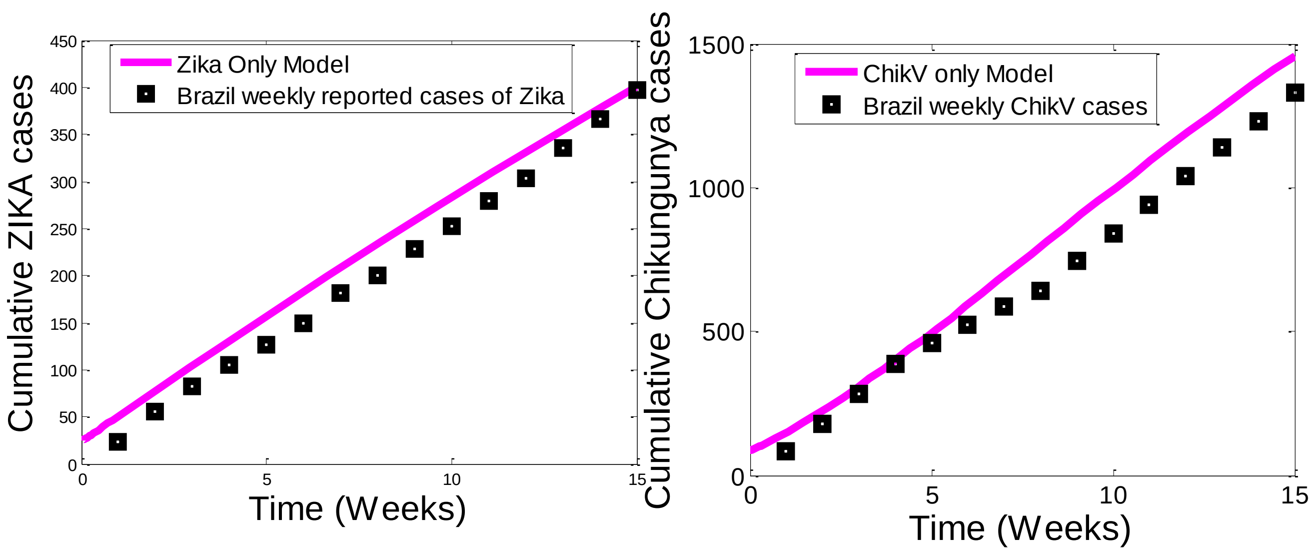


Figure 4: Data fitting for Zika and Chikungunya virus

5.0 Conclusion

In conclusion, we developed a fractional-order compartmental model to explore the transmission dynamics of Zika and Chikungunya viruses within the human population. Utilizing the Laplace Adomian Decomposition Method, we solved and analyzed a thirteen-compartment epidemiological model, deriving co-infection solutions for Zika and Chikungunya viruses that converged accurately. Our investigations highlighted that reducing mosquito biting rates and promoting compliance with treated bed net usage can substantially mitigate Zika-Chikungunya co-infection dynamics. To validate our simulations, we incorporated real-life data into our models to estimate key parameters, confirming that lowering infectious mosquito activity significantly alleviates the burden of Zika-Chikungunya co-infection.

Data availability

All data used in the course of this work are well cited in the work and referenced accordingly.

Conflict of interest

The authors declared no conflict of interest.

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