# DESIGN OPTIMIZATION OF SERIAL ROBOT MANIPULATOR

**Dr. Ahmed A.A.**
**Univ. of Baghdad**
**College of Eng.**

**MS.c Alyaa H.A**
**Univ. of Baghdad**
**College of Eng.**

## ABSTRACT

Optimal design of three links and four links serial manipulator involves striking a balance between an appropriate link length, radius, link exact end effecter deflection and the amount of stress induced in each link. Optimization has been applied for getting a minimum robot weight through making the robot arm section tapered while keeping the first link as cylindrical tube as it represent the robot base only. The synthesis optimization problem involves setting up guess values for links length and radius subjected to constraints of deflection, stress and geometric constraints of total robot length. The optimization process focuses on minimization of robot weight as an objective function, the guess values has taken from three links manipulator and the industrial robot as four links serial manipulator. The results of optimization has been plotted and represented through the different relations between the design parameters (Link radius, length and total robot deflection, total robot weight, stress…etc). The results shows a good agreement minimizing the total deflection to($2x10^{-5}$ m) with this degree of precision an optimum design features may be obtained that gives a robot structure with high stiffness and minimum weight that enables the robot to do its tasks with minimum inertia effect.

## الخلاصة

التصميم الهندسي الأمثل لنظام الاذرع الالية لنوعين من الروبوت ذوثلاثة أذرع والاخر ذوأربعة أذرع يتضمن الأختيار الأمثل لطول وقطر الذراع مع السيطرة على مقدار التشوه الكلي الحاصل للروبوت ومقدار الأجهاد المتولد في كل ذراع.تم تطبيق الأمثلية للحصول على اخف وزن ممكن للروبوت من خلال جعل مقطع ذراع الروبوت مسلوب مع ابقاء الذراع الأول اسطواني بما أنه يمثل قاعدة الروبوت فقط،تمت عملية الأمثلية من خلال اعطاء قيم اولية لكل من طول ونصف قطر الأذرع مأخوذة من نوعين من الروبوت أحدهما يمثل روبوت ذو ثلاثة أذرع أما الأخر يمثل روبوت ذو أربعة أذرع وتم تمثيل نتائج الأمثلية بيانياً من خلال العلاقات المختلفة بين الأبعاد(نصف القطر،الطول)والتشوه الكلي والوزن والأجهاد...إلخ تم تمثيلهاخلال هذا العمل والتي تظهر ان الهدف من العمل قد تحقق. النتائج أظهرت نسبة توافق عند قيمة التشوه ($2x10^{-5}$) حيث مثلت هذه القيمة النتيجة المثلى للتصميم التي أعطت لهيكل الروبوت المتانة العالية مع الوزن الأقل مما يمكن الروبوت من تأدية مهامه بصورة مثالية.

## Introduction

Optimal design of robots is important as it influences the system performance such as the cost of manufacturing, accuracy, related deflection and so on.

Ever since the robotic manipulators were introduced in the automation industry, robotic manipulators have been refined to have better energy efficiency, faster operation and higher payload to arm weight ratio. These technical goals have been achieved up to a certain levels by designing the low inertia and stiff structure is relative to the motion speed and control accuracy. The dynamic effect of the payload is much larger in the light weight flexible manipulators than in the conventional rigid manipulators. One of the main points of designing a robust and versatile robot is to develop a solid geometry are the link shape and weight.

Shiakolas et al (2002)made a comparison of three evolutionary optimization approaches which are Simple Genetic Algorithm, Genetic Algorithm Elitism and Differential Evolution for designing serial link robotic manipulators based on task specification and constraints. The design process considered the kinematic, dynamic and structural characteristics of the manipulator links and the end effectors' payload. The objective function was minimizing the torque required to perform the defined motion subject to constraints on link parameters which are length and cross section area. Hollow square section, hollow circular section and hollow rectangular section were considered. The analysis procedure was first defining the problem, the design variables; assign values to all parameters and the constraint vector. The defined values are used in the analysis routines to obtain values for design variables, the design variables are checked for constraints violation and then used in evaluating the objective function.

These evaluations are used in optimization routine where new values for design variables are generated.Differential Evolution Method gives more accurate results than the other two methods.

Matlab Optimization Toolbox was used for simulation for the various sections.

**Ceccarelli et al** (2005) presented a design procedure for manipulators both of serial and parallel architectures taking into account the several aspects and behaviors for optimum solutions both in design and optimization.

The optimality criteria are focused on the well organized main aspects of workspace, singularity and stiffness. Optimality criteria and computational aspects have been elaborated by taking into account the peculiarity and constraints of each other.

Multi objective function of workspace position, workspace orientation, velocity response, static behavior and the angular compliant displacements is presented and solved by the numerical technique which is advised for solving the proposed multi objective optimization problem.

A six degree of freedom PUMA- like manipulator has been considered to test the engineering feasibility of the optimum design of manipulators as specifically applied to serial architecture.

The CaPaMan manipulator has been considered to test the engineering feasibility of the optimum design of manipulators as specifically applied to parallel architectures.

**J.P.Merlet**(2002) presents a dimensional synthesis approach based on the design requirements that allows one to obtain almost all the feasible design solutions that are guaranteed to satisfy the requirements. The research presents(The Parameters Space Approach)as the design methodology consist of defining the parameters space as a n-dimensional space in which each dimension corresponds to one of the n-design parameters of the robot and a list of requirements that define minimal or maximal allowed values of some robots performances(such as accuracy, stiffness,…) or some required properties.The methodology is applied to the micro-robot MIPS for medical application and it implemented in C++ using BIAS/profil package.

**Edward Mebarak** (2003) addressed the optimal design of robots which are designed for minimum weight, which still withstand the highest levels of allowable stresses while carrying design payload. A commercial robot(Schilling robot)was analyzed as a case study, implementing an automated interaction between the Matlab and Pro/Engineer software packages was made that the optimization was carried out within Matlab, and the optimal design results were automatically shipped to Pro/Engineer to regenerate the three dimensions graphical representation of the final robot design.

An evaluation for optimal design of robotic manipulators to achieve high stiffness to weight ratio, this had been done through making the links of robot tapered to minimize its weight and also to make full use of the metal used in the construction of the robot links. The effect of varying the dimension of the robot structure had been addressed in this work to find its effect on the total deflection of the end point effectors. This work presents a geometric design of serial robot manipulator through optimizing link parameters which are length and radius so that the robot weight is minimized while keeping the link deflection and stress at acceptable design limits, this had been achieved through making the link cross section tapered along its length. A flow chart of optimization technique used in this work is shown in fig (1).

**Theoretical Analysis**

Types of robots studied in this work are three links and four links manipulator where the first link is considered as cylindrical tube while the other links are tapered cylinders, the four links manipulator is shown in fig(2),all tubes have a constant thickness.

The Load that the robot will manipulate is assumed to be a concentrated at the free end of the last manipulator link.
Manipulator's link section is circular which has radius(r) and thickness (t),
Where:
r: Distance from Neutral Axis to the Inside Surface
I: Second Moment of Area of Link's Cross Section
The second moment of area of link's cross section is shown in fig (3)

$$I_x = \int_0^{2\pi} \left(r^2 \sin^2 \theta\right).rtd\theta$$

$$I_x = \int_0^{2\pi} r^3 t \sin^2 \theta d\theta$$

$$I_x = r^3 t \int_0^{2\pi} \sin^2 \theta d\theta$$

$$I_x = \pi r^3 t \qquad (1)$$

**Calculation of Maximum Deflection due to Concentrate Load**

$$M_{AA} = EI \frac{d^2 y}{dx^2}$$

$$M_{AA} = Px$$

$$\therefore \quad EI \frac{d^2 y}{dx^2} = Px$$

$$I = \pi r^3 t \qquad See \quad Eq(1)$$

$$\therefore \quad E\frac{dy}{dx} = P\int \frac{xdx}{\pi r^3 t}$$

As the relation between moment and length of the section is relation of square root of the length(x) so this function is approximated to straight line this means that the end diameter of manipulator arm section will be half of the diameter of the other end.

$$r = \frac{R}{2} \left(1 + \frac{x}{L}\right)$$

$$\therefore \quad E\frac{dy}{dx} = \frac{8P}{\pi R^3 t} \int \frac{xdx}{\left(1+\frac{x}{L}\right)^3}$$

$$E\frac{dy}{dx} = \frac{8P}{\pi R^3 t}\left[\frac{-x \quad L}{2\left(1+\frac{x}{L}\right)^2} - \frac{L^2}{2\left(1+\frac{x}{L}\right)} + \frac{3L^2}{8}\right]$$

$$Ey = \frac{+8P}{\pi R^3 t} \int \left[\frac{-xL}{2\left(1+\frac{x}{L}\right)^2} - \frac{L^2}{2\left(1+\frac{x}{L}\right)} + \frac{3L^2}{8}\right]dx$$

$$Ey = \frac{+8P}{\pi R^3 t} *$$

$$\left[\frac{xL^2}{2\left(1+\frac{x}{L}\right)} - L^3 \ln\left|1+\frac{x}{L}\right| + \frac{3L^2}{8}x + 0.068147L^3\right] \quad (2)$$

$$y = y_{max} \quad \text{at} \quad x = 0$$

$$y_{max} = 0.54517 \frac{PL^3}{\pi E R^3 t} \qquad (3)$$

**Calculations of Maximum Deflection due to Link's Weight**

The center of the tapered section occurs at distance of $\left(\frac{4}{9}x\right)$ from the first end, thus the weight is centered at this distance.

$$M_{AA} = EI \frac{d^2 y}{dx^2}$$

$$M_{xx} = W\frac{4}{9}x$$

$$\therefore EI\frac{d^2 y}{dx^2} = \frac{4}{9}Wx$$

$$W = \rho V g$$

$$V=\frac{\pi R t\sqrt{L^2+\frac{R^2}{4}}}{2L}\left[x\left(1+\frac{x}{L}\right)+x\right]$$

$$\therefore W=\frac{\pi\rho g R t\sqrt{L^2+\frac{R^2}{4}}}{2L}\left[x\left(1+\frac{x}{L}\right)+x\right]$$

$$I=\pi r^3 t$$

$$I=\frac{\pi R^3 t}{8}\left(1+\frac{x}{L}\right)^3 \qquad (4)$$

$$E\frac{d^2y}{dx^2}=\frac{4\pi\rho g R t\sqrt{L^2+\frac{R^2}{4}}}{18L}\cdot \frac{8}{\pi R^3 t\left(1+\frac{x}{L}\right)^3}\left[x\left(1+\frac{x}{L}\right)+x\right]x$$

$$E\frac{d^2y}{dx^2}=\frac{16\rho g\sqrt{L^2+\frac{R^2}{4}}}{9R^2L}\left[x\left(1+\frac{x}{L}\right)+x\right]\frac{x}{\left(1+\frac{x}{L}\right)^3}$$

$$\text{let}\ \frac{16\rho g\sqrt{L^2+\frac{R^2}{4}}}{9R^2L}=\text{const.}$$

$$E\frac{dy}{dx}=\text{const.}\int\frac{x\left[x\left(1+\frac{x}{L}\right)+x\right]}{\left(1+\frac{x}{L}\right)^3}dx$$

$$E\frac{dy}{dx}=const.\left[\int\frac{x^2dx}{\left(1+\frac{x}{L}\right)^2}+\int\frac{x^2dx}{\left(1+\frac{x}{L}\right)^3}\right]$$

$$\therefore E\frac{dy}{dx}=const.\left[\frac{-x^2L}{\left(1+\frac{x}{L}\right)}+2xL^2-2L^3\ln\left|1+\frac{x}{L}\right|-\frac{x^2L}{2\left(1+\frac{x}{L}\right)^2}+\frac{L^3}{\left(1+\frac{x}{L}\right)}+L^3\ln\left|1+\frac{x}{L}\right|-1.18185L^3\right]$$

$$Ey=const.\int\left[\frac{-x^2L}{\left(1+\frac{x}{L}\right)}+2xL^2-L^3\ln\left|1+\frac{x}{L}\right|-\frac{x^2L}{2\left(1+\frac{x}{L}\right)^2}+\frac{L^3}{\left(1+\frac{x}{L}\right)}-1.18185L^3\right]dx$$

$$\therefore y=\frac{16\rho g\sqrt{L^2+\frac{R^2}{4}}}{9ER^2L}\Big[2L^4\left(1+\frac{x}{L}\right)-\frac{L^4}{2}\left(1+\frac{x}{L}\right)^2-L^4\ln\left|1+\frac{x}{L}\right|+x^2L^2-L^3x\ln\left|1+\frac{x}{L}\right|$$

$$L^3x-L^4\ln\left|1+\frac{x}{L}\right|+\frac{x^2L^2}{2\left(1+\frac{x}{L}\right)}-xL^3$$

$$+L^4\ln\left|1+\frac{x}{L}\right|+L^4\ln\left|1+\frac{x}{L}\right|-1.181815L^3x-1.375L^4\Big]$$

$y=y_{max}$ at $x=0$

$$y_{max}=\frac{2\rho g L^3\sqrt{L^2+\frac{R^2}{4}}}{9ER^2} \qquad (5)$$

**Calculation of Maximum Deflection of Robot Link due to Moment**

$$M_{AA}=EI\frac{d^2y}{dx^2}$$

$$M_{AA}=M_O$$

$$EI\frac{d^2y}{dx^2} = M_O$$

$$E\frac{dy}{dx} = \frac{8M_O}{\pi R^3 t}\int \frac{dx}{\left(1+\frac{x}{L}\right)^3}$$

$$E\frac{dy}{dx} = \frac{8M_O}{\pi R^3 t}\left[\frac{-L\left(1+\frac{x}{L}\right)^{-2}}{2} + \frac{L}{8}\right]$$

$$E\frac{dy}{dx} = -\frac{4M_O L}{\pi R^3 t\left(1+\frac{x}{L}\right)^2} + \frac{M_O L}{\pi R^3 t}$$

$$Ey = \frac{4M_O L^2}{\pi R^3 t\left(1+\frac{x}{L}\right)} + \frac{M_O L}{\pi R^3 t}x - \frac{3M_O L^2}{\pi R^3 t}$$

$y = y_{max}$ at $x=0$

$$y_{\max} = \frac{M_O L^2}{\pi E R^3 t} \tag{6}$$

**Robot Manipulator Links Equations**

The system of forces applied on the robot Links are shown in Fig (4)

Where:

$m_1, m_2, m_3, m_4$= Masses of Gear Boxes (kg)
$W_1, W_2, W_3, W_4$= Weight of Each Arm (N)
$L_1, L_2, L_3, L_4$= Length of Each Arm (m)
$W_{Load}$= Manipulated load (N)
$f_1, f_2, f_3, f_4$= Reaction forces at the Joints (N)
$M_{O1}, M_{O2}, M_{O3}, M_{O4}$ = Reaction Moment at the Joints (N.m)

**<u>Fourth Arm</u>**

The Fourth arm section and its system of forces are shown in Fig (12)

$w_4 = \rho V g$

$$W_4 = \frac{3}{2}\pi\rho g t R_4\sqrt{L_4^2 + \frac{R_4^2}{4}} \tag{7}$$

$$\sigma = \frac{My}{I}$$

$$\sigma \le \sigma_{allowable}$$

$$\sigma_4 = \frac{M_{O4}(R_4 + t)}{\pi R_4^3 t} \tag{8}$$

$$MO_4 = W_{load} L_4 + W_4\left(\frac{4}{9}L_4\right) + m_4 g L_4 \tag{9}$$

$$f_4 = W_4 + W_{load} + m_4 g \tag{10}$$

$\delta_4$ = Deflection of Fourth Arm

$$\delta_4 = \delta_{\text{arm weight}} + \delta_{\text{load}}$$

$$\delta_4 = \frac{2\rho g\sqrt{L_4^2 + \frac{R_4^2}{4}}}{9ER_4^2}L_4^3 + \frac{0.54517L_4^3}{\pi E R_4^3 t}\left(W_{Load} + m_4 g\right) \tag{11}$$

**Third Arm**

$$W_3 = \frac{3}{2}\pi\rho g t R_3\sqrt{L_3^2 + \frac{R_3^2}{4}} \tag{12}$$

$$\sigma_3 = \frac{M_{O3}(R_3 + t)}{\pi R_3^3 t} \tag{13}$$

$$M_{O3} = f_4 L_3 + W_3\left(\frac{4L_3}{9}\right) + m_3 g L_3 + M_{O4} \tag{14}$$

$$f_3 = W_3 + f_4 + m_3 g \tag{15}$$

$$\delta_3 = \delta_{\text{arm weight}} + \delta_{\text{load}} + \delta_{\text{moments}}$$

$$\delta_3 = \frac{2\rho g\sqrt{L_3^2 + \frac{R_3^2}{4}}}{9ER_3^2}L_3^3 + \frac{0.54517L_3^3}{\pi E R_3^3 t} + \frac{M_{O4}L_3^3}{\pi E R_3^3 t} \tag{16}$$

**Second Arm**

$$W_2 = \frac{3}{2}\pi\rho g t R_2\sqrt{L_2^2 + \frac{R_2^2}{4}} \tag{17}$$

$$\sigma_2 = \frac{M_{O2}(R_2 + t)}{\pi R_2^3 t} \tag{18}$$

$$M_{O2} = f_3 L_2 + W_2\left(\frac{4L_2}{9}\right) + m_2 g L_2 + M_{O3} \tag{19}$$

$$f_2 = W_2 + f_3 + m_2 g \tag{20}$$

$$\delta_2 = \delta_{\text{arm weight}} + \delta_{\text{load}} + \delta_{\text{moments}}$$

$$\delta_2 = \frac{2\rho g\sqrt{L_2^2 + \frac{R_2^2}{4}}}{9ER_2^2} L_2^3 + \frac{0.54517(m_2 g + f_3)L_2^3}{\pi E R_2^3 t} + \frac{M_{O3}L_2^2}{\pi E R_2^3 t} \quad (21)$$

**First Arm**

The dimensions of the first link are calculated by equating the maximum stress induced in it with the maximum allowable stress. This maximum stress is found by the Rankin-Gordon formula which is a combination of the Euler and crushing loads for a strut.

$$\frac{1}{F_R} = \frac{1}{F_e} + \frac{1}{F_c} \quad (22)$$

Where:

$F_R$ : Rankin Load

$F_e$ : Euler Load

$F_c$ : Compressive Load

$\sigma_e$ : Euler Stress

For very short struts $F_e$ is very large, $\frac{1}{F_e}$ can therefore be neglected and $F_R = F_c$, for very large struts $F_e$ is very small and $\frac{1}{F_e}$ is very large so that $\frac{1}{F_c}$ can be neglected, thus $F_R = F_e$

The Rankin formula is therefore valid for extreme values of slenderness ratios. It is also found to be fairly accurate for the intermediate values.

For a strut with one end free and the other fixed

$$F_e = \frac{\pi^2 EI}{4L^2} \quad (23)$$

$$\sigma_e = \frac{\pi^2 EI}{4L^2 A} \quad (24)$$

The crushing load on the first arm is:

$$f_c = f_1 = f_2 + W_1 + m_1 g \quad (25)$$

$$\sigma_y = \frac{f_c}{A} = \frac{f_1}{A} \quad (26)$$

The final stress $\sigma_1$ on the first arm is the sum of direct stress calculated by Rankin formula and that due to bending generated by the exerted moment

$$\therefore \sigma_1 = \sigma_R + \sigma_{bending} = \frac{\sigma_y}{1 + \frac{\sigma_y}{\sigma_e}} + \frac{M_{O1}\cdot(R_1 + t)}{I_1} \quad (27)$$

$$\delta_1 = \frac{M_{O1}L_1^2}{2E\pi R_1^3 t} \quad (28)$$

**Results and Conclusions**

A geometric optimization of three and four links serial robot manipulator is addressed in this work, the goal of the optimization was reducing the robot manipulator weight through making the second, third and fourth links tapered while keeping the first link cylindrical. The objective function(robot weight) of the optimization was subjected to several constraints that keep the end effecter deflection as minimum as possible and the links stresses at the allowable values, the optimization process is hold on using two software tools which are Mathcad and Matlab Optimization Toolbox, a comparison between results of each tool had been done and are shown in figures, the results shows convergence between both tools with better accuracy and smoothness in results of Matlab Optimization Toolbox.The optimization focused on the links length and radius while the thickness is considered to be constant through the whole robot length.

In the case of three links manipulator, the stress in the third link calculated by (Matlab Optimization Toolbox) is larger than that by Mathcad for the same radius. This is referred to the length of the third link which is longer in the calculated results of (Matlab Optimization Toolbox) than in Mathcad which makes the moment arm longer and therefore increasing the stress. also it is clear that the radius in both methods of calculation begins to approach one another when minimizing the limit of deflection. In the first arm, the stress calculated by (Matlab Optimization Toolbox)is bigger than that by Mathcad although the moment arms($L_2$+$L_3$) are almost equal, this is because the radius of the first link calculated by Mathcad is bigger than that calculated by (Matlab Optimization Toolbox)for the same total deformation limit. It is clear that the stress in the second link which is calculated by both (Matlab Optimization Toolbox) and Mathcad are so close this is referred to the link radius which is so close in both programs. This can be said also for the third link stress and this referred to the third link length which is so close as calculated by both programs, this make the moment arm almost the same, those results are shown in figures (5-8). the links length effect was more than the links radii effect on the stress values this is referred to the moments values keeping in mind that the moment involves the actuators moment and as the actuator mass is bigger and the link length is long this makes the moment value larger and therefore the stress value increases even though the radius is big.

The sum of robot links length and first link length were considered as constraints in the optimization problem, we chose to fix the first link length because it represent the base in which its section is cylindrical while the optimization is focused on the tapered sections and its effect so we can see that each figure which represent the relation between total links length robot deflection is ended at the same point. We prefer the presentation of the total robot links length rather than each link separately so as to show the change in length of each link corresponding to the other, this can be seen in figures(14-15),it is obvious that the second link length($L_2$) calculated by(Matlab Optimization Toolbox) is less than that calculated by Mathcad while the third link length($L_3$) calculated by(Matlab Optimization Toolbox) is bigger than that calculated by Mathcad. In the case of four links manipulator, the stress in the first link calculated by(Matlab Optimization Toolbox) is larger than in Mathcad, this is referred to the length of the third and fourth links which are bigger in the calculated results of (Matlab Optimization Toolbox)while the second link length is smaller in the calculated results of (Matlab Optimization Toolbox) ,this make the moment arm bigger and therefore the stress is higher. It can seen that the first and second links radii were the same in figures(16-17) and this may referred to the first link which represent the robot base which handle the maximum part of the robot structure weight in addition to the extra components so the first link is fixed and has no degree of freedom, this makes the robot movement to be done by the other three links and as the second link is the nearest to the first link and it carries reasonable part of the robot movement and transport it to the neighbor links so its dimensions must not differ so much from the first link. The third link radius calculated by (Matlab Optimization Toolbox) and Mathcad was also so close to each other and there was a little difference for the fourth link radius which proves that the optimization process is reaching the feasible design. This results can be seen in figures (18-19).It can be noticed that the first and fourth links act at the same manner which can be noticed also for the second and third links, even though the values of each link parameters such as (radius,stress,length,…etc.) is different. This may referred to that the first link handle the maximum part of the robot structure weight and the fourth link handle the load capacity which make them related in the weight effect issue and overcome the inertia effect and keeping the deformation at the acceptable range while the second and third links are related in transporting the movement and the power between the robot links and controlling the final robot movement, this can be seen in figures (29-30).

Whenever the deflection was minimized the results of both programs began to approach which shows the optimization process procedure that is having an objective design to get and constraints to keep so the program of optimization is acting like a search engine looking for the feasible area for the designer to start from to get the optimum performance and because of this it has been chosen to run the optimization process through two programs to check our results because any optimization method must lead us to the same result of the other, the difference will be in the accuracy, speed of optimization only.

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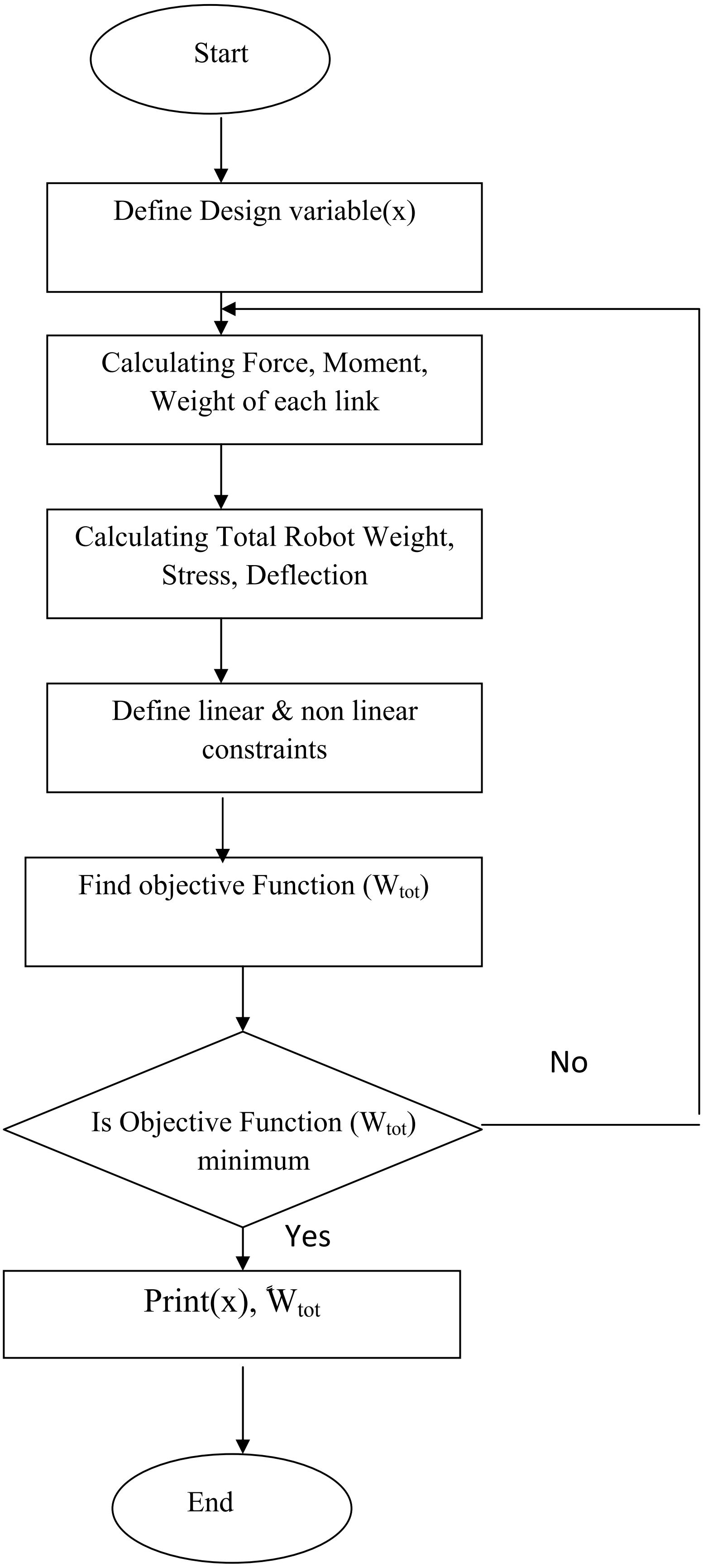

Start
Define Design variable(x)
Calculating Force, Moment, Weight of each link
Calculating Total Robot Weight, Stress, Deflection
Define linear & non linear constraints
Find objective Function ($W_{tot}$)
Is Objective Function ($W_{tot}$) minimum
No
Yes
Print(x), $W_{tot}$
End

**Fig (1)**

**Flowchart of Optimization Problem**

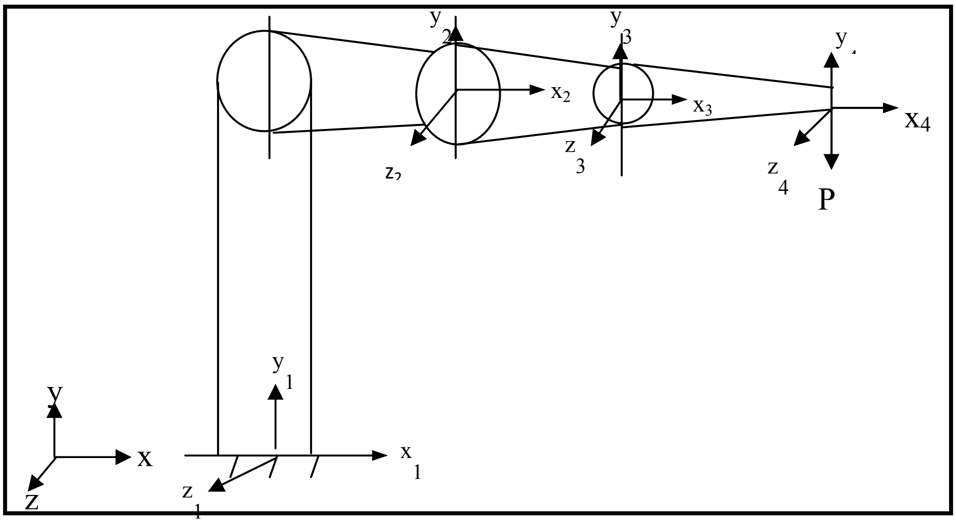



**Fig (2) Four Links Manipulator Construction**

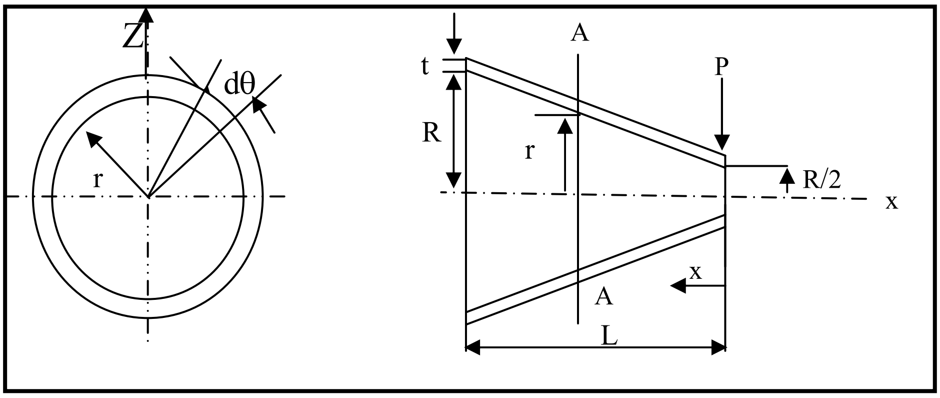



**Fig (3) Link Cross Section**

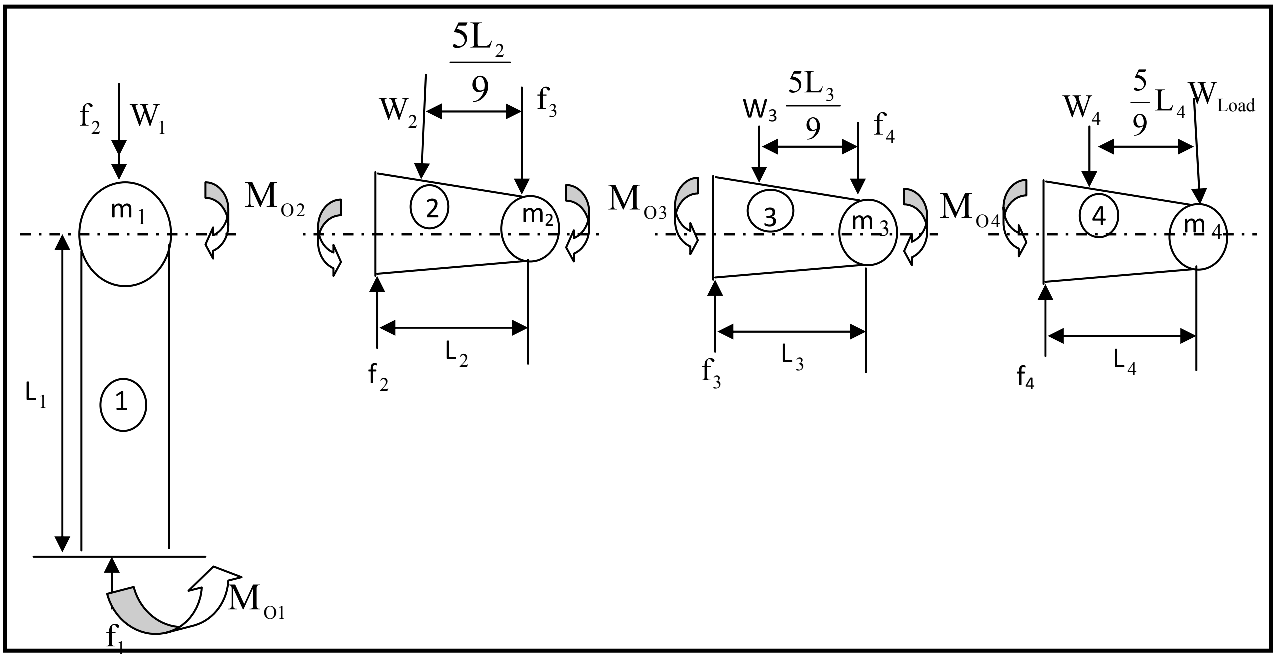
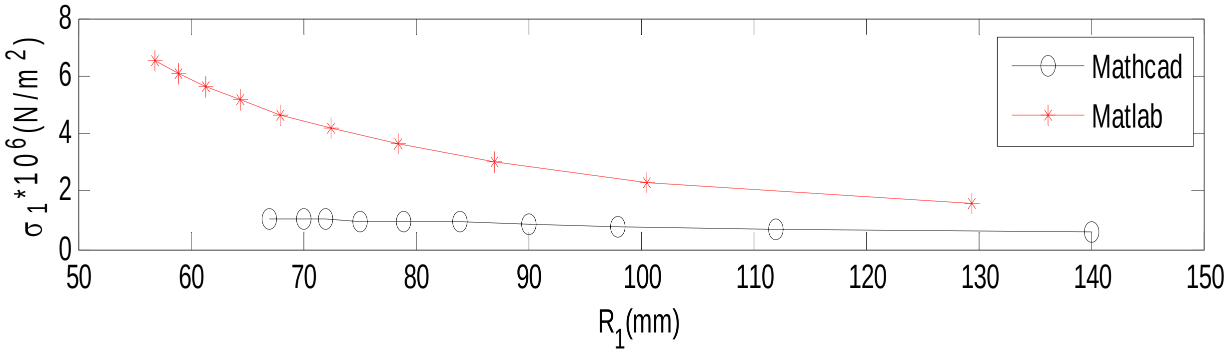
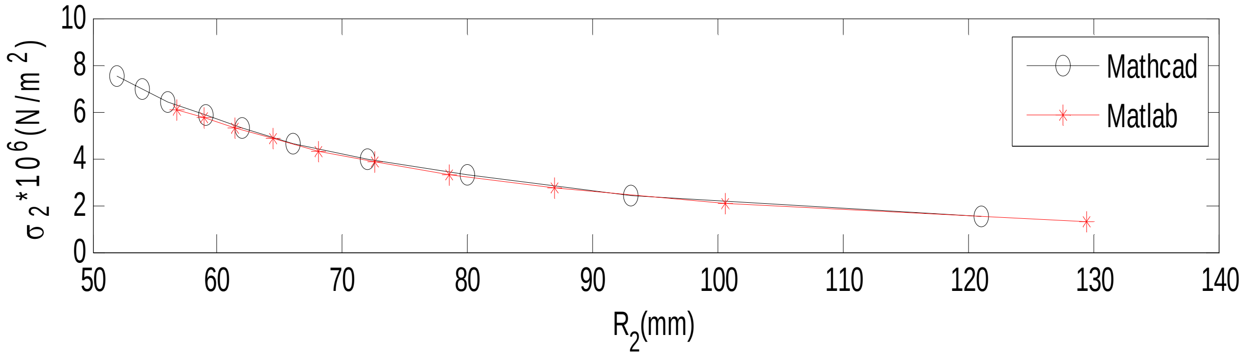



**Fig (4) Robot Links Forces System**

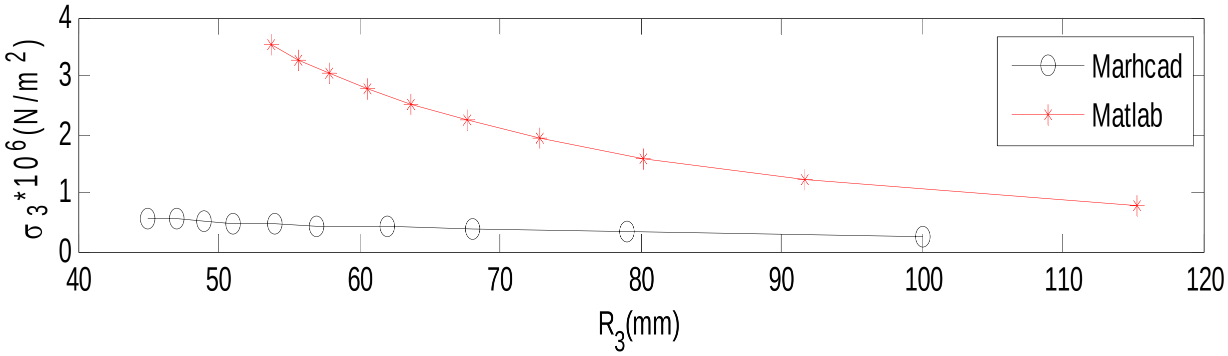
## Results of Three Links Manipulator



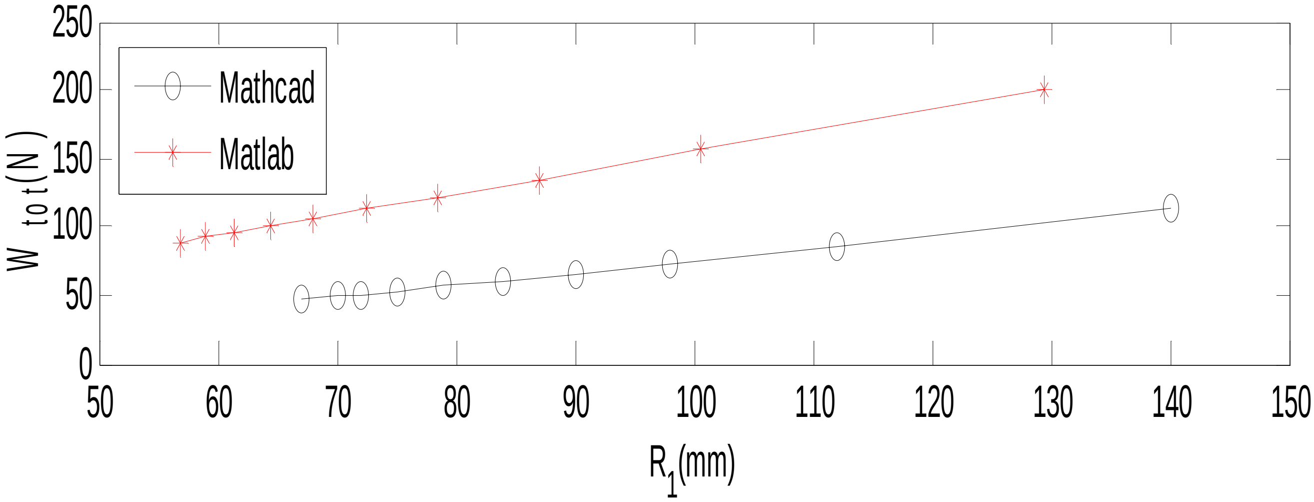

**Fig(5) Relation between First Link Radius and its Bending Stress**



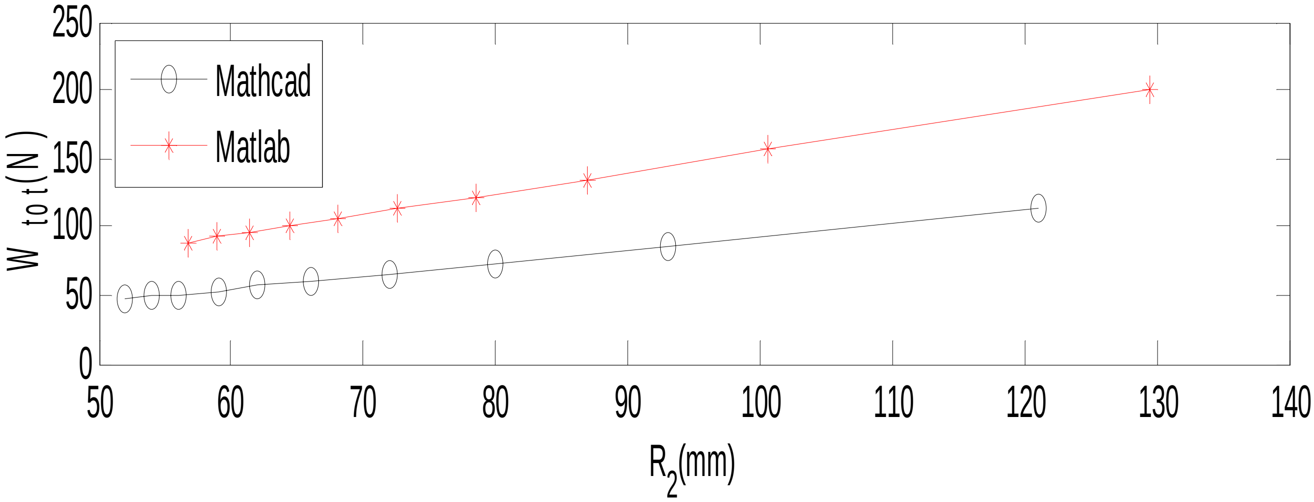

**Fig(6) Relation between Second Link Radius and its Stress**



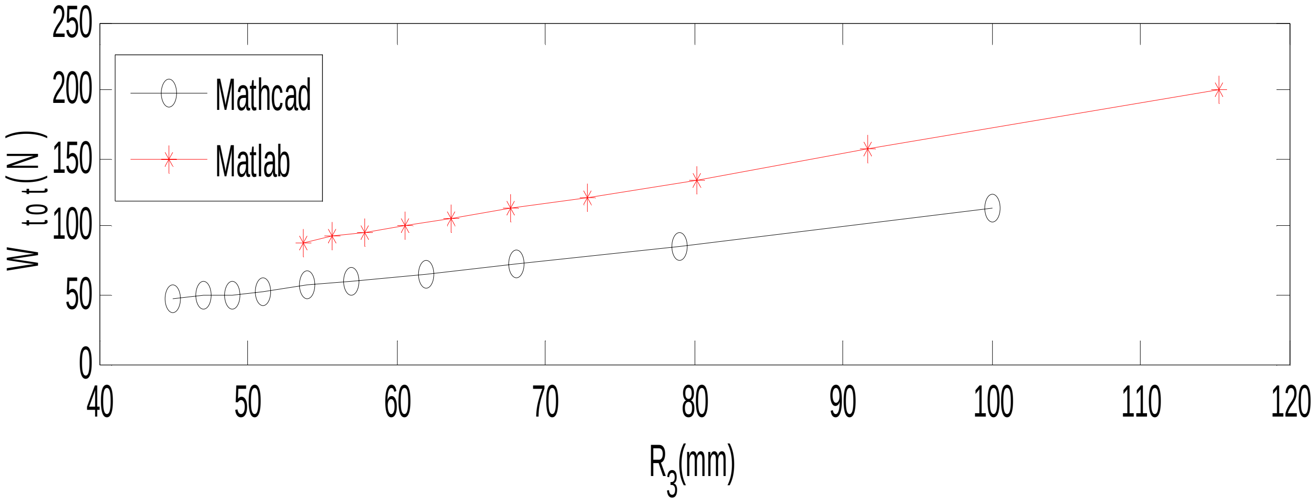

**Fig(7) Relation between Third Link Radius and its Stress**



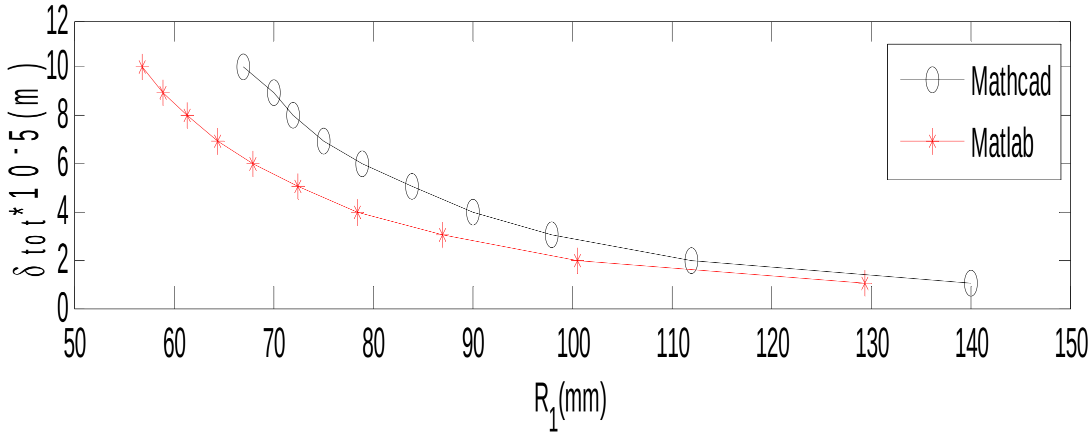

**Fig(8) Relation between First Link Radius and Total Robot Weight**



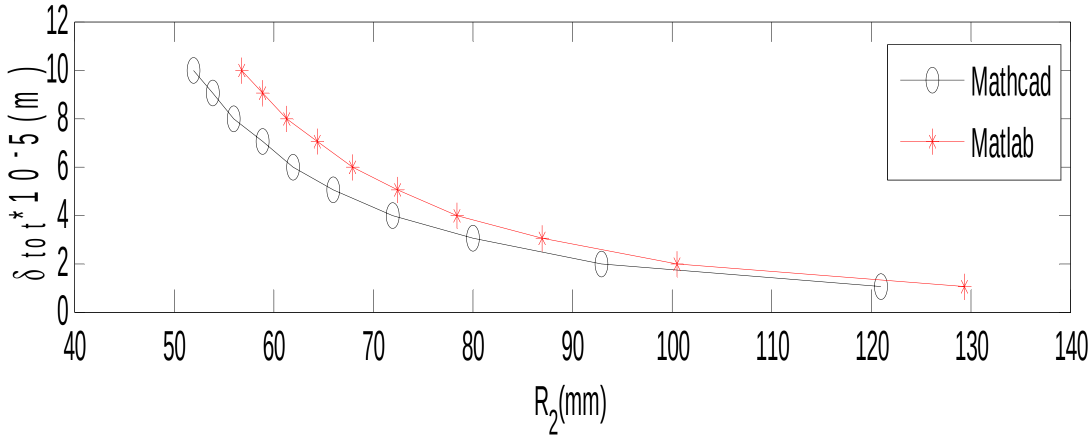

**Fig(9) Relation between Second Link Radius and Total Robot**



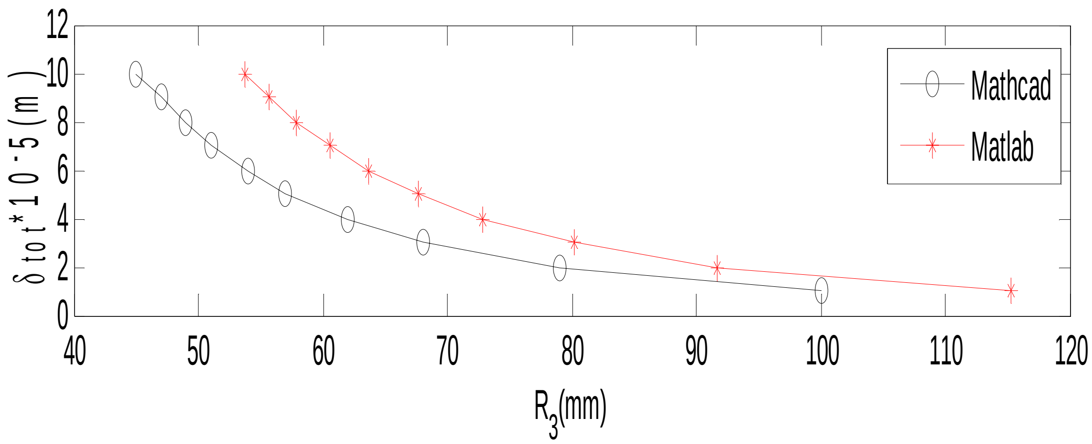

**Fig(10) Relation between Third Link Radius and Total Robot Weight**




Fig(11) Relation between First Link Radius and Total Robot




Fig(12) Relation between Second Link Radius and Total Robot Deflection



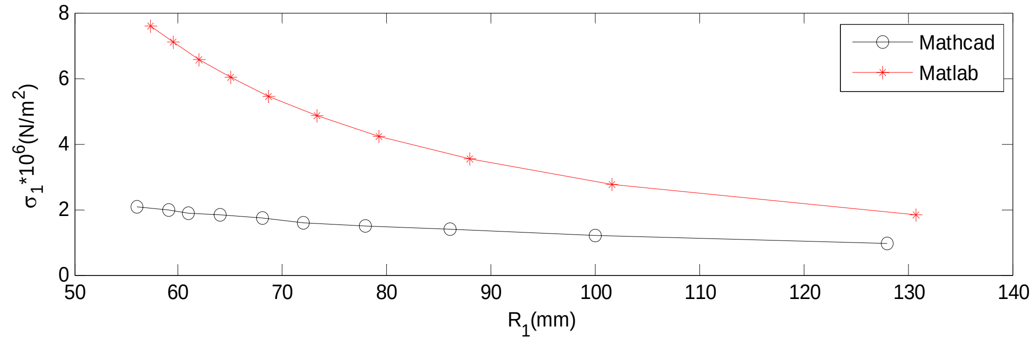

Fig(13) Relation between Third Link Radius and Total Robot

**Table (1) Comparison between Links Length using Matlab Optimization Toolbox and Math CAD for Three**

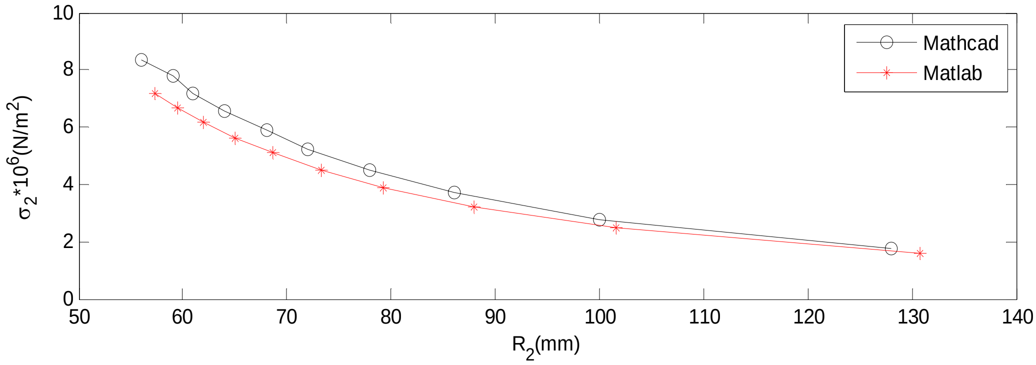
**Links Manipulators**

| Deflections $\times 10^{-5}$(mm) | Matlab and MathCAD First Link Length (mm) | Matlab Second Link Length (mm) | MathCAD Second Link Length (mm) | Difference (mm) | Matlab Third Link Length (mm) | MathCAD Third Link length (mm) | Difference (mm) |
|---|---|---|---|---|---|---|---|
| 1 | 475 | 328 | 375 | -47 | 502 | 455 | 47 |
| 2 | 475 | 320 | 373 | -53 | 510 | 457 | 53 |
| 3 | 475 | 316.1 | 373 | -56.9 | 513.9 | 457 | 56.9 |
| 4 | 475 | 313.6 | 372 | -58.4 | 516.4 | 457 | 58.4 |
| 5 | 475 | 311.9 | 372 | -60.1 | 518.1 | 458 | 60.1 |
| 6 | 475 | 310.6 | 372 | -61.4 | 519.4 | 458 | 61.4 |
| 7 | 475 | 309.5 | 372 | -62.5 | 520.5 | 458 | 62.5 |
| 8 | 475 | 308.6 | 371 | -62.4 | 521.4 | 459 | 62.4 |
| 9 | 475 | 307.9 | 371 | -63.1 | 522.1 | 459 | 63.1 |
| 10 | 475 | 307.2 | 371 | -63.8 | 522.8 | 459 | 63.8 |

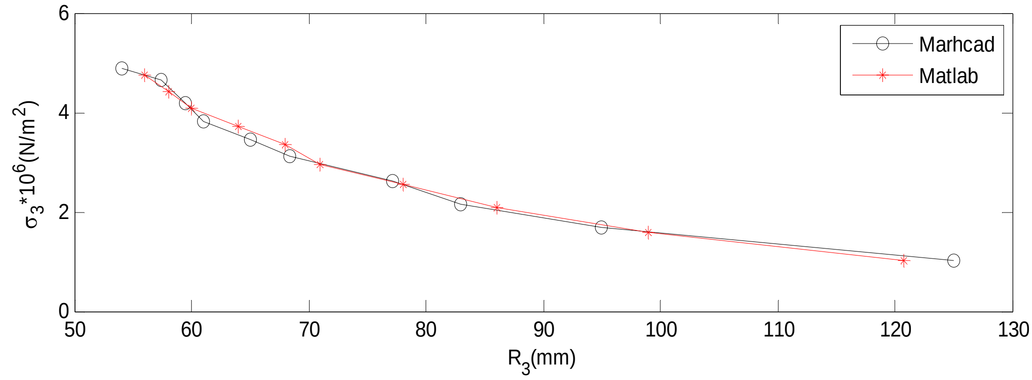
**Results of Four Links Manipulator**



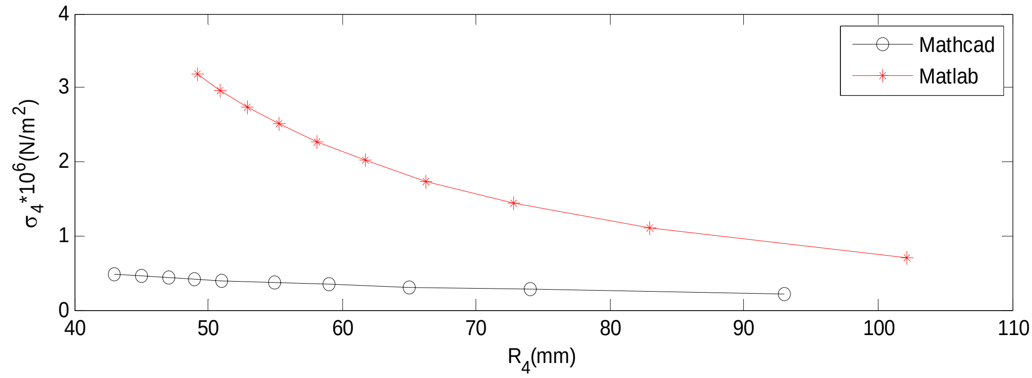

**Fig(16) Relation between First Link Radius and its Bending Stress**



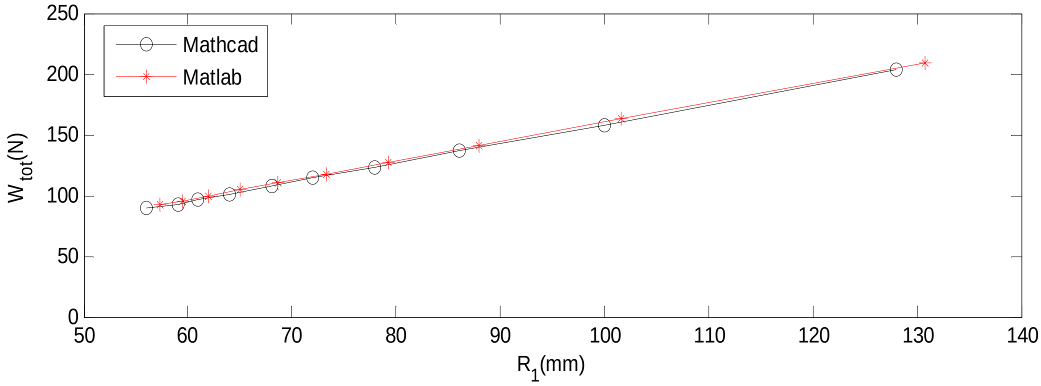

**Fig(17) Relation between Second Link Radius and its Bending Stress**



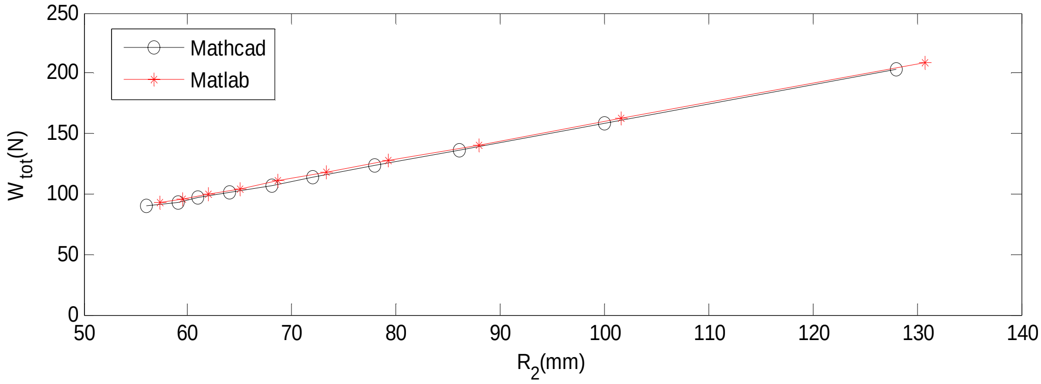

**Fig(18) Relation between Third Link Radius and its Bending Stress**



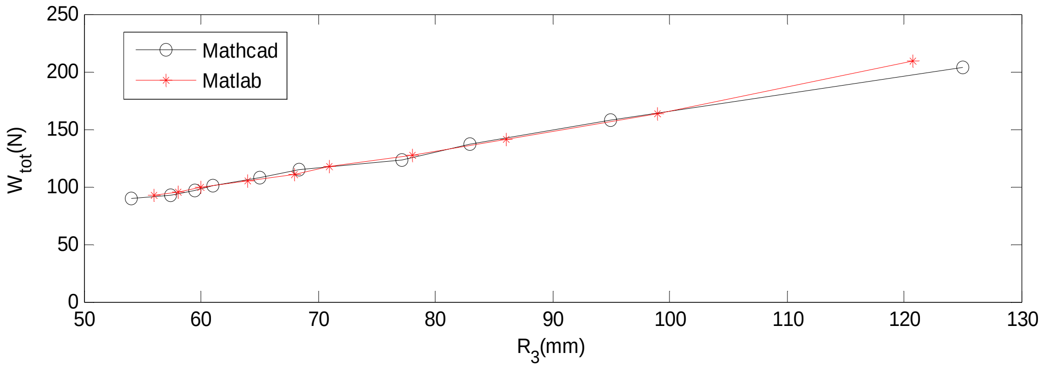

**Fig(19) Relation between Fourth Link Radius and its Bending Stress**



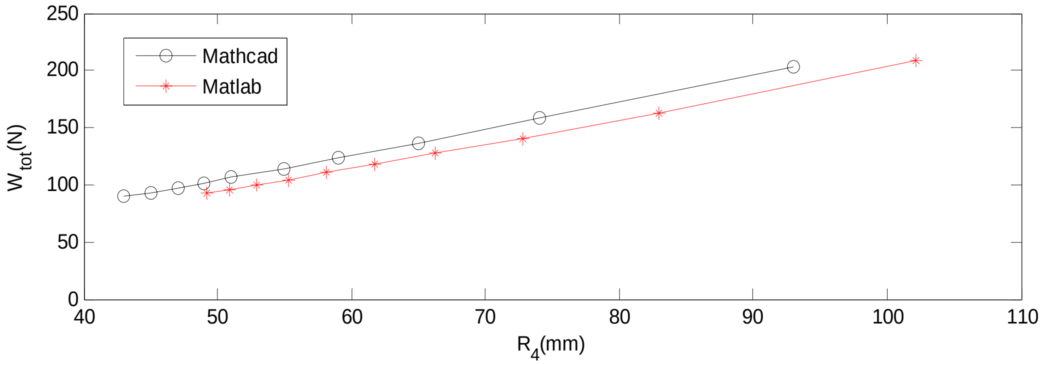

**Fig(20) Relation between First Link Radius and Total Robot Weight**



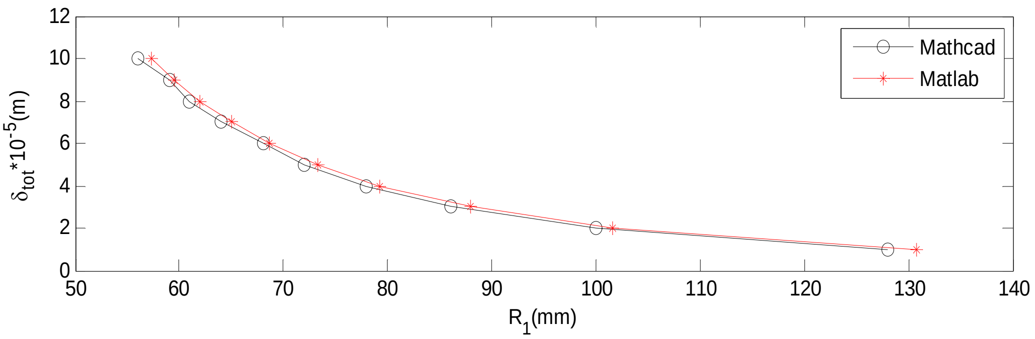

**Fig(21) Relation between Second Link Radius and Total Robot Weight**



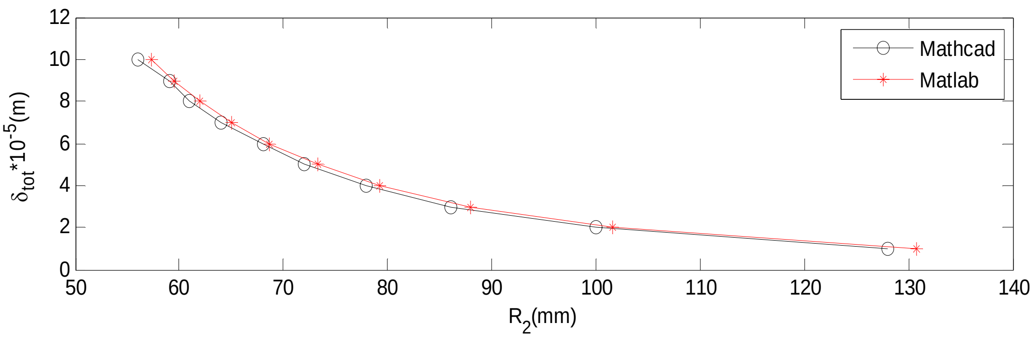

**Fig(22) Relation between Third Link Radius and Total Robot Weight**



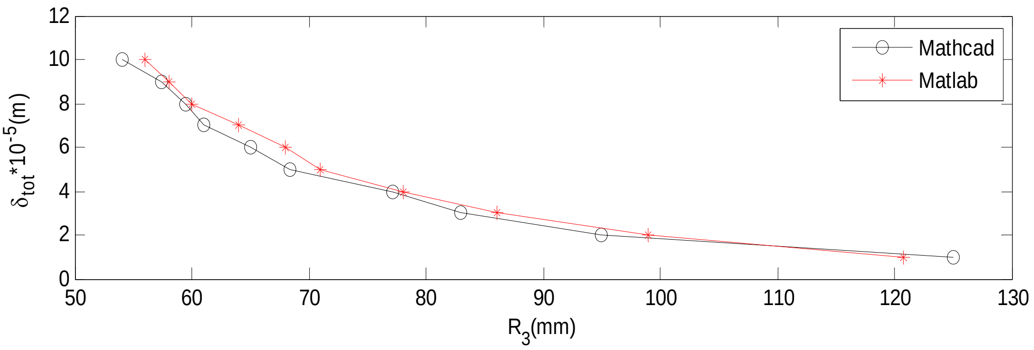

**Fig(23) Relation between Fourth Link Radius and Total Robot Weight**



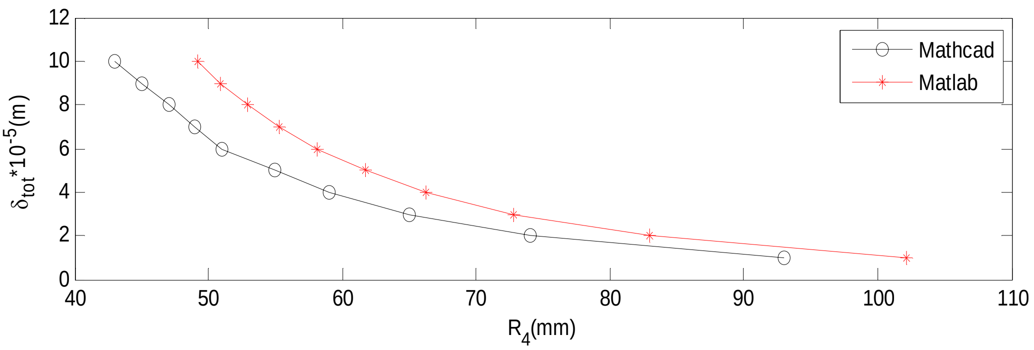

**Fig(24) Relation between First Link Radius and Total Robot Deflection**




**Fig(25) Relation between Second Link Radius and Total Robot Deflection**




**Fig(26) Relation between Third Link Radius and Total Robot Deflection**




**Fig(27) Relation between Fourth Link Radius and Total Robot Deflection**

**Table(2) Comparison between Links Lengths using Matlab and Mathcad for Four Links Manipulator**

| **Deflections** x $10^{-5}$ | Matlab and MathCAD First Link Length (mm) | Matlab Second Link Length (mm) | Math CAD Second Link Length (mm) | Difference (mm) | Matlab Third Link Length (mm) | MathCAD Third Link Length (mm) | Difference (mm) | Matlab Fourth Link Length (mm) | MathCAD Fourth Link Length (mm) | Difference (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 420 | 229.1 | 293 | -63.9 | 325.3 | 323 | 2.3 | 425.6 | 364 | 61.6 |
| 2 | 420 | 224.7 | 30.2 | -77.3 | 319.1 | 307 | 12.1 | 436.2 | 368 | 68.2 |
| 3 | 420 | 222.5 | 303 | -80.5 | 316 | 308 | 8 | 441.5 | 369 | 72.5 |
| 4 | 420 | 221.2 | 305 | --83.8 | 314 | 305 | 9 | 444.9 | 370 | 74.9 |
| 5 | 420 | 220.2 | 306 | -85.8 | 312.6 | 303 | 9.6 | 447.3 | 371 | 76.3 |
| 6 | 420 | 219.4 | 307 | -78.6 | 311.5 | 301 | 10.5 | 449.1 | 372 | 77.1 |
| 7 | 420 | 218.8 | 308 | -89.2 | 310.6 | 301 | 9.6 | 450.6 | 371 | 79.6 |
| 8 | 420 | 218.3 | 309 | -90.7 | 309.9 | 299 | 10.9 | 451.9 | 372 | 79.9 |
| 9 | 420 | 217.8 | 310 | -92.2 | 309.2 | 298 | 11.2 | 452.9 | 372 | 80.9 |
| 10 | 420 | 217.5 | 310 | -92.9 | 308.7 | 297 | 11.7 | 453.8 | 373 | 80.8 |