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Wide-spectrum optical synthetic aperture imaging via spatial intensity interferometry

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Section 1: The spatial field autocorrelation of the point-spread function (PSF)

When imaging in wide spectrum, the point-spread function (PSF) of optical synthetic aperture imaging via spatial intensity interferometry is

$$h_{\mathrm{E}}(\boldsymbol{u}) = \int_{-\infty}^{\infty} W(k) h_{\mathrm{E}}(\boldsymbol{u}, k) \mathrm{d}k \ , \tag{S1}$$

where $h_{\mathrm{E}}(\boldsymbol{u}, k)$ is the point-spread function (PSF) for the wave number $k = 1/\lambda$ (λ is the lambda), $W(k)$ is the spectral distribution of the wide spectrum. Then, the spatial field autocorrelation of h_{E} is

$$R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}') = \overline{h_{\mathrm{E}}(\boldsymbol{u}) h_{\mathrm{E}}^{*}(\boldsymbol{u}')} = \iint_{-\infty}^{\infty} W(k) W(k') R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') \mathrm{d}k \mathrm{d}k' \ , \tag{S2}$$

where $R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') = \overline{h_{\mathrm{E}}(\boldsymbol{u}, k) h_{\mathrm{E}}^{*}(\boldsymbol{u}', k')}$.

According to Fresnel diffraction, we have

$$h_{E}(\boldsymbol{u}, k) = \int_{-\infty}^{\infty} P(\boldsymbol{r}, k) \exp\left[-\mathrm{j}2\pi c_0 k (\boldsymbol{r} - c_1 \boldsymbol{u})^2\right] \mathrm{d}\boldsymbol{r} \ , \tag{S3}$$

where we have omitted a constant term outside the integration in the following derivation to simplify the expression, $c_0 = \dfrac{(z_1 + z_2)}{2 z_1 z_2}$, $c_1 = \dfrac{z_1}{z_1 + z_2}$, and

$$P(\boldsymbol{r}, k) = A(\boldsymbol{r}) S(\boldsymbol{r}, k) \ , \tag{S4}$$

is the pupil function of optical synthetic aperture imaging via spatial intensity interferometry, $A(\boldsymbol{r})$ is the aperture function, $S(\boldsymbol{r}, k) = \exp\{\mathrm{j}\varphi(\boldsymbol{r}, k)\}$ is the modulation phase transfer function, and $\varphi(\boldsymbol{r}, k)$ is the modulation phase function. Letting $\tilde{\boldsymbol{r}} = \boldsymbol{r} - c_1 \boldsymbol{u}$, we find

$$h_{\mathrm{E}}(\boldsymbol{u}, k) = \int_{-\infty}^{\infty} P(\tilde{\boldsymbol{r}} + c_1 \boldsymbol{u}, k) \exp\left(-\mathrm{j}2\pi c_0 k \tilde{\boldsymbol{r}}^2\right) \mathrm{d}\tilde{\boldsymbol{r}} \ . \tag{S5}$$

Since the ensemble statistics are introduced by the pupil function, interchanging the orders of ensemble statistics and double integration, we obtain

$$\begin{aligned} R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') = & \iint_{-\infty}^{\infty} \mathrm{d}\tilde{\boldsymbol{r}}_1 \mathrm{d}\tilde{\boldsymbol{r}}_2 \exp\left[-\mathrm{j}2\pi c_0 \left(k \tilde{\boldsymbol{r}}_1^2 - k' \tilde{\boldsymbol{r}}_2^2\right)\right] \\ & * \overline{P(\tilde{\boldsymbol{r}}_1 + c_1 \boldsymbol{u}, k) P^{*}(\tilde{\boldsymbol{r}}_2 + c_1 \boldsymbol{u}', k')} \\ \approx & \iint_{-\infty}^{\infty} \mathrm{d}\tilde{\boldsymbol{r}}_1 \mathrm{d}\tilde{\boldsymbol{r}}_2 \exp\left[-\mathrm{j}2\pi c_0 k \left(\tilde{\boldsymbol{r}}_1^2 - \tilde{\boldsymbol{r}}_2^2\right)\right] \\ & * \overline{P(\tilde{\boldsymbol{r}}_1 + c_1 \boldsymbol{u}, k) P^{*}(\tilde{\boldsymbol{r}}_2 + c_1 \boldsymbol{u}', k')} \ , \end{aligned} \tag{S6}$$

where the approximation is valid for $|\Delta k| = |k - k'| \ll k$. For further simplification, we define

$$R_{\mathrm{P}}(\boldsymbol{u}, \boldsymbol{u}'; k, k')) \stackrel{\mathrm{def}}{=} \overline{P(\boldsymbol{u}, k) P^{*}(\boldsymbol{u}', k')} \ . \tag{S7}$$

Assuming the width of the aperture is much greater than the correlation width of the pupil function, we have

$$R_{\mathrm{P}}(\boldsymbol{u}, \boldsymbol{u}'; k, k')) \approx A(\breve{\boldsymbol{u}}) R_{\mathrm{s}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') \ , \tag{S8}$$

with $R_{\mathrm{s}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') = \overline{S(\boldsymbol{u}, k) S^{*}(\boldsymbol{u}', k')}$, and $\breve{\boldsymbol{u}} = \dfrac{\boldsymbol{u} + \boldsymbol{u}'}{2}$. Making the following definitions:

$$\tilde{\boldsymbol{r}}_3 = (\tilde{\boldsymbol{r}}_1 + \tilde{\boldsymbol{r}}_2)/2, \quad \tilde{\boldsymbol{r}}_4 = \tilde{\boldsymbol{r}}_1 - \tilde{\boldsymbol{r}}_2 \ ,$$

to simply the expressions of Eq. (S6) and Eq. (S2), we have

$$R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}'; k, k') \approx \iint_{-\infty}^{\infty} \mathrm{d}\tilde{\boldsymbol{r}}_3 \mathrm{d}\tilde{\boldsymbol{r}}_4 \exp\left(-\mathrm{j}4\pi c_0 k \tilde{\boldsymbol{r}}_3 \tilde{\boldsymbol{r}}_4\right) A(\tilde{\boldsymbol{r}}_3 + c_1 \breve{\boldsymbol{u}}) R_s\left(\tilde{\boldsymbol{r}}_3 + \frac{\tilde{\boldsymbol{r}}_4}{2} + c_1 \boldsymbol{u}, \tilde{\boldsymbol{r}}_3 - \frac{\tilde{\boldsymbol{r}}_4}{2} + c_1 \boldsymbol{u}'; k, k'\right) \ , \tag{S9}$$

and

$$R_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}') \approx \iiiint_{-\infty}^{\infty} \mathrm{d}k \mathrm{d}k' \mathrm{d}\tilde{\boldsymbol{r}}_3 \mathrm{d}\tilde{\boldsymbol{r}}_4 W(k) W(k') A(\tilde{\boldsymbol{r}}_3 + c_1 \breve{\boldsymbol{u}}) \exp\left(-\mathrm{j}4\pi c_0 k \tilde{\boldsymbol{r}}_3 \tilde{\boldsymbol{r}}_4\right) R_s\left(\tilde{\boldsymbol{r}}_3 + \frac{\tilde{\boldsymbol{r}}_4}{2} + c_1 \boldsymbol{u}, \tilde{\boldsymbol{r}}_3 - \frac{\tilde{\boldsymbol{r}}_4}{2} + c_1 \boldsymbol{u}'; k, k'\right) \ , \tag{S10}$$

Equation (S10) shows the relationship of the spatial field autocorrelation function $R_{h_{\mathrm{E}}}$ and the correlation function R_s of mdulation phase transfer function.

Section 2: The correlation of modulation phase transfer function

The modulation transfer phase S is assumed to be wide-sense stationary, and its relationship with the surface height η is

$$S(\boldsymbol{r},k)=\exp\left[\mathrm{j}2\pi k(n-1)\eta(\boldsymbol{r})\right] \ . \tag{S11}$$

Then, the autocorrelation function of the modulation phase function is given by

$$R_{\mathrm{s}}(\boldsymbol{r},\boldsymbol{r}';k,k')=\overline{\exp\left\{\mathrm{j}2\pi(n-1)\left(k\eta(\boldsymbol{r})-k'\eta(\boldsymbol{r}')\right)\right\}} \ . \tag{S12}$$

Considering the characteristic function of the random variable η, the above equation can be written as

$$R_{\mathrm{s}}(\boldsymbol{r},\boldsymbol{r}';k,k')=M_{\eta(r)}\left(2\pi(n-1)k,-2\pi(n-1)k'\right) \ , \tag{S13}$$

here $M_{\eta(r)}$ is the second-order characteristic function of the surface-height function η, which is assumed to be statistically stationary. Assuming η is a Gaussian random variable, in this case, its characteristic function is

$$M_{\eta(r)}\left(\omega,\omega'\right)=\exp\left\{-\frac{\left[\left(\omega^2+\omega'^2\right)\sigma_\eta^2+2\omega\omega' R_\eta(\Delta\boldsymbol{r})\right]}{2}\right\} \ , \tag{S14}$$

where σ_η^2 is the height variance, R_η is the height correlation function. Substituting this equation into Eq. (S13), the result is

$$\begin{aligned}
R_{\mathrm{s}}(\boldsymbol{r},\boldsymbol{r}';k,k') &= \exp\left\{-\frac{\left[2\pi(n-1)\sigma_\eta\right]^2}{2}\left[k^2+k'^2-\frac{2kk'R_\eta(\Delta\boldsymbol{r})}{\sigma_\eta^2}\right]\right\} \\
&= \exp\left\{-\frac{\left[2\pi(n-1)\sigma_\eta\right]^2}{2}\left[\Delta k^2+2kk'\left(1-\frac{R_\eta(\Delta\boldsymbol{r})}{\sigma_\eta^2}\right)\right]\right\} \\
&\approx \exp\left\{-\frac{\left[2\pi(n-1)\sigma_\eta\right]^2}{2}\left[\Delta k^2+2k^2\left(1-\frac{R_\eta(\Delta\boldsymbol{r})}{\sigma_\eta^2}\right)\right]\right\} \\
&= R_{\mathrm{sr}}(\Delta\boldsymbol{r},k)R_{\mathrm{sk}}(\Delta k) \ ,
\end{aligned} \tag{S15}$$

where

$$R_{\mathrm{sr}}(\Delta\boldsymbol{r},k)=\exp\left\{-\left[2\pi(n-1)\sigma_\eta k\right]^2\left[1-\frac{R_\eta(\Delta\boldsymbol{r})}{\sigma_\eta^2}\right]\right\} \ , \tag{S16}$$

and

$$R_{\mathrm{sk}}(\Delta k)=\exp\left\{-\frac{\left[2\pi(n-1)\sigma_\eta\Delta k\right]^2}{2}\right\} \ , \tag{S17}$$

For Gaussian height, the height correlation function is

$$R_\eta(\Delta\boldsymbol{r})=\sigma_\eta^2\exp\left(-\frac{\Delta\boldsymbol{r}^2}{r_{\mathrm{c}}^2}\right) \ , \tag{S18}$$

where the transverse correlation length r_{c} is the radius at which the normalized height correlation falls to $1/\mathrm{e}$. So, we obtain the result

$$R_{\mathrm{sr}}(\Delta\boldsymbol{r},k)=\exp\left\{-\left[2\pi(n-1)\sigma_\eta k\right]^2\left[1-\exp\left(-\frac{\Delta\boldsymbol{r}^2}{r_{\mathrm{c}}^2}\right)\right]\right\} \ . \tag{S19}$$

Section 3: The spatial intensity autocorrelation of the incoherent intensity impulse response function

Now to get the spatial field autocorrelation function, substitute Eq. (S15) in Eq. (S10):

$$\begin{aligned}
R_{h_{\mathrm{E}}}(\boldsymbol{u},\boldsymbol{u}') = & \iiiint_{-\infty}^{\infty}\mathrm{d}k\mathrm{d}\Delta k\mathrm{d}\tilde{\boldsymbol{r}}_3\mathrm{d}\tilde{\boldsymbol{r}}_4 W(k)W(k-\Delta k)A(\tilde{\boldsymbol{r}}_3+c_1\breve{\boldsymbol{u}}) \\
& \cdot\exp\left(-\mathrm{j}4\pi c_0 k\tilde{\boldsymbol{r}}_3\tilde{\boldsymbol{r}}_4\right)R_{\mathrm{sr}}(\tilde{\boldsymbol{r}}_4+c_1\Delta\boldsymbol{u},k)R_{\mathrm{sk}}(\Delta k) \\
= & \iiint_{-\infty}^{\infty}\mathrm{d}k\mathrm{d}\Delta k\mathrm{d}\tilde{\boldsymbol{r}}_4 W(k)W(k-\Delta k)\mathfrak{F}_{\mathrm{a}}\left(2c_0k\tilde{\boldsymbol{r}}_4\right)\exp\left(\mathrm{j}4\pi c_0c_1k\breve{\boldsymbol{u}}\tilde{\boldsymbol{r}}_4\right)R_{\mathrm{sr}}(\tilde{\boldsymbol{r}}_4+c_1\Delta\boldsymbol{u},k)R_{\mathrm{sk}}(\Delta k) \\
= & \int_{-\infty}^{\infty}\mathrm{d}kW(k)\int_{-\infty}^{\infty}\mathrm{d}\Delta kW(k-\Delta k)R_{\mathrm{sk}}(\Delta k)\int_{-\infty}^{\infty}\mathrm{d}\tilde{\boldsymbol{r}}_4'\mathfrak{F}_{\mathrm{a}}\left(\tilde{\boldsymbol{r}}_4'\right)\tilde{R}_{\mathrm{sr}}\left(\tilde{\boldsymbol{r}}_4'+\frac{k\Delta\boldsymbol{u}}{z_2},k\right) \ ,
\end{aligned} \tag{S20}$$

where $\mathfrak{F}_{\mathrm{a}}$ is the Fourier transform of the aperture function A, $\Delta k = k - k'$, $\Delta \boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}'$, and

$$\tilde{R}_{\mathrm{sr}}(\Delta \boldsymbol{r}, k) = \frac{1}{2c_0 k} R_{\mathrm{sr}}\left(\frac{\Delta \boldsymbol{r}}{2c_0 k}, k\right) = \frac{1}{2c_0 k}\exp\left\{-(2\pi(n-1)\sigma_\eta k)^2\left[1-\exp\left(-\left(\frac{\Delta \boldsymbol{r}}{2c_0 k r_{\mathrm{c}}}\right)^2\right)\right]\right\} , \tag{S21}$$

and

$$\tilde{\tilde{\mathfrak{F}}}_{\mathrm{a}}(\tilde{\boldsymbol{r}}) = \mathfrak{F}_{\mathrm{a}}(\tilde{\boldsymbol{r}})\exp\left(\mathrm{j}2\pi c_1 \breve{\boldsymbol{u}}\tilde{\boldsymbol{r}}\right) . \tag{S22}$$

Thus, the normalized spatial field autocorrelation of h_E is expressed as

$$\Gamma_{h_{\mathrm{E}}}(\boldsymbol{u}, \boldsymbol{u}') = \tau \int_{-\infty}^{\infty} \mathrm{d}k W(k)\left\{W \otimes R_{\mathrm{sk}}\right\}_k \left\{\tilde{\tilde{\mathfrak{F}}}_{\mathrm{a}} \otimes \tilde{R}_{\mathrm{sr}}\right\}_{-\frac{k\Delta \boldsymbol{u}}{z_2}} , \tag{S23}$$

where τ is the normalized factor.

According to the complex Gaussian moment theorem, the expression of the normalized intensity auto-correlation reduces to

$$\Gamma_{h_{\mathrm{I}}}(\Delta \boldsymbol{u}) = 1 + \left|\Gamma_{h_{\mathrm{E}}}(\Delta \boldsymbol{u})\right|^2 . \tag{S24}$$

Since the spatial fluctuations of h_I are wide-sense stationary, its spatial intensity autocorrelation $G_{h_{\mathrm{I}}}^{(2)}(\Delta \boldsymbol{u})$ equals to the normalized intensity autocorrelation $\Gamma_{h_{\mathrm{I}}}(\Delta \boldsymbol{u})$. Substituting Eq. (S23) and Eq. (S24), we find the following result,

$$G_{h_{\mathrm{I}}}^{(2)}(\Delta \boldsymbol{u}) = 1 + \left|\tau \int_{-\infty}^{\infty} \mathrm{d}k W(k)\left\{W \otimes R_{\mathrm{sk}}\right\}_k \left\{\tilde{\tilde{\mathfrak{F}}}_{a} \otimes \tilde{R}_{\mathrm{sr}}\right\}_{-\frac{k\Delta \boldsymbol{u}}{z_2}}\right|^2 . \tag{S25}$$

Section 4: The design conditions of sub-aperture spatial random phase modulators

Now, consider Taylor's expansion of R_η in Eq. (S18) up to order one, we have

$$R_\eta(\Delta \boldsymbol{r}) \approx \sigma_\eta^2\left(1 - \frac{\Delta \boldsymbol{r}^2}{r_{\mathrm{c}}^2}\right) , \tag{S26}$$

and

$$\tilde{R}_{\mathrm{sr}}(\Delta \boldsymbol{r}, k) \approx \exp\left\{-\left[\frac{4\pi(n-1)z_1 z_2 \sigma_\eta \Delta \boldsymbol{r}}{(z_1+z_2) r_{\mathrm{c}}}\right]^2\right\} = \tilde{R}_{\mathrm{sr}}(\Delta \boldsymbol{r}) . \tag{S27}$$

If the radius of $\tilde{\tilde{\mathfrak{F}}}_{\mathrm{a}}$ is designed much larger than the radius of $\tilde{R}_{\mathrm{sr}}$, namely

$$\begin{gathered}\frac{1}{D/2} \gg \frac{(z_1+z_2) r_c}{4(n-1) z_1 z_2 \sigma_\eta} \approx \frac{r_{\mathrm{c}}}{4(n-1) z_2 \sigma_\eta} \\ \Updownarrow \\ \text{Condition 1:} \quad D \ll \frac{8(n-1) z_2 \sigma_\eta}{r_{\mathrm{c}}}\end{gathered} , \tag{S28}$$

where the fact $z_1 \gg z_2$ has been used, then the result of Eq. (S25) is

$$G_{h_{\mathrm{I}}}^{(2)}(\Delta \boldsymbol{u}) \approx 1 + \left|\tau \int_{-\infty}^{\infty} \mathrm{d}k W(k)\left\{W \otimes R_{\mathrm{sk}}\right\}_k \mathfrak{F}_{\mathrm{a}}\left(-\frac{k\Delta \boldsymbol{u}}{z_2}\right)\right|^2 . \tag{S29}$$

When the radius Δ_{w} of the spectral distribution W is much smaller than the radius of the R_{sk}, namely

$$\text{Condition 2:} \quad \Delta_{\mathrm{w}} \ll \frac{1}{\sqrt{2}\pi(n-1)\sigma_\eta} , \tag{S30}$$

thus,

$$G_{h_{\mathrm{I}}}^{(2)}(\Delta \boldsymbol{u}) \approx 1 + \left|\tau \int_{-\infty}^{\infty} \mathrm{d}k W(k) R_{\mathrm{sk}}(k) \mathfrak{F}_{\mathrm{a}}\left(-\frac{k\Delta \boldsymbol{u}}{z_2}\right)\right|^2 . \tag{S31}$$

To further analysis Eq. (S31), W is given by

$$W(k) = \Gamma_{\mathrm{w}}(k - k_0) , \tag{S32}$$

where k_0 is the center wave number. Equation (S31) becomes

$$G_{h_{\rm I}}^{(2)}(\Delta\boldsymbol{u}) \approx 1 + \left|\tau \int_{-\infty}^{\infty} {\rm d}k \Gamma_{\rm w}(k-k_0) R_{\rm sk}(k) \mathfrak{F}_{\rm a}\left(-\frac{k\Delta\boldsymbol{u}}{z_2}\right)\right|^2 . \tag{S33}$$

In addition, if $\Delta_{\rm w}$ is assumed much smaller then the radius of $\tilde{\mathfrak{F}}_{\rm a}\left(-\frac{k\Delta\boldsymbol{u}}{z_2}\right)$, namely

$$\text{Condition 3:} \quad \Delta_{\rm w} \ll \frac{2z_2}{DD_{\rm t}} , \tag{S34}$$

where $D_{\rm t}$ is the detected width of CCD. Then the function $G_{h_{\rm I}}^{(2)}(\Delta\boldsymbol{u})$ can be expressed as

$$G_{h_{\rm I}}^{(2)}(\Delta\boldsymbol{u}) \approx 1 + \left|\tau \mathfrak{F}_{\rm a}\left(-\frac{k_0\Delta\boldsymbol{u}}{z_2}\right)\right|^2 , \tag{S35}$$

where the condition 1–3 in Eq. (4, 5) is considered.