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*Published in:*

Proceedings of American Control Conference, 2007. ACC' 07

*DOI (link to publication from Publisher):*

[10.1109/ACC.2007.4282624](https://doi.org/10.1109/ACC.2007.4282624)

*Publication date:*

2007

*Document Version*

Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Wu, Z., Rajamani, M. R., Rawlings, J. B., & Stoustrup, J. (2007). Application of Autocovariance Least - Squares Method for Model Predictive Control of Hybrid Ventilation in Livestock Stable. In Proceedings of American Control Conference, 2007. ACC' 07 (pp. 3630-3635). DOI: 10.1109/ACC.2007.4282624

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# Application of an Autocovariance Least - Squares Method for Model Predictive Control of Hybrid Ventilation in Livestock Stables

Zhuang Wu and Murali R. Rajamani and James B. Rawlings and Jakob Stoustrup

**Abstract**—In this paper, the implementation of a new Autocovariance Least-Square (ALS) technique for livestock hybrid ventilation systems and associated indoor climate with a Model Predictive Control (MPC) strategy is presented. The design is based on thermal comfort parameters for poultry in barns and a combined dynamic model describing the entire system knowledge. Reference offset-free tracking is achieved using target calculation and quadratic programming and adding a disturbance model that accommodates unmeasured disturbances entering through the process input. The unknown noise covariances are diagnosed and corrected by applying the ALS estimator with the closed loop process data. The comparative simulations show the performance improvement with the ALS estimator in the presence of disturbances and moderate amount of error in the model parameters. The results demonstrate the high potential of ALS methods in improving the best practice of process control and estimation.

## I. INTRODUCTION

Environmental control for living systems differs greatly from comparable control for physical systems. Environmental requirements for living systems are typically more complex and nonlinear, and the biological system is likely to have significant and numerous effects on its physical surroundings. The design objective of this work is hybrid ventilation system and associated indoor environment for livestock barn, where hybrid ventilation systems combine the natural ventilation and mechanical ventilation, and have been widely used for livestock stables. Based on a so called conceptual multi-zone method, the horizontal variation of the indoor temperature and ventilation rate are taken into account and the entire system becomes a strongly coupled Multiple Input and Multiple Output (MIMO) dynamic nonlinear system. The system is exposed to external disturbances with random noises and has actuators with saturation.

As stated in books [1] and [2], papers [17], [4] and [5], Model Predictive Control (MPC) has become the advanced control strategy of choice by industry mainly for the economically important, large-scale, multi-variable processes in the plant. The rationale for MPC in these applications is that it can deal with strong non-linearities, handle constraints and modeling errors, fulfill offset-free tracking, and it is

easy to tune and implement. Consequently, applying MPC technology to allow a trade-off between the thermal comfort and energy consumption within constraints is necessary.

The heat dissipation from living animals such as pigs or poultry is one of the major influencing factors to the indoor comfort conditions, and lack of the knowledge about these disturbances makes the implementation of the control algorithm complicated, especially when covariances of the disturbance are unknown. A variety of methods have been proposed to solve this problem. A new Autocovariance Least-Squares (ALS) method for estimating noise covariances using routine operating data is employed to recover the covariances and adaptively determine an optimal filter gain. Odelson, Lutz, Rawlings [6] and Odelson, Rajamani, Rawlings [7] have researched and proved the superior advantages of ALS method convincingly through comparing with previous work.

In this paper, the livestock indoor environment and its control system will be regarded as a feedback loop. Through regulation, target calculation and state estimation, the predictive controller provides the optimal control actions involved with operating the valves and the fans. The ALS technique is not only expected to give an optimal estimator gain, but also to improve the closed loop performance in the presence of disturbances and model/plant mismatch. The comparative simulation results with the nominal controller and the ALS method are illustrated.

## II. PROCESS DYNAMIC MODELING

The schematic diagram of a large scale livestock barn equipped with hybrid ventilation system analyzed with conceptual multi-zone method is shown in Fig. 1(1), 1(2) and 1(3). The system consists of evenly distributed exhaust units mounted in the ridge of the roof and fresh air inlet openings installed on the walls. From the view of direction A and B, Fig. 1(a) and 1(b) provide a description of the dominant air flow map of the building including the airflow interaction between each conceptual zone.

As stated in [8] and [9], the differential algebraic equations govern sensible heat for indoor thermal comfort is shown in (1). The subscript  $i$  represents the zone number.

$$M_i c_{p,i} \frac{dT_i}{dt} = \dot{Q}_{i+1,i} + \dot{Q}_{i,i+1} + \dot{Q}_{in,i} + \dot{Q}_{out,i} + \dot{Q}_{conve,i} + \dot{Q}_{source,i}, \quad (1)$$

where,  $T_i$  is the zonal air temperature ( $^{\circ}C$ ),  $c_{p,i}$  is the specific heat of the air ( $J \cdot kg^{-1} \cdot K^{-1}$ ),  $M_i$  is the mass of the air ( $kg$ ),  $\dot{Q}_{i+1,i}$ , indicate the heat exchange ( $J/s$ ) due to the air flow

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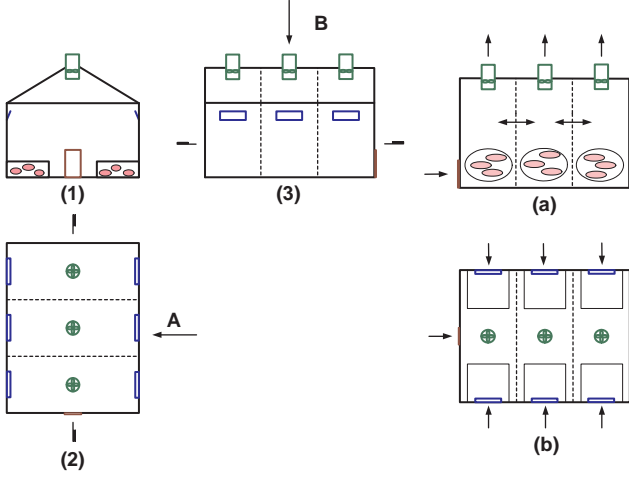


Fig. 1. Large Scale Livestock Barn and the Dominant Airflow Map of the Barn

across the conceptual boundary of zone  $i$  and zone  $i + 1$ .  $\dot{Q}_{in,i}$ ,  $\dot{Q}_{out,i}$  represent the heat transfer ( $J/s$ ) by air flow through inlet and outlet respectively. The convective heat loss through the building envelope is denoted by  $\dot{Q}_{conve,i}$  ( $J/s$ ). The heat source  $\dot{Q}_{source,i}$  includes the heat gain from animal heat production and heating system.

The volume flow rate through the inlet is calculated by (2), where  $C_d$  is the discharge coefficient,  $A$  is the geometrical opening area ( $m^2$ ),  $\Delta P$  is the pressure difference across the opening ( $Pa$ ) and can be computed by a set of routines solving thermal buoyancy and wind effect as (3). The subscript  $ref$  stands for the value at reference height,  $NPL$  stands for the Neutral Pressure Level (NPL). The internal pressure at NPL is denoted by  $P_i$ .

$$q_{in} = C_d \cdot A \cdot \sqrt{\frac{2 \cdot \Delta P}{\rho}}, \quad (2)$$

$$\Delta P = \frac{1}{2} C_p \rho_o V_{ref}^2 - P_i + \rho_o g \frac{T_i - T_o}{T_i} (H_{NPL} - H_{in}). \quad (3)$$

The exhaust unit consists of an axial-type fan and a swivel shutter. We introduce a fan law, as a relationship between the total pressure difference  $\Delta P_{fan}$ , volume flow rate  $q_{out}$  and supplied voltage  $V_{volt}$  with a specific shutter opening angle which can be expressed in (4) and (5), where the parameters  $a_0$ ,  $a_1$ ,  $a_2$  are empirically determined.

$$\Delta P_{fan} = a_0 \cdot (V_{volt})^2 + a_1 \cdot q_{out} \cdot (V_{volt}) + a_2 \cdot q_{out}^2, \quad (4)$$

$$\Delta P_{fan} = \frac{1}{2} \rho_o C_{p,r} V_{ref}^2 - P_i - \rho_i g \frac{T_i - T_o}{T_o} (H_{NPL} - H_{fan}). \quad (5)$$

For a detailed description and necessary simplifying assumptions of those system models development, we refer to [10].

### III. MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) refers to a class of control algorithms that compute a sequence of manipulated variable adjustments by utilizing a process model to forecast

process behavior and optimize based on a linear or quadratic open-loop performance objective, subject to equality or inequality constraints over a future time horizon.

#### A. Model Transformation

We regard the livestock ventilation system as two parts by noting that the overall system consists of a static air distribution system (inlet-exhaust air flow system) and a dynamic thermal system (animal environmental zones). Both of these two systems are mildly nonlinear with MIMO. However, representing or approximating a nonlinear model's dynamic response with some form of linear dynamics is an easy and illuminating way to analyze and solve on-line optimization, and especially, for processes maintained at nominal operating conditions and subject to small disturbances, the potential improvement of using a nonlinear model in MPC would appear small.

Through substitution and multiplication as described in [9], the general form of a combined Linear Time Invariant (LTI) state space model as (6) connecting the airflow model with thermal model, and representing the entire system dynamics around the equilibrium point is obtained.

$$x(k+1) = A \cdot x(k) + B \cdot u(k) + B_d \cdot \begin{bmatrix} d_{umd}(k) \\ d_{md}(k) \end{bmatrix}, \quad (6a)$$

$$y(k) = C \cdot x(k) + D \cdot u(k) + D_d \cdot \begin{bmatrix} d_{umd}(k) \\ d_{md}(k) \end{bmatrix}, \quad (6b)$$

where,

$$B_d = [B_{dumd} \quad B_{dmd}], D_d = [D_{dumd} \quad D_{dmd}]. \quad (7)$$

and,  $A \in \mathfrak{R}^{3 \times 3}$ ,  $B \in \mathfrak{R}^{3 \times 9}$ ,  $C \in \mathfrak{R}^{3 \times 3}$ ,  $D \in \mathfrak{R}^{3 \times 9}$ ,  $B_d \in \mathfrak{R}^{3 \times 8}$ ,  $D_d \in \mathfrak{R}^{3 \times 8}$  are the coefficient matrices at the equilibrium point.  $x, y, u, d_{umd}, d_{md}$  denote the sequences of vectors representing deviation variable values of the process state for the indoor temperature of each conceptual zone, the controlled output which is equal to the state, the manipulated input which consists of the valve openings and voltage supplied to the fans, the disturbances of the heat generated from animals and heating system, and the disturbances of external wind speed, wind direction and ambient temperature respectively.

$$x = [\bar{T}_1 \quad \bar{T}_2 \quad \bar{T}_3]_{3 \times 1}^T, \quad (8a)$$

$$u = [\bar{A}_{in,i=1\dots6} \quad \bar{V}_{volt,j=1\dots3}]_{9 \times 1}^T, \quad (8b)$$

$$d_{umd} = [\bar{Q}_1 \quad \bar{Q}_2 \quad \bar{Q}_3]_{3 \times 1}^T, \quad (8c)$$

$$d_{md} = [\bar{V}_{ref} \quad \bar{c}_{P,w} \quad \bar{c}_{P,l} \quad \bar{c}_{P,r} \quad \bar{T}_o]_{5 \times 1}^T. \quad (8d)$$

The pair  $(A, B)$  is controllable and the pair  $(A, C)$  is observable. Thus, the nonlinear plant model has been transformed into a series of LTI state space models and well prepared for solving the optimization problem in the predictive control scheme as will be discussed in the following sections.

## B. Disturbance Model and State Estimation

To achieve offset-free control of the output to their desired targets at steady state, in the presence of plant/model mismatch and/or un-modeled disturbances, the system model expressed in (6) is augmented with an integrating disturbance according to the general methodology proposed in [11] and [12]. The process states are influenced by the input disturbances from animal heat production, heating system and external weather condition. The animal productivity heat which is affected by various factors, will be modeled by integrating a random white noise. The resulting augmented system with process noise  $n_w$  and measurement noise  $n_v$  is:

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}u(k) + \tilde{G}n_w(k), \quad (9a)$$

$$y(k) = \tilde{C}\tilde{x}(k) + n_v(k), \quad (9b)$$

$$n_w(k) \sim \mathcal{N}(0, Q_w(k)), \quad (9c)$$

$$n_v(k) \sim \mathcal{N}(0, R_v(k)), \quad (9d)$$

in which the augmented state and system matrices are defined as follows,

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ x_{umd}(k) \end{bmatrix}_{6 \times 1}, \tilde{A} = \begin{bmatrix} A & B_{dumd} C_{umd} \\ 0 & A_{umd} \end{bmatrix}_{6 \times 6},$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}_{6 \times 9}, \tilde{C} = [C \quad 0]_{3 \times 6}, \tilde{G} = \begin{bmatrix} B_{dmd} & 0 \\ 0 & B_{umd} \end{bmatrix}_{6 \times 11}. \quad (10)$$

The full process state  $x \in \mathfrak{R}^3$  and unmeasurable disturbance state  $x_{umd} \in \mathfrak{R}^3$  are estimated from the plant measurement  $y$  by means of a steady state Kalman filter. The process and measurement noise  $n_w$  and  $n_v$  are assumed to be uncorrelated zero-mean Gaussian noise sequences with covariance  $Q_w$  and  $R_v$ . The determination of these covariances for an optimal filter gain is addressed in the ALS estimator section. The measurable deterministic disturbance  $d_{umd} \in \mathfrak{R}^8$  is assumed to remain unchanged within the prediction horizon and equal to the constant at the last measured value, namely  $d_{umd}(k) = \hat{d}_{dumd}(k+1/k) = \dots = \hat{d}_{dumd}(k+H_p-1/k)$ . The detectability of the augmented system in (9d) is guaranteed when the condition holds:

$$\text{Rank} \begin{bmatrix} (I-A) & -G \\ C & 0 \end{bmatrix} = n + s_d, \quad (11)$$

in which,  $n$  is the number of the process states,  $s_d$  is the number of the augmented disturbance states. This condition ensures a well-posed target tracking problem. For detailed explanation about the proof refer to [13] and [14].

## C. Target Calculation

We now formulate the target tracking optimization as the quadratic program formulation in (12), subjected to the constraints in (13), in which the steady state target of input and state vector  $u_s$  and  $x_s$  can be determined from the solution of the following computation when tracking a nonzero target vector  $z_t$ . The objective of the target calculation is to find the feasible triple  $(z_s, x_s, u_s)$  such that  $z_s$  and  $u_s$  are as close as possible to  $z_t$  and  $u_t$ , where  $u_t$  is the desired value of the

input vector at steady state, and,  $z_s = Cx_s$ .

$$\min_{[x_s, u_s]^T} \Psi = (u_s - u_t)^T R_s (u_s - u_t) \quad (12)$$

$$s.t. \begin{cases} \begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ z_t \end{bmatrix} \\ u_{\min} \leq u_s \leq u_{\max} \end{cases} \quad (13)$$

In this quadratic program,  $R_s$  is a positive definite weighting matrix for the deviation of the input vector from  $u_t$ . The equality constraints in (13) guarantee a steady-state solution and offset free tracking of the target vector.

## D. Constrained Receding Horizon Regulation

Given the calculated steady state, the constrained optimization problem is formulated by a quadratic cost function (14) on finite horizon, subjected to the following linear equality and inequalities (15) formed by the system dynamics (6) and equipment limitation and the constraints on the controlled variables.

$$\min_{u^N} \Phi_k = \hat{w}_{k+N}^T \tilde{Q} \hat{w}_{k+N} + \Delta v_{k+N}^T S \Delta v_{k+N} + \quad (14)$$

$$+ \sum_{j=0}^{N-1} [\hat{w}_{k+j}^T C^T Q C \hat{w}_{k+j} + v_{k+j}^T R v_{k+j} + \Delta v_{k+j}^T S \Delta v_{k+j}]$$

$$s.t. \begin{cases} w_{k+j} = x_{k+j} - x_s, \\ v_{k+j} = u_{k+j} - u_s, \\ w_{k+j+1} = A w_{k+j} + B v_{k+j}, \\ y_{\min} - y_s \leq C w_{k+j} \leq y_{\max} - y_s, j = j_1, j_1 + 1, \dots, j_2 \\ u_{\min} - u_s \leq v_{k+j} \leq u_{\max} - u_s, j = 0, 1, \dots, N-1 \\ \Delta u_{\min} \leq \Delta v_{k+j} \leq \Delta u_{\max}, j = 0, 1, \dots, N \end{cases} \quad (15)$$

where,  $\Phi$  is the performance index to be minimized by penalizing the deviations of the predictive state  $\hat{x}_{k+j}$ , control input  $u_{k+j}$  and the rate of change  $\Delta u_{k+j}$ , at time  $j$ , from the desired steady states.  $Q \in \mathfrak{R}^{3 \times 3}$  and  $S \in \mathfrak{R}^{9 \times 9}$  are symmetric positive semi-definite penalty matrices,  $R \in \mathfrak{R}^{9 \times 9}$  is symmetric positive definite penalty matrix. It is commonly taken that  $Q$  comprises terms of the form  $C^T C$  where  $r_{k+j} - y_{k+j} = C(x_s - x_{k+j})$ . The vector  $u^N$  contains the  $N$  future open-loop control moves as shown below

$$u^N = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix} \quad (16)$$

At time  $k+N$ , the input vector  $u_{k+j}$  is set to zero and kept at this value for all  $j \geq N$  in the open-loop objective function value calculation. As discussed in previous section, the plant is stable, therefore, according to [15],  $Q_N$  is defined as the infinite sum:  $Q_N = \sum_{i=0}^{\infty} A^{Ti} Q A^i$ , which will be determined from the solution of the discrete Lyapunov equation:  $Q_N = C^T Q C + A^T Q_N A$ . This regulator formulation guarantees nominal stability for all choices of tuning parameters satisfying the conditions outlined above [16], [17].

The output constraints are applied from time  $k+j_1$ ,  $j_1 \geq 1$ , through time  $k+j_2$ ,  $j_2 \geq j_1$ . The value of  $j_2$  is chosen such

that feasibility of the output constraints up to time  $k + j_2$  implies feasibility of these constraints on the infinite horizon. The value of  $j_1$  is chosen such that the output constraints are feasible at time  $k$ . The constrained regulator will remove the output constraints at the beginning of the horizon up to time  $k + j_1$  in order to obtain feasible constraints and a solution to the quadratic program. Muske and Rawlings in [16] and [18] explain the existence of finite values for both  $j_1$  and  $j_2$ .

Through on-line constrained dynamic optimization, we could obtain a sequence of optimal control signals  $u^N$  through a state and disturbance estimator, and the first input value in  $u^N$ ,  $u_k$ , is injected into the plant. This procedure is repeated by using the plant measurements to update the state vector at time  $k$ .

#### IV. ALS ESTIMATOR

The technique described in this section is originated in [7]. Consider the LTI discrete-time model of the augmented system as (9d), estimates of the states of the system are constructed using the standard Kalman filter as (17)

$$\hat{x}_{k+1/k} = A\hat{x}_{k/k-1} + Bu_k + AL_k(y_k - C\hat{x}_{k/k-1}). \quad (17)$$

The estimate error is defined as  $\varepsilon_k = x_k - \hat{x}_{k/k-1}$ , with covariance  $P_{k/k-1}$ . This covariance  $P_{k/k-1} = E[\varepsilon_k \varepsilon_k^T]$  is the solution to the Riccati equation (18)

$$P_{k+1/k} = AP_{k/k-1}A^T + GQ_wG^T - AP_{k/k-1}C^T [CP_{k/k-1}C^T + R_v]^{-1} CP_{k/k-1}A^T, \quad (18)$$

and the Kalman gain  $L_k$  is defined as (19)

$$L_k = P_{k/k-1}C^T [CP_{k/k-1}C^T + R_v]^{-1}. \quad (19)$$

Assume we process the  $y_k$  to obtain state estimates using a linear filter with gain  $L$ , which is not necessarily the optimal  $L$  for the system. The state estimation error  $\varepsilon_k$  evolves according to (20)

$$\varepsilon_{k+1} = (A - ALC)\varepsilon_k + \begin{bmatrix} G & -AL \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix}. \quad (20)$$

The state space model of the innovations  $\mathcal{Y} = y_k - C\hat{x}_{k/k-1}$  is defined as (21)

$$\varepsilon_{k+1} = \bar{A}\varepsilon_k + \bar{G}\bar{w}_k, \quad (21a)$$

$$\mathcal{Y}_k = C\varepsilon_k + v_k, \quad (21b)$$

in which,

$$\bar{A} = [A - ALC]_{n \times n}, \bar{G} = [G \quad -AL]_{n \times (g+p)}, \quad (22)$$

$$\bar{w} = \begin{bmatrix} w_k \\ v_k \end{bmatrix}_{(g+p) \times 1}.$$

$n$  is the number of states in (9d),  $p$  is the number of outputs,  $g$  is the number of independent noises.  $(A, C)$  is detectable,  $\bar{A} = A - ALC$  is stable, the initial estimate error is distributed with mean  $m_0$  and covariance  $P_0^-$ . We choose  $k$  sufficiently large so that the effects of the initial condition can be neglected, or equivalently, we choose the steady-state distribution as the initial condition:

$$E(\varepsilon_0) = m_0 = 0, \text{cov}(\varepsilon_0) = P_0^- = P^-. \quad (23)$$

Now we consider the autocovariance which is defined as the expectation of the data with some lagged version of itself [19]

$$\mathcal{C}_j = E[\mathcal{Y}_k \mathcal{Y}_{k+1}^T], \quad (24)$$

and the symmetric autocovariance matrix (ACM) is then defined as (25)

$$\mathcal{R}(N) = \begin{bmatrix} \mathcal{C}_0 & \cdots & \mathcal{C}_{N-1} \\ \vdots & \ddots & \vdots \\ \mathcal{C}_{N-1}^T & \cdots & \mathcal{C}_0 \end{bmatrix}, \quad (25)$$

where,  $N$  is the user-defined number of lags used in ACM. Accordingly, an ACM of the innovations can be written as follows:

$$\begin{aligned} & [\mathcal{R}(N)]_s \\ &= [(\mathcal{O} \otimes \mathcal{O})(I_{n^2} - \bar{A} \otimes \bar{A})^{-1} + (\Gamma \otimes \Gamma)\mathcal{J}_{n,N}] (G \otimes G)(Q_w)_s \\ &+ \left\{ [(\mathcal{O} \otimes \mathcal{O})(I_{n^2} - \bar{A} \otimes \bar{A})^{-1} + (\Gamma \otimes \Gamma)\mathcal{J}_{n,N}] (AL \otimes AL) \right. \\ &+ \left. [\Psi \oplus \Psi + I_{p^2 N^2}] \mathcal{J}_{p,N} \right\} (R_v)_s, \end{aligned} \quad (26)$$

in which

$$\mathcal{O} = \begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^{N-1} \end{bmatrix}, \Psi = \Gamma \begin{bmatrix} N \\ \oplus_{j=1} (-AL) \end{bmatrix}, \quad (27)$$

$$\Gamma = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ C\bar{A}^{N-2} & \cdots & C & 0 \end{bmatrix}.$$

$\mathcal{J}_{n,N}$  is a permutation matrix that converts the direct sum to a vector, i.e.  $\mathcal{J}_{n,N}$  is the  $(pN)^2 \times p^2$  matrix of zeros and ones satisfying

$$\begin{pmatrix} N \\ \oplus_{i=1} R_v \end{pmatrix}_s = \mathcal{J}_{p,N} (R_v)_s, \quad (28)$$

where, the subscript  $s$  denotes the outcome of applying the *vec* operator. Practically, the estimate of the autocovariance from real data is computed as

$$\hat{\mathcal{C}}_j = \frac{1}{N_d - j} \sum_{i=1}^{N_d - j} \mathcal{Y}_i \mathcal{Y}_{i+j}^T, \quad (29)$$

where,  $N_d$  is the sample size. Therefore, the estimated ACM  $\hat{\mathcal{R}}(N)$  is analogously defined using the computed  $\hat{\mathcal{C}}_j$ .

We define the ALS estimate as

$$\hat{x} = [(\hat{Q}_w)_s^T (\hat{R}_v)_s^T]^T = \arg \min_x \left\| \mathcal{A} \cdot \hat{x} - \hat{\mathcal{R}}(N)_s \right\|_2^2, \quad (30)$$

and the solution for estimating  $Q_w$ ,  $R_v$  is the well-known

$$\hat{x} = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T \cdot \hat{b}, \quad (31)$$

where,  $\mathcal{A}$  indicates the left hand side matrix to the least square problem, and

$$\mathcal{A} = \left[ D(G \otimes G) \quad D(AL \otimes AL) + [\Psi \oplus \Psi + I_{p^2 N^2}] \mathcal{J}_{p,N} \right], \quad (32)$$

$$D = [(\mathcal{O} \otimes \mathcal{O})(I_{n^2} - \bar{A} \otimes \bar{A})^{-1} + (\Gamma \otimes \Gamma)\mathcal{J}_{n \times N}], \quad (33)$$

$$x = \left[ \begin{matrix} (Q_w)_s^T & (R_v)_s^T \end{matrix} \right]^T, b = \mathcal{R}(N)_s. \quad (34)$$

The uniqueness of the estimate is a standard result of least-squares estimation [20]. The covariance can be found uniquely when the matrix  $\mathcal{A}$  has full column rank. However, in the augmented system as (9d), the dimension of the driving noise is  $w \in \mathfrak{R}^{11}$ , according to [6] and [7], it is unlikely to find unique estimates of the covariance  $(Q_w, R_v)$ , and the solution may not be positive semi-definite. In order to avoid leading to any meaningless solution, adding the semi-definite constraint directly to the estimation problem to maintain a convex program as (35) will ensure uniqueness of the covariance estimation.

$$V = \min_{Q_w, R_v} \left\| \mathcal{A} \begin{bmatrix} (Q_w)_s \\ (R_v)_s \end{bmatrix} - \hat{b} \right\|_2^2 \quad (35)$$

$$s.t. \begin{cases} Q_w \geq 0 \\ R_v \geq 0 \end{cases}$$

The constraints in (35) are convex, and the optimization is in the form of a semi-definite programming (SDP) problem, which can be solved efficiently with Newton's method [21].

## V. SIMULATION RESULTS

In order to demonstrate the benefits of using ALS method, the comparisons between using the ALS and nominally tuned estimator combined with MPC are presented. Since we have introduced an integrated white noise model for the input disturbance which could account for the model/plant mismatch, the following simulation results are derived in presence of a step change of deterministic un-modeled output disturbance. We assume that the state noise covariance  $Q_w = 0.01$  and measurement noise  $R_v = 0.001$ . The data set used for ALS computation is collected from open loop nonlinear plant simulation. Let  $N_d = 200$  and  $N = 12$ . The first 30 points are used as the training set, and the rest are used as a validation set. For the control system, the sampling time step is  $T_s = 120(s)$ , the prediction horizon is  $H_N = 20$ .

The estimator gain determined from the known covariances is conventionally regarded as a good tuning choice. However, as demonstrated in Fig. 2, in the presence a step increase of output disturbance, there are some marked contrasts in the closed loop output performances between using the ALS estimator (solid curves) and the conventional estimator. Using the ALS estimator, the regulator is able to reject the disturbances, tracking the reference faster and further reduce the steady state variances. The frequency distribution for the actuator's changes are shown in Fig. 3 and Fig. 4 respectively. The changing frequency of the six inlet vents openings and supplied voltages for three fans are about the same and prove that the improved closed loop performance does not require more aggressive manipulated inputs through using ALS estimator.

The covariance estimation techniques are based on the properties of the process innovations. Implementing ALS has high potential for improving the quality of estimation in comparison with the original estimator. This may be illustrated as Fig. 5 by comparing the frequency distribution

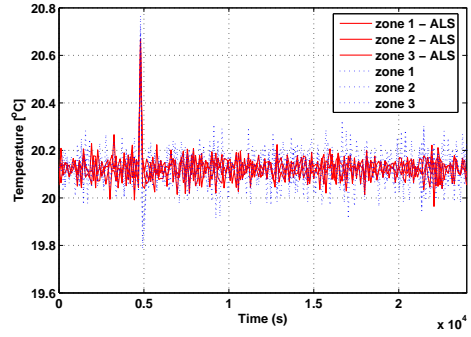


Fig. 2. Comparison of Closed loop Performances for Set-point Tracking with ALS and Nominal Estimator

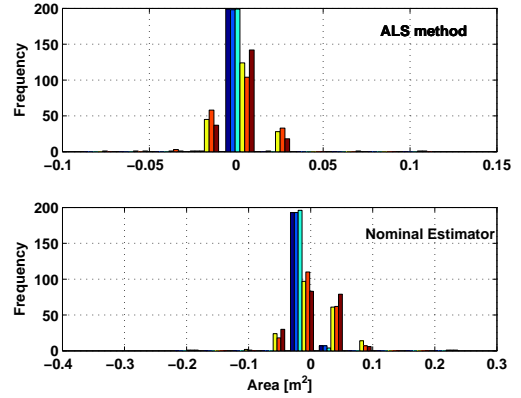


Fig. 3. Histogram of the Changes in Manipulated Inputs - Inlet Vent Openings

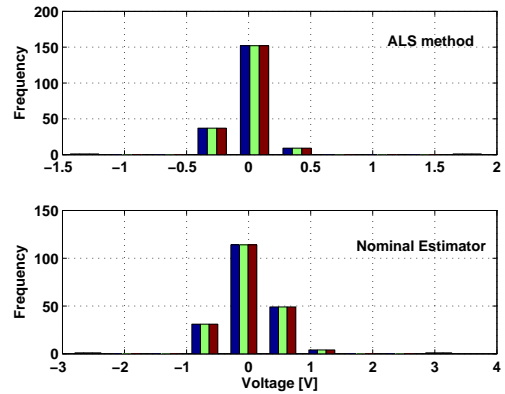


Fig. 4. Histogram of the Changes in Manipulated Inputs - Supplied Fan Voltages

of the innovations  $\mathcal{Y} = y_k - C\hat{x}_{k/k-1}$  for ALS with that of a nominal estimator.

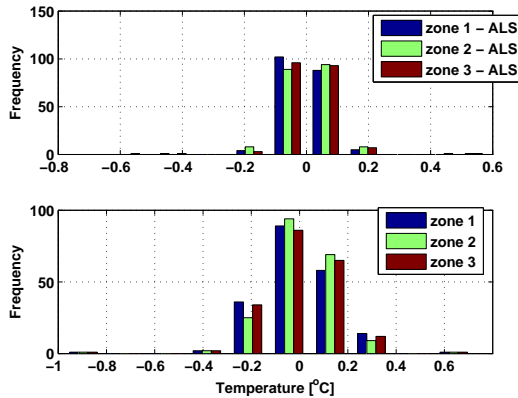


Fig. 5. Histogram of the Innovations with ALS method and Nominal Estimator

In conclusion, the normal tuning approach for estimator gain is time consuming and probably prone to failure especially when the real covariances are not known. The predictive controller combined with the ALS estimator is able to not only achieve off-set free tracking, but also design an optimal estimator to compensate model/plant mismatch and un-modeled disturbances without sacrificing more control actions.

## VI. CONCLUSIONS AND FUTURE WORK

### A. Conclusions

The main achievement of this work is the efficient application of the ALS method to design an adaptive estimation filter for Model Predictive Control of livestock ventilation systems. Through linearization of the nonlinear system, an LTI model in terms of state space representation which connected the thermal system and air distribution system is derived, and augmented by the integrated white noise disturbance model to achieve offset-free control. The presented simulation results show the significant advantages and performance improvement when using MPC over linear models for control and ALS method for estimation.

### B. Future Works

The entire control and estimation system will be implemented and identified in a real scale livestock barn equipped with hybrid ventilation systems in Syvsten, Denmark. The result will be compared with those obtained with the currently used control and estimation system.

## VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the contribution and financial support from the Danish Ministry of Science and Technology (DMST) and Center for Model Based Control (CMBC) with Grant number: 2002-603/4001-93.

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