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A SOLUTION PROCEDURE TO SOLVE MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM IN NEUTROSOPHIC FUZZY ENVIRONMENT

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Abstract. In this paper, an attempt has been taken to develop a method to solve the neutrosophic multiobjective linear fractional programming (NMOLFPP) problem. In the first step of our method, the problem is linearized based on some transformations. Then, the linearized model is reduced to a crisp multi-objective programming problem with the help of the accuracy function for each objective. In the following, we extend Zimmerman's approach to maximize the truth membership and minimize the indeterminacy and falsity membership functions in the solution procedure. Finally, to illustrate the proposed approach, a numerical example is included.

Keywords: Multiobjective programming problem, Neutrosophic set, Single valued trapezoidal neutrosophic number, Indeterminacy membership functions.
2020 MSC: Primary 90C29, 90C70

1. Introduction

In the real world, many problems involve two or more objective functions that address certain constraints. These problems are called multi-objective programming problems. In this type of problem, the objective functions are often in conflict with each other, that is, the improvement of one objective function may lead to the deterioration of at least one of the other functions. Thus, there is no single optimal solution in multi-objective problems to optimize all of the objective functions contemporaneously, rather there is a set of inferior (Pareto optimal) solutions which the Decision Maker (DM) must choose a preferred solution or best compromise solution as a satisfactory solution. The multi-objective linear fractional (MOLF) optimization problem is a generalization of the multi-objective linear (MOL) optimization problems in which there are multiple ratios of physical quantities and/or economic quantities that must be optimized simultaneously.

Recently, authors such as Nuran Guzel [17], Chakraborty and Gupta [7], D.

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Dutta et. al. [9], Hosseinzadeh et. al. [10] and Luhandjula [14] have proposed several methods to solve MOLFP. Because real-world problems are so complex, one of the major challenges faced by DMs and practitioners is that DM may not be able to accurately determine parameter values in MOLF. Therefore, Zadeh [27] initially presented the theory of fuzzy sets (FSs) to solve these problems, in which it is possible to describe observations imprecisely and uncertainly. FS theory has also been developed in many fields and different generalizations have been presented that are able to deal with different types of uncertainties. Among the generalizations of fuzzy sets are type-2 fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets and etc. Then, several authors used these concepts of FSs theory for solving MOLF optimization problems in the environment of ambiguous and hesitance. Such as, Yang and Li presented an algorithm to solve fuzzy MOLF problems through an approach based on superiority and inferiority measures method [25]. The MOLF problems with fuzzy number were analyzed in [18–20], and under the concept of α-Pareto optimality, nonsmooth multi-objective nonlinear fractional programming problems with fuzzy numbers were analyzed by Ammar [2]. The solving of MOLF optimization problem with fuzzy variables and parameters is most difficult. To the best of our knowledge, Rubi Arya et. al. [3], for the first, presented an algorithm for solving fully fuzzy multi-objective linear fractional (FFMOLF) optimization problem where all the coefficients and variables are assumed to be the triangular fuzzy numbers (TFNs).

In the real world, suppose that when choosing a candidate in a voting system, in addition to confirming the choice or not choosing, there are also options to opt out or not to decide. Such situations are not governed by intuitionistic characters. Therefore, in these cases, neutrosophic set (NS) and neutrosophic logical concept originated and were successfully applied by Smarandache [22]. It is a generalization of intuitionistic fuzzy set [4]. The complexity of real life often creates uncertain situations or neutral thoughts when making optimal decisions. Apart from the degrees of acceptance and rejection in the decision-making process, the degree of indeterminacy is also very important. Therefore, to mask neutral thoughts or the degree of indeterminacy of the element in the set of feasible solutions, Smarandach [22] investigated a neutrosophic set. NS considers three types of membership functions such as truth, indeterminacy and falsity degrees in the set of feasible solutions. Due to the inclusion of the idea of independent and neutral thoughts in Neutrosophic sets, this set is different from other uncertain decision sets such as FS and IFS. The NS is growing rapidly and it is used in different directions. Some of the contributions of neutrosophic set mentioned in [5, 6, 11, 16, 21]. Recently, the concepts of extended fuzzy such as intuitionistic, hesitant, Pythagorean, and bipolar fuzzy environments have been applied to a wide range of decision-making problems [1, 12, 15, 24, 28].

Although the main concept of our proposed approach is based on the Zimmerman's approach, it is extended to neutrosophic environment which, the best of our knowledge, is not investigated in the literature. The novelty of the paper

is due to that, any paper does not exist to obtain a solution to the general multiobjective linear fractional programming problems in neutrosophic fuzzy environment by optimizing truth-membership, indeterminacy-membership, and falsity-membership functions, simultaneously.
In this paper, we present a method for solving the fuzzy multi-objective linear fractional (FMOLF) optimization problem in neutrosophic environment where all the coefficients are assumed to be the neutrosophic. The paper is classified as follows: After the introduction, Section 2 presents some concepts and preliminaries. In the next section, mathematical formulation of neutrosophic multiobjective linear fractional programming problem (NMOLFPP) was introduced. Section 3 was divided into three subsections. Section (3.1) deals with linearization process of of (NMOLFPP) to (NMOLPP) and Section 3.2 deals with the conversion process of (NMOLPP) to (crisp MOLPP) using accuracy function. In Section (3.3), we discussed Zimmermann fuzzy technique to convert (MOLPP) to (LPP). The development of the proposed method and an algorithm for solution procedures are described in Section 4. Section 5 presents a numerical example to demonstrate the applicability of the proposed approach. Finally, some results are included in Section 6.

2. Preliminaries

In this section, some basic concepts and definitions about neutrosophic sets and single-valued trapezoidal numbers from the literature are reviewed.

Definition 2.1. [22] Let X be a non-empty set. A neutrosophic set (NS) $\tilde{A}^N$ is defined as:

$$\tilde{A}^N = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle : x \in X, T_A(x), I_A(x), F_A(x) \in]0^-, 1^+[\}$$

where $T_A(x), I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively, and there is no restriction on the summation of them, so $0^- \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+$ and $]0^-, 1^+[$ is non-standard unit interval.

As NSs are difficult to apply to practical problems, Wang et. al. [25] introduced the concept of a single-valued neutrosophic set (SVNS), which is an example of a NS and can be used in real scientific and engineering applications.

Definition 2.2. A single valued trapezoidal neutrosophic number (SVTNN) $\tilde{a}^N = \langle (a, b, c, d); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a special neutrosophic set on the real number set $\mathbb{R}$, whose truth-membership, indeterminacy-membership, and falsity-membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{w_{\tilde{a}}(x-a)}{b-a} & a \leq x \leq b \\ w_{\tilde{a}} & b \leq x \leq c \\ \frac{w_{\tilde{a}}(d-x)}{d-c} & c < x \leq d \\ 0 & otherwise, \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \frac{b-x+u_{\tilde{a}}(x-a)}{b-a} & a \leq x \leq b \\ u_{\tilde{a}} & b \leq x \leq c \\ \frac{x-c+u_{\tilde{a}}(d-x)}{d-c} & c < x \leq d \\ 1 & otherwise, \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{b-x+y_{\tilde{a}}(x-a)}{b-a} & a \leq x \leq b \\ y_{\tilde{a}} & b \leq x \leq c \\ \frac{x-c+y_{\tilde{a}}(d-x)}{d-c} & c < x \leq d \\ 1 & otherwise, \end{cases}$$

where, $w_{\tilde{a}}, u_{\tilde{a}}$ and $y_{\tilde{a}}$ are the maximum truth, minimum indeterminacy, and minimum falsity membership degrees, respectively. The set of all neutrosophic numbers is denoted by $NF(\mathbb{R})$.

Here is an example (see Figure **??**) of a graphical representation of a single-valued trapezoidal neutrosophic number $\tilde{a}^N = \langle(1, 3, 5, 7); 0.9, 0.2, 0.4\rangle$.

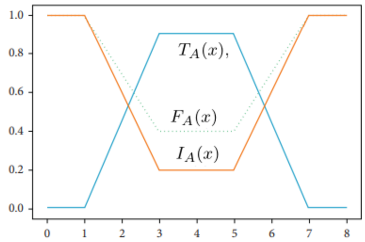

FIGURE 1. Graphical representation of a single-valued trapezoidal neutrosophic set

Definition 2.3. Let $\tilde{a}^N = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ and $\tilde{b}^N = \langle(b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}\rangle$ be two arbitrary SVTNNs and $\gamma \neq 0$ be any real number. Then

- $\tilde{a}^N + \tilde{b}^N = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$
- $\tilde{a}^N - \tilde{b}^N = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$
- $\gamma\tilde{a}^N = \begin{cases} \langle(\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle & \gamma > 0 \\ \langle(\gamma a_4, \gamma a_3, \gamma a_2, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle & \gamma < 0 \end{cases}$

Definition 2.4. [13] Let $\tilde{a}^N = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ be a SVTNN. Then, the score function $\psi(\tilde{a}^N)$ and accuracy function $\phi(\tilde{a}^N)$ of a SVTNN are defined as follows:

- $\psi(\tilde{a}^N) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4)(w_{\tilde{a}} + (1 - u_{\tilde{a}}) + (1 - y_{\tilde{a}}))$
- $\phi(\tilde{a}^N) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4)(w_{\tilde{a}} + (1 - u_{\tilde{a}}) + (1 + y_{\tilde{a}}))$

Definition 2.5. Suppose $\tilde{a}^N = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ and $\tilde{b}^N = \langle(b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}\rangle$ be any two SVTNNs. Then, we define a ranking method as follows:

- If $\psi(\tilde{a}^N) > \psi(\tilde{b}^N)$ then $\tilde{a}^N > \tilde{b}^N$.

- If $\psi(\tilde{a}^N) = \psi(\tilde{b}^N)$, and if $\phi(\tilde{a}^N) > \phi(\tilde{b}^N)$ then $\tilde{a}^N > \tilde{b}^N$,
$\phi(\tilde{a}^N) < \phi(\tilde{b}^N)$ then $\tilde{a}^N < \tilde{b}^N$,
$\phi(\tilde{a}^N) = \phi(\tilde{b}^N)$ then $\tilde{a}^N = \tilde{b}^N$,

3. Neutrosophic multiobjective linear fractional programming problem

The general form of neutrosophic multiobjective linear fractional programming problem (NMOLFPP) can be written as:

$$\begin{aligned} &Maximize \qquad Z_r(x) = \frac{F_r(x)}{G_r(x)} = \frac{\tilde{c}_r^N x + \tilde{p}_r^N}{\tilde{d}_r^N x + \tilde{q}_r^N}, \qquad r = 1, \dots, k \\ &subject\ to \qquad x \in \Delta = \{x : \ \tilde{A}^N x \leq \tilde{b}^N, x \geq 0\} \end{aligned} \tag{1}$$

where $\tilde{c}_r^N, \tilde{d}_r^N \in NF(\mathbb{R}^n)$, $\tilde{p}_r^N, \tilde{q}_r^N, \tilde{b}^N \in NF(\mathbb{R})$ for all $r = 1, \dots, k$, and $\tilde{A}^N = \left[\tilde{a}_{ij}^N\right] \in NF(\mathbb{R}^{m\times n})$ for all $i = 1, \dots, m$, $j = 1, \dots, n$.
To improve the presentation and easy understanding of the topic, this section was divided into three subsections. Section (3.1) deals with linearization process of (NMOLFPP) to (NMOLPP) and Section (3.2) deals with the conversion process of (NMOLPP) to (crisp MOLPP) using accuracy function. In Section (3.3), we discussed Zimmermann fuzzy technique to convert (MOLPP) to (LPP).

3.1. **Linearization of (NMOLFPP) to (NMOLPP).** In this part, a brief description of Charnes and Cooper [8] linear transformation is presented and a linear programming problem equivalent to NMOLPP is also presented.
For this purpose, let Δ be the set of all feasible solution of (1). For some value of $x \in \Delta$, $G_r(x) = \tilde{d}_r^N x + \tilde{q}_r^N$ may be equal to zero. To avoid this case we required that either $L = \{r : G_r(x) > 0 \ for \ x \in \Delta\}$ or $L^c = \{r : G_r(x) < 0 \ for \ x \in \Delta\}$. In this circumstance, we consider the least value of $\frac{1}{G_r(x)}$ and $\frac{-1}{F_r(x)}$ is t for $r \in L$ and $r \in L^c$, respectively, i.e.,

$$\bigcap_{r\in L} \frac{1}{\tilde{d}_r^N x + \tilde{q}_r^N} = t \quad , \quad \bigcap_{r\in L^c} \frac{-1}{\tilde{c}_r^N x + \tilde{p}_r^N} = t \tag{2}$$

which is equivalent to

$$t \leq \frac{1}{\tilde{d}_r^N x + \tilde{q}_r^N} \quad \text{for } r \in L \quad , \quad t \leq \frac{-1}{\tilde{c}_r^N x + \tilde{p}_r^N} \quad \text{for } r \in L^c \tag{3}$$

With the help of the transformation $t = \frac{1}{G_r(x)}$, $y = tx$, and using the above mentioned inequalities (3), NMOLFP problem (1) may be written as follows:

$$(4)\quad \begin{array}{ll} Maximize & Z_r(y,t) = \{tF_r(\frac{y}{t}), \text{for } r \in L; tG_r(\frac{y}{t}), \text{for } r \in L^c\} \\ \text{Subject to} & \qquad tG_r(\frac{y}{t}) \leq 1, \quad \text{for } r \in L \\ & \qquad -tF_r(\frac{y}{t}) \leq 1, \quad \text{for } r \in L^c \\ & \qquad \tilde{A}^N(\frac{y}{t}) - \tilde{b}^N \leq 0, \\ & \qquad t, y \geq 0. \end{array}$$

Definition 3.1. [20] Let D be the set of feasible solution of (4). A feasible solution $(\bar{y}, \bar{t}) \in D$ is said to be efficient solution of (4) if there is no $(y^*, t^*) \in D$ such that $Z_r(y^*, t^*) \geq Z_r(\bar{y}, \bar{t})$, $r = 1, \ldots, k$ and $Z_s(y^*, t^*) > Z_s(\bar{y}, \bar{t})$ for at least one $s \in D$.

Theorem 3.2. *The solution x^* is an efficient solution for problem (1) if and only if the solution (y^*, t^*) with $x^* = \frac{y^*}{t^*}$ is an efficient solution for problem (4).*

Proof. Contrarily, suppose that $x^* = \frac{y^*}{t^*}$ is an efficient solution for problem (1), but (y^*, t^*) is not an efficient solution for the problem (4). Then, there must exist $(\hat{y}, \hat{t}) \in D$ (where D is the feasible space of problem (4)) such that $t^*F_r(\dfrac{y^*}{t^*}) \leq \hat{t}F_r(\dfrac{\hat{y}}{\hat{t}})$ for $r \in L$ and $t^*F_s(\dfrac{y^*}{t^*}) < \hat{t}F_s(\dfrac{\hat{y}}{\hat{t}})$ for at least one index $s \in L$, also $t^*G_r(\dfrac{y^*}{t^*}) \leq \hat{t}G_r(\dfrac{\hat{y}}{\hat{t}})$ for $r \in L^C$ and $t^*G_s(\dfrac{y^*}{t^*}) < \hat{t}G_s(\dfrac{\hat{y}}{\hat{t}})$ for at least one index $s \in L^C$.
Using the definition of chakraborty and Gupta [7], that extended by Arya et. al. [3], we conclude that $t^*F_r(x^*) \leq \hat{t}F_r(\dfrac{\hat{y}}{\hat{t}})$ for $r \in L$ and $t^*G_r(x^*) \leq \hat{t}G_r(\dfrac{\hat{y}}{\hat{t}})$ for $r \in L^C$, and $t^*F_s(x^*) < \hat{t}F_s(\dfrac{\hat{y}}{\hat{t}})$ for at least one index $s \in L$ and $t^*G_s(x^*) < \hat{t}G_s(\dfrac{\hat{y}}{\hat{t}})$ for at least one index $s \in L^C$. This implies that $\dfrac{F_r(x^*)}{G_r(x^*)} \leq \dfrac{F_r(\hat{x})}{G_r(\hat{x})}$ for $r = 1, 2, \ldots, k$ and $\dfrac{F_r(x^*)}{G_r(x^*)} < \dfrac{F_r(\hat{x})}{G_r(\hat{x})}$ for at least one $s \in \{1, \ldots, k\}$.
This proves that the solution x^* is not efficient solution for problem (1). This contradicts our assumption and hence concludes that the solution (y^*, t^*) is efficient for the problem (4).
Conversely, let (y^*, t^*) be an efficient solution to the problem (4) and x^* is not efficient solution to the problem (1). Then, there must exist $\hat{x} \in D$ such that $\dfrac{F_r(x^*)}{G_r(x^*)} \leq \dfrac{F_r(\hat{x})}{G_r(\hat{x})}$ for $r = 1, \ldots, k$, and $\dfrac{F_r(x^*)}{G_r(x^*)} \leq \dfrac{F_r(\hat{x})}{G_r(\hat{x})}$ for at least one $s \in \{1, \ldots, k\}$.
Using the transformation $y^* = t^*x^*$ we deduce that $tF_r(\dfrac{y^*}{t^*}) \leq \dfrac{F_r(\hat{x})}{G_r(\hat{x})}$ for $r \in L$

and $tF_s(\frac{y^*}{t^*}) < \frac{F_s(\hat{x})}{G_s(\hat{x})}$ for at least one $s \in L$, and $tG_r(\frac{y^*}{t^*}) \leq \frac{F_r(\hat{x})}{G_r(\hat{x})}$ for $r \in L^c$ and $tG_s(\frac{y^*}{t^*}) < \frac{F_s(\hat{x})}{G_s(\hat{x})}$ for at least one index $s \in L^c$.

This conflicts the reality that the solution (y^*, t^*) of problem (4) is efficient . Hence, the solution x^* of problem (1) is efficient. This demonstrates the theorem. □

3.2. **Conversion of (NMOLPP) to (crisp MOLPP).** After, linearizing the problem based on the transformation presented in the previous section, by introducing a new intuitionistic fuzzy variable, we apply the accuracy function for each objective function and constraints. Thus, the NMOLPP (4) can be converted into the following deterministic MOLPP:

$$\begin{array}{lll} Maximize & O_r(y,t) = \{c'_r y + p'_r t, \text{for } r \in L; d'_r y + q'_r t, \text{for } r \in L^c\} & \\ \text{Subject to} & d'_r y + q'_r t \leq 1, & \text{for } r \in L \\ & -c'_r y - p'_r t \leq 1, & \text{for } r \in L^c \\ & \sum_{j=1}^n a'_{ij} y_j - b'_i t \leq 0, & i = 1, \ldots, m \\ & t, y \geq 0. & \end{array} \tag{5}$$

where $c'_r = \phi(\tilde{c}_r^N)$, $p'_r = \phi(\tilde{p}_r^N)$, $d'_r = \phi(\tilde{d}_r^N)$ and $q'_r = \phi(\tilde{q}_r^N)$ for $r = 1, \ldots, k$. Also $a'_{ij} = \phi(\tilde{a}_{ij}^N)$ and $b'_i = \phi(\tilde{b}_i^N)$ for all $i = 1, \ldots, m,\ j = 1, \ldots, n$ are the crisp version of all parameters.

Theorem 3.3. *An efficient solution of problem (5) is an efficient solution for problem (4).*

Proof. Assume that (y, t) is an efficient solution of problem (5). Then x is feasible for problem (5), it means that the following conditions will hold:

$$\begin{array}{ll} d'_r y + q'_r t \leq 1, & \text{for } r \in L \\ -c'_r y - p'_r t \leq 1, & \text{for } r \in L^c \\ \sum_{j=1}^n a'_{ij} y_j - b'_i t \leq 0, & i = 1, \ldots, m \\ t, y \geq 0. & \end{array}$$

Since the function ϕ is linear [11], then we have

$$\begin{array}{ll} \phi(\tilde{d}_r^N) y + \phi(\tilde{q}_r^N) t \leq 1, & \text{for } r \in L \\ -\phi(\tilde{c}_r^N) y - \phi(\tilde{p}_r^N) t \leq 1, & \text{for } r \in L^c \\ \sum_{j=1}^n \phi(\tilde{a}_{ij}^N) y_j - \phi(\tilde{b}_i^N) t \leq 0, & i = 1, \ldots, m \\ t, y \geq 0. & \end{array}$$

Consequently, we have

$$\begin{array}{ll} \tilde{d}_r^N y + \tilde{q}_r^N t \leq 1, & \text{for } r \in L \\ -\tilde{c}_r^N y - \tilde{p}_r^N t \leq 1, & \text{for } r \in L^c \\ \sum_{j=1}^n \tilde{a}_{ij}^N y_j - \tilde{b}_i^N t \leq 0, & i = 1, \ldots, m \\ t, y \geq 0. & \end{array}$$

Hence, (y, t) is a feasible solution for problem (4).
Moreover, since (y, t) is an efficient solution for problem (5), there does not exist any (y^*, t^*) such that $O_r(y^*, t^*) \geq O_r(y, t)$ for $r \in L$ and $O_r(y^*, t^*) > O_r(y, t)$ for at least one index $s \in L$, also $O_r(y^*, t^*) \geq O_r(y, t)$ for $r \in L^c$ and $O_r(y^*, t^*) > O_r(y, t)$ for at least one index $s \in L^c$. Thus we have no (y^*, t^*) such that: max $\phi(tF_r(\frac{y^*}{t^*})) \geq$ max $\phi(tF_r(\frac{y}{t}))$ for $r \in L$ and max $\phi(tF_r(\frac{y^*}{t^*})) >$ max $\phi(tF_r(\frac{y}{t}))$ for at least one $s \in L$, also, there does not exist any (y^*, t^*) such that: max $\phi(tG_r(\frac{y^*}{t^*})) \geq$ max $\phi(tG_r(\frac{y}{t}))$ for $r \in L^c$ and max $\phi(tG_r(\frac{y^*}{t^*})) >$ max $\phi(tG_r(\frac{y}{t}))$ for at least one $s \in L^c$.
Since ϕ is a linear function, we have no (y^*, t^*) such that max $Z_r(y^*, t^*) \geq$ max $Z_r(y, t)$ for $r \in L$ and max $Z_r(y^*, t^*) >$ max $Z_r(y, t)$ for at least one $s \in L$, also, there does not exist any (y^*, t^*) such that max $Z_r(y^*, t^*) \geq$ max $Z_r(y, t)$ for $r \in L^c$ and max $Z_r(y^*, t^*) >$ max $Z_r(y, t)$ for at least one $s \in L^c$. Therefore, (y, t) is an efficient solution for problem (4) □

The above model is the linearized version of the NMOLFPP. Thus, the obtained NMOLPP can be handed by different approaches to get the efficient solution. Since the objective functions $O_r(y, t),\ r = 1, \ldots, k$ are to be maximized, the level of satisfaction of the DM will increases as the solution tends towards upper bound to each objective and hence the DM will satisfy fully if the objective reach to their upper bound. Therefore, we construct a model by introducing NF constraints $O_r(y, t) \approx U_r,\ r = 1, \ldots, k$ with constraints of (5). where, the minimum and maximum values of each objective functions have been represented by $U_r = \max\{O_r(y, t)\}$ and $L_r = \min\{O_r(y, t)\}$ for all $r = 1, \ldots, k$.
Here, the constraint $O_r(y, t) \approx U_r,\ r = 1, \ldots, k$ is an neutrosophic fuzzy constraint that includes neutrosophic fuzzy equality that can be treated employing a membership function. This membership function may be linear, parabolic, or hyperbolic based on the choice of the DM. In this paper, since linear membership functions are used in the literature and practice more than other types of membership functions [26], we choose the linear membership functions.
The bounds for r-th objective function under the neutrosophic environment can be obtained as follows:

$$U_r^\mu = U_r, \quad L_r^\mu = L_r \quad \textit{for truth membership} \tag{6}$$

$$U_r^\lambda = U_r^\mu + s_r, \quad L_r^\lambda = L_r^\mu \quad \textit{for indeterminacy membership} \tag{7}$$

$$U_r^\nu = U_r^\mu, \quad L_r^\nu = L_r^\mu + t_r \quad \textit{for falsity membership} \tag{8}$$

where s_r and $t_r \in (0,1)$ are real numbers prescribed by decision-makers.
If $r \in L$, then truth-membership function of each objective function can be stated as:

$$\mu(O_r(y,t)) = \begin{cases} 0 & \phi(tF_r(\frac{y}{t})) \leq L_r^{\mu} \\ \dfrac{c_r' y + p_r' t - L_r^{\mu}}{U_r^{\mu} - L_r^{\mu}} & L_r^{\mu} \leq \phi(tF_r(\frac{y}{t})) \leq U_r^{\mu} \\ 1 & \phi(tF_r(\frac{y}{t})) \geq U_r^{\mu}, \end{cases} \tag{9}$$

If $r \in L^c$, then truth-membership function of each objective function can be stated as:

$$\mu(O_r(y,t)) = \begin{cases} 0 & \phi(tG_r(\frac{y}{t})) \leq L_r^{\mu} \\ \dfrac{d_r' y + q_r' t - L_r^{\mu}}{U_r^{\mu} - L_r^{\mu}} & L_r^{\mu} \leq \phi(tG_r(\frac{y}{t})) \leq U_r^{\mu} \\ 1 & \phi(tG_r(\frac{y}{t})) \geq U_r^{\mu}, \end{cases} \tag{10}$$

If $r \in L$, then indeterminacy-membership function of each objective function can be written as:

$$\lambda(O_r(y,t)) = \begin{cases} 0 & \phi(tF_r(\frac{y}{t})) \leq L_r^{\lambda} \\ \dfrac{U_r^{\lambda} - (c_r' y + p_r' t)}{U_r^{\lambda} - L_r^{\lambda}} & L_r^{\lambda} \leq \phi(tF_r(\frac{y}{t})) \leq U_r^{\lambda} \\ 1 & \phi(tF_r(\frac{y}{t})) \geq U_r^{\lambda}, \end{cases} \tag{11}$$

If $r \in L^c$, then indeterminacy-membership function of each objective function can be written as:

$$\lambda(O_r(y,t)) = \begin{cases} 0 & \phi(tG_r(\frac{y}{t})) \leq L_r^{\lambda} \\ \dfrac{U_r^{\lambda} - (d_r' y + q_r' t)}{U_r^{\lambda} - L_r^{\lambda}} & L_r^{\lambda} \leq \phi(tG_r(\frac{y}{t})) \leq U_r^{\lambda} \\ 1 & \phi(tG_r(\frac{y}{t})) \geq U_r^{\lambda}, \end{cases} \tag{12}$$

If $r \in L$, then falsity-membership function of each objective function can be written as:

$$\nu(O_r(y,t)) = \begin{cases} 1 & \phi(tF_r(\frac{y}{t})) \leq L_r^{\nu} \\ \dfrac{U_r^{\nu} - (c_r' y + p_r' t)}{U_r^{\nu} - L_r^{\nu}} & L_r^{\nu} \leq \phi(tF_r(\frac{y}{t})) \leq U_r^{\nu} \\ 0 & \phi(tF_r(\frac{y}{t})) \geq U_r^{\nu}, \end{cases} \tag{13}$$

If $r \in L^c$, then falsity-membership function of each objective function can be written as:

$$\nu(O_r(y,t)) = \begin{cases} 1 & \phi(tG_r(\frac{y}{t})) \leq L_r^{\nu} \\ \dfrac{U_r^{\nu} - (d_r' y + q_r' t)}{U_r^{\nu} - L_r^{\nu}} & L_r^{\nu} \leq \phi(tG_r(\frac{y}{t})) \leq U_r^{\nu} \\ 0 & \phi(tG_r(\frac{y}{t})) \geq U_r^{\nu}, \end{cases} \tag{14}$$

Now our problem is reduced to increase the range of acceptance and to decrease the range of rejection subject to the given constraint. For this we can adopt the following Zimmermann's technique.

3.3. **Zimmerman's technique.** Let $\delta = \min\{\mu(O_r(y,t)), r = 1, \ldots, k\}$, $\gamma = \max\{\lambda(O_r(y,t)), r = 1, \ldots, k\}$ and $\vartheta = \max\{\nu(O_r(y,t)), r = 1, \ldots, k\}$. In other words $\mu(O_r(y,t)) \geq \delta$, $\lambda(O_r(y,t)) \leq \gamma$ and $\nu(O_r(y,t)) \leq \vartheta$.
Based on Zimmerman's approach [29] which provides simultaneous maximization of the minimum truth degree of acceptance and minimization of the maximum indeterminacy degree of rejection upto some extent and minimization of the falsity degree of rejection. Thus, we formulate the model using max-min as the operator as follows:

$$
\begin{array}{lll}
\max & \delta - \gamma - \vartheta & \\
s.t. & c'_r y + p'_r t - \delta(U^\mu_r - L^\mu_r) \geq L^\mu_r, & r \in L \\
 & d'_r y + q'_r t - \delta(U^\mu_r - L^\mu_r) \geq L^\mu_r, & r \in L^c \\
 & c'_r y + p'_r t + \gamma(U^\lambda_r - L^\lambda_r) \geq U^\lambda_r, & r \in L \\
 & d'_r y + q'_r t + \gamma(U^\lambda_r - L^\lambda_r) \geq U^\lambda_r, & r \in L^c \\
 & c'_r y + p'_r t + \vartheta(U^\nu_r - L^\nu_r) \geq U^\nu_r, & r \in L \\
 & d'_r y + q'_r t + \vartheta(U^\nu_r - L^\nu_r) \geq U^\nu_r, & r \in L^c \\
 & \delta \geq \gamma,\ \delta \geq \vartheta,\ \delta + \gamma + \vartheta \leq 3, & \delta, \gamma, \vartheta \in (0,1) \\
 & \text{all the constraints of (5)} &
\end{array}
\tag{15}
$$

Theorem 3.4. *A unique optimal solution of (15) is an efficient solution of problem (5).*

Proof. Let $(\hat{y}, \hat{t}, \hat{\delta}, \hat{\gamma}, \hat{\vartheta})$ be the unique optimal solution of (15). Then, $(\hat{\delta} - \hat{\gamma} - \hat{\vartheta}) \geq (\delta - \gamma - \vartheta)$ for any feasible solution $(y, t, \delta, \gamma, \vartheta)$ of theproblem (15). Contrarily, suppose that $(\hat{y}, \hat{t}, \hat{\delta}, \hat{\gamma}, \hat{\vartheta})$ is not an efficient solution to model (5). Then, there exist feasible solution $(y^*, t^*) \neq (\hat{y}, \hat{t})$ such that $O_r(\hat{y}, \hat{t}) \leq O_r(y^*, t^*)$ for $r = 1, \ldots, k$ and $O_r(\hat{y}, \hat{t}) < O_r(y^*, t^*)$ for at least one index r. Therefore, $\frac{U^\nu_r - O_r(\hat{y}, \hat{t})}{U^\nu_r - L^\nu_r} \geq \frac{U^\nu_r - O_r(y^*, t^*)}{U^\nu_r - L^\nu_r}$ and $\frac{U^\nu_r - O_r(\hat{y}, \hat{t})}{U^\nu_r - L^\nu_r} > \frac{U^\nu_r - O_r(y^*, t^*)}{U^\nu_r - L^\nu_r}$, for at least one r. Thus $\max_r \frac{U^\nu_r - O_r(\hat{y}, \hat{t})}{U^\nu_r - L^\nu_r} > (\geq) \max_r \frac{U^\nu_r - O_r(y^*, t^*)}{U^\nu_r - L^\nu_r}$. Let $\vartheta^* = \max_r \frac{U^\nu_r - O_r(y^*, t^*)}{U^\nu_r - L^\nu_r}$, then $\hat{\vartheta} > (\geq) \vartheta^*$.

Similarly, consider that $\gamma^* = \max_r \frac{U^\lambda_r - O_r(y^*, t^*)}{U^\lambda_r - L^\lambda_r}$, then $\hat{\gamma} > (\geq) \gamma^*$.

In the same manner, $\frac{O_r(\hat{y}, \hat{t}) - L^\mu_r}{U^\mu_r - L^\nu_r} \leq \frac{O_r(y^*, t^*) - L^\mu_r}{U^\mu_r - L^\mu_r}$ and $\frac{O_r(\hat{y}, \hat{t}) - L^\mu_r}{U^\mu_r - L^\mu_r} < \frac{O_r(y^*, t^*) - L^\mu_r}{U^\mu_r - L^\mu_r}$, for at least one r. Thus $\min_r \frac{O_r(\hat{y}, \hat{t}) - L^\mu_r}{U^\mu_r - L^\mu_r} < (\leq) \min_r \frac{O_r(y^*, t^*) - L^\mu_r}{U^\mu_r - L^\mu_r}$. Let $\delta^* = \min_r \frac{O_r(y^*, t^*) - L^\mu_r}{U^\mu_r - L^\mu_r}$, then $\hat{\delta} < (\leq) \delta^*$. Therefore, $(\hat{\delta} - \hat{\gamma} - \hat{\vartheta}) < (\delta^* - \gamma^* - \vartheta^*)$ which means that the solution is not unique optimal.

This conflict the reality that $(\hat{y}, \hat{t}, \hat{\delta}, \hat{\gamma}, \hat{\vartheta})$ is the unique optimal solution of (15), hence, it is an efficient solution of model (5). □

4. Algorithm for solving NFMOFLPP

The step-wise solution algorithm discussed in Section (3.2) is summarized as follows:

- Step 1. Formulate the FNMOLPP as given in problem (4).
- Step 2. Using accuracy function, obtain the crisp version of NMOLPP as given in problem (5).
- Step 3. Solve the crisp multi-objective linear programming problem by considering one objective function at a time with all constraints and ignoring all other objective function. Repeat this process k times for k different objective function. Suppose that the corresponding optimal solutions are $(y,t)^{(1)}, (y,t)^{(2)}, \ldots, (y,t)^{(k)}$.
- Step 4. Find the value of the objective function $O_r(y,t)$ for $r = 1, 2, \ldots, k$ at each point in (y,t). Form a payoff matrix O of order $K \times K$, whose (i,j)th element is equal to $O_j(y,t)^{(i)} = O_{ij}$.
- Step 5. Find the minimum and maximum value of each objective function, then evaluate $U_r = \max\{O_{1r}, \ldots, O_{kr}\}$ and $L_r = \min\{O_{1r}, \ldots, O_{kr}\}$ for $r = 1, \ldots, k$.
- Step 6. With the aid of U_r and L_r, calculate the upper and lower bound for truth, indeterminacy and a falsity membership under neutrosophic envirnment as given in Eqs (6)–(8).
- Step 7. Elicit the membership functions under neutrosophic environment by using Eqs (9)-(14).
- Step 8. Utilize Zimmerman's approach to formulate and solve model (15) to determine the optimal compromise outcomes by applying the appropriate methods or different optimization software packages.

5. Numerical illustration

In this section, we present a numerical example to demonstrate the steps of the proposed methodology. Consider the following problem having three objectives:

$$
\begin{aligned}
\max \quad & \tilde{Z}_1(x) = \frac{\tilde{6}^N x_1 + \tilde{7}^N x_2 + \tilde{5}^N x_3 + \tilde{4}^N}{\tilde{7}^N x_1 - \tilde{5}^N x_2 - \tilde{4}^N x_3 - \tilde{1}^N} \\
\max \quad & \tilde{Z}_2(x) = \frac{\tilde{7}^N x_1 + \tilde{8}^N x_2 + \tilde{9}^N x_3}{\tilde{5}^N x_1 + \tilde{4}^N x_2 + \tilde{6}^N x_3 + \tilde{3}^N} \\
\max \quad & \tilde{Z}_3(x) = \frac{\tilde{9}^N x_1 + \tilde{8}^N x_2 + \tilde{7}^N x_3 + \tilde{7}^N}{\tilde{4}^N x_1 + \tilde{6}^N x_2 + \tilde{5}^N x_3 + \tilde{6}^N} \\
\text{subject to} \quad & \tilde{4}^N x_1 + \tilde{5}^N x_2 + \tilde{7}^N x_3 \geq \tilde{4}^N \\
& \tilde{5}^N x_1 + \tilde{8}^N x_2 + \tilde{6}^N x_3 \leq \tilde{20}^N \\
& \tilde{6}^N x_1 + \tilde{7}^N x_2 + \tilde{4}^N x_3 \leq \tilde{15}^N \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
\tag{16}
$$

where $\tilde{1} = \langle (0, 1, 2, 3); 0.8, 0.5, 0.3 \rangle$, $\tilde{3} = \langle (2, 3, 4, 5); 0.9, 0.4, 0.5 \rangle$, $\tilde{4} = \langle (3, 4, 5, 6); 0.6, 0.4, 0.5 \rangle$, $\tilde{5} = \langle (4.5, 5, 6.2, 7); 0.3, 0.4, 0.8 \rangle$, $\tilde{6} = \langle (5, 6, 7, 8); 0.75, 0.5, 0.25 \rangle$, $\tilde{7} = \langle (6, 7, 10, 12); 0.8, 0.6, 0.5 \rangle$, $\tilde{8} = \langle (7, 8, 10, 11); 0.8, 0.5, 0.3 \rangle$, $\tilde{9} = \langle (8, 9, 10, 12); 0.8, 0.1, 0.4 \rangle$, $\tilde{20} = \langle (18, 20, 21, 22); 0.9, 0.2, 0.6 \rangle$, $\tilde{15} = \langle (14, 15, 16, 18); 0.4, 0.3, 0.6 \rangle$
Firstly, using the transformation $y = tx$ the NMOLFPP (16) is converted into NMOLPP as: (Note that $r = \{2, 3\} \in L,\ r = \{1\} \in L^c$)

$$
\begin{aligned}
\max \quad & \tilde{Z}_1(y,t) = \tilde{7}^N y_1 - \tilde{5}^N y_2 - \tilde{4}^N y_3 - \tilde{1}^N, \\
\max \quad & \tilde{Z}_2(y,t) = \tilde{7}^N y_1 + \tilde{8}^N y_2 + \tilde{9}^N y_3, \\
\max \quad & \tilde{Z}_3(y,t) = \tilde{9}^N y_1 + \tilde{8}^N y_2 + \tilde{7}^N y_3 + \tilde{7}^N t, \\
\text{subject to} \quad & -\tilde{6}^N y_1 - \tilde{7}^N y_2 - \tilde{5}^N y_3 - \tilde{4}^N t \leq \tilde{1}^N \\
& \tilde{5}^N y_1 + \tilde{4}^N y_2 + \tilde{6}^N y_3 + \tilde{3}^N t \leq \tilde{1}^N \\
& \tilde{4}^N y_1 + \tilde{6}^N y_2 + \tilde{5}^N y_3 + \tilde{6}^N t \leq \tilde{1}^N \\
& \tilde{4}^N y_1 + \tilde{5}^N y_2 + \tilde{7}^N y_3 - \tilde{4}^N t \geq \tilde{0}^N \\
& \tilde{5}^N y_1 + \tilde{8}^N y_2 + \tilde{6}^N y_3 - \tilde{20}^N t \leq \tilde{0}^N \\
& \tilde{6}^N y_1 + \tilde{7}^N y_2 + \tilde{4}^N y_3 - \tilde{15}^N t \leq \tilde{0}^N \\
& y_1, y_2, y_3, t \geq 0
\end{aligned}
\tag{17}
$$

Then we defuzzify problem (17) using the accuracy function and arithmetic operations to obtain the crisp model as follows:

$$
\begin{aligned}
&\max \quad O_1(y,t) = 5.9063y_1 - 3.831y_2 - 3.0375y_3 - 1.625,\\
&\max \quad O_2(y,t) = 5.9063y_1 + 5.85y_2 + 7.556y_3,\\
&\max \quad O_3(y,t) = 7.556y_1 + 5.85y_2 + 5.9063y_3 + 5.9063t,\\
&\text{subject to} \quad -4.0625y_1 - 5.9063y_2 - 3.831y_3 - 3.0375t \le 1\\
&\qquad 3.831y_1 + 3.0375y_2 + 4.0625y_3 + 2.625t \le 1\\
&\qquad 3.0375y_1 + 4.0625y_2 + 3.831y_3 + 4.0625t \le 1\\
&\qquad 3.0375y_1 + 3.831y_2 + 5.9063y_3 - 3.0375t \ge 0\\
&\qquad 3.831y_1 + 5.85y_2 + 4.0625y_3 - 16.706t \le 0\\
&\qquad 4.0625y_1 + 5.9063y_2 + 3.0375y_3 - 10.631t \le 0
\end{aligned} \tag{18}
$$

Solving each objective function with all constraints in problem (18) at a time, we get the pay-off matrix as follows:

Table 1. Pay-off matrix

	y_1	y_2	y_3	t	$O_1(y,t)$	$O_2(y,t)$	$O_3(y,t)$
$\max O_1$	0.2069	0	0	0.0791	1.093	1.2218	2.03
$\max O_2$	0	0	0.2003	0.0572	-0.7015	1.5137	1.5213
$\max O_3$	0.1894	0	0	0.1045	0.9487	1.1186	2.0485

From the pay-off matrix (Table 1) lower bound and upper bound are estimated as $L_1 = -0.7015$, $U_1 = 1.093$, $L_2 = 1.1186$, $U_2 = 1.5137$, $L_3 = 1.5213$, $U_3 = 2.0485$. After constructing the truth, indeterminacy and falsity membership functions defined in relations (9-14) and using Zimmermann's approach, Problem (18) reduces to

$$
\begin{aligned}
\max \quad & \delta - \gamma - \vartheta\\
s.t. \quad & O_1(y,t) - \delta(1.7945) \ge -0.7015,\\
& O_2(y,t) - \delta(0.3951) \ge 1.1186,\\
& O_3(y,t) - \delta(0.5273) \ge 1.5213,\\
& O_1(y,t) + \gamma(1.7945 + s_1) \ge 1.093 + s_1,\\
& O_2(y,t) + \gamma(0.3951 + s_2) \ge 1.5137 + s_2,\\
& O_3(y,t) + \gamma(0.5273 + s_3) \ge 2.0485 + s_3,\\
& O_1(y,t) + \vartheta(1.7945 - t_1) \ge 1.093,\\
& O_2(y,t) + \vartheta(0.3951 - t_2) \ge 1.5137,\\
& O_3(y,t) + \vartheta(0.5273 - t_3) \ge 2.0485,\\
& \delta \ge \gamma,\ \delta \ge \vartheta,\ \delta + \gamma + \vartheta \le 3, \quad \delta, \gamma, \vartheta \in (0,1)\\
& \text{all the constraints of (18)}
\end{aligned} \tag{19}
$$

For different values of s_r and t_r, the comparative study of the obtained optimal solutions are given in Table 2. It is clear that for $s_r = t_r = 0.5$ we obtained better solution for the first and second objective function, whereas for $s_r = t_r = 0.1$, we find better solution for the third objective function. Hence it depends on the DM that which objective has higher priority to be achieved. Accordingly, the method should be applied.

TABLE 2. Optimal solution results

s_r, t_r	$X = (x_1, x_2, x_3)$	$Z_1(x)$	$Z_2(x)$	$Z_3(x)$	δ	γ	ϑ
0.1	$(2.6169, 0, 0)$	0.8844	1.2218	2.1379	1	1	1
0.3	$(2.5948, 0, 0.0295)$	0.8987	1.2257	2.1304	1	1	1
0.5	$(1.8201, 0, 1.0657)$	1.9319	1.35	1.898	0.89	0.89	0.6338
0.7	$(1.9381, 0, 0.9078)$	1.6559	1.3326	1.9298	0.5904	0.5904	0.279
0.9	$(1.9708, 0, 0.8641)$	1.5921	1.3276	1.9388	0.1789	0.6632	0.6632

6. Conclusion

The formulation and analysis of an optimization problem under neutrosophic fuzzy scenario is more realistic and has high practical applicability in comparison with the problem under fuzzy environment. In this paper an effective procedure has been suggested to solve the NMOLFP problem, where all the coefficients and right-hand side parameters are single-valued trapezoidal neutrosophic numbers. In the first phase of our approach, the problem is converted into a linear one using Charnes-Cooper transformation method. Then the linearized model is reduced to a crisp multi-objective problem using the accuracy function for each objective. We extended Zimmerman's approach to maximize the truth-membership and minimize the indeterminacy-membership and falsity-membership functions in the solution procedure. Moreover, an algorithm that indicates the procedures for tackling NMOLFPP is presented, the applicability and efficiency of our approach is also indicated numerically. The proposed approach can be used to solve real-world problems that have imprecise and contradictory information. As future researches, the proposed approach can be applied to various optimization problems such as supplier selection problems, inventory control, portfolio optimization, etc.

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