Unified Approach to The Electromagnetic Field: The Role of Sources, Causality and Wave Propagation

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Abstract

The question of the sources of electric and magnetic fields and their causes has been discussed extensively in the literature over the last decades. In this article, we approach this problem from the unified treatment of electromagnetic fields emphasizing the role of their sources in accordance with the cause-effect relationships. First, we analyze whether this unified treatment contributes to a better understanding of these phenomena. Then, we discuss the implications for teaching of a correct understanding of electromagnetic field sources and causality in Maxwell's equations. In particular, we present a series of examples at the introductory physics level that allow us to recognize the reasons why considering electric and magnetic fields as disjoint entities can lead to contradictions.

Keywords: Cause-effect, Maxwell's equations, physics education research, university education, Jefimenko's equations.

INTRODUCTION

The electromagnetic theory presented by J. C. Maxwell (Maxwell, 1873) in his "*A Treatise on Electricity and Magnetism"* is one of the most important scientific constructs in physics. There is an undeniable need for students at different educational levels to acquire an adequate understanding of the concepts involved. Considerable research in the field of Physics Education Research (PER) has addressed the teaching of electromagnetism in regard to conceptual understanding, problem solving, and curriculum and instruction (Docktor and Mestre, 2014). However, the advances in PER have generally had less impact in introductory physics textbooks than in other areas such as mechanics (Galili and Goihbarg, 2005; Chabay and Sherwood, 2006; Zuza, Guisasola, Michelini and Santi, 2012).

One aspect in particular that has been discussed extensively in the literature over the last 50 years is the sources of electric and magnetic fields. A traditional treatment of Maxwell's equations in general physics textbooks usually leads to the interpretation that electric fields can be generated by charged particles or time-varying magnetic fields, and, at the same time, that magnetic fields are produced by time-varying currents or electric fields (Serway and Jewett, 2019; Young and Freedman, 2019). However, studies on the nature of classical electrodynamics show that constant and time-varying charge and current densities are the generators of electromagnetic fields (Rosser, 1963; Jefimenko, 1989 and 2004; French, 2000; Heald and Marion, 2012; Griffiths, 2013). This approach has had little impact on general physics textbooks. An approach neglecting these aspects could lead to students inadequately interpreting the sources of electromagnetic fields and the meaning of Maxwell's equations (Rainson, Tranströmer and Viennot, 1994; Guisasola, Almudi and Zubimendi, 2004; Hill, 2010 and 2011; Campos, Zavala, Zuza and Guisasola, 2019). In this article, we address the problem of field sources and cause-effect relations in Maxwell's equations at the introductory physics level and present a series of situations highlighting an interpretation compatible with the unified treatment of the electromagnetic field.

In the following section, we develop the main arguments supporting the fact that constant and time-varying charge and current densities are the generators of electric and magnetic fields, after which we discuss whether or not the displacement current generates a magnetic field. Then, we present a series of examples at the undergraduate introductory physics level that show how misinterpretations can be avoided with a unified treatment of the fields and the emphasis on their sources, which contributes to a better understanding of Maxwell's equations. Finally, we conclude with a discussion and implications for teaching.

Sources of Electromagnetic Fields

In their integral form, Maxwell's equations, a formalism that allows us to predict the time evolution of electromagnetic fields, are expressed as follows:

Gauss's law for electric fields,

$$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{q}{\varepsilon_0} \tag{1}$$

Gauss's law for magnetic fields
$$\oint \vec{B} \cdot \overrightarrow{dA} = 0 \tag{2}$$

Faraday's law
$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\int \vec{B} \cdot \overrightarrow{dA} \tag{3}$$

Ampère-Maxwell's law
$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot \overrightarrow{dA} \tag{4}$$

where $\vec{E}$ is the electric field, $\vec{B}$ the magnetic field, q the electric charge, I the conduction current intensity, μ_0 the magnetic permeability of vacuum, and ε_0 the electric permittivity.

Generally, in undergraduate introductory physics courses, the chapters dealing with electric and magnetic fields initially present the Gaussian laws and the stationary implications of Faraday's and Ampère-Maxwell's laws. These laws allow us to describe static fields and they reveal that charged particles generate electric fields, whereas electric currents generate magnetic fields. This framework presents the first obstacles for students to adequately interpret the sources of the fields (Campos, Hernández, Barniol and Zavala, 2021). Regarding Gauss's law for the electric field, all the charges in the space contribute to the electric field and must be considered for the calculation of the flux through the closed surface and not only those enclosed by the surface (Wangsness, 1979). On the other hand, according to Ampère's law, the magnetic field used to determine the circulation along a closed curve is the one corresponding to all the conduction currents and not only to those enclosed by the curve. Guisasola, et al. (2003 and 2008) found that, when asking students which charges generate the electric field of an infinite plane obtained by Gauss's law, many understand that only the charges enclosed by the Gaussian surface contribute. Similarly, when students are shown a typical Amperian loop to find the magnetic field of an infinite solenoid and asked about the currents that generate the field, many believe that only those enclosed by the curve contribute. These results show that students frequently exhibit a lack of understanding about the nature of the relationships between the different elements of Gauss's and Ampère's laws, that they do not adequately consider all variables and that they simplify cause and effect relationships. Therefore, when interpreting laws, they perform functional reductions typical of linear causal reasoning (Guisasola, 2003 and 2008).

When studying situations with time-dependent fields, Faraday's and Ampère-Maxwell's laws come into play, as they describe non-electrostatic electric fields and magnetic fields associated with displacement currents. In these equations, the fields are related to each other through the terms of the time derivatives of the flows (Alonso and Finn, 1983). In this scenario, we encounter another obstacle to properly interpret the relationships between the different

variables in Maxwell's equations and the sources of the fields, probably stemming from the way in which the terms that related them are interpreted.

A superficial analysis of Faraday's and Ampère-Maxwell's laws would allow us to infer that a time-varying magnetic field generates an electric field and vice versa. This is the dominant interpretation in many introductory physics textbooks. For example, in the case of Faraday's law, Tipler and Mosca (2007, p. 1031) state that "*According to Faraday's law, a changing magnetic flux produces an electric field whose line integral around a closed curve is proportional to the rate of change of the magnetic flux across any surface bounded by the curve*". Serway and Jewett (2019, p. 809) conclude that "*Equation 30.8 is the general form of Faraday's law. It represents all situations in which a changing magnetic field generates an electric field.*" Resnick, Halliday and Krane (2003, p. 784), on the other hand, argue that "*It is in this form that Faraday's law appears as one of the four basic Maxwell equations of electromagnetism. In this form, it is apparent that Faraday's law implies that a changing magnetic field produces an electric field.*" Similar interpretations can be found in the literature regarding Ampère-Maxwell's law. Tipler and Mosca (2007, p. 1031) assert that "*We thus have the interesting reciprocal result that a changing magnetic field produces an electric field (Faraday's law) and a changing electric field produces a magnetic field (generalized form of Ampère's law)*". Serway and Jewett (2019, p. 876) argue that Ampère-Maxwel's law "*describes the creation of a magnetic field by a changing electric field and by electric current*". Finally, Resnick, Halliday and Krane (2003, p. 862) argue that "*a magnetic field is set up by a changing electric field.*" We can also find the idea that a time-varying electric field generates a magnetic field and vice versa in other introductory physics textbooks (Bauer and Westfall, 2014; Fishbane, Gasiorowicz and Thorton, 2005; Giancoli, 2008; Knight, 2013; Lea, 1996; Ohanian, 2007; Young and Freedman, 2019). However, Faraday's and Ampère-Maxwell's laws cannot imply cause-effect relationships, since they link two quantities that are simultaneous and, therefore, neither of these quantities can be the source of the other (Tran, 2018).

According to the restricted version of the principle of causality in agreement with the principle of delayed action, "*there is always a time delay between the cause and the effect, the former being prior in time to the latter, so that (relatively to a given physical system, such as a reference system), C and E cannot be both distant in space and simultaneous*" (Bunge, 2009, p. 62). In accordance with this principle, the electric and magnetic fields at a given instant and point in space must depend on the constant and time-varying charge and current densities at an earlier time. These fields can be determined from their sources using the Jefimenko equations (Jefimenko, 1989):

$$\vec{E}(r,t) = \frac{1}{4\pi\varepsilon_0}\left[\int\left(\frac{\rho(r',t')}{|\vec{r}-\vec{r'}|^3} + \frac{\dot{\rho}(r',t')}{c|\vec{r}-\vec{r'}|^2}\right)(\vec{r}-\vec{r'})dv' - \int\frac{\dot{\vec{j}}(r',t')}{c^2|\vec{r}-\vec{r'}|}dv'\right] \tag{5}$$

$$\vec{B}(r,t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{j}(r',t')}{|\vec{r}-\vec{r'}|^3} + \frac{\dot{\vec{j}}(r',t')}{c|\vec{r}-\vec{r'}|^2}\right] \times (\vec{r}-\vec{r'})dv' \tag{6}$$

where $\vec{E}$ and $\vec{B}$ are evaluated at position $\vec{r}$ at time instant t, being $\vec{r'}$ the distance from the origin of coordinates to charge density ρ and current density $\vec{J}$, c is the speed of light and $t' = t - |\vec{r} - \vec{r'}|/c$.

According to this analysis, the only sources of the fields are the constant or time-varying charge and current densities (Hill, 2010 and 2011; Rosser, 2013). To clarify this matter, let us consider an electromagnetic wave propagating in space, with the electric and magnetic fields in phase. The cause of the wave is simply a distribution of charges oscillating, with electric and magnetic fields at a certain position r and time t linked to the movement of these charges over a previous time t'. It impossible to affirm the existence of an "electric wave" that subsequently produces a magnetic field.

Electric and magnetic fields form a single entity, the electromagnetic field (Griffiths, 2013). Thus, to conceive that if we are given one field, we can obtain the evolution of the other ignores the fact that they are components of the same entity and, therefore, they cannot interact with each other (Jefimenko, 2004).

The Case of The Displacement Current

Let us now consider the case of the displacement current and how to interpret its possible role as a source of magnetic fields. To do this, let us analyze the current density term in the Jefimenko equation for the magnetic field. In equation (6), $\vec{J}$ includes, in addition to the current density of free charges, the polarization current density $\partial\vec{P}/\partial t$ and the magnetization current density $\nabla \times \vec{M}$ (Griffiths and Heald, 1991; Jefimenko, 1992), being $\vec{P}$ and $\vec{M}$ the polarization and magnetization vectors, respectively. Although it may come as a surprise, the density of the vacuum displacement current ($\varepsilon_0 \, \partial\vec{E}/\partial t$) is not a source of magnetic field (Rosser, 1976). To properly understand this result, we must go back to Maxwell's early work.

The displacement current was first introduced by James Clerk Maxwell in *"On Physical Lines of Force",* published in 1861, where he succeeded in developing an electromagnetic theory based on a mechanical model of the ether (Lima, 2019). By introducing the displacement current, Maxwell arrived at a version of the continuity equation similar to the one used today (Maxwell, 1890, p. 496), although he considered it as a component of the conduction current (Darrigol, 2000). In that same work, Maxwell deduced for the first time the speed of propagation of electromagnetic waves through hypotheses linked to the mechanism of the electromagnetic field, suggesting that *"light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena"* (Maxwell, 1890, p. 500). Aware of the limitations and difficulties associated with his mechanical model of the ether, he decided to make it independent of the electromagnetic field. Thus, in 1864, he published *"A Dynamical Theory of the Electromagnetic Field"*, in which he presents eight equations of the electromagnetic field and an electromagnetic theory of light that can be tested experimentally (Berkson, 2014). It is in this work that he clearly describes his view of the physical meanings of electrical displacement and displacement current. *"Electrical displacement consists in the*

opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body… The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity…" (Maxwell, 1890, p. 554). Two fundamental conclusions can be drawn from this fragment: on the one hand, that in *"A Dynamical Theory of the Electromagnetic Field"* Maxwell considers the displacement current as another type of current that contributes to the total current (Chalmers, 1975; Darrigol, 2000) and, on the other hand, that the electric displacement was for him what the polarization vector is for us today (Arthur, 2009).

In 1873, Maxwell published his main work, *"A Treatise on Electricity and Magnetism",* in which he presents in detail the whole electromagnetic theory. Maxwell clearly states his position regarding the temporal variation of the electric displacement as current, asserting that this magnitude must generate a magnetic field, just like conduction currents. *"The current produces magnetic phenomena in its neighborhood… We have reason for believing that even when there is no proper conduction, but merely a variation of electric displacement, as in the glass of a Leyden jar during charge or discharge, the magnetic effect of the electric movement is precisely the same"* (Maxwell, 1873, p. 144-145).

From the above, we see that, from Maxwell's point of view, the displacement current generates a magnetic field, just like the conduction current. Moreover, given Maxwell's conception of space, in particular his conviction about the existence of the ether, he considers the displacement current a consequence of the variation of the electric displacement in any mechanical medium, and therefore it is always associated with a movement of bound charges, unlike the currently accepted view.

The suppression of the mechanical ether presents several problems for the interpretation of the displacement current, especially when analyzing the problems of the generation and propagation of electromagnetic fields in a vacuum. The absence of the ether makes the vacuum displacement current a simple term directly proportional to the rate of change of the electric field (Arthur, 2009). To understand this, it should be noted that when a medium is polarized by the effect of an electric field, the electric displacement $\vec{D}$ is given by:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad (7)$$

whereas the displacement current results as follows:

$$\frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} \qquad (8)$$

It should also be noted that while the term $\partial \vec{P}/\partial t$ is associated with a real motion of bound charges, the last term, $\varepsilon_0 \, \partial \vec{E}/\partial t$, presented in introductory university physics courses, corresponds to the vacuum's contribution to the displacement current. Therefore, and contrary to the view of Maxwell and his contemporaries, we can have a displacement current in a vacuum, due to the variation of the electric field over time. This last point is crucial to

understand why the vacuum displacement current is not a source of magnetic fields. In the vacuum, a variation of vector $\vec{D}$ is solely due to a variation of the electric field, and, as we discussed in the previous section, a time-varying electric field is not a source of magnetic fields (French, 2000; Griffiths, 2013).

Maxwell's Equations Imply Relationships Between The Fields

As we have seen in the previous sections, Maxwell's equations do not imply cause and effect relationships between their terms. Through a careful analysis of different situations, even in introductory physics courses at the university level, we can observe that they simply correspond to relationships between different magnitudes at the same instant of time. In the following section, we present a series of examples that allow us to recognize the reasons why considering electric and magnetic fields as separate entities can lead to contradictions.

A Point Charge in Motion

When a point charge moves with constant velocity, it generates electromagnetic fields around it. Let us analyze the magnetic field, which is usually calculated by the Biot-Savart law for a moving charge, but in this case calculating it by Ampère-Maxwell's law. For this purpose, let us consider a point charge performing a uniform rectilinear motion (URM) with respect to an inertial frame of reference *R* and a closed curve *C*, as shown in Figure 1.
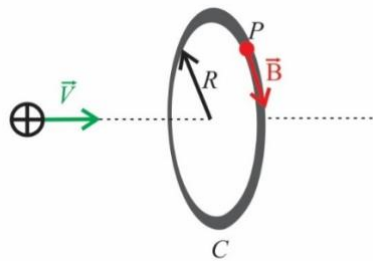

Figure 1. Point charge performing a URM with respect to an inertial frame of reference R and a closed curve C

The magnetic field circulation along the curve *C* is non-zero. Although we know this simply from plotting the magnetic field at each point on the curve, we should be able to predict it using Ampère-Maxwell's law. The surface S, delimited by *C*, is not traversed by conduction currents, therefore, there is a displacement current that appears because the point charge is approaching curve *C* and the flux of electric field is getting larger. Thus, the magnetic field circulation along the curve *C* is given by:

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{dA} \tag{9}$$

If the velocity of the point charge is much smaller than the speed of light, and the electric field flux and its time variation are determined, we can find an expression for the magnetic field that evidently coincides with the Biot-Savart law for a moving charge (Buschauer, 2013; Suarez, 2013). At this point, we must be careful with the physical interpretation of the phenomenon. We are faced with a situation where we can determine the magnetic field at a point in space directly by the Biot-Savart law for a moving charge or by Ampère-Maxwell's law. If we assign a cause-effect relationship to both laws and analyze the situation applying the former, we would say that, because the point charge is in motion, it generates a magnetic field around it, whereas if we apply the latter, we would conclude that the time-varying electric field is the cause of the magnetic field. If both interpretations were valid, we would have to consider both contributions for the calculation of the magnetic field.

When a point charge is moving with constant velocity it generates an accompanying electromagnetic field. It is not accurate to say that the electric field generates the magnetic field, because both are parts of the same entity. As we mentioned earlier, the sources of electric and magnetic fields are the charge and current densities, either constant or time-varying.

Ampère-Maxwell's Law and Charging a Capacitor

Consider the best-known example for introducing the displacement current, showing a capacitor being charged, a closed curve C, and two surfaces S_1 y S_2 (Figure 2).
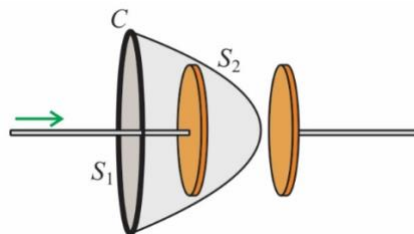

Figure 2. Surfaces S_1 and S_2 are delimited by the same curve C

A typical interpretation of the problem is that the current that generates the magnetic field changes depending on the surface taken for its calculation. For example, Resnick, Halliday and Krane (2003, p. 862) argue that *"In the first case, it is the current through the surface (S_1) that gives the magnetic field, and in the second case, it is the changing electric flux through the surface (S_2) that gives the magnetic field"*.

The actual source of a magnetic field at a point cannot depend on the surface taken to calculate circulation. In that sense, assertions such as the one transcribed above may lead students to believe that the magnetic field appearing in the line integral of Ampère-Maxwell's law is due only to the currents traversing the surface, which may result in cause-effect interpretations of Maxwell's equations (Hill, 2011).

As in the previous example, other analyses could also lead to conceptual errors among students, promoting causal linear reasoning in the interpretation of Ampère-Maxwell's law. For

example, Young and Freedman (2019, p. 972) determine an expression for the magnetic field between the plates of a capacitor being charged with the displacement current. In that situation, the authors (Young and Freedman, 2019, p. 972) assert that *"When we measure the magnetic field in this region (*between the plates of a capacitor*), we find that it really is there and that it behaves just as Eq. (29.17) predicts. This confirms directly the role of displacement current as a source of magnetic field"*. This statement could reinforce the erroneous interpretation of the different terms of Maxwell's equations as field generators. The magnetic field between the plates of a capacitor is entirely due to the conduction currents in the plate's surfaces and wires (French, 2000; Milsom, 2020).

A Magnet in Motion

Consider a bar magnet moving with a constant velocity with respect to an inertial frame of reference *R* and a closed curve *C*, as shown in Figure 3. From Faraday's law, we know that there is a non-conservative electric field at all points on the curve. The standard explanation for the appearance of this electric field is that it originates as a consequence of the temporal variation of the magnetic field. However, as we know, the time-varying magnetic field does not generate the electric field. From Faraday's law, we can affirm that if the magnetic field flux through a surface *S* is varying with time, in the curve that delimits the surface there must exist an electric field whose circulation is the opposite of the time derivative of the magnetic field flux.
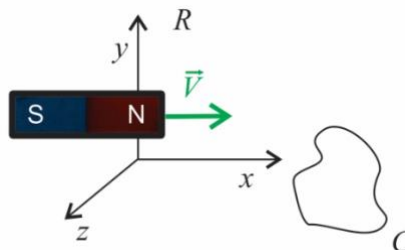

Figure 3. Bar magnet performing a URM with respect to an inertial reference frame R and a closed curve C.

Imagine now that the magnet is inside a black box and the only information we have are the measurements of the electric and magnetic field in a small area outside the box. If we apply Faraday's law or Ampère-Maxwell's law to an arbitrary closed curve we would find that the fields are related, as implied by the equations, but we could not say that one of them was generated by the other. A magnet moving at a constant speed generates an accompanying electromagnetic field around it. To affirm that the magnetic field generates the electric field is a conceptual simplification, because they are parts of the same entity.

For both a magnet and a point charge performing a URM with respect to an inertial frame of reference *R*, there is a system *R'* where they are at rest and only the magnetic or electric aspect of their electromagnetic field is detected. A simple change in the reference system enables us to observe different aspects of the electromagnetic field. We can determine the

relation between the fields in R and R' with the Lorentz transformations for $\vec{E}$ and $\vec{B}$ or with their weakly relativistic approximation (Galili and Kaplan, 1997). The introduction of some of these transformations in basic physics courses makes it possible to emphasize the fact that the electric and magnetic fields are parts of the electromagnetic field.

Faraday's Law and The Fields Generated By a Solenoid

Now, let us analyze a typical electromagnetic induction problem, which consists of determining the electric field in a region of space where a circular solenoid of length L and radius R, whose current varies with time following a certain function $I(t)$, is located.
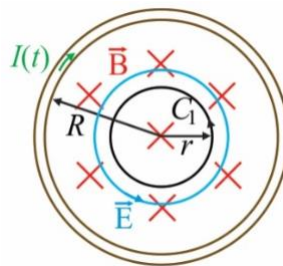

Figure 4. Cross section of a solenoid through which a current I is flowing.

To determine the electric field at a distance r smaller than the radius R of the solenoid, we apply Faraday's law to the closed curve C_1 illustrated in Figure 4, where a cross section is shown. If the solenoid is very long, we can assume that the magnetic field inside it is uniform and the magnetic field flux results as follows:

$$\Phi_B = \mu_0 I(t) r^2 \pi \tag{10}$$

If we substitute (10) in Faraday's law (3) and operate, we obtain the electric field at a distance r from the center of the solenoid.

$$E(t) = \frac{\mu_0 n r}{2} \frac{dI(t)}{dt} \tag{11}$$

The expression obtained for the electric field is known, emphasizing that it is directly proportional to the time derivative of the current intensity in the solenoid. Equations 10 and 11 are strictly valid in situations where I is constant or increases linearly with time. Although a discussion about its framework of validity is usually presented in higher electromagnetism courses, if we apply the Ampère- Maxwell's law to the electric field obtained through Faraday's law, we can relatively easily understand its framework of applicability and reveal the way in which Maxwell's equations are linked.
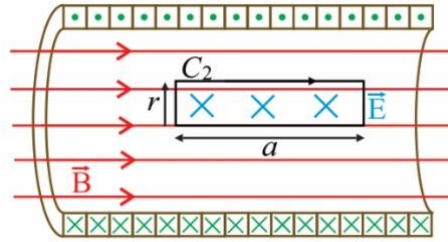
Figure 5. Longitudinal section of the solenoid. The magnetic field is horizontal to the right.

Figure 5 shows a longitudinal section of the solenoid, together with the directions of the magnetic and electric fields, under the assumption that the current intensity increases with time. If we apply Ampère-Maxwell's law to the closed curve C_2, since there are no conduction currents through a surface delimiting it, we have that:

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \vec{dA} = \mu_0 \varepsilon_0 \frac{d \int_0^r E a \, dr}{dt} \tag{12}$$

If we substitute (11) in (12) and operate:

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0^2 \varepsilon_0 n a r^2}{4} \frac{d^2 I(t)}{dt^2} \tag{13}$$

This equation reveals that the magnetic field circulation is directly proportional to the second time derivative of the intensity of the current in the solenoid. In the case in which the magnetic field is uniform, the circulation along C_2 should vanish. Then by applying the Ampère-Maxwell law, we directly find the framework of validity of Eqs. 10 and 11. The only way that the electric field obtained through Faraday's law is a solution of Maxwell's equations is $d^2 I / dt^2 = 0$, or equivalent, the current intensity varies linearly with time.

The problem posed has several interesting points to discuss, the first of which is the fact that, when dealing with electromagnetic induction problems in introductory physics courses, electric field functions are generally determined using Faraday's law, without checking or discussing whether the solution also verifies Ampère-Maxwell's law. This could lead students to believe that the solutions of the fields only have to verify one of Maxwell's equations. On the other hand, the analysis performed allows us to understand why the equation for the electric field found is only valid when $d^2 I / dt^2 = 0$. Faraday's and Ampère-Maxwell's laws are coupled, since the electric field circulation depends on the rate of change of the magnetic field flux, and the magnetic field circulation depends on the rate of change of the electric field flux. When we affirm that $d^2 I / dt^2 = 0$, the rate of change of the electric field flux becomes zero, decoupling Faraday's and Ampère-Maxwell's laws, which then allows us to determine the electric field with Faraday's law alone.

Let us now consider a situation in which $d^2 I / dt^2 \neq 0$. In this case, Eq. 13 gives the first order correction of the magnetic field circulation. A typical approach to the problem of why Faraday's law is not enough to determine the electric field is to argue that if the electric

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field changes with time it generates a new magnetic field that overlaps with the original one. This would result in another configuration of magnetic fields inside the solenoid that would have to be taken into account to recalculate the electric field and so on. This reasoning, which may be reinforced by an approach of successive approximations to determine the fields, contradicts the way of approaching the problem to find a general solution (Batell and Ferstl, 2003). To find a solution to the fields, one must solve a system of coupled differential equations for the electromagnetic fields that arises from applying Faraday's and Ampère-Maxwell's laws to the solenoid. Electric and magnetic fields must verify both laws simultaneously at any instant of time. It is not correct to point out that one of the fields is the cause of the other.

DISCUSSION AND IMPLICATIONS FOR TEACHING

The discussion raised so far has important implications for the teaching of electromagnetism. A unified approach to electric and magnetic fields, in which the actual sources of the fields are the charge and current densities, either constant or time-varying, would show students a more accurate and probably more consistent picture of field sources, causality, and electromagnetic wave propagation. Likewise, presenting an adequate relationship between the different magnitudes, particularly in Faraday's law, would help to avoid conceptual errors. W. Rosser, one of the main advocates of emphasizing the correct "unified" relationship between electric and magnetic fields, calls the causal interpretation between $\vec{E}$ and $\vec{B}$ *"obsolete"* and suggests that when speaking of electromagnetic induction it should be said that *"…if we set up the experimental conditions to get a varying magnetic flux through the stationary circuit, then the varying charge and current distributions that give rise to the varying magnetic flux also give rise to an induction electric field, which gives rise to an induced emf $\oint \vec{E} \cdot \vec{dl}$ in the stationary circuit, which is numerically equal to minus the rate of change of the magnetic flux through the circuit"* (Rosser, 2013, p. 125-126). On the other hand, and although the introductory physics textbooks mentioned so far consider that a time-varying magnetic field produces an electric field and vice versa, we find that two of the texts based on PER research, namely *"Matter and interactions"* by Chabay and Sherwood (2015) and *"Six ideas that shaped physics"* by Moore (2017), expound the idea that Maxwell's equations imply associations and not cause-effect relationships. Regarding Faraday's law, Chabay and Sherwood (2015, p. 902) state that *"The historical term 'magnetic induction' is often used to describe this phenomenon, and one says that the time-varying magnetic field 'induces' the curly electric field. This is somewhat misleading. It is more correct to say that anywhere we observe a time-varying magnetic field, we also observe a curly electric field. Faraday's law relates these observations quantitatively"*. Moore (2017, p. 271), on the other hand, is more emphatic: *"I have been very careful to state that this is the electric field that is correlated with the changing magnetic field, not created by that field. Electromagnetic fields are created only by stationary or moving charged particles… So though E and B are correlated by Faraday's law in a given reference frame, correlation is not causation"*.

Ideally, a unified treatment of the electromagnetic field would avoid conceptual confusions such as those presented in the previous sections. To this end, it is key to develop didactic sequences that allow students to understand that Maxwell's equations do not give information about the sources of the fields but rather describe relationships between different quantities and determine their temporal evolution. This can be done not only in situations where time-varying electric and magnetic fields are present, as in the example given in the previous section, but also when analyzing magnetostatic and electrostatic problems where field expressions are determined by Gauss's or Ampère's laws. Thus, emphasis should be placed on the fact that the fields must simultaneously verify all four Maxwell's equations and not one in particular, reaffirming the fact that they form a system of coupled equations. Along these lines, both in static and time-varying field situations, efforts should be made to discuss problems where hypothetical configurations of electric and magnetic fields are analyzed to verify Maxwell's equations. Another possible approach to Gauss's and Ampère's laws, which can be easily extended to Ampère-Maxwell's law, is to analyze the field line configurations for different distributions of charge and current. These problems would stimulate students to reflect on field sources and avoid the emergence of causal linear reasoning (Guisasola, et al. 2008).

Finally, we point out the convenience of designing activities that allow students to understand that electric and magnetic fields are not independent but parts of the same entity, the electromagnetic field. This unified vision helps us to avoid the appearance of contradictory situations, such as those presented in this article, and to develop a better understanding of electromagnetic phenomena.

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