

Resilient Sliding Mode Control via Direct Pole Placement Technique for Large Scale Systems

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Abstract—This paper introduces an innovative method for determining the switching surface within an equivalent pole placement control framework designed specifically for multivariable systems. Moreover, it showcases adaptability across both minimum phase and non-minimum phase systems. The sliding mode control system offers significant flexibility in shaping the closed-loop eigenvalues, providing considerable freedom in customizing the system's characteristics. The formulation of the sliding surface plays a crucial role in defining the performance features of the control system, as the representative point of the system is constrained to follow a predetermined switching surface. Researchers are actively addressing the pole assignment problem within sliding mode control, presenting a unique geometric approach for designing sliding surfaces. This contribution introduces a fresh perspective to the domain of control system design, and the inclusion of numerical examples serves to highlight the simplicity and effectiveness of the proposed method.

Keywords- VSC, pole placement approach, large-scale multivariable system

I. INTRODUCTION

The Variable Structure Control (VSC) design comprises two essential steps. The initial phase involves defining the sliding surfaces to ensure that the system behaves like a linear system and remains impervious to parameter changes and disturbances during the sliding operation. Secondly, identify the control law that facilitates the system in reaching and progressing through the crossings of the sliding surfaces.

The effectiveness of the control system hinges on the design of the sliding surface [1]. The representative point of the system is constrained to move exclusively along a predetermined switching surface. Considerable research attention has been devoted to the sliding mode pole assignment problem [2]. These techniques facilitate the implementation of geometric and algebraic sliding surface designs, employing equivalent control methodologies, especially within the domain of large-scale multivariable systems [2]. The algebraic determination of all sliding surface characteristics is a distinctive feature.

This paper introduces a systematic approach for determining the switching surface in an equivalent pole placement control system applied to multivariable systems. This method is applicable to systems with both minimum and non-minimum phases, showcasing its versatility. Furthermore, in the sliding mode, the control system's closed-loop eigenvalues exhibit remarkable flexibility, allowing for allocation in a highly customizable manner. To showcase the

effectiveness of the proposed approach, two illustrative cases are examined.

The chapter unfolds in the following sequence: Section 2 introduces algebraic methods for the design of sliding surfaces. Section 3 illustrates the effectiveness of the proposed method through its application to the oblique-winged research aircraft. Section 4 presents an additional example related to the fixed-winged research aircraft. Finally, Section 5 encapsulates the conclusion of this chapter.

II. DESIGN OF SLIDING SURFACES

Examine a dynamic multivariable system subject to uncertainties, described by a differential equation, given as

$$\dot{x} = [A + \Delta A(x,t)]x + f(t,x,u) + B[u + w(x,t,u)] \quad (1)$$

Here, the state vector is denoted by x , the control input by u , and the matrix B is specified to be of full rank. The term $\Delta A(x,t)$ and $f(t,x,u)$ represent the linear uncertainty and nonlinearities inherent in the plant, while $w(x,t,u)$ can be considered as input disturbances. For ease of analysis, we assume, ΔA , f , and w are bounded. Furthermore, we impose matching requirements as follows

Assumption 1: There exist functions $h(\cdot)$ and $d(\cdot)$ such that

$$f(t, x, u) = B h(t, x, u) \quad (2)$$

$$A(t, x, u) = B d(t, x, u) \quad (3)$$

Under the matching conditions (2) and (3), system (1) can be simplified to

$$\dot{x} = Ax + B[u + v(x, t, u)] \quad (4)$$

Where $v(t, x, u)$ represents the lumped uncertainties and/or nonlinearities;

$$v(t, x, u) = h(t, x, u) + d(t, x)x + w(t, x, u). \quad (5)$$

Assumption 2: The pair (A, B) is completely controllable.

Assumption 3: There exists a continuous positive bounded real-valued function $\rho(\cdot)$ such that

$$|v(x, t, u)| < \rho(t, u) \quad (6)$$

Variable Structure Controller design involves two primary steps:

1. Defining the sliding surface to dictate specific system behavior while on the surface.
2. Implementing a control strategy to guide the system onto the sliding surface and subsequently sustain it.

In general, the sliding surfaces are defined as

$$\sigma \equiv Gx = 0, \sigma \in R^p \quad (7)$$

Upon reaching the sliding surfaces, it is stated that the system has entered the sliding mode. In the sliding mode, the linear control equivalent control u_{eq} is obtained by setting $\dot{\sigma} = 0$, that is

$$\sigma = GAx + Gbu = 0, \text{ and hence}$$

$$u_{eq} = -(GB)^{-1}GAx - v. \quad (8)$$

The closed loop equivalent system becomes

$$\dot{x} = [I_n - B(GB)^{-1}G]Ax \equiv A_{eq}x \quad (9)$$

At the origin, it encompasses p eigenvalues [17]. It is feasible to diminish the system's order to $n-p$, a crucial aspect inherent to Variable Structure Control (VSC) in sliding mode [2]. To achieve the desired system performance in the closed-loop characteristic polynomial, $n-p$ eigenvalues are arbitrarily assigned, with p eigenvalues positioned at the origin.

λ_i denotes the closed-loop poles in the sliding mode in equation (10).

$$\det(sI_n - A_{eq}) = s^p (s - \lambda_1)(s - \lambda_2) \dots (s - \lambda_{n-p}) \equiv s^p \alpha(s) \quad (10)$$

Lemma 1[12]: The controllability of the n -dimensional linear time-invariant system (4) is established if and only if any of the subsequent equivalent conditions are met.

(a) The linear independence of all rows of matrix $(sI_n - A)^{-1}B$ over C .

(b) The controllability matrix $U \equiv [B: AB: A^2B: \dots: A^{n-1}B]$ of dimensions $n \times np$ has a rank equal to n .

Examining the multivariable system in (4), assume that B is of full rank where $B = [b_1, b_2, \dots, b_p]$. Following the 'crate search by rows' method [19], a crate diagram can be depicted as illustrated in Figure 1.

Sequential exploration of the rows allows identification of a vector $A^{k_i}b_i$ that is linearly dependent on all preceding vectors in that column. Subsequently, eliminating all linearly dependent vectors results in a collection of n linearly independent vectors, which can be represented as a matrix.

$$M = [b_1, Ab_1, \dots, A^{m_1}b_1; b_2, Ab_2, \dots, A^{m_2}b_2; \dots; b_p, Ab_p, \dots, A^{m_p}b_p] \quad (12)$$

where $\sum_{j=1}^p m_j = n$. Since the matrix M is non-singular, the inverse of M exists.

$m_1=4$	$m_2=3$...	$m_i=k_i+1$...	$m_p=2$
b_1	b_2	...	b_i	...	b_p
Ab_1	Ab_2	...	Ab_i	...	Ab_p
A^2b_1	A^2b_2	...	A^2b_i	...	x
A^3b_1	x		.		x
X	x		$A^k b_i$.
X	.		x		.
.	.		.		.

Figure 1: A typical crate diagram corresponding to the controllability matrix.

The formulation of sliding surfaces can be effectively accomplished by applying the principles outlined in the following theorem.

Theorem 1: When the multivariable system in (1) enters the sliding mode and the linear equivalent control is applied, the closed loop system in (9) has p eigenvalues at zero and $n - p$ eigenvalues exactly located at the roots of $\alpha(s)=0$ as given in (10) if the sliding surfaces (7) are chosen according to

$$G = [g_1, g_2, \dots, g_p]^T \quad (15a)$$

$$g_i^T = q_i^T f_i(A), \quad (15b)$$

$$f_i(A) = A^{m_i-1} + f_{i,1}A^{m_i-2} + \dots + f_{i,m_i-1}I_n. \quad (15c)$$

Proof: Initially, it can be confirmed that, when adopting the selection (13) and constructing the M matrix as described in (12), the vectors q_i satisfy the following conditions.

$$q_i^T A^k B = 0, \quad k=0,1,\dots,m_i-2 \quad (16a)$$

$$q_i^T A^{m_i-1} b_i = 1, \quad (16b)$$

$$q_i^T A^{m_i-1} b_j = 0, \text{ for } m_i \leq m_j, i \neq j. \quad (16c)$$

Emphasize that the matrix product GB is a pivotal component in this strategy and is demonstrated to be non-singular, as indicated below. Utilizing (15) and (16a), the expression for GB can be reformulated as:

$$GB = \begin{bmatrix} q_1^T A^{m_1-1} B \\ q_2^T A^{m_2-1} B \\ \vdots \\ q_p^T A^{m_p-1} B \end{bmatrix} \quad (17)$$

Let the column vectors in the matrix M be recorded and renamed as

$$\tilde{M} = [\tilde{b}_1, A\tilde{b}_1, \dots, A^{\tilde{m}_1-1}\tilde{b}_1; \tilde{b}_2, A\tilde{b}_2, \dots, A^{\tilde{m}_2-1}\tilde{b}_2; \dots; \tilde{b}_p, A\tilde{b}_p, \dots, A^{\tilde{m}_p-1}\tilde{b}_p] \quad (18)$$

where \tilde{m}_i must satisfy the condition $1 \leq \tilde{m}_1 \leq \tilde{m}_2 \leq \dots \leq \tilde{m}_p$ with $\sum_{j=1}^p \tilde{m}_j = n$.

Then (17) for the recorded system becomes

$$\tilde{G}\tilde{B} = \begin{bmatrix} \tilde{q}_1^T A^{\tilde{m}_1-1} \tilde{B} \\ \tilde{q}_2^T A^{\tilde{m}_2-1} \tilde{B} \\ \vdots \\ \tilde{q}_p^T A^{\tilde{m}_p-1} \tilde{B} \end{bmatrix} \quad (19)$$

This condition (16), which is applied to the reordered system, remains valid as both GB and differ solely in row and column reordering while retaining the same absolute value for the determinant.

$$q_i^T A^{m_i-1} b_i = 1, \quad (20)$$

$$q_i^T A^{m_i-1} b_j = 0, \quad 1 \leq i \leq j \leq p \quad (20b)$$

As an illustration, each diagonal element of GB is unity, and the upper triangular part consists entirely of zeros. Consequently, both $\tilde{G}\tilde{B}$ and GB are non-singular.

Next, the closed-loop characteristic polynomial of (9) is considered:

$$d(s) = \det[sI_n - A + B(GB)^{-1}GA] \\ = \det(sI_n - A) \det[I_n + (sI_n - A)^{-1}B(GB)^{-1}GA]. \quad (21)$$

Employing the matrix properties $\det(I_n + XY) = \det(I_p + YX)$, where X and Y are $n \times p$ and $p \times n$ matrices respectively [9], the following is obtained:

$$d(s) = \det(sI_n - A) \det[I_n + (GB)^{-1}GA(sI_n - A)^{-1}B] \\ = \det(sI_n - A) \det(GB)^{-1} \times \det[GB + GA(sI_n - A)^{-1}B]. \quad (22)$$

For convenience, let

$$\Phi(s) = [\Phi_1(s) \quad \Phi_2(s) \quad \dots \quad \Phi_p(s)]^T = GB + GA(sI_n - A)^{-1}B \quad (23)$$

Then according to (15a), (15b), and (17), the $\Phi(s)$ element is seen to be

$$\Phi_i^T(s) = q_i^T A^{m_i-1} B + q_i^T f_i(A) A(sI_n - A)^{-1} B. \quad (24)$$

which can be further arranged as [17]

$$\Phi_i^T(s) = s f_i(s) q_i^T (sI_n - A)^{-1} B. \quad (25)$$

Using (23) and (25), (22) can be written as

$$d(s) = \det(sI_n - A) \det(GB)^{-1} \\ \times \det \begin{bmatrix} s f_1(s) q_1^T (sI_n - A)^{-1} B \\ s f_2(s) q_2^T (sI_n - A)^{-1} B \\ \vdots \\ s f_p(s) q_p^T (sI_n - A)^{-1} B \end{bmatrix} \\ d(s) = \det(sI_n - A) \det(GB)^{-1} \\ \times \det \left\{ \begin{bmatrix} s f_1(s) \\ s f_2(s) \\ \vdots \\ s f_p(s) \end{bmatrix} Q (sI_n - A)^{-1} B \right\} \quad (26)$$

where $Q = [q_1, q_2, \dots, q_p]^T$.

By exploiting (15c) and the matrix property

$$\det \begin{bmatrix} X & Y \\ Z & 0 \end{bmatrix} = \det X \cdot \det(-Z X^{-1} Y), \quad (27)$$

the characteristic polynomial d(s) becomes

$$d(s) = s^p \alpha(s) \det(GB)^{-1} \det \begin{bmatrix} sI_n - A & B \\ -Q & 0 \end{bmatrix} \quad (28)$$

As both sides of equation (28) represent polynomials, and since (28) holds true for an infinite set of s values, equating coefficients on both sides reveals that the product can be expressed as .

$$\det(GB)^{-1} \det \begin{bmatrix} sI_n - A & B \\ -Q & 0 \end{bmatrix}$$

must be unity (note that d(s) and $s^p \alpha(s)$ are monic polynomials of degree n). Hence

$$d(s) = s^p \alpha(s). \quad (29)$$

Corollary 1: In the specific scenario of a single-input system ($p=1$), is simplified to a row matrix which is expressed as $g^T = q^T \alpha(A)$.

$$(30)$$

where q^T is the last row of U^{-1} , in which U is the controllability matrix (11) of the open-loop system.

Example 1: Oblique-winged Research Aircraft

The state vector (x) for the tenth-order linear model of the oblique-winged research aircraft can be defined as follows

$$x^T = [u \quad h \quad \alpha \quad \beta \quad \phi \quad \theta \quad \varphi \quad p \quad q \quad r] \quad (31)$$

where, u denotes the forward velocity,

- h denotes the height,
- α denotes the angle of attack,
- β denotes the side-slip angle,
- ϕ denotes the roll angle,
- θ denotes the pitch angle,
- φ denotes the yaw angle,
- p denotes the roll rate,
- q denotes the pitch rate ,
- r denotes the yaw rate.

This state vector captures the key dynamic variables for the tenth-order linear model of the oblique-winged research aircraft.

The aircraft has been provided with three controls, such as

$$u^T = [\delta_E \ \delta_A \ \delta_R], \quad (32)$$

where, δ_E denotes the stabilizer deflection, δ_A denotes the aileron deflection, δ_R denotes the rudder deflection.

For a particular flight condition, the corresponding matrices A and B are

$$A = \begin{bmatrix} -0.0075 & 0 & 0.19 & 0 & 0 & -32.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -634.4 & 0 & 0 & -634.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.24 & 0.05 & 0 & 0 & 0.006 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -24.39 & 0 & 0 & 0 & -5.86 & -0.03 & 0.84 \\ 0 & 0 & -6.3 & 0 & 0 & 0 & 0 & 0.002 & -0.71 & 0.1 \\ 0 & 0 & 0 & 6.14 & 0 & 0 & 0 & -0.13 & -0.1 & -0.67 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.734 & -0.77 & 0 \\ 0 & 0 & 0 \\ -0.09 & -0.04 & 0 \\ 0 & 0 & 0.054 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6.1 \\ -6.53 & -0.012 & 0 \\ 0 & 0 & -4.3 \end{bmatrix} \quad (34)$$

Given that the pair (A, B) is controllable, multiple options exist for the free choices of M in (12). As an illustration, one can opt for $m_1 = 4$, $m_2 = 3$, and $m_3 = 3$. Utilizing MATLAB software facilitates obtaining the following results easily:

$$M = \begin{bmatrix} 1.73 & -0.03 & 209.04 & -166.9 & -0.77 & -0.0018 & 0.39 & 0 & 0 & 0 \\ 0 & 57.10 & -57.10 & -4085.5 & 0 & 25.376 & -25.38 & 0 & 0 & 0 \\ -0.09 & -6.44 & 11.64 & 25.3 & -0.04 & 0.028 & 0.23 & 0 & 0 & -0.42 \\ 0 & 0 & -0.65 & 1.145 & 0 & 0 & -0.0012 & 0.054 & 4.32 & -3.40 \\ 0 & 0 & 0.20 & -0.76 & 0 & 0 & 0.0004 & 0 & 6.10 & -40.68 \\ 0 & -6.53 & 5.20 & 36.94 & 0 & -0.012 & 0.26 & 0 & 0 & -0.42 \\ 0 & 0 & 0.65 & -0.98 & 0 & 0 & 0.0012 & 0 & -4.3 & 2.42 \\ 0 & 0.20 & -0.76 & 18.39 & 0 & 0.0004 & -0.0089 & 6.10 & -40.68 & 134.95 \\ -6.53 & 5.20 & 36.94 & -99.68 & -0.012 & 0.26 & -0.36 & 0 & -0.42 & 0.46 \\ 0 & 0.65 & -0.98 & -6.94 & 0 & 0.0012 & -0.027 & -4.30 & 2.42 & 30.26 \end{bmatrix} \quad (35)$$

Suppose that we want to place the eigenvalues of the closed loop system (9) at $\{0, 0, 0, -10, -10, -20, -20, -20, -30, -30\}$.

Then, $\alpha(s)$ of the closed loop system is

$$\alpha(s) = (s+10)^2 (s+20)^3 (s+30)^2 \quad (36)$$

There are many free choices for $f_i(A)$ according to (15c). One may freely choose

$$f_i(A) = (A+10I_{10}) (A+20I_{10}) (A+30I_{10}) \quad (37)$$

$$f_2(A) = (A+20I_{10}) (A+30I_{10}), \quad (38)$$

$$f_3(A) = (A+10I_{10}) (A+20I_{10}), \quad (39)$$

such that

$$f_1(s) f_2(s) f_3(s) = \alpha(s).$$

Restoring to (13) and (14), the sliding surface parameters are calculated easily using MATLAB as

$$G = \begin{bmatrix} 0 & 1.2469 \times 10^5 & 0 \\ 0 & 3.1657 \times 10^3 & 0 \\ 0 & -2.4203 \times 10^6 & 0 \\ 1.1279 \times 10^5 & -1.4256 \times 10^9 & 534.7132 \\ -3.6688 \times 10^2 & 7.7950 \times 10^7 & -34.9931 \\ 22.0405 & 3.0481 \times 10^6 & -0.3709 \\ 1.1505 \times 10^5 & -1.4528 \times 10^9 & 537.3439 \\ -648.0273 & 1.3798 \times 10^7 & -5.9299 \\ -0.1531 & 6.6469 \times 10^4 & 0 \\ 504.9037 & 1.6712 \times 10^6 & -1.9298 \end{bmatrix} \quad (40)$$

The validity of the result can be confirmed by employing (9) and (40). Indeed, the closed-loop poles align with the desired positions at $\{0, 0, 0, -10, -10, -20, -20, -20, -30, -30\}$.

Example 2: Fixed-wing Research Aircraft

The state vector (x) for the seventh-order linear model of the fixed-wing aircraft [17][18] can be defined as follows:

$$x^T = [v \ p \ r \ \phi \ \psi \ \zeta \ \xi] \quad (41)$$

Where, v defines the sideslip velocity, (m/s),

p defines the roll rate, (rad/s),

r defines the yaw rate, (rad/s),

ϕ defines the roll angle, (rad),

ψ defines the heading angle, (rad),

ζ defines the rudder angle, (rad),

ξ defines the aileron angle, (rad).

This state vector captures the key dynamic variables for the seventh-order linear model of the fixed-wing aircraft in a stick-linearized form

The aircraft has been provided with two controls, such as

$$u^T = [\zeta_c \ \xi_c], \quad (42)$$

Where, ζ_c denotes the rudder angle demand, (rad),

ξ_c denotes the aileron angle demand, (rad),

For a particular flight condition, the corresponding matrices A and B are

$$A = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 & -5.432 & 0 \\ -0.1033 & -8.325 & 3.75 & 0 & 0 & 0 & -28.64 \\ 0.3649 & 0 & -0.639 & 0 & 0 & -9.49 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 \end{bmatrix} \quad (43)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 10 \end{bmatrix} \quad (44)$$

Since the pair (A, B) is controllable, there are various permissible choices for the transformation matrix M in equation

(12). As an example, we select $m_1=4$ and $m_2=3$. Utilizing MATLAB software facilitates the straightforward computation of the transformed system as:

$$M = \begin{bmatrix} 0 & -108.64 & 7360.9 & -78033 & 0 & 0 & 0 \\ 0 & 0 & -700.5275 & 12495 & 0 & -286.4 & 3816.3 \\ 0 & -189.8 & 1979.6 & -17559 & 0 & 0 & 0 \\ 0 & 0 & 0 & -700.5275 & 0 & 0 & -286.4 \\ 0 & 0 & -189.8 & 1979.6 & 0 & 0 & 0 \\ 20 & -200 & 2000 & -20000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & -50 & 250 \end{bmatrix} \quad (45)$$

Suppose that we want to place the eigenvalues of the closed loop system (9) at $\{0, 0, -1, -2, -2.5, -9, -12\}$. Then, $\alpha(s)$ of the closed loop system is

$$\alpha(s) = (s+1)(s+2)(s+2.5)(s+9)(s+12) \quad (46)$$

There are many free choices for $f_i(A)$ according to (15c). One may freely choose

$$f_1(A) = (A+I_7)(A+2I_7)(A+2.5I_7), \quad (47)$$

$$f_2(A) = (A+9I_7)(A+12I_7), \quad (48)$$

such that

$$f_1(s)f_2(s) = \alpha(s). \quad (49)$$

Reverting to equations (13) and (14), the parameters for the sliding surface can be effortlessly computed using MATLAB as follows:

$$G = \begin{bmatrix} -0.0021 & 0.0107 & -0.0349 & -0.0229 & -0.0542 & 0.0500 & 0.0929 \\ 0.0796 & -0.0363 & -0.0456 & -0.2144 & 2.8646 & 0 & 0.1000 \end{bmatrix} \quad (50)$$

The validity of the obtained result can be confirmed by comparing it to equations (9) and (50). Notably, the closed-loop poles are found to be precisely positioned at the desired locations: $\{0, 0, -1, -2, -2.5, -9, -12\}$.

III. CONCLUSION

In conclusion, this paper presents a novel methodology for the design of a switching surface in Variable Structure Control (VSC) by pole placement approach, enabling the arbitrary placement of n-p closed-loop eigenvalues within a multivariable system. Leveraging the presumption of complete state controllability, our employed design algorithm is not only direct and algebraic but also offers a stark departure from the intricate algorithms required for designing sliding surfaces in linear multivariable structural systems [4]. The flexibility in selecting n linearly independent vectors in the M matrix affords substantial freedom in defining each segment of the desired closed-loop characteristic polynomial. A numerical example is provided to underscore the efficacy and straightforward nature of the proposed method.

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