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Maximizing the survival probability in a cash flow inventory problem with a joint service level constraint

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ABSTRACT

This paper investigates a multi-period stochastic cash flow inventory problem with the aim of maximizing the long-term survival probability, which may be the objective of some retailers especially in periods of economic distress. Demand in each period is stochastic and can be non-stationary. In order to avoid too many lost sales under this objective, we introduce a joint chance constraint requiring the probability of no stockouts during the planning horizon to be higher than a specified service level. We develop a scenario-based model and a sample average approximation (SAA) model to solve the problem. A statistical upper bound on the survival probability based on SAA is provided and we discuss upper and lower bounds for the problem based on stochastic dynamic programming. We also propose a rolling horizon approach with service rate updating to test the out-of-sample performance of the two stochastic models and solve problems with long planning horizons. We test the two methods in large numerical tests and find that the rolling horizon approach together with the stochastic models can solve realistically sized problems in reasonable time.

1. Introduction

While many retailers set the objective of maximizing profits or minimizing costs, they may still want to maximize their survival probabilities in certain periods. For example, for some newly created firms, the probability of survival rather than profit is the main objective until they become established (Archibald et al., 2002, 2007, 2015). For some so-called nanostores in emerging markets, survival can also have a very important influence on operational decisions (Boulaksil and van Wijk, 2018). Moreover, in periods of economic distress such as the economic crisis caused by the Covid-19 pandemic in recent years, some industries like tourism, hospitality and retailing can be greatly affected. During the crisis, survival becomes more important than maximizing profit for some businesses in these industries. Nevertheless, we find that placing too much emphasis on maximizing survival probabilities, as in some related works (e.g., Archibald et al., 2002), may result in a high lost sale rate. This is illustrated in Fig. 1 with a numerical example when initial inventory is 0.

Fig. 1 shows the survival probability, service level and optimal ordering quantity in the first period of a survival maximizing strategy determined by the model of Archibald et al. (2002) as the initial cash position of the retailer varies. As the initial cash position increases from 0 to 75, we observe a gradual increase in the both survival probability and service level. However, as the initial cash position increases beyond 75, the survival probability continues to increase towards 100%, but the service level declines to 0%. This phenomenon is attributed to the ordering quantity decisions illustrated in Fig. 1(b): the optimal ordering quantity remains consistently positive when the cash position is below 160, gradually diminishing to zero as the initial cash position increases further. This pattern is due to the fact that, when the initial cash is sufficiently large, the retailer can sustain itself without ordering items to satisfy demand. A similar pattern arises in subsequent periods, but the cash position at which the order quantity reaches zero decreases as the end of the planning horizon becomes closer. This clearly demonstrates that survival maximization models can lead to significant lost sales despite the retailer having sufficient cash reserves. Importantly, this phenomenon is not unique to this specific example, as the original survival maximization model tends to generate conservative ordering policies (Archibald et al., 2007). To provide a comprehensive understanding of the relationship between service level and ordering quantities for various parameter values, we conduct extensive numerical tests in Section 7.4. These tests further illustrate the impact of different parameter values on the survival probability, service level and ordering decisions.

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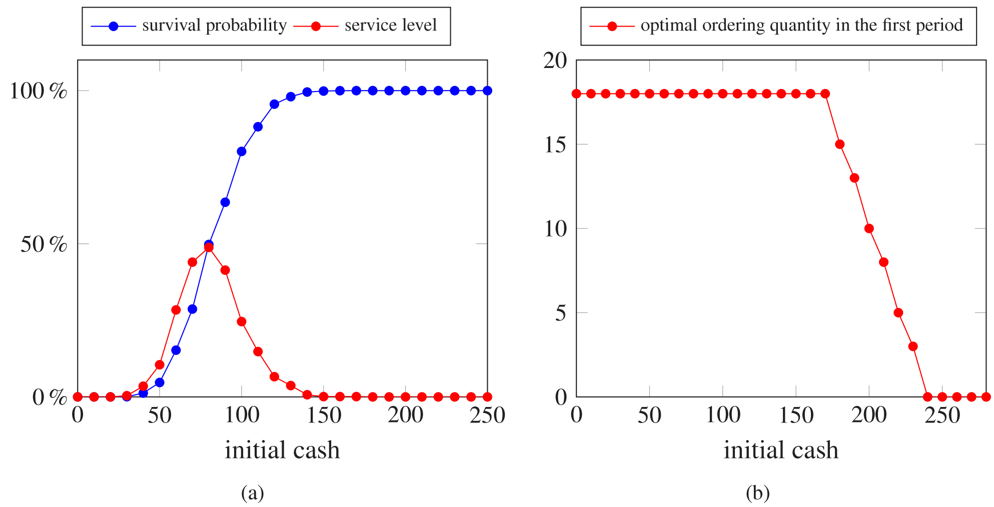


Fig. 1. Typical example of survival probability, service level and optimal ordering quantity in the first period as functions of the initial cash position.

In retail businesses, on-time delivery or service rate is often very import. Research has found that 70% of consumers may not shop with a retailer again after receiving a late shipment (Logistics, 2017). Empirical data shows a one percentage point increase in service rate is associated with a statistically significant 11% increase in retailer demand (Craig et al., 2016). In some industries, service-level guarantees are often contract-enforced and very strict (Silver et al., 1998). If a retailer focuses on survival without considering service level, it may lose customer goodwill due to too many lost sales and still go bankrupt in the end.

During the pandemic, numerous retailers and restaurants prioritized managing their cash flow to avoid bankruptcy due to the severe impact of the COVID-19 crisis on the economic environment. However, they also strived to maintain a low stock-out rate in order to retain as many customers as possible in the future, as they anticipated the eventual end of the pandemic and a return to normalcy.

Therefore, maintaining a positive cash position and meeting customer demand at the same time is a trade-off for a retail business. The aim of this paper is to maximize the survival probability in a multi-period stochastic inventory management problem while trying to satisfy the required service level. Considering the real-life background of the problem, the periods can be days, weeks, or even months. The service level is represented by a joint chance constraint requiring the lost sale rate in the planning horizon to be lower than a specified proportion. The main contributions of this study are listed below.

- We consider an operational research problem, namely maximizing the survival probability with a joint service level constraint, which has not been considered in the literature before. Two stochastic modelling methods are applied to solve the problem: scenario modelling and SAA modelling.
- We construct a statistical upper bound on the survival probability for this problem based on SAA and we also provide a lower bound by stochastic dynamic programming with Bonferroni approximation of the service level constraint.
- We develop a service rate updating technique in a rolling horizon approach to solve realistically sized problems efficiently.
- Through comparison with the model in Archibald et al. (2002), we show that our models can ensure a good chance of survival while avoiding overly conservative ordering decisions and low service rates.

The rest of this paper is structured as follows. Section 2 reviews the related literature and Section 3 describes the problem. Section 4 formulates the scenario and SAA models. The bounds on the objective function are presented in Section 5. Section 6 explains the rolling horizon approach and the service rate updating method. A computational study and its results are detailed in Section 7. Finally, conclusions and future research directions are outlined in Section 8.

2. Literature review

The literature associated with our work can be classified into two main streams: cash flow inventory management problems and chance-constrained programming in inventory management. Each stream is reviewed below.

2.1. Cash flow inventory management problems

Cash flow is often taken into account in the operational research literature. For example, Comelli et al. (2009) consider cash flow in the supply chain planning problem; Benedito et al. (2016) study a manufacturing system with production capacity renewal, tax and cash flow management. Nevertheless, the cash flow does not affect ordering or production quantities in their models. With regard to the cash constraint, Chao et al. (2008) investigate a cash constrained multi-period newsvendor problem and prove the optimality of an approximate base stock policy. Katehakis et al. (2016) discuss short-term financing and prove that the optimal ordering policy is characterized by a pair of threshold parameters. Boulaksil and van Wijk (2018) propose a cash constrained stochastic inventory model with consumer loans and supplier credits and give some managerial insights by simulating numerical cases. Fu et al. (2021) address inventory-based financing and partially characterize the optimal inventory policy controlled by two state-dependent levels. Chen and Rossi (2021) propose an $(s, C(x), S)$ policy for a stochastic lot sizing problem with cash constraints. Kajjoune et al. (2021) consider short-term financing in a dynamic lot sizing problem with deterministic demands and solve the problem with dynamic programming. Chen and Zhang (2021) build two multi-stage stochastic models for a multi-product problem with order-based loan.

While many research papers set the objective to be either maximizing profits or minimizing costs, there are also some works that aim to maximize the survival probability. Archibald et al. (2002) analyse an inventory control model maximizing the survival probability and show the optimal ordering policy is more conservative than the profit maximizing policy. Archibald et al. (2007) compare various inventory models and give conditions under which their policies are equivalent. Archibald et al. (2015) extend the problem for managing inventory and production capacity. There are also some works that address a firm's survival probability by a bankruptcy threshold. For example, Xu and Birge (2006) build a multi-stage scenario tree model incorporating both production and external financing decisions and solve it to maximize the present value of future cash flow subject to bankruptcy risk; Tanrısever et al. (2012) assume there is a minimum profit level required for survival as a constraint in a two-stage model; Kouvelis and Zhao (2012) assume there is a minimum demand as bankruptcy threshold. Similar assumptions are also adopted by Yang et al. (2015), Wu et al. (2019), and Esenduran et al. (2022). Several other works compare models with profit maximization and survival probability maximization objectives. For example, Swinney et al. (2011) investigate capacity investment timing decisions by a duopoly model, where the start-up maximizes its survival probability and the established firm maximizes profits. A similar model is built in Xing et al. (2022) for quality investment decisions. There are also a few works considering bankruptcy cost and credit in the profit maximization model. For example, Yan and He (2020) discuss a cash constrained problem with trade credit, service level and bankruptcy cost.

2.2. Joint chance-constrained programming in inventory management

Chance-constrained optimization problems were introduced by Charnes and Cooper (1959). A chance constraint places an upper bound on the probability an event occurs in a period. When the probability constraint applies to the whole planning horizon rather than individual periods, it is called a joint chance constraint (Miller and Wagner, 1965). In the following, we focus on the literature about joint chance-constrained programming in inventory management along with solution methods and analysis related to our paper.

The main challenge for linear programs with joint chance constraints is that the feasible region is non-convex. Nemirovski and Shapiro (2007) build a convex approximation for chance-constrained problems. Luedtke and Ahmed (2008) provide conditions for lower and upper bounds for the SAA method of solving joint chance-constrained problems. Pagnoncelli et al. (2009) discuss the convergence properties of the SAA method. Luedtke et al. (2010) provide a strong extended formulation of the SAA method for joint chance-constrained problems in which only the right-hand side is random and this random vector has a finite distribution. Since the extended SAA method reduces the number of constraints and makes the computation faster, it is adopted by some later works such as a transmission switching problem with guaranteed wind power utilization (Qiu and Wang, 2014) and a single-item capacitated lot-sizing problem (Gicquel and Cheng, 2018).

In solving multi-period problems, many works model the problems with a static joint chance constraint (e.g., Gicquel and Cheng, 2018), in which decisions are made before random demands are known. Zhang et al. (2023) provide valid inequalities for the static joint chance-constrained lot-sizing problem. Andrieu et al. (2010) use a dynamic joint chance constraint, where decisions can be made after the uncertainty is revealed, in a hydro power reservoir problem. Zhang et al. (2014) develop a branch-and-cut method for dynamic joint chance-constrained problems and show significant cost savings can be achieved compared with static joint chance-constrained models.

From the above literature review, we can see that there are few works that consider dynamic joint chance constraints in inventory management problems. In particular, we could find no work that maximizes the survival probability with chance constraints to control service level. These aspects together with the practical relevance of the problem inspire us to investigate the topic in our paper.

3. Problem description

For convenience, the main notations adopted in this paper are listed in Table 1. Other relevant notations will be introduced as needed.

In our problem, a cash constrained retailer periodically purchases a product from its supplier for sale to its customers. The planning horizon is comprised of T periods. Customer demand in each period is stochastic and can be non-stationary. Demand is assumed to be independently distributed from period to period. The retailer incurs purchasing and overhead costs and earns revenue by selling the product. The retailer is assumed to have failed if its cash position is negative at the end of any period in the planning horizon. Similar to Archibald et al. (2002), we assume the retailer's objective is to maximize its survival probability.

Let I_t denote the inventory available at the end of period t. Moreover, let the initial inventory at the beginning of the planning horizon be I_0. At the beginning of period t, the retailer must decide the period's order quantity Q_t, where $Q_t \geq 0$. Demand in period t is represented by a random variable D_t with probability density function f_t and cumulative distribution function F_t. Demand in a period can be met by inventory held at the beginning of the period or items ordered in the period. Unmet demand is assumed to be lost with no additional penalty costs. Excess stock is transferred to the next period as inventory. The selling of excess stock back to the supplier is not allowed before the end of the planning horizon. Suppliers require immediate payment and they supply items immediately, i.e. the order delivery lead time is zero. The inventory balance equation can then be expressed as

$$I_t = (I_{t-1} + Q_t - D_t)^+, \tag{1}$$

where $(x)^+$ denotes $\max\{x, 0\}$. A variable ordering cost v is charged on every ordered unit. We do not consider fixed ordering cost in the problem. The selling price of the product is p per unit, and the retailer receives payments only when items are delivered to customers. The sales quantity in period t is $\min\{D_t, I_{t-1} + Q_t\}$, and revenue is thus $p\min\{D_t, I_{t-1} + Q_t\} = p(I_{t-1} + Q_t - I_t)$.

Any inventory left over at the end of the planning horizon has a salvage value of γ per unit. It is reasonable to assume $0 \leq \gamma \leq v < p$. At the end of each period, overhead costs H such as wages or rents, are required to be paid irrespective of the retailer's activity. Let C_t denote the cash position at the end of period t. The retailer's cash position at the beginning of the planning horizon is C_0. The full expression for C_t is given by Eq. (2), where there is salvage value γI_T for the remnant inventory at the end of period T.

$$C_t = \begin{cases} C_{t-1} + p(I_{t-1} + Q_t - I_t) - vQ_t - H & 1 \leq t \leq T-1 \\ C_{T-1} + p(I_{T-1} + Q_T - I_T) - vQ_T - H + \gamma I_T & t = T. \end{cases} \tag{2}$$

In the above formula, the end-of-period cash position C_t for period t $(1 \leq t \leq T)$ is defined as the period's initial cash position C_{t-1}, plus the period's revenue, minus the period's total ordering costs, overhead cost and, for $t = T$, plus the salvage value of any remnant inventory. The retailer's objective of maximizing its survival probability can be expressed as

$$\max \quad \Pr\{C_t \geq 0, \forall t\}, \tag{3}$$

where Pr denotes probability. To avoid too many lost sales, the retailer requires a service level constraint in the model: the probability of no stockouts in the planning horizon should be higher than $1 - \epsilon$. The joint chance constraint formulation can be expressed by the following inequality:

$$\Pr\{I_{t-1} + Q_t \geq D_t, \forall t\} \geq 1 - \epsilon. \tag{4}$$

Note that if the retailer wishes to enforce the service level constraint in each period rather than for the whole planning horizon, individual chance-constraints can be added to the model. This approach is used to construct a lower bound for the survival probability in Section 5.2. Since the retailer is a small business or a start-up firm, we assume that its access to finance is limited and consequently its ordering decision for a period is constrained by the available cash, i.e.,

$$vQ_t \leq \max\{0, C_{t-1}\}, \quad \forall t. \tag{5}$$

It is worth noting that the model proposed by Archibald et al. (2002) does not include a strict cash constraint on the order quantity because they assume that the supplier grants the retailer credit within a period.

Table 1
Main notations adopted in the paper.

Indices:	
T	Length of the planning horizon.
t	Index of a period, $t = 1, \dots, T$.
Constants:	
C_0	Initial cash position.
I_0	Initial inventory level.
p	Selling price for the product.
v	Unit variable ordering cost for the product.
γ	Unit salvage value for the remnant inventory.
H	Overhead costs in each period (e.g., wages or rents).
Stochastic variable:	
D_t	Demand for the product in period t, which follows a known probability distribution.
Decision variables:	
Q_t	Ordering quantity for the product in period t.
I_t	End-of-period inventory level in period t.
C_t	End-of-period cash position in period t.

If the retailer's cash position is negative at the end of a period, the retailer is said to have failed and it is unable to order from the supplier during the remainder of the planning horizon. This leads to the following constraint on the order quantities:

$$Q_t + \dots + Q_T = 0, \quad \text{if } C_{t-1} < 0. \tag{6}$$

After failure, the retailer will liquidate its assets to try to repay creditors. Hence, we assume that the retailer can still satisfy customer demand using existing inventory until the end of the planning horizon.

4. Stochastic modelling

This paper adopts a dynamic joint chance constraint which allows ordering decisions in one period to be made after the random demands of previous periods have been realized. In contrast, with a static joint chance constraint, all the ordering decisions are made at the beginning of the planning horizon prior to the realization of the uncertain demand. The dynamic joint chance constraint is more flexible and can obtain better objectives in numerical tests than the static joint chance constraint (Zhang et al., 2014).

Let Γ_t represent the random events that occur before the end of period t, i.e., $\Gamma_t = (D_1, D_2, \dots, D_t)$. In period t, the ordering quantity is decided after the realization of demands in the first $t-1$ periods. This is denoted by $Q_t(\Gamma_{t-1})$. The end-of-period inventory level and cash position in period t are determined after demands in the first t periods are observed. We denote these as $I_t(\Gamma_t)$ and $C_t(\Gamma_t)$, respectively. The dynamic joint chance constraint is thus expressed by:

$$\Pr\begin{bmatrix} I_0 + Q_1 \geq D_1 \\ I_1(\Gamma_1) + Q_2(\Gamma_1) \geq D_2 \\ \vdots \\ I_{T-1}(\Gamma_{T-1}) + Q_T(\Gamma_{T-1}) \geq D_T \end{bmatrix} \geq 1 - \epsilon, \tag{7}$$

where Q_1 is not related to Γ_t because the ordering decision in the first period is made before any random demand is realized. In a similar way, the objective function is reformulated as:

$$\max \quad \Pr\begin{bmatrix} C_1(\Gamma_1) \geq 0 \\ C_2(\Gamma_2) \geq 0 \\ \vdots \\ C_T(\Gamma_T) \geq 0 \end{bmatrix}. \tag{8}$$

The multi-dimensional integration of (7) and (8) make the problem difficult to solve. We provide two stochastic modelling techniques to solve this dynamic joint chance-constrained problem in the following subsections.

4.1. Scenario modelling

By assuming that the random vector Γ_T has finitely many realizations, the main idea of scenario modelling is to represent the distribution by random samples in the form of a scenario tree. A scenario s is a possible realization of the demands in the T periods, the set of all scenarios is represented by S and the number of scenarios is N. Decision variables Q_t, I_t and C_t are scenario-specific and are represented by Q_t^s, I_t^s and C_t^s, respectively, in the model. Note that $I_0^s = I_0$ and $C_0^s = C_0$. Additional notations used in the stochastic models are detailed in Table 2. The scenario model is formulated as follows.

Scenario model:

$$\max \quad \sum_s \pi^s z^s \tag{9}$$

s.t.

$$I_t^s \leq I_{t-1}^s + Q_t^s - D_t^s + \delta_t^s M_1, \qquad \forall t, \forall s, \tag{10}$$

$$I_t^s \geq I_{t-1}^s + Q_t^s - D_t^s, \qquad \forall t, \forall s, \tag{11}$$

$$I_{t-1}^s + Q_t^s - D_t^s \leq (1 - \delta_t^s) M_1, \qquad \forall t, \forall s, \tag{12}$$

$$I_t^s \leq (1 - \delta_t^s) M_1, \qquad \forall t, \forall s, \tag{13}$$

$$C_t^s = \begin{cases} C_{t-1}^s + p(I_{t-1}^s + Q_t^s - I_t^s) - vQ_t^s - H & 1 \leq t \leq T-1 \\ C_{T-1}^s + p(I_{T-1}^s + Q_T^s - I_T^s) - vQ_T^s - H + \gamma I_T^s, & t = T \end{cases} \quad \forall s, \tag{14}$$

$$C_t^s \leq \alpha_t^s M_2, \qquad \forall t, \forall s, \tag{15}$$

$$C_t^s \geq -(1 - \alpha_t^s) M_3, \qquad \forall t, \forall s, \tag{16}$$

$$z^s \leq \alpha_t^s, \qquad \forall t, \forall s, \tag{17}$$

$$vQ_t^s \leq C_{t-1}^s + (1 - \alpha_{t-1}^s) M_2, \qquad \forall t, \forall s, \tag{18}$$

$$\sum_{k=t}^{T} Q_k^s \leq \alpha_t^s M_1 \qquad \forall t = 1, 2, \dots, T, \forall s \tag{19}$$

$$\delta_t^s \leq \beta^s, \qquad \forall t, \forall s, \tag{20}$$

$$\sum_s \pi^s \beta^s \leq \epsilon, \tag{21}$$

$$\sum_{s' \in J_{t-1}^s} \pi^{s'} Q_t^{s'} = Q_t^s \sum_{s' \in J_{t-1}^s} \pi^{s'}, \qquad \forall t, \forall s, \tag{22}$$

$$I_t^s, Q_t^s \geq 0, \delta_t^s, \alpha_t^s, \beta^s, z^s \in \{0, 1\} \qquad \forall t, \forall s. \tag{23}$$

The objective function is the expected survival probability among the N scenarios where scenario s has probability π^s. Binary variable z^s equals 1 if there is no negative cash position in scenario s, which means the retailer survives the planning horizon under this scenario.

Table 2
Additional notations for the stochastic models.

Indices and index sets:	
N	Number of scenarios.
S	Set of scenarios ($s \in \{1, 2, \ldots, N\}$).
J_t^s	Set of scenarios that share the same demand history with scenario s in the first t periods.
Scenario-specific parameters:	
d_t^s	Realized demand in period t for scenario s.
π^s	Probability of scenario s occurring where $\sum_s \pi_s = 1$.
Auxiliary binary decision variables:	
z^s	$z^s = 1$ if there is no negative cash position for scenario s.
β^s	$\beta^s = 1$ if a lost sale occurs for scenario s.
α_t^s	$\alpha_t^s = 1$ if the cash position is non-negative in period t for scenario s.
δ_t^s	$\delta_t^s = 1$ if a lost sale occurs in period t for scenario s.

Constraints (10)–(13) are the linear formulation of the inventory balance equation Eq. (1): $\delta_t^s = 1$ if a lost sale occurs in period t for scenario s and M_1 is a sufficiently large number. Constraint (14) is the scenario-related cash flow formula. Constraints (15)–(16) enforce $\alpha_t^s = 0$ when the cash position $C_t^s < 0$, where M_2 and M_3 are sufficiently large numbers. Constraint (17) together with the maximizing objective (9) enforce $z^s = 1$ when $\alpha_t^s = 1$ for all t, meaning there is no negative cash position in scenario s. Constraint (18)–(19) are the linear formulations of the cash constraint (5)–(6). Constraint (19) ensures the ordering quantities are zeros if the cash position at period t is negative ($\alpha_t^s = 0$). Constraint (20) forces $\beta^s = 1$ if $\delta_t^s = 1$ for any t, indicating lost sales occur in scenario s. Constraint (21) is the linear expression for the joint-chance constraint which requires that the probability of lost sales is no greater than ϵ.

Constraints (22) enforce the non-anticipativity constraints, where J_t^s represents the sets of scenarios that share the same history with scenario s up to period t ($J_0^s = \emptyset$). The non-anticipativity constraints imply that if two scenarios share the same demand history up to period t, then the values of the decision variables up to period t should also be the same in the two scenarios. There are no non-anticipativity constraints for I_t^s, C_t^s, α_t^s and δ_t^s because these decision variables are functions of Q_t^s. The subscript for J in Constraint (22) is $t-1$ because the ordering decision is made before the demand in a period is known. Constraint (23) imposes the conditions that I_t^s, Q_t^s are non-negative and δ_t^s, α_t^s, β^s, z^s are binary variables.

Choosing appropriate big-M values generally yields a tighter formulation of the linear model and improves the performance of solvers. In this problem, we set the values of M_1, M_2, M_3 in computation as follows:

$$M_1 = \max_s \sum_{t=1}^{T} D_t^s \tag{24}$$

$$M_2 = C_0 + p \max_s \sum_{t=1}^{T} D_t^s = C_0 + pM_1 \tag{25}$$

$$M_3 = v(M_1 - I_0) + TH - C_0 \tag{26}$$

The value of M_1 reflects the fact that the inventory level at one period need not be larger than the maximum cumulative demand among all scenarios. M_2 is an upper bound for the cash position based on sales of the maximum cumulative demand without any costs. Similarly, $-M_3$ is a lower bound for the cash position, which would occur if the retailer orders up to the maximum cumulative demand, but does not make any sales.

4.2. SAA modelling

The SAA model relaxes the non-anticipativity constraints in the scenario model and assumes that every scenario has the same probability, i.e., $\pi^s = 1/N$ in the SAA model. The SAA model can be considered an approximation to the scenario model that is much easier to solve for problems with long planning horizons.

SAA model:

$$\max \quad \sum_s \frac{z^s}{N} \tag{27}$$

s.t. (10)–(20), (23)

$$\sum_s \beta^s \leq \lfloor N\epsilon \rfloor, \tag{28}$$

$$Q_1^s = Q_1^{s'} \qquad \forall s, s' \in S. \tag{29}$$

Constraint (28) guarantees that the number of scenarios with lost sales should not exceed $\lfloor N\epsilon \rfloor$, where $\lfloor N\epsilon \rfloor$ denotes the integer part of $N\epsilon$. Constraint (29) ensures that the first period order quantity is the same for all scenarios because the ordering quantity for period 1 is confirmed before any customer demand is observed. We still use the superscript s in Q_1 for convenience. In fact, SAA reduces the original multi-stage problem to a two-stage problem, in which the first period order quantity is the first stage decision.

Note that when solving the SAA model, the value of ϵ used can be different from the value implied by the service level required (Luedtke and Ahmed, 2008; Pagnoncelli et al., 2009). In other words, one may set a smaller value of ϵ, resulting in a stricter service level requirement in the model, in order to obtain a solution that achieves the desired service level for the problem. This method is also suitable for the scenario model in our numerical tests. As noted by Pagnoncelli et al. (2009), how to set the values for ϵ and N usually depends on the underlying joint chance-constrained problem. We will discuss this issue for our problem in Section 7.1.

5. Upper and lower bounds on the survival probability

In this section, we provide a statistical upper bound for the survival probability in our problem based on the properties of SAA and upper and lower bounds using stochastic dynamic programming.

5.1. A statistical upper bound

Nemirovski and Shapiro (2007) propose a method to compute a statistical upper bound for a chance-constrained maximization problem. The upper bound is suitable for both two-stage and multi-stage problems. This method was slightly modified in Luedtke and Ahmed (2008) and later employed by Pagnoncelli et al. (2009) and Zhao et al. (2014) among others. We briefly summarize the steps of this method for computing the upper bound in our problem as follows.

- Take M sets of N independent scenarios and solve the corresponding SAA model for each set. Denote the optimal objective values for the SAA model in descending order by $\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_M$.
- For a required confidence level $1-\tau$, denote L as the largest integer such that

$$B(L-1; \theta_N, M) \leq \tau, \tag{30}$$

where $\theta_N = B(\lfloor \epsilon N \rfloor; \epsilon, N)$ and $B(k,q,n) = \sum_{i=0}^{k} \binom{n}{i} q^i (1-q)^{n-i}$ denotes the cumulative distribution function of a binomial distribution.

- Pick the Lth value denoted as $\hat{v}_L$ among $\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_M$. By Luedtke and Ahmed (2008), $\hat{v}_L$ gives an upper bound for the true optimal value with a confidence level $1-\tau$.

5.2. Upper and lower bounds based on stochastic dynamic programming

Note that after adding the constraint (5) and (6) to the model in Archibald et al. (2002), which does not impose constraints on the service level, actually provides an upper bound for the survival probability in our problem. Any feasible solution to the problem in Section 3 is a lower bound for the optimal objective value. However, a feasible solution to the problem may not always exist. For example, if the initial cash position is very low, the retailer can only order a few items due to the cash constraint (5), and it will be impossible to satisfy a high service level if the demands in all periods are very large.

To compute a possible feasible solution by stochastic dynamic programming, we first transform the joint chance constraint into a set of constraints with one constraint for each time period:

$$\Pr\{I_{t-1} + Q_t - D_t < 0\} \leq \eta, \qquad \forall t \tag{31}$$

where η is a self-assigned lost sale rate. This gives a lower bound on the ordering quantity for a period given the initial inventory level I:

$$\underline{Q_t}(I) = F^{-}_{D_t}(1-\eta) - I, \tag{32}$$

where $F^{-}_{D_t}(\cdot)$ is the inverse cumulative distribution function of D_t. When the lower bound is an infeasible order quantity because of the cash constraint (5) (i.e., $v\underline{Q_t}(I_{t-1}) > C_{t-1}$), we define $Q_t = \lfloor C_{t-1}/v \rfloor$, i.e., the largest possible order quantity. Otherwise, the retailer chooses an order quantity between $\underline{Q_t}(I_{t-1})$ and the maximum order quantity permitted given the cash position. Therefore, the feasible region for Q_t under the cash constraint is:

$$\mathbb{Q}_t(I,C) = \begin{cases} \{Q_t \mid Q_t \geq \underline{Q_t}(I),\ vQ_t \leq C\} & \text{if } v\underline{Q_t}(I) \leq C \\ \{\lfloor C_{t-1}/v \rfloor\} & \text{otherwise.} \end{cases} \tag{33}$$

For given inventory level and cash position at the beginning of the planning horizon, the problem with individual chance constraints can be solved by stochastic dynamic programming. Let $q(t,I,C)$ be the maximum survival probability when the retailer has inventory level I and cash position C at the beginning of period t. The dynamic programming optimality equation when $t \leq T$ and $C \geq 0$ is:

$$q(t,I,C) = \max_{Q_t \in \mathbb{Q}_t(I,C)} \left(\int_0^{I+Q_t} f_t(D_t) q(t+1, I+Q_t-D_t, C+pD_t - vQ_t - H) dD_t \right. \\ \left. + q(t+1, 0, C + p(I+Q_t) - vQ_t - H) \int_{I+Q_t}^{\infty} f_t(D_t) dD_t \right), \tag{34}$$

with boundary conditions:

$$\begin{cases} q(T+1,I,C) = 1 & \text{if } C + \gamma I \geq 0 \\ q(T+1,I,C) = 0 & \text{if } C + \gamma I < 0 \\ q(t,I,C) = 0 & \text{if } C < 0, \quad t = 1,2,\ldots,T. \end{cases} \tag{35}$$

In the numerical tests, we use Eqs. (34) and (35) to compute a possible feasible solution as a lower bound. We start from a Bonferroni approximation with $\eta = \epsilon/T$, which was also used by Nemirovski and Shapiro (2007) in a joint chance constrained problem, then gradually decrease η while the solution is infeasible to see if a feasible solution can be found. The feasibility is checked by computing the service level from the dynamic programming formulation below.

We introduce a binary variable b_t to indicate whether the retailer fails before the beginning of period t ($b_t = 1$) or not ($b_t = 0$). If the retailer fails before the beginning of period t, then $C_k < 0$ for some k satisfying $1 \leq k \leq t-1$ and $Q_t = 0$ by our assumption in Constraint (6). Let $Q^*(t,I,C,0)$ be the order quantity that maximizes the right hand side of Eq. (34) for $q(t,I,C)$ and define $Q^*(t,I,C,1) = 0$. Let $s(t,I,C,b)$ denote the probability of no stockouts during the remainder of the planning horizon when the retailer has initial inventory level I and cash position C at the beginning of period t and b indicates whether or not the retailer fails before the beginning of period t.

$$s(t,I,C,b) = \int_0^{I+Q^*(t,I,C,b)} f_t(D_t) s(t+1, I+Q^*(t,I,C,b) - D_t, C + pD_t \\ - vQ^*(t,I,C,b) - H, b') dD_t, \tag{36}$$

with the transition function for the failure status given by:

$$b' = \begin{cases} 1 & \text{if } C + pD_t - vQ^*(t,I,C,b) - H < 0 \\ b & \text{otherwise.} \end{cases} \tag{37}$$

and boundary condition:

$$s(T+1,I,C,b) = 1. \tag{38}$$

6. Rolling horizon approach

One of the main drawbacks for the scenario-based multi-stage stochastic programming approach is that the models are generally computationally expensive (Fattahi and Govindan, 2022). The so-called rolling horizon approach is often applied to overcome the computational burden. Moreover, the objective value obtained by SAA for our problem is likely to be overly optimistic because SAA in fact reduces the original multi-stage problem to a two-stage problem by relaxing the non-anticipativity constraints. Therefore, we provide a rolling horizon approach to implement the ordering decisions in out-of-sample scenarios. The basic idea is as follows:

- Generate some out-of-sample scenarios, i.e., scenarios that are different from those used in computing the scenario model and the SAA model.
- Implement the first-stage decision from the solution to a stochastic model with a possibly shorter planning horizon of K periods ($K \leq T$).
- Update the initial cash position, initial inventory level and joint chance constraint for the next period t, re-solve the stochastic model for the next $\min\{K, T-t\}$ periods and implement the new solution for period t.
- Repeat this process until the last period T in the planning horizon is reached.

When updating the joint chance constraint in a scenario, assume the required maximum lost sales rate from period t to period $t+K$ in scenario s is $\epsilon^s_{t\sim t+K}$, which means that the required service level from period t to period $t+K$ is $1-\epsilon^s_{t\sim t+K}$ and $\epsilon^s_{1\sim T} = \epsilon$. When implementing the rolling horizon approach, the service level requirement is updated as follows:

$$1 - \epsilon_{t\sim t+K} = (1-\eta)^{\overline{D}_{t\sim t+K}/\overline{D}_{1\sim T}}. \tag{39}$$

where $\overline{D}_{t\sim t+K}$ is the sum of mean demands in periods t to $t+K$, $\overline{D}_{1\sim T}$ is the sum of mean demands over the whole planning horizon and $1-\eta$ is a self assigned service level value. The above formula is a heuristic step which assumes that the joint chance level from period t to period $t+K$ is a geometric proportion of the total joint chance level. Note that sometimes one may self assign the value of $1-\eta$ to be close to 1 in order to get feasible solutions. The rolling horizon process can be applied to independent sample sets and the average objective value obtained can be used to approximate the optimal objective value. Details of this process are presented in Algorithm 1.

Algorithm 1: Rolling horizon algorithm

Data: Required service rate $1-\epsilon$ and other parameter values.
Result: Approximate survival probability $\widehat{SP}$ for the problem.
Initialize: Generate M sets of N independent scenarios for the entire planning horizon. Each set is represented by $\mathcal{S}^m (m = 1, 2, \dots, M)$; Rolling horizon length K.
for $m \leftarrow 1$ **to** M **do**
  for $s \in \mathcal{S}^m$ **do**
    $\epsilon^s_{1\sim T} \leftarrow \eta$, $z^s \leftarrow 1$;
    $Q_0 \leftarrow 0$, $I^s_0 \leftarrow I_0$, $C^s_0 \leftarrow C_0$;
    for $t \leftarrow 1$ **to** T **do**
      Update service rate requirement according to Eq. (39);
      Build and solve the scenario or SAA model for period t to $\min\{t+K, T\}$ $(K \le T)$ to obtain Q^s_t;
      $I^s_t \leftarrow \max\{0, I^s_{t-1} + Q^s_t - D^s_t\}$, update C^s_t by Eq. (14);
      if $C^s_t < 0$ **then**
        $z^s \leftarrow 0$;
      end
    end
  end
  $\widehat{SP}_m \leftarrow \sum_s \pi^s z^s$;
end
$\widehat{SP} \leftarrow \sum_m \widehat{SP}_m / M$;

7. Numerical analysis

In this section, we first discuss a numerical example to illustrate how to set the values for N and ϵ and compute the upper and lower bounds for the problem. Detailed results of the scenario and SAA models and comparisons of their computational effectiveness are also discussed. Next numerical tests are conducted to investigate the performance of the rolling horizon approach. Finally, we compare the survival probabilities and service levels of the models with and without the service level constraint to obtain some managerial insights.

The computational studies were coded in Java and run on a notebook computer with an Apple M1 Pro CPU, 16 GB of RAM, and macOS Monterey operating system. We use Gurobi 10.0.1 to solve the linear programming models.

7.1. Setting the values of N and ϵ

In the numerical example for the tests: planning horizon $T = 3$, initial inventory $I_0 = 0$, selling price $p = 5$, unit variable ordering cost $v = 1$, overhead cost $H = 80$, unit salvage value for remnant inventory $\gamma = 0.5$, initial cash position $C_0 = 130$, demand in each period is independent and Poisson distributed with mean values [10, 20, 10]. Random samples are generated by Latin hypercube sampling (McKay et al., 2000) for both methods. The sample detail for scenario size N is $[N, N, N]$, meaning that N random demand samples are generated in each period. We consider scenario sizes 3, 5, 7, 9 and 11 with sample detail [3, 3, 3], [5, 5, 5], [7, 7, 7], [9, 9, 9] and [11, 11, 11], respectively. Since the service level in the stochastic models can be assigned a value that is different from the required value (Pagnoncelli et al., 2009), we use η instead of ϵ in Constraint (21) as well as in the upper bound computation in (31). The required service level is $1-\epsilon = 70\%$ and we set the service level $1-\eta$ in the stochastic models to be 70%, 80%, 85% and 95%. We report the objective values for the stochastic models as the average of 10 runs of the model.

The results for different values of $1-\eta$ and scenario sizes N are shown as box plots in Fig. 2. We also report the average simulated objective values and average service levels for the solutions obtained from the two methods in Fig. 3 by applying the rolling horizon approach with $K = T$ to 1000 out-of-sample scenarios. Based on Figs. 2 and 3, we make the following observations:

- as the scenario size increases, the objective values obtained by the scenario model are slightly higher than SAA (see Fig. 2); but there a few exceptional situations; the objective values of both models are relatively stable with smaller fluctuations, as shown in Fig. 2;
- for the smallest scenario size (size3) or low assigned service level value $1-\eta$ (70%), some of the simulated service levels are lower than the required joint service level 70% (see Fig. 3), demonstrating that both methods may obtain infeasible solutions when the scenario size is too small or the value of service level assigned is not high enough although the problem itself is feasible as we show in the next subsection;
- when the service level is set high enough for the required service level and the scenario size is not too small, both methods can obtain feasible solutions (see Fig. 3); note that assigning the service level a high value in the stochastic models generally results in a lower survival probability (see Fig. 2);
- the objective values obtained by the scenario model are generally more stable than those of SAA (see Fig. 2); however, the simulated objective values of both methods, obtained by the rolling horizon approach, are very close (see Fig. 3).

7.2. Illustrations of the upper and lower bounds on the survival probability

In the following numerical tests, the required service level is $1-\epsilon = 70\%$ and the value of $1-\eta$ is set to be 95%. We compare the computational efficiency of the two methods and calculate the upper bound and lower bound on the objective function of the problems. In addition to the numerical example in Section 7.1, the two methods are applied to solve a 4-period problem with mean demand values [10, 20, 20, 10]. All the other parameter values are the same as Section 7.1. We run each method 10 times for each numerical case, so $M = 10$ when computing the statistical upper bound with confidence level $1-\tau = 95\%$.

The results are presented in Table 3, in which columns 1, 5, 6, 7, 8, 9, 10 and 11 show the scenario size, the average objective value, the simulated objective value, the simulated service rate for the scenario or SAA model, the statistical upper bound, the upper bound, the lower bound and the service rate for the model in Archibald et al. (2002), respectively. The lower bounds are obtained by initially setting η to be ϵ/T in Eq. (31) as explained in Section 5.2. There are several points illustrated by Table 3:

- the running time grows sharply as the length of the planning horizon increases from 3 periods to 4 periods especially for the scenario model, which runs out of memory in the solver for the 4-period case when the scenario size is 9;
- the statistical upper bound obtained by SAA is generally higher than the upper bound by dynamic programming for the problem;
- the service rate for the model in Archibald et al. (2002) is very low (3.69% for the 3-period problem and 6.79% for the 4-period problem).

7.3. Assessment of the rolling horizon framework

For multi-stage stochastic problems with large planning horizons, rolling horizon computation is usually applied. As noted by Glomb et al. (2022), increasing the rolling horizon length does not always lead to better solutions. The design of the following numerical tests is two-fold: first we investigate this issue by varying the rolling horizon length and assigned service level values $(1-\eta)$ within the two stochastic methods for the 4-period problem of the previous subsection (see Table 4); second, we test the performance of the SAA model on larger problems

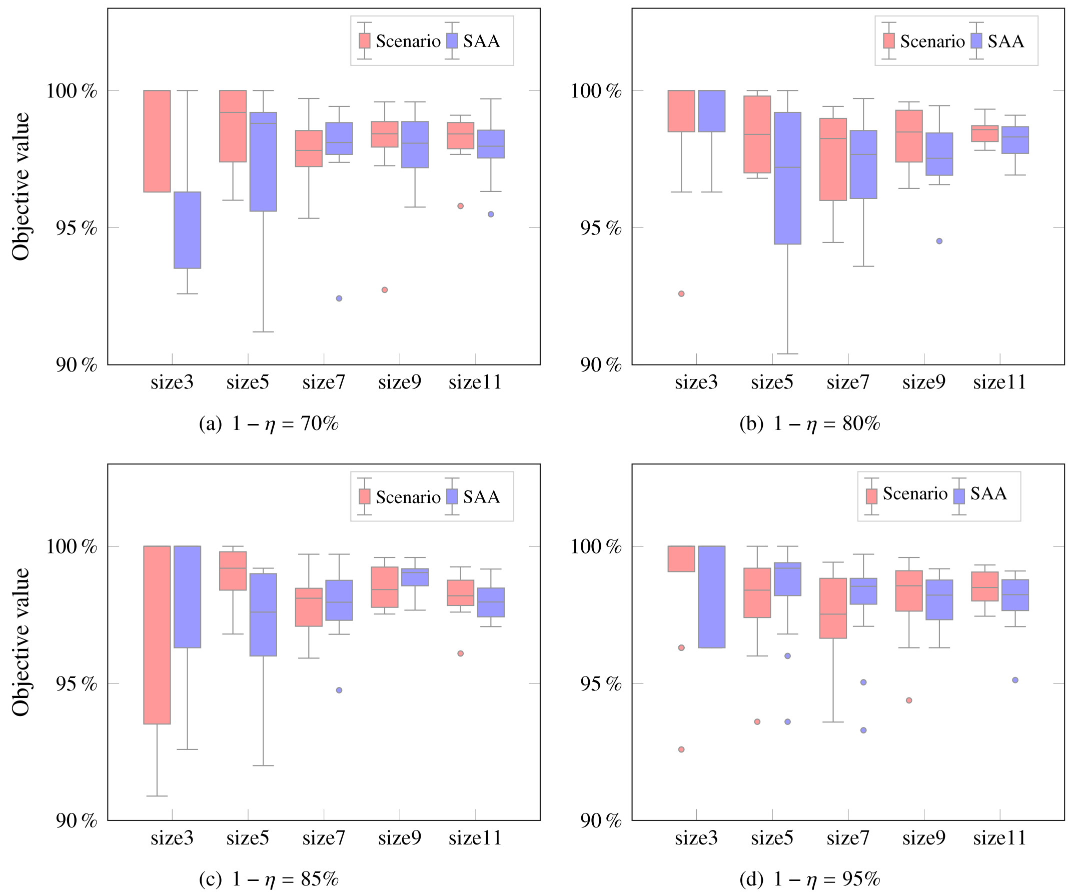


Fig. 2. Objective values for the scenario and SAA methods with different scenario sizes and assigned service levels.

Table 3
The results of 3-period and 4-period numerical cases.

size	case	method	time	avg. obj.	sim. obj.	sim. ser1.[a]	sta. ub.	ub.	lb.	ser2.[b]
5	3-period	Scenario	0.11s	98.08%	97.70%	87.73%	98.40%	98.06%	91.48%	3.69%
		SAA	0.08s	98.52%	97.00%	81.82%				
	4-period	Scenario	2.65s	97.70%	96.60%	75.36%	97.68%	95.07%	81.16%	6.79%
		SAA	1.13s	95.17%	96.02%	73.06%				
7	3-period	Scenario	0.57s	97.45%	97.75%	85.50%	98.54%	98.06%	91.48%	3.69%
		SAA	0.51s	98.06%	97.45%	85.70%				
	4-period	Scenario	876.73s	96.04%	94.12%	93.23%	96.50%	95.07%	81.16%	6.79%
		SAA	6.08s	96.21%	95.89%	92.40%				
9	3-period	Scenario	30.13s	98.12%	97.60%	89.85%	99.17%	98.06%	91.48%	3.69%
		SAA	6.98s	98.01%	97.70%	88.81%				
	4-period	Scenario	—[c]	–	–	–	97.65%	95.07%	81.16%	6.79%
		SAA	39.82s	96.08%	96.15%	83.37%				

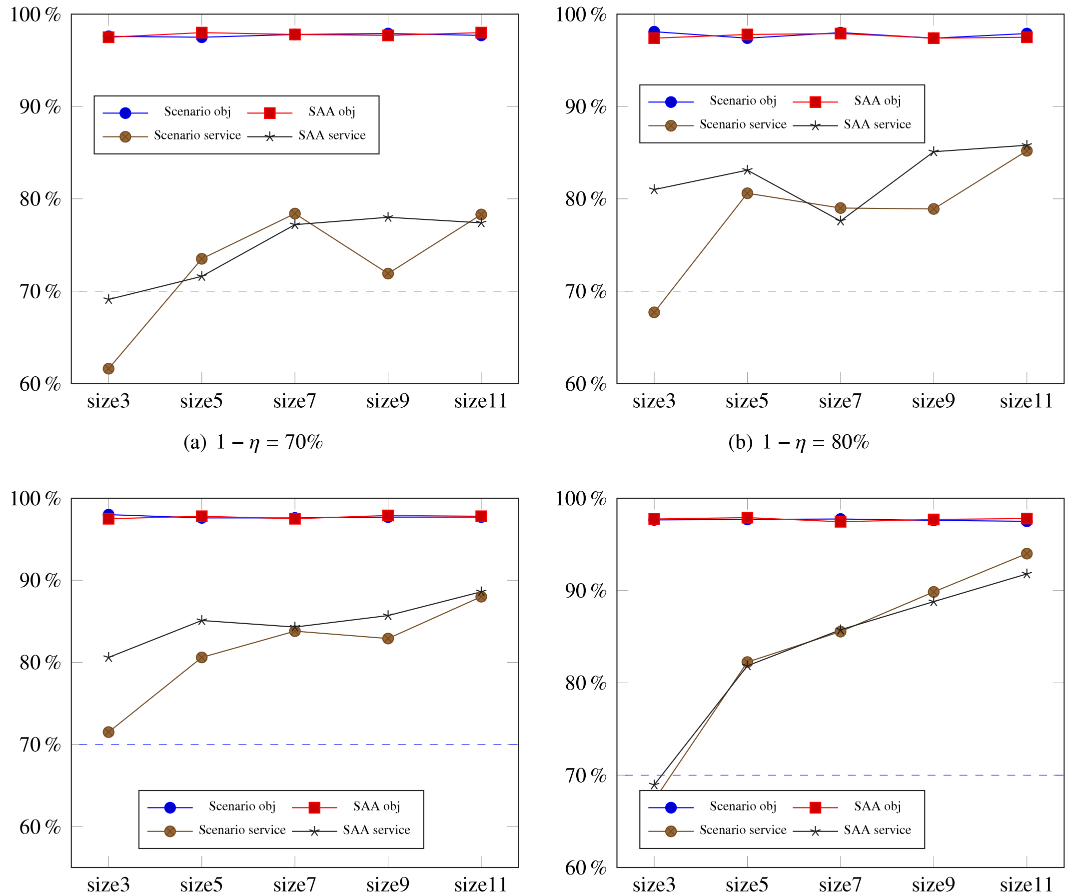
[a] Simulated service rate for the scenario or SAA model.
[b] Service rate for the model in Archibald et al. (2002).
[c] — means the solver is out of memory.

with planning horizons of 12 periods (see Table 5) and compare it with the model without service constraint (see Table 6).

For the tests with the 4-period problem, the scenario size is 9^4 and the required joint service level is $1 - \epsilon = 70\%$. As shown in Table 4, there is no apparent advantage for the scenario model over SAA in the rolling horizon approach among the different rolling lengths and different values of $1 - \eta$.

We focus on applying SAA when conducting numerical tests with larger problem sizes for the rolling horizon approach because SAA solves problems faster with no noticeable reduction in the quality of the solutions. The test beds are adopted from Rossi et al. (2015). There are 10 demand patterns for numerical analysis: 1 stationary pattern (STA), 2 life cycle patterns (LCY1 and LCY2), 2 sinusoidal patterns (SIN1 and SIN2), 1 random pattern (RAND), and 4 empirical patterns (EMP1, EMP2, EMP3, EMP4). Demands follow Poisson distributions and expected demands for different patterns are shown in Fig. 4. Details of the expected demand data are given in Appendix.

In the 12-period problems, we set $1 - \eta = 95\%$ in the stochastic model and the parameter values are the same as Section 7.1. The simulated service rates, simulated objectives, running time for different



(a) $1 - \eta = 70\%$ (b) $1 - \eta = 80\%$

(c) $1 - \eta = 85\%$ (d) $1 - \eta = 95\%$

Fig. 3. Average simulated objectives and service levels for the scenario and SAA methods with different scenario sizes and assigned service levels.

Table 4
The results of different rolling horizon lengths and assigned service levels in a 4-period problem.

Method	Rolling length	$1 - \eta = 70\%$		$1 - \eta = 80\%$		$1 - \eta = 95\%$	
		sim. obj.	ser.	sim. obj.	ser.	sim. obj.	ser.
SAA	1	95.92%	86.92%	95.70%	85.10%	95.64%	85.02%
	2	95.12%	90.61%	95.80%	95.21%	95.52%	93.60%
	3	96.23%	83.42%	96.21%	91.33%	96.32%	90.30%
	4	96.32%	70.72%	96.62%	85.31%	96.15%	83.37%
Scenario	1	95.50%	85.53%	96.00%	84.90%	96.10%	84.31%
	2	96.11%	90.63%	95.13%	93.40%	96.21%	94.53%
	3	96.15%	83.10%	94.81%	90.90%	96.07%	90.61%
	4	–	–	–	–	–	–

scenario sizes and different rolling horizon lengths in 100 out-of-sample scenarios are shown in Table 5, where size N means there are N samples in each period. From the table, we can see that:

- as the scenario size increases, the rolling horizon method demonstrates notable improvements in terms of the objective and service rate for almost all numerical cases; however, this enhancement comes at the expense of increased running time;
- for demand pattern EMP1, the rolling horizon method does not achieve high objectives and service rates across all three rolling lengths; this can be attributed to the initial demands for EMP1 being very low; it should be noted that the scale values differ for different sub-figures in Fig. 4; due to the limited initial demand, the retailer faces challenges in generating sufficient cash positions to sustain future operations while maintaining a high service level;
- increasing the rolling horizon does not always result in improved performance; our findings reveal that in certain numerical cases characterized by significant demand fluctuations or low initial cash position (e.g., LCY2, EMP1, EMP2), rolling lengths of 2 or 3 may lead to worse performance; however, for other cases where the sample sizes in each period are the same (size 10 in Table 5), larger rolling lengths generally exhibit better performance;
- the running time significantly increases as the rolling length grows; in particular, for rolling length 3, the solver encounters the time limit (we set 20,000 s for the solver) and is unable to complete certain cases, as indicated by the notation "—".

7.4. Comparison of results with and without the service level constraint

As the rationale for this paper is that the model maximizing survival probability without a service level constraint may result in a high lost

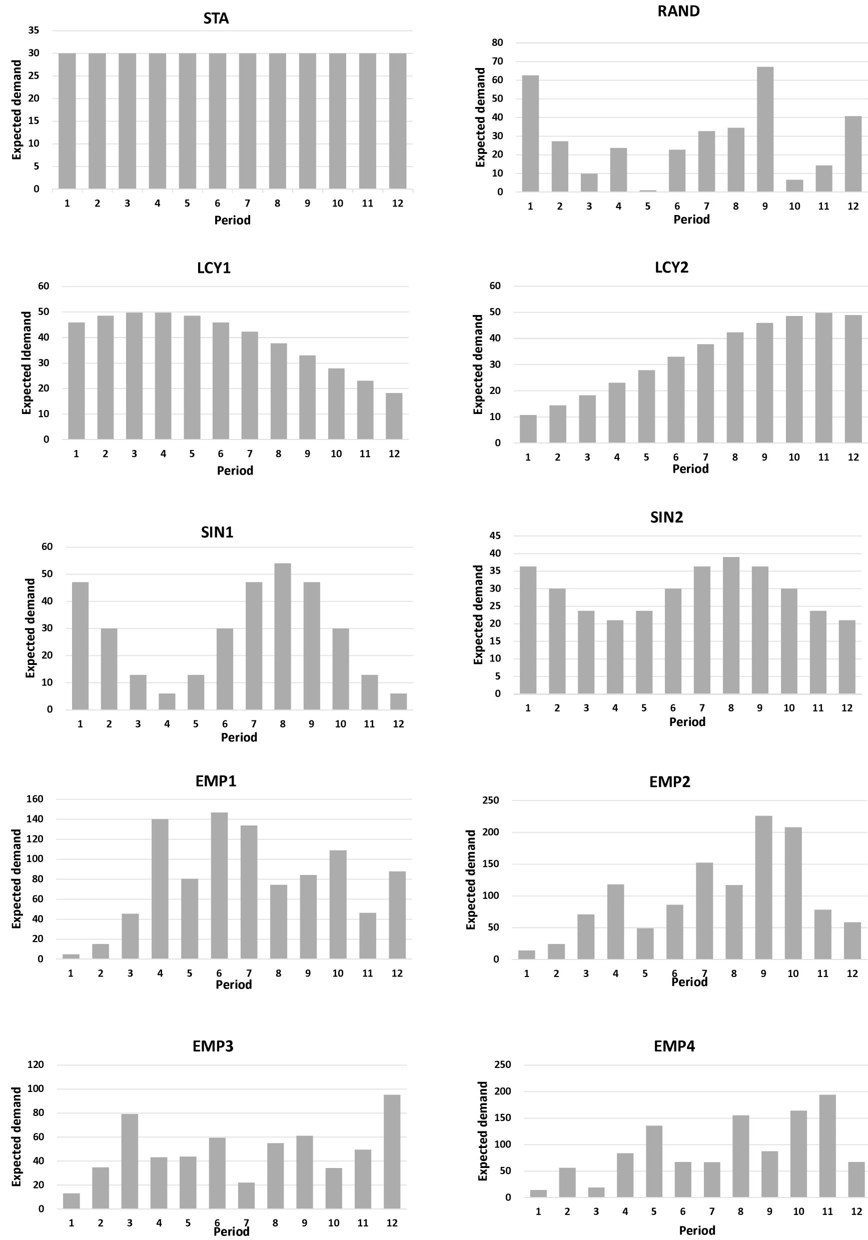


Fig. 4. Demand patterns used in the numerical tests.

sales rate, we compare the results from Archibald et al. (2002) with our rolling horizon approach with a joint chance constraint.

The test beds are the same as in Section 7.3, but we scale the planning horizon length to 5 periods to make the problems easier for computation. We introduce some parameter variations in the tests: the initial cash position C_0 takes values in the range of (80, 100, 120), price p takes values in the range of (4, 5, 6), overhead cost H takes values in the range of (60, 80, 100), unit variable ordering cost v is set to 1, and unit salvage value γ is set to 0.5. In the rolling horizon approach, the rolling length is set to 1, the scenario size is 300, and the required service level $(1-\eta)$ in the model is set to 95%.

Table 6 presents the average objectives, average service rates, and average ordering quantities in the first period (Q_1) for both the model without a service level constraint, as in Archibald et al. (2002), and our approach with the joint service level constraint.

It is evident from Table 6 that the average service rates are very low (0.85%) for the model without a service level constraint, indicating a significantly high lost sale rate. In some cases with high expected profits (i.e., high prices or low overhead costs), the service rates are almost 0.00%. On the contrary, our approach provides solutions with high service rates, particularly for cases with high expected profits. The reason for this is reflected in the ordering quantities of the two approaches. Without the service level constraint, the survival probability maximization model leads to conservative decisions compared to the model with the joint service level constraint. For example, among the 270 numerical cases, the average Q_1 is 27.63 for the former approach, while it is 44.27 for the latter approach. This difference is more pronounced in cases with high expected profits, such as when the price is high or the overhead cost is low. For instance, when the price is 6, the average Q_1 is only 20.96 for the former approach, whereas it is 44.42 for the latter approach.

From the findings presented in Table 6 and our model formulation, it becomes evident that the overhead cost has a major impact on the retailer's survival probability, irrespective of the presence of a service level constraint. To investigate its influence in more detail, we varied the overhead cost from 0 to 100 and show the average objectives

Table 5
Results of rolling horizon method for different demand patterns ($1 - \eta = 95\%$).

Demand pattern	Rolling length 1				Rolling length 2				Rolling length 3			
	size	obj.	ser.	time	size	obj.	ser.	time	size	obj.	ser.	time
	10	100%	55%	1s	10	100%	63%	7s	10	100%	93%	450s
STA	100	100%	94%	3s	30	100%	97%	430s	15	100%	95%	2048s
	300	100%	99%	9s	50	100%	98%	960s	20	–	–	–
	10	100%	64%	1s	10	100%	68%	7s	10	100%	90%	339s
LCY1	100	100%	95%	4s	30	100%	98%	380s	15	100%	93%	936s
	300	100%	99%	10s	50	100%	98%	820s	20	–	–	–
	10	80%	44%	1s	10	46%	35%	6s	10	68%	62%	249s
LCY2	100	81%	72%	4s	30	53%	51%	50s	15	70%	60%	1416s
	300	81%	74%	9s	50	68%	68%	464s	20	65%	46%	16319s
	10	100%	75%	1s	10	99%	63%	6s	10	100%	92%	409s
SIN1	100	100%	95%	4s	30	100%	92%	255s	15	100%	93%	2326s
	300	100%	99%	10s	50	100%	95%	1094s	20	–	–	–
	10	100%	63%	1s	10	100%	63%	5s	10	100%	88%	391s
SIN2	100	100%	97%	4s	30	100%	99%	406s	15	100%	95%	2126s
	300	100%	97%	10s	50	100%	96%	1049s	20	–	–	–
	10	100%	66%	1s	10	100%	59%	6s	10	100%	77%	353s
RAND	100	100%	92%	4s	30	100%	92%	181s	15	100%	88%	2153s
	300	100%	98%	9s	50	100%	94%	715s	20	–	–	–
	10	38%	14%	1s	10	38%	19%	5s	10	24%	18%	48s
EMP1	100	19%	13%	2s	30	25%	17%	41s	15	39%	24%	504s
	300	20%	13%	9s	50	15%	10%	120s	20	23%	17%	3041s
	10	80%	56%	1s	10	93%	50%	6s	10	99%	89%	210s
EMP2	100	86%	84%	4s	30	92%	79%	50s	15	97%	92%	971s
	300	86%	84%	10s	50	91%	81%	148s	20	92%	88%	5956s
	10	100%	51%	1s	10	98%	73%	6s	10	100%	74%	303s
EMP3	100	100%	94%	4s	30	100%	86%	50s	15	95%	97%	1191s
	300	100%	94%	10s	20	100%	97%	818s	20	–	–	–
	10	100%	51%	1s	10	100%	57%	5s	10	100%	84%	276s
EMP4	100	100%	87%	4s	30	100%	88%	50s	15	100%	98%	1644s
	300	100%	96%	10s	50	100%	96%	688s	20	–	–	–

Table 6
Impact of the service level constraint.

	Without service constraint			Joint service constraint			Cases
	avg. obj.	avg. ser.	avg. Q_1	avg. obj.	avg. ser.	avg. Q_1	
Initial cash							
80	86.67%	0.84%	29.95	65.33%	60.79%	43.72	90
100	93.18%	0.88%	27.83	86.69%	63.98%	44.57	90
120	96.71%	0.83%	25.11	93.37%	73.67%	44.51	90
Price							
4	81.30%	2.47%	34.88	58.32%	54.00%	44.37	90
5	96.13%	0.08%	27.05	80.51%	76.47%	44.01	90
6	99.12%	0.00%	20.96	90.54%	81.82%	44.42	90
Overhead cost							
60	99.99%	0.00%	16.09	94.81%	91.02%	44.26	90
80	97.49%	0.12%	29.40	79.68%	74.13%	44.12	90
100	79.08%	2.44%	37.40	54.87%	53.53%	44.42	90
Demand pattern							
STA	97.70%	0.21%	28.07	96.21%	95.06%	47.81	27
LCY1	99.98%	0.01%	28.89	99.97%	99.31%	72.30	27
LCY2	85.22%	0.00%	22.03	72.39%	71.07%	26.56	27
SIN1	95.56%	0.00%	44.37	92.91%	88.14%	69.70	27
SIN2	79.92%	0.87%	17.26	55.86%	54.47%	19.04	27
RAND	91.31%	7.44%	62.22	90.36%	98.34%	85.30	27
EMP1	99.76%	0.00%	17.70	86.24%	70.25%	41.70	27
EMP2	87.08%	0.00%	18.78	46.47%	45.26%	26.67	27
EMP3	92.46%	0.00%	17.85	68.36%	58.49%	25.00	27
EMP4	92.97%	0.00%	19.11	55.81%	48.55%	28.59	27
General	92.19%	0.85%	27.63	76.46%	72.89%	44.27	270

and service levels in Fig. 5. When the overhead cost is zero, the cash position at the end of a period can never be negative and the retailer's survival is assured. While the overhead cost remains low, the retailer's survival probability remains close to 1 in both models. However as the overhead cost increases, a point is reached at which the retailer's survival probability is significantly affected. This point is reached earlier, and is more pronounced, for the model with the service level constraint. Service level follows a similar pattern for the model

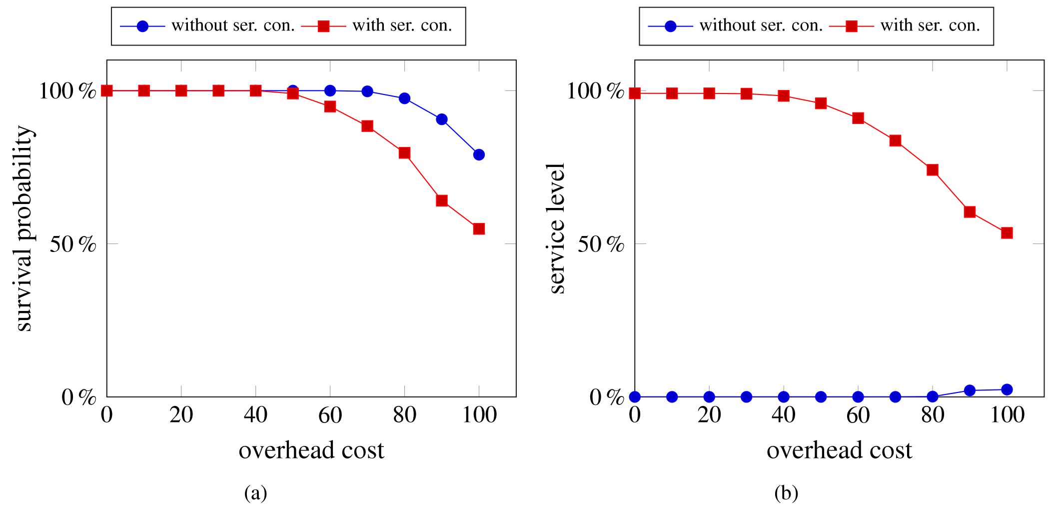


Fig. 5. Survival probability and service level as functions of the overhead cost.

Table A.1
Detailed mean demand data of the 12-period demand patterns.

Demand pattern	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
STA	30	30	30	30	30	30	30	30	30	30	30	30
LCY1	46	49	50	50	49	46	42	38	33	28	23	18
LCY2	11	14	18	23	28	33	38	42	46	49	50	49
SIN1	47	30	13	6	13	30	47	54	47	30	13	6
SIN2	36	30	24	21	24	30	36	39	36	30	24	21
RAND	63	27	10	24	1	23	33	35	67	7	14	41
EMP1	5	15	46	140	80	147	134	74	84	109	47	88
EMP2	14	24	71	118	49	86	152	117	226	208	78	59
EMP3	13	35	79	43	44	59	22	55	61	34	50	95
EMP4	15	56	19	84	136	67	67	155	87	164	19	67

Table A.2
Detailed mean demand data of the 5-period demand patterns.

Demand pattern	Period				
	1	2	3	4	5
STA	30	30	30	30	30
LCY1	50	46	38	28	14
LCY2	14	23	33	46	50
SIN1	47	30	6	30	54
SIN2	9	30	44	30	8
RAND	63	27	10	24	1
EMP1	25	46	140	80	147
EMP2	14	24	71	118	49
EMP3	13	35	79	43	44
EMP4	15	56	19	84	136

with the service level constraint. In contrast, the service levels in the model without a service level constraint consistently hover near 0% across the range of overhead cost values.

8. Conclusions

While several previous works aim to maximize the survival probability of a retailer facing a multi-period inventory problem, the lost sales rates of the solutions found are usually too large to be acceptable in practice. This paper addresses this issue by adding a joint chance constraint on service level to the problem. Two stochastic models: a scenario-based model and the SAA model are formulated to solve the problem. We also provide a statistical upper bound for the survival probability of the retailer based on SAA and upper and lower bounds based on stochastic dynamic programming. When the planning horizon is large, a rolling horizon approach with service rate updating is also developed. We find that the rolling horizon approach together with the stochastic models can solve realistically sized problems in large numerical tests.

Our numerical results show that, across a range of parameter settings, the proposed model with a service level constraint is able to strike a balance between maximizing the probability of survival and achieving an acceptable minimum service level. Our results also show that without the constraint on service level, the optimal survival strategy can result in service levels close to zero. It is important for managers to recognize that even when carefully managing cash flow during periods when cash is heavily constrained, such as during an economic crisis, the minimum service level achieved also needs careful consideration. Otherwise low customer satisfaction may result in failure anyway.

Future research may consider distributionally robust optimization methods to solve the problem when the demand distribution is ambiguous. A second possible future research direction is to investigate supply chain financing in the problem.

CRediT authorship contribution statement

Zhen Chen: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Writing – original draft, Writing – review & editing. **Thomas W. Archibald:** Supervision, Writing – review & editing.

Data availability

Data will be made available on request.

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Appendix. Detailed demand data

See Tables A.1 and A.2.

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