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METRIC DIMENSION OF LEXICOGRAPHIC PRODUCT OF SOME KNOWN GRAPHS

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Abstract. For an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of vertices and a vertex v in a connected graph G, the ordered k-vector $r(v|W) := (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$ is called the (metric) representation of v with respect to W, where $d(x, y)$ is the distance between the vertices x and y. The set W is called a resolving set for G if distinct vertices of G have distinct representations with respect to W. The minimum cardinality of a resolving set for G is its metric dimension. In this paper, we investigate the metric dimension of the lexicographic product of graphs G and H, $G[H]$, for some known graphs.

Keywords: Lexicographic product; Resolving set; Metric dimension; Basis; Adjacency dimension.
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1. Introduction

In this section, we present some definitions and known results which are necessary to prove our main results. Throughout this paper, G is a simple graph with vertex set $V(G)$, edge set $E(G)$ and order $n(G)$. We use $\overline{G}$ for the complement of graph G. The distance between two vertices u and v in a connected graph G, denoted by $d_G(u, v)$, is the length of a shortest path between u and v in G. Also, $N_G(v)$ is the set of all neighbors of vertex v in G. We write these simply $d(u, v)$ and $N(v)$, when no confusion can arise. The diameter of a connected graph G is $\text{diam}(G) = \max_{u,v\in V(G)} d(u, v)$. The symbols $(v_1, v_2, \ldots, v_n)$ and $(v_1, v_2, \ldots, v_n, v_1)$ represent a path of order n, P_n, and a cycle of order n, C_n, respectively. We also use notation $\mathbf{1}$ for the vector $(1, 1, \ldots, 1)$ and $\mathbf{2}$ for $(2, 2, \ldots, 2)$.

For an ordered subset $W = \{w_1, \ldots, w_k\}$ of $V(G)$ and a vertex v of a connected graph G, the *metric representation* of v with respect to W is

$$r(v|W) = (d(v, w_1), \ldots, d(v, w_k)).$$

The set W is a *resolving set* for G if the distinct vertices of G have different metric representations, with respect to W. A resolving set W for G with

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minimum cardinality is a *metric basis* of G, and its cardinality is the *metric dimension* of G, denoted by $\dim(G)$.

The concepts of resolving sets and metric dimension of a graph were introduced independently by Slater [13] and by Harary and Melter [7]. Resolving sets have several applications in diverse areas such as coin weighing problems [12], network discovery and verification [2], robot navigation [10], mastermind game [5], problems of pattern recognition and image processing [11], and combinatorial search and optimization [12]. For more results about resolving sets and metric dimension see [1, 3–6, 8].

The *lexicographic product* of graphs G and H, denoted by $G[H]$, is a graph with vertex set $V(G) \times V(H) := \{(v, u) \mid v \in V(G), u \in V(H)\}$, where two vertices (v, u) and (v', u') are adjacent whenever, v is adjacent to v', or $v = v'$ and u is adjacent to u'. When the order of G is at least 2, it is easy to see that $G[H]$ is a connected graph if and only if G is a connected graph.

Jannesari and Omoomi [9] studied the metric dimension of the lexicographic product of graphs using *adjacency dimension* of graphs. They defined the concepts of adjacency resolving sets and adjacency dimension in graphs a the following.

Let G be a graph, and $W = \{w_1, \ldots, w_k\} \subseteq V(G)$. For each vertex $v \in V(G)$, the *adjacency representation* of v with respect to W is the k-vector

$$r_2(v|W) = (a_G(v, w_1), \ldots, a_G(v, w_k)),$$

where $a_G(v, w_i) = min\{2, d_G(v, w_i)\}; 1 \le i \le k$. The set W is an *adjacency resolving set* for G if the vectors $r_2(v|W)$ for $v \in V(G)$ are distinct. The minimum cardinality of an adjacency resolving set is the adjacency dimension of G, denoted by $\dim_2(G)$. An adjacency resolving set of cardinality $\dim_2(G)$ is an *adjacency basis* of G.

We say that a set W *(adjacency) resolves* a set T of vertices in G if the (adjacency) metric representations of vertices in T with respect to W are distinct. To determine whether a given set W is a (an adjacency) resolving set for G, it is sufficient to look at the (adjacency) metric representations of vertices in $V(G)\backslash W$, because $w \in W$ is the unique vertex of G for which $d(w, w) = 0$.

The main goal of this paper is to investigate the metric dimension of the lexicographic product, for some known graphs, such as paths, cycles, complete bipartite graphs and kneser graphs. The metric dimension of the lexicographic product is studied in [9] for the first time. In that work the authors obtained the metric dimension of $G[H]$ in terms of the adjacency dimension of H and some parameters in G. In this paper we get these parameters for the mentioned graphs and then we get the exact value of the metric dimension of $G[H]$ where G and H are paths, cycles, complete bipartite graphs and kneser graphs. To express the results of [9] we need some definitions.

Two distinct vertices u, v are said *twins* if $N(v)\backslash\{u\} = N(u)\backslash\{v\}$. It is called that $u \equiv v$ if and only if $u = v$ or u, v are twins. In [8], it is proved that "$\equiv$" is an equivalent relation. The equivalence class of a vertex v is denoted by v^*. Hernando et al. [8] proved that v^* is a clique or an independent set in G. As in [8], we say v^* is of type (1), (K), or (N) if v^* is a class of size 1, a clique of size at least 2, or an independent set of size at least 2. We denote the number of equivalence classes of G with respect to "$\equiv$" by $\iota(G)$. We mean by $\iota_{_K}(G)$ and $\iota_{_N}(G)$, the number of classes of type (K) and type (N) in G, respectively. We also use $a(G)$ and $b(G)$ for the number of all vertices in G which have at least an adjacent twin and a none-adjacent twin vertex in G, respectively. On the other way, $a(G)$ is the number of all vertices in the classes of type (K) and $b(G)$ is the number of all vertices in the classes of type (N). Clearly, $\iota(G) = n(G) - a(G) - b(G) + \iota_{_N}(G) + \iota_{_K}(G)$.

Observation 1.1. [8] Suppose that u, v are twins in a graph G and W resolves G. Then u or v is in W. Moreover, if $u \in W$ and $v \notin W$, then $(W \setminus \{u\}) \cup \{v\}$ also resolves G.

Lemma 1.1. [6] Let G be a connected graph of order n. Then,

: (i) $\dim(G) = 1$ if and only if $G = P_n$,

: (ii) $\dim(G) = n - 1$ if and only if $G = K_n$.

Proposition 1.2. [9] For every connected graph G, $\dim(G) \leq \dim_2(G)$.

Proposition 1.3. [9] For every graph G, $\dim_2(G) = \dim_2(\overline{G})$.

Let G be a graph of order n. It is easy to see that, $1 \leq \dim_2(G) \leq n-1$. In the following proposition, all graphs G with $\dim_2(G) = 1$ and all graphs G of order n and $\dim_2(G) = n - 1$ are characterized.

Proposition 1.4. [9] If G is a graph of order n, then

: (i) $\dim_2(G) = 1$ if and only if $G \in \{P_1, P_2, P_3, \overline{P}_2, \overline{P}_3\}$.

: (ii) $\dim_2(G) = n - 1$ if and only if $G = K_n$ or $G = \overline{K}_n$.

Proposition 1.5. [9] If $n \geq 4$, then $\dim_2(C_n) = \dim_2(P_n) = \lfloor \frac{2n+2}{5} \rfloor$.

Proposition 1.6. [9] If $K_{m_1,m_2,\ldots,m_t}$ is the complete t-partite graph, then

$$\dim_2(K_{m_1,m_2,\ldots,m_t}) = \dim(K_{m_1,m_2,\ldots,m_t}) = \begin{cases} m - r - 1 & \text{if } r \neq t, \\ m - r & \text{if } r = t, \end{cases}$$

where $m_1, m_2, \ldots, m_r$ are at least 2, $m_{r+1} = \cdots = m_t = 1$, and $\sum_{i=1}^{t} m_i = m$.

The metric dimension of the lexicographic product of graphs are obtained in [9], through the following four theorems.

Theorem 1.7. [9] Let G be a connected graph of order n and H be an arbitrary graph. If there exist two adjacency bases W_1 and W_2 of H such that, there is no vertex with adjacency representation $\mathbf{1}$ with respect to W_1 and no vertex with adjacency representation $\mathbf{2}$ with respect to W_2, then $\dim(G[H]) = \dim(G[\overline{H}]) = n \dim_2(H)$.

Theorem 1.8. [9] Let G be a connected graph of order n and H be an arbitrary graph. If for each adjacency basis W of H there exist vertices with adjacency representations **1** and **2** with respect to W, then $\dim(G[H]) = \dim(G[\overline{H}]) = n(\dim_2(H) + 1) - \iota(G)$.

Theorem 1.9. [9] Let G be a connected graph of order n and H be an arbitrary graph. If H has the following properties

: (i) for each adjacency basis of H there exists a vertex with adjacency representation **1**,

: (ii) there exists an adjacency basis W of H such that there is no vertex with adjacency representation **2** with respect to W,

then $\dim(G[H]) = n\dim_2(H) + a(G) - \iota_K(G)$.

Theorem 1.10. [9] Let G be a connected graph of order n and H be an arbitrary graph. If H has the following properties

: (i) for each adjacency basis of H there exists a vertex with adjacency representation **2**,

: (ii) there exists an adjacency basis W of H such that there is no vertex with adjacency representation **1** with respect to W,

then $\dim(G[H]) = n\dim_2(H) + b(G) - \iota_N(G)$.

Corollary 1.11. [9] If G has no pair of twin vertices, then $\dim(G[H]) = n\dim_2(H)$.

2. Metric dimension of the lexicographic product of some important graphs

In this section we investigate metric dimension of the lexicographic product of graphs for some families of graphs. Theorems 1.7, 1.8, 1.9 and 1.10 imply that to find the exact value of $\dim(G[H])$, we need to compute parameters a, b, ι_N and ι_K for G and adjacency dimension of H. Let start with bipartite graphs.

Observation 2.1. It is easy to see that if G is a bipartite graph of order at least 3, then it does not have any pair of adjacent twins. Therefore, by Theorem 1.10, $\dim(G[H]) = n\dim_2(H) + b(G) - \iota_N(G)$.

By Corollary 1.11 to compute the $\dim(G[H])$, where G has no any pair of twin vertices, it is enough to obtain the value of $\dim_2(H)$. Now we consider the family of Kneser graphs.

Definition 2.1. The Kneser graph $KG(k, r)$ is the graph whose vertices correspond to the r-element subsets of a set of k elements, and where two vertices are adjacent if and only if the two corresponding sets are disjoint.

Lemma 2.2. *If $G = KG(k, r)$, $k \geq 2r + 1$ is the Kneser graph, then G has not any pair of twin vertices.*

Proof. If A and B are distinct twin vertices in G, then $A \cap C = \emptyset$ if and only if $B \cap C = \emptyset$, for each $C \in V(G)$. Since $k \geq 2r+1$, there exists a vertex $C \in V(G) \backslash \{A, B\}$. Let $C \in V(G) \backslash \{A, B\}$, $A \cap C = \emptyset$, $x \in A \backslash B$, and $y \in C$. Let $D = C \cup \{x\} \backslash \{y\}$. Therefore, $A \cap D \neq \emptyset$ and $B \cap D = \emptyset$, which is a contradiction. □

The Kneser graph $KG(5, 2)$ is called the Petersen graph. In the next example we compute the adjacency dimension of the Petersen graph.

Example 2.3. *If P is the Petersen graph, then* $\dim_2(P) = \dim(P) = 3$.

Proof. Since the diameter of P is 2, we have $\dim_2(P) = \dim(P)$. As we see in Figure **??**, set $\{a, b, c\}$ is an adjacency resolving set for P, therefore $\dim_2(P) \leq 3$. On the other hand, each set of at most two vertices can provide at most 8 different metric representation, hence $\dim_2(P) = \dim(P) = 3$. □

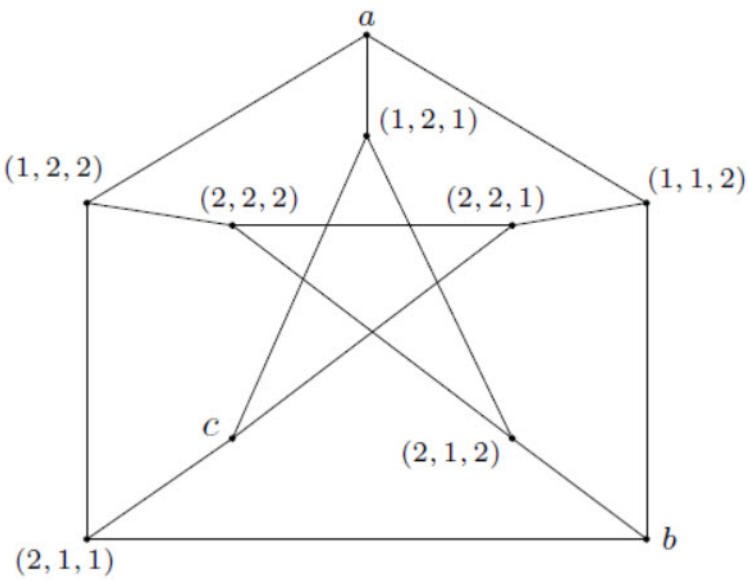

FIGURE 1. Caption.

Note that the line graph of the complete graph K_n is the complement of $KG(n, 2)$. Since all twin vertices of a graph are twins in its complement, as well; by Lemma 2.2, $L(K_n)$, $n \geq 5$, have no any pair of twin vertices. Also, the path P_n, $\overline{P}_n$, $n \geq 4$, and the cycle C_n, $\overline{C}_n$, $n \geq 5$, have no any pair of twin vertices. Thus, by Theorems 1.8 the exact value of $\dim G[H]$, when $G \in \{\overline{P}_n \ (n \geq 4), \overline{C}_n (n \geq 5), L(K_n) \ (n \geq 5), KG(k, r)\}$ and $H \in \{P_m, C_m, \overline{P}_m, \overline{C}_m, K_m, \overline{K}_m, P, K_{m_1,\ldots,m_t}\}$ are obtained.

To study the adjacency basis of a graph H, we need the following definitions. Let S be a subset of vertices of H, where $|s| \geq 2$. The set of vertices of a nonempty connected component of the induced subgraph by $V(H)\backslash S$ of H is called a *gap* of S. This definition agrees with the one in [4] which is given for the cycle C_m. If Q_1, Q_2 are two gaps of S for which there exists a vertex $x \in S$ such that the induced subgraph by $Q_1 \cup Q_2 \cup \{x\}$ is connected, then Q_1 and Q_2 are called *neighboring gaps*. In [4], the following observation is expressed for the gaps of a basis of the join graph of a cycle and an isolated vertex. Particularly, it is true for an adjacency basis of C_m.

Observation 2.2. If B is an adjacency basis of C_m, then

: (1) Every gap of B contains at most three vertices, otherwise there are two vertices in that gap that have the same adjacency representation **2**.

: (2) At most one gap of B contains three vertices, otherwise the middle vertex of both of them has adjacency representation **2**.

: (3) If a gap of B contains at least two vertices, then any neighboring gaps of which contain one vertex, otherwise the second vertex of both of them has adjacency representation **2**.

It is easy to see the following observation for P_m.

Observation 2.3. Let B be an adjacency basis of $P_m = (w_1, w_2, \ldots, w_m)$. If R_1 and R_2 are gaps of P_n with $w_1 \in R_1$ and $w_m \in R_2$, then

: (1) Every gap of B contains at most three vertices and $|R_i| \leq 2$, where $1 \leq i \leq 2$, otherwise there are two vertices in that gap that have the same adjacency representation **2**.

: (2) At most one gap of B contains three vertices and at most one of the gaps R_1 and R_2 contains two vertices, otherwise the middle vertex of both of them has adjacency representation **2**.

: (3) If $|R_i| = 2$ for some i, $1 \leq i \leq 2$, then all gaps of B contains at most two vertices, otherwise the second vertex of both of them has adjacency representation **2**.

: (4) If a gap of B contains at least two vertices, then any neighboring gap of which is neither R_1 nor R_2 and contains one vertex, otherwise the second vertex of both of them has adjacency representation **2**.

Theorem 2.4. *Let G be a connected graph of order n and $H \in \{P_m, C_m\}$, $m = 5k + r$, where $m \geq 4$ and $0 \leq r < k$.*

: *(i) If r is even, then* $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor$.

: *(ii) If $m = 6$, then* $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor + a(G) + b(G) - \iota_{_K}(G) - \iota_{_N}(G)$.

: *(iii) If r is odd and $m \neq 6$, then* $\dim(G[H]) = n\lfloor \frac{2m+2}{5} \rfloor + b(G) - \iota_{_N}(G)$ *and* $\dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor + a(G) - \iota_{_K}(G)$.

Proof. Let $P_m = (w_1, w_2, \ldots, w_m)$ and $C_m = (w_1, w_2, \ldots, w_m, w_1)$. If $m = 4$, then the set $B_4 = \{w_1, w_4\} \subseteq V(H)$ is an adjacency basis of H and $r_2(w_i|B_4)$ is neither $\mathbf{1}$ nor $\mathbf{2}$, for each i, $1 \leq i \leq 4$. Therefore, by Proposition 1.5 and Theorem 1.7, $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor$. If $m = 5$, then the sets $B_1 = \{w_1, w_5\}$ and $B_2 = \{w_2, w_4\}$ are adjacency bases of H and for each i, $1 \leq i \leq 5$, $r_2(w_i|B_1)$ is not $\mathbf{1}$ and $r_2(w_i|B_2)$ is not $\mathbf{2}$. Hence, by Lemma 1.5 and Theorem 1.7, $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor$. If $m = 6$, then it is easy to check that for each adjacency basis A of H there exist vertices $x_A, y_A \in V(H)$ such that x_A is adjacent to w and y_A is not adjacent to w for each $w \in A$. Consequently $r(x_A|A) = \mathbf{1}$ and $r(y_A|A) = \mathbf{2}$, so by Theorem 1.8, $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor + a(G) + b(G) - \iota_K(G) - \iota_N(G)$.

Now, let $m \geq 7$. By Proposition 1.5, $\dim_2(H) \geq 3$. Let B be an adjacency basis of H. Since each vertex of H has at most two neighbors, $r_2(w|B)$ is not $\mathbf{1}$, for each $w \in V(H)$. If r is even, then, let $S_0 = \{w_{5q+2}, w_{5q+4}|\ 0 \leq q \leq k-1\}$, $S_2 = S \cup \{w_{5k+1}\}$, and $S_4 = S \cup \{w_{5k+1}, w_{5k+3}\}$. Thus, the set S_t, is an adjacency basis of H when $r = t$, $t \in \{0, 2, 4\}$. Also, $r_2(w|S_t)$ is neither $\mathbf{1}$ nor $\mathbf{2}$, for each $w \in V(H)$ and $t \in \{0, 2, 4\}$. Hence, by by Lemma 1.5 and Theorem 1.7, $\dim(G[H]) = \dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor$.

If r is odd, then Observations 2.2 and 2.3 imply that for each adjacency basis A of H there exists a vertex $y_A \in V(H)$ such that $y_A \nsim w$ for each $w \in A$. Therefore, by Theorem 1.10, $\dim(G[H]) = n\lfloor \frac{2m+2}{5} \rfloor + b(G) - \iota_N(G)$. Since the adjacency bases of H and $\overline{H}$ are the same, for each adjacency basis Q of $\overline{H}$ there exists a vertex $x_Q \in V(\overline{H})$ such that $x_Q \sim u$ for each $u \in Q$. Hence, by Theorem 1.9, $\dim(G[\overline{H}]) = n\lfloor \frac{2m+2}{5} \rfloor + a(G) - \iota_K(G)$. □

By computing parameters a, b, ι_N and ι_K for the graphs K_n, P_n, C_n and $K_{n_1,n_2,\ldots,n_t}$ we have the following example.

Example 2.5. *Let $m = 5k + r$. If $H \in \{P_m, C_m\}$, then for all $n \geq 2$,*

$$: (1)\ \dim(K_n[H]) = \begin{cases} 2n-1 & \text{if } H = P_2 \text{ or } H = P_3, \\ 3n-1 & \text{if } H \in \{C_3, P_6, C_6\}, \\ n\lfloor \frac{2m+2}{5} \rfloor & \text{otherwise.} \end{cases}$$

$$: (2)\ \dim(P_n[H]) = \begin{cases} 5 & \text{if } n = 2 \text{ and } H = C_3, \\ 2n & \text{if } n \neq 2 \text{ and } H = C_3, \\ n\lfloor \frac{2m+2}{5} \rfloor + 1 & \text{if } n = 2 \text{ and } H \in \{P_2, P_3, P_6, C_6\}, \\ n\lfloor \frac{2m+2}{5} \rfloor + 1 & \text{if } n = 3,\ r \text{ is odd, and } H \neq C_3, \\ n\lfloor \frac{2m+2}{5} \rfloor & \text{otherwise.} \end{cases}$$

$$: (3)\ \dim(C_n[H]) = \begin{cases} 8 & \text{if } n = 3 \text{ and } H = C_3, \\ 2n & \text{if } n \neq 3 \text{ and } H = C_3, \\ n\lfloor \frac{2m+2}{5} \rfloor + 2 & \text{if } n = 3 \text{ and } H \in \{P_2, P_3, P_6, C_6\}, \\ n\lfloor \frac{2m+2}{5} \rfloor + 2 & \text{if } n = 4,\ r \text{ is odd, and } H \neq C_3, \\ n\lfloor \frac{2m+2}{5} \rfloor & \text{otherwise.} \end{cases}$$

: *(4)* $\dim(K_{n_1,n_2,\ldots,n_t}[H]) =$

$$= \begin{cases} n\lfloor \frac{2m+2}{5} \rfloor + t - j - 1 & \text{if } H = P_2 \text{ and } j \neq t, \\ n(m-1) + t - j - 1 & \text{if } H = C_3 \text{ and } j \neq t, \\ n(m-1) & \text{if } H = C_3 \text{ and } j = t, \\ n\lfloor \frac{2m+2}{5} \rfloor + n - j - 1 & \text{if } H \in \{P_3, P_6, C_6\} \text{ and } j \neq t, \\ n\lfloor \frac{2m+2}{5} \rfloor + n - t & \text{if } H \in \{P_3, P_6, C_6\} \text{ and } j = t, \\ n\lfloor \frac{2m+2}{5} \rfloor + n - t & \text{if } m \geq 7 \text{ and } r \text{ is odd}, \\ n\lfloor \frac{2m+2}{5} \rfloor & \text{otherwise.} \end{cases}$$

where $n_1, n_2, \ldots, n_j$ *are at least* 2, $n_{j+1} = \ldots = n_t = 1$, *and* $\sum_{i=1}^{t} n_i = n$.

Proof. Since K_n does not have any pair of non-adjacent twin vertices, by Proposition 2.4, $\dim(K_n[H]) = n\lfloor \frac{2m+2}{5} \rfloor$ for $m \notin \{2, 3, 6\}$. If $H = P_2$ or $H = C_3$, then $K_n[H]$ is the complete graph and hence, $\dim(K_n[P_2]) = 2n - 1$ and $\dim(K_n[C_3]) = 3n - 1$.

Now let $H \in \{P_3, P_6, C_6\}$. Also, let $P_m = (w_1, w_2, \ldots, w_m)$, $C_m = (w_1, \ldots, w_m, w_1)$, and B is a basis of $K_n[H]$. It is easy to see that B contains at least $\dim_2(H)$ vertices from each set $R_i = \{v_{rs} \in V(K_n[H]) | r = i\}$, and $B \cap R_i$ resolves R_i, $1 \leq i \leq n$. Let $J = \{i | \dim_2(H) = |B \cap R_i|\}$. If $|J| \geq 2$, then there exist i, j, $1 \leq i, j \leq n$, such that $|B \cap R_i| = |B \cap R_j| = \dim_2(H)$. Let $A_i = \{w_s | v_{is} \in B \cap R_i\}$ and $A_j = \{w_s | v_{js} \in B \cap R_j\}$. Since $d_{K_n[H]}(v_{rs}, v_{rq}) = a_H(w_s, w_q)$ for each r, s, q, $1 \leq r \leq n$, $1 \leq s, q \leq m$, we conclude that A_i and A_j are adjacency bases of H. On the other hand, for each adjacency basis A of H there exist a vertex $w \in V(H)$ such that $r_2(w|A) = (1, 1, \ldots, 1)$. Therefore, there exist vertices $w_1, w_2 \in V(H)$ such that $r_2(w_1|A_i) = r_2(w_2|A_j) = (1, 1, \ldots, 1)$. Consequently, $r(v_{i1}|B \cap R_i) = r(v_{j2}|B \cap R_j) = (1, 1, \ldots, 1)$. Also, we have $r(v_{i1}|B \backslash R_i) = r(v_{j2}|B \backslash R_j) = (1, 1, \ldots, 1)$. Hence, $r(v_{i1}|B) = r(v_{j2}|B)$, which is a contradiction. Thus, $|J| \leq 1$. Therefore, $\dim(K_n[H]) \geq n \dim_2(H) + n - 1$. On the other hand, the set $\{v_{rs} \in V(K_n[P_3]) | s \neq 3\} \backslash \{v_{12}\}$ is a resolving set for $K_n[P_3]$ with cardinality $n \dim_2(H) + n - 1 = 2n - 1$. Also, the set $\{v_{rs} \in V(K_n[H]) | 2 \leq s \leq 4\} \backslash \{v_{13}\}$ is a resolving set for $K_n[H]$, for $H \in \{P_6, C_6\}$. Consequently, $\dim(K_n[P_6]) = \dim(K_n[C_6]) = 3n - 1$. □

3. Conclusion

By the results of [9] to get the exact value of $G[H]$ it is needed to compute the adjacency dimension of H and parameters a, b, ι_N and ι_K for G. In this paper we do this for some known families of graphs such as paths, cycles, complete multipartite graphs and kneser graphs. For a future work one can do this for other families of graphs. Specially studying the adjacency dimension of graphs is an interesting and important work in this contex.

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