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# ON THE NORM OF JORDAN ELEMENTARY OPERATOR IN TENSOR PRODUCT OF C*-ALGEBRAS

By

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## ABSTRACT

**The norm property of different types of Elementary operators has attracted a lot of researchers due to its wide range applications in functional analysis. From available literature the norm of Jordan elementary operator has been determined in C*-algebras, JB*-algebras, standard operator algebra and prime JB*-triple but not much has been done in tensor product of C*-algebras. This paper, dealt with the norm of Jordan elementary operator in a tensor product of C*-algebras. More precisely, the paper investigated the bounds of the norm of Jordan elementary operator in a tensor product of C*-algebras and obtained that $\| U_{A\otimes B,C\otimes D} \| = 2 \| A \| \| B \| \| C \| \| D \|$.**

**The concept of finite rank operator and properties of tensor product of Hilbert spaces and operators and vectors in Hilbert spaces were used to achieve the paper's objective**

## KEYWORDS

**Jordan Elementary Operator, Finite Rank Operator, Tensor Product, Operators and C*-algebras.**

## 1. INTRODUCTION

**Definition 1.1:** Let $H = \{x, \ldots\ldots\ldots\}$and $K = \{y, \ldots\ldots\ldots\}$be fixed Hilbert spaces with inner/scalar products $\langle x, x\rangle$and $\langle y, y\rangle$respectively. The *tensor product*of $H$and $K$is a Hilbert space $H \otimes K$,where $\otimes : H \times K \rightarrow H \otimes K, (x, y) \rightarrow (x \otimes y)$ is a bilinear mapping such that;

(a) The set of all vectors $(x \otimes y) : x \in H, y \in K$form a total subset of $H \otimes K$, that is it's closed linear span( the minimum closed set containing the intersection of all subspaces containing the set) is equal to $H \otimes K$.
(b) $\langle x_1 \otimes y_1, x_2 \otimes y_2\rangle = \langle x_1, x_2\rangle\langle y_1, y_2\rangle \forall x_1, x_2 \in H$and $\forall y_1, y_2 \in K$.

In condition above putting $x_1 = x_2 = x$and $y_1 = y_2 = y$, in (b) above we have $\| x \otimes y \| = \| x \| \| y \|$.

**Definition 1.2:** Let H and K be complex Hilbert spaces and$H \otimes K$ be tensor product of $H$ and $K$. Let $B(H \otimes K$ be the set of bounded linear operators on $H \otimes K$. Then the *Jordan elementary operator*, $U_{A\otimes B, C\otimes D}: B(H \otimes K) \rightarrow B(H \otimes K)$, is defined by;

$U_{A\otimes B, C\otimes D}(X \otimes Y) = A \otimes B(X \otimes Y)C \otimes D + C \otimes D(X \otimes Y)A \otimes B, \forall X \otimes Y \in B(H \otimes K)$, $A \otimes B$,$C \otimes D$being fixed elements of $B(H \otimes K)$,where $A, C \in B(H)$, the set of bounded linear operators on $H$and $B, D \in B(K)$,the set of bounded linear operators on $K$

**Definition 1.3:** let $u : H \rightarrow \mathbb{R}$be functional, for a complex Hilbert space , With dual $H^*$. A *finite rank operator*$u \otimes x : H \rightarrow H$is defined by $(u \otimes x)y = u(y)x, \forall y \in H$, where $u \in H^*$and $x \in H$is a unit vector, with;

$\| u \otimes x \| = sup\{\| (u \otimes x)y \| : y \in H\}$

$= sup\{\| u(y)x \| : y \in H\}$

$= sup\{| u(y) | \| x \| : y \in H\}$

$= | u(y) |$

## 2. NORM OF JORDAN ELEMENTARY OPERATOR

Previous studies in the norm of elementary operator have shown that the norm can be determined in different spaces using different types of elementary operators. While working on prime C*-algebras, Mathieu (1990) determined the lower bound of the norm of the Jordan elementary operator and proved the following theorem 2.1;

**Theorem 2.1: (Mathieu, 1990)**

Let $\Lambda$ be a prime C*-algebra and $A, B$its elements. Then $\| AXB + BXA \| \geq \frac{2}{3} \| A \| \| B \| \forall X \in \Lambda$.

Also Garcia and Palacios (1994) determined the norm of Jordan operator for JB*-algebras as shown in theorem 2.2;

**Theorem 2.2: (Garcia and Palacios, 1994)**

Let $Z$be a JB*-algebra and $A, B$its elements. Then $\| AXB + BXA \| \geq \frac{1}{20412} \| A \| \| B \| \forall X \in Z$.

Stacho and Zalar (1996) also determined the norm of Jordan elementary operator for standard operator algebras and proved the following theorem 2.3;

**Theorem 2.3: (Stacho and Zalar, 1996)**

Let P be standard operator algebra on a Hilbert space$H$. If $A, B \in$ P , then the uniform estimate

$$\| AXB + BXA \| \geq 2(\sqrt{2}-1) \| A \| \| B \| \ \forall X \in P.$$

Moreover, Bunce et al. (1997) both sharped and extended the result of Garcia and Palacios (1994) for any prime JB*-triple. They arrived at the result in the next proposition 2.4.

**Proposition 2.4: (Bunce et al., 1997)**

Let $\Lambda$ be a finite dimensional cartan factor. Then $\| MAB \| \geq \frac{1}{6} \| A \| \| B \| \ \forall A, B \in \Lambda$.

Stacho and Zalar(1998) also determined the norm of Jordan elementary operator for the algebra of symmetric operators acting on a Hilbert space and proved result as shown in the theorem 2.5 below.

**Theorem 2.5: ( Stacho and Zalar, 1998 )**

Let$A, B \in B(H)$. Then$\| MA, B + MB, A \| \geq \| A \| \| B \|$.

In the above result, Stacho and Zalar (1998) were interested in the quantum mechanical Jordan algebra symm$(H) = \{A \in B(H); A^* = A\}$

On their part, Baraa and Boumazgour(2001) used the concept of the maximal numerical range and finite rank operators in prime C*-algebras to determine norm of Jordan elementary operator and managed to arrive at the result as shown in the theorem 2.6 below;

**Theorem 2.6: (Baraa and Boumazgour, 2001)**

Let $A, B \in B(H)$ with $B \neq 0$ then: $\| U_{A,B} \| \geq sup_{\lambda \in WB(A^*B)} \{\| \| B \| A + \frac{\bar{\lambda}}{\|B\|} B \|\}$

where $W_B(A^*B) = \{\lambda \in \mathbb{C} : \exists x_n \in H, \| x_n \| = 1, \lim_{n\to\infty} \langle A^*Bx_n, x_n \rangle = \lambda, \lim_{n\to\infty} \| x_n \| = \| B \|\}$.

On their part, Blanco et al. (2004) determined norm of a Jordan elementary operator on a C*-algebra using the notion of matrix valued numerical ranges and a kind of geometrical mean for positive matrices and tracial geometrical mean as shown in the next theorem.

**Theorem 2.8: (Blanco et al., 2004)**

Let H be a two dimensional complex Hilbert space, B(H) the algebra of bounded linear operators on H. Let $M_{A,B} : B(H) \to B(H)$ be defined by $M_{A,B}(X) = AXB + BXA, \forall X \in B(H)$ where $A$and $B$ are fixed in $B(H)$ and $e_1, e_2$are orthonormal basis of $H$then $\| AXB + BXA \| = \| A \| \| B \|$.

## 3. NORM OF ELEMENTARY OPERATOR IN TENSOR PRODUCT OF C*-ALGEBRAS

Muiruriet al. (2018) determined the norm of basic elementary operator in a tensor product of C∗-algebras using the finite rank operator and properties of tensor product and proved the following theorem 3.1;

**Theorem 3.1: (Muiruriet al., 2018).**

Let $H \otimes K$ be tensor product of complex Hilbert spaces $H$and $K$and$B(H \otimes K)$ be the set of bounded linear operators on $H \otimes K$. Then $\forall X \otimes Y \in B(H \otimes K)$with $\| X \otimes Y \| = 1$, we have $\| M_{A\otimes B,C\otimes D} \| = \| A \| \| B \| \| C \| \| D \|$, where $A, C$and $B, D$are fixed elements in $B(H)$ and $B(K)$ respectively.

As a consequence of theorem 3.1 Muiruriet al. (2018) related the norm of basic elementary operator in tensor product and the usual norm in this elementary operator in C*-algebra and arrived at the corollary 3.2 below;

**Corollary 3.2: (Muiruriet al., 2018).**

Let $H$and $K$be complex Hilbert spaces and $B(H \otimes K)$ be the set of bounded linear operators on $H \otimes K$. Then $\forall X \otimes Y \in B(H \otimes K)$with $\| X \otimes Y \| = 1$, we have $\| M_{A\otimes B,C\otimes D} \| = \| M_{A,C} \| \| M_{B,D} \|$, where $M_{A,C}$and $M_{B,D}$ are basic elementary operators in $B(H)$ and $B(K)$ respectively.

Daniel et al. (2022) used the stampli's maximal numerical range to determine the norm of basic elementary operator in a tensor product and they obtained the following theorem 3.3;

**Theorem 3.3: (Daniel et al., 2022)**.

Let $H$and $K$be Hilbert spaces and let $M_{A\otimes B,C\otimes D}$be basic elementary operator on $B(H \otimes K)$ the set of complex Hilbert space $H \otimes K$.If $\forall U \otimes V \in B(H \otimes K)$with $\| U \otimes V \| = 1$, $A, C \in B(H)$,$B, D \in B(K)$ $\zeta \in W_0(C), \xi \in W_0(D)$ then we have $\| M_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| = Sup_{\zeta \in W_0(C)} Sup_{\xi \in W_0(D)} \{|\zeta||\xi| \| A \| \| B \|\}$

Also, Daniel et al., (2023) determined the bounds of the norm of elementary operator of length two in tensor product using the Stampli's maximal numerical range and obtained theorem 3.4

**Theorem 3.4: (Daniel et al., (2023).**

Let $H \otimes K$ be tensor product of Hilbert spaces $H$and $K$and$B(H \otimes K)$ be the set of complex Hilbert space $H \otimes K$. Let $M_{2A\otimes B,C\otimes D}$be basic elementary operator on $B(H \otimes K)$. If $\forall U \otimes V \in B(H \otimes K)$with $\| U \otimes V \| = 1$, $A_i, C_i \in B(H)$,$B_i, D_i \in B(K)$ $\zeta_i \in W_0(C_i), \xi_i \in W_0(D_i)$ then we have $\| M_{2A\otimes B,C\otimes D} \backslash B(H \otimes K) \| = Sup_{\zeta_i \in W_0(C_i)} Sup_{\xi_i \in W_0(D_i)} \{|\zeta_i||\xi_i| \| A_i \| \| B_i \|\}$

Muiruri et al. (2024) investigated the bounds of the norm of an elementary operator of finite length in a tensor product of C*-algebras using the concept of finite rank operator and properties of tensor product of C*-algebras and obtained the theorem 3.5.

**Theorem 3.5 (Muiruri et al., 2024)**

If $H$ and $K$are complex Hilbert spaces and $B(H \otimes K)$, the set of bounded linear operator on $H \otimes K$. If

$\forall X \otimes Y \in B(H \otimes K)$and$\| X \otimes Y \| = 1$ then;

$$\| T_n \| = \sum_{i=1}^{n} \| A_i \| \| B_i \| \| C_i \| \| D_i \|$$

, where $T_n$ is the Elementary operator of finite length as defined earlier and $A_i, C_i \in B(H)$and $B_i, D_i \in B(K)$.

The above literature forms the basis of the statement and methodology of our main result.

## 4. NORM OF JORDAN ELEMENTARY OPERATOR IN TENSOR PRODUCT OF C*-ALGEBRAS.

As the paper's main result we determine the bounds of the norm of Jordan elementary operator in tensor product of C*-algebras.

**Theorem 4.1**

Let $H \otimes K$ be tensor product of Hilbert spaces$H$ and $K$and$B(H \otimes K)$ be the set of bounded linear operator on $H \otimes K$. Then

$\forall\ X \otimes Y \in B(H \otimes K)$with$\| X \otimes Y \| = 1$ we have;

$$\| U_{A\otimes B, C\otimes D} \| = 2 \| A \| \| B \| \| C \| \| D \|$$

, where $U_{A\otimes B, C\otimes D}$ is the Jordan Elementary operator as defined earlier and $A, C \in B(H)$and $B, D \in B(K)$.

Proof

By definition $\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| = Sup\{\| U_{A\otimes B, C\otimes D}(X \otimes Y) \| : \forall X \otimes Y \in B(H \otimes K), \| X \otimes Y || = 1\}$

Thus;

$$\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| \geq \| U_{A\otimes B, C\otimes D}(X \otimes Y) \| : \forall X \otimes Y \in B(H \otimes K), \| X \otimes Y \| = 1$$

Therefore $\forall \varepsilon \geq 0$ we have;

$$\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| - \varepsilon < \| U_{A\otimes B, C\otimes D}(X \otimes Y) \| : \forall X \otimes Y \in B(H \otimes K), \| X \otimes Y \| = 1$$

$$\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| - \varepsilon < \| A \otimes B(X \otimes Y) C \otimes D + C \otimes D(X \otimes Y) A \otimes B \|$$

$$: \forall X \otimes Y \in B(H \otimes K), \| X \otimes Y \| = 1$$

Since $\varepsilon \geq 0$ was arbitrary taken then;

$$\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| \leq \| A \otimes B(X \otimes Y) C \otimes D + C \otimes D(X \otimes Y) A \otimes B \| : \forall X \otimes Y \in B(H \otimes K), \| X \otimes Y \| = 1$$

Now using the properties of tensor product $(A \otimes B)(C \otimes D) = (AC \otimes BD)$ then;

$$\| U_{A\otimes B, C\otimes D} \backslash B(H \otimes K) \| \leq \| AXC \otimes BYD + CXA \otimes DYB \|.$$

By triangular inequality we have;

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq \| AXC \otimes BY D || + ||CXA \otimes DYB \|.$

By properties of tensor product of operators $||A \otimes B|| = ||A||||B||$ we have;

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq \| AXC||||BY D || + ||CXA||||DYB \|.$

Also it is clear that;

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq \| A \| ||X||||C||||B||||Y|||| D || + ||C||||X||||A||||D||||Y||||B \|$

Clearly;

$\| X \otimes Y \| = \| X \| \| Y \| = 1$

Therefore;

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq \| A \| \| C \| \| B \| \| D \| + \| C \| \| A \| \| D \| \| B \|$

Since $||.||$ is a scalar then

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq \| A \| \| B \| \| C \| \| D \| + \| A \| \| B \| \| C \| \| D \|$

Finally then;

$\| U_{A\otimes B,C\otimes D} \backslash B(H \otimes K) \| \leq 2 \| A \| \| B \| \| C \| \| D \|$ (4.1)

Conversely, for a unit vector $\mu \otimes \nu$ in $H \otimes K$ where $\mu \in H$ and $\nu \in K$

Then we have that;

$\| U_{A\otimes B,C\otimes D}(X \otimes Y)(\mu \otimes \nu) \| \leq$

$\| U_{A\otimes B,C\otimes D}(X \otimes Y) \| \| \mu \otimes \nu \| \leq$

$\| U_{A\otimes B,C\otimes D} \| \| X \otimes Y \| \| \mu \otimes \nu \|$

Using the properties of tensor product in operators and vectors $\| X \otimes Y \| = \| X \| \| Y \|$ and since $X, Y, \mu$ and $\nu$ are unit operators and vectors respectively we have;

$= \| U_{A\otimes B,C\otimes D} \| \| X \| \| Y \| \| \mu \| \| \nu \| = \| U_{A\otimes B,C\otimes D} \|$ (4.2)

Thus reversing inequality (4.2)

$\| U_{A\otimes B,C\otimes D} \| \geq \| U_{A\otimes B,C\otimes D}(X \otimes Y)(\mu \otimes \nu) \| =$

$\| (A \otimes B(X \otimes Y)C \otimes D + C \otimes D(X \otimes Y)A \otimes B)(\mu \otimes \nu) \|$

By the properties of tensor product on operators $(A \otimes B)(X \otimes Y) = (AX \otimes BY)$ then;

$= \| AXC \otimes BYD(\mu \otimes \nu) + CXA \otimes DYB(\mu \otimes \nu) \|$

$= \| AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \|$

Therefore

$$\| U_{A\otimes B,C\otimes D} \| \geq \| AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \| \quad (4.3)$$

Squaring both sides of (4.3) we get

$\{\| U_{A\otimes B,C\otimes D} \|\}^2 \geq \{\| AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \|\}^2$

$= \langle AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu, AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \rangle$

$= \langle AXC\mu \otimes BYD\nu, AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \rangle + \langle CXA\mu \otimes DYB\nu, AXC\mu \otimes BYD\nu + CXA\mu \otimes DYB\nu \rangle$

$= \langle AXC\mu \otimes BYD\nu, AXC\mu \otimes BYD\nu \rangle + \langle AXC\mu \otimes BYD\nu, CXA\mu \otimes DYB\nu \rangle + \langle CXA\mu \otimes DYB\nu, AXC\mu \otimes BYD\nu \rangle +$

$\langle CXA\mu \otimes DYB\nu, CXA\mu \otimes DYB\nu \rangle$

$= \| AXC\mu \otimes BYD\nu \|^2 + \langle AXC\mu \otimes BYD\nu, CXA\mu \otimes DYB\nu \rangle + \langle CXA\mu \otimes DYB\nu, AXC\mu \otimes BYD\nu \rangle + \| CXA\mu \otimes DYB\nu \|^2$

By properties of tensor product $\langle x_1 \otimes y_1, x_2 \otimes y_2 \rangle = \langle x_1, x_2 \rangle \langle y_1, y_2 \rangle \forall x_1, x_2 \in H$ and $\forall y_1, y_2 \in K$, we have

$= \| AXC\mu \|^2 \| BYD\nu \|^2 + \langle AXC\mu \otimes BYD\nu, CXA\mu \otimes DYB\nu \rangle + \langle CXA\mu \otimes DYB\nu, AXC\mu \otimes BYD\nu \rangle + \| CXA\mu \|^2 \| DYB\nu \|^2$

$= \| AXC\mu \|^2 \| BYD\nu \|^2 + \langle AXC\mu, CXA\mu \rangle \langle BYD\nu, DYB\nu \rangle + \langle CXA\mu, AXC\mu \rangle \langle DYB\nu, BYD\nu \rangle +$
$\| CXA\mu \|^2 \| DYB\nu \|^2$(4.4)

We now let $e, f : H \to \mathbb{R}^+$ be functionals. From above, choosing unit vectors $y, z \in H$ and define the finite rank operators

$A = e \otimes y, y \in H, \| y \| = 1$ by $A\mu = (e \otimes y)\mu = e(\mu)y$ and

$C = f \otimes z, z \in H, \| z \| = 1$ by $C\mu = (f \otimes z)\mu = f(\mu)z$

We observe that the norm of S is

$\| A \| = sup\{\| (e \otimes y)\mu \| : \mu \in H\}$

$= sup\{\| e(\mu)y \| : \mu \in H\}$

$= sup\{| e(\mu) | \ \| y \| : \mu \in H\}$

$= sup\{| e(\mu) |: \mu \in H \ \|\} = | e(\mu) |$

That is $\| A \| = | e(\mu)|$ for any $\mu \in H$.

Likewise using the concept above, the norm of $C$is $\| C \| = | f(\mu) |$for any $\mu \in H$

Therefore, from (4.4) we have $\| AXC\mu \|^2 = \| (e \otimes y)X(f \otimes z))\mu \|^2$

$= \| (e \otimes y)X(f(\mu)z) \|^2 = | f(\mu) |^2 \| (e \otimes y)X(z) \|^2$

$= | f(\mu) |^2 | e(X(z)) |^2 \| y \|^2 = \| A \|^2 \| C \|^2$

Then this shows that:

$\| AXC\mu \|^2 = \| A \|^2 \| C \|^2$(4.5)

Using the same concept we have;

$\| BY Dv \|^2 = \| B \|^2 \| D \|^2$(4.6)

$\| DY Bv \|^2 = \| B \|^2 \| D \|^2$(4.7)

$\| CXA\mu \|^2 = \| A \|^2 \| C \|^2$(4.8)

Also $\langle AXC\mu, CXA\mu \rangle = \langle (e \otimes y)X(f \otimes z))\mu, (e \otimes y)X(f \otimes z))\mu \rangle$

$= \langle (e \otimes y)X(f(\mu)z), (e \otimes y)X(f(\mu)z) \rangle$

$= \langle f(\mu)(e \otimes y)X(z), f(\mu)(e \otimes y)X(z) \rangle$

$= \langle f(\mu)e(X(z))y, f(\mu)e(X(z))y \rangle$

$= f(\mu)e(X(z))f(\mu)e(X(z))\langle y, y \rangle$

$= f(\mu)e(X(z))f(\mu)e(X(z))$

Since $f(\mu)$ and $e(X(z))$ are positive real numbers, we have:

$f(\mu) = | f(\mu) | = \| C \|, e(X(z)) = | e(X(z)) | = \| A \|$

Thus we have $\langle AXC\mu, CXA\mu \rangle = f(\mu)e(X(z))f(\mu)e(X(z)) = \| C \| \| A \| \| C \| \| A \|$

Since the norms of $A$ and $C$ are scalars then:

$\langle AXC\mu, CXA\mu \rangle = \| A \|^2 \| C \|^2$(4.9)

Then following the same concept we have;

$\langle CXA\mu, AXC\mu \rangle = \| A \|^2 \| C \|^2$(4.10)

$\langle BYDv, DYBv \rangle = \| B \|^2 \| D \|^2$(4.11)

$\langle DYBv, BYDv \rangle = \| B \|^2 \| D \|^2$ (4.12)

Then applying equations (4.5) to (4.12) to equation (4.4) we have

$= \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 + \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 +$

$\| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 + \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2$

Thus

$$\{\| U_{A\otimes B,C\otimes D} \|\}^2 \geq \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 + \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 +$$

$\| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 + \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2$

Then

$$\{\| U_{A\otimes B,C\otimes D} \|\}^2 \geq 4 \| A \|^2 \| B \|^2 \| C \|^2 \| D \|^2 \qquad (4.13)$$

Obtaining square root on both sides of (4.13) we obtain;

$$\| U_{A\otimes B,C\otimes D} \| \geq 2 \| A \| \| B \| \| C \| \| D \| \qquad (4.14)$$

Hence from (4.1) and (4.14)

$$\| U_{A\otimes B,C\otimes D} \| = 2 \| A \| \| B \| \| C \| \| D \|$$

## CONCLUSION

From the main result the paper has determined the bounds of the norm of Jordan elementary operator in tensor product of C*-algebras. The norm of other types of elementary operator in tensor product of C*-algebras can be determined.

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