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Modeling and Simulation of Dynamics in Soft Robotics: a Review of Numerical Approaches

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Abstract

Purpose of review In this review, we briefly summarize the numerical methods commonly used for the nonlinear dynamic analysis of soft robotic systems. The underlying mechanical principles as well as the geometrical treatment tailored for soft robots are introduced with particular emphasis on one-dimensional models. Additionally, the review encompasses three-dimensional frameworks, available simulation packages, and various types of interaction models, shedding light on the design, actuation, motion control, and internal and external forces of soft robots.

Recent findings Reduced-order models can offer high efficiency in characterizing nonlinear deformations, allowing convenient tailoring based on specific structural and material configurations. For pursuing high simulation accuracy and detailed mechanics, the finite element method proves to be a valuable tool through numerous off-the-shelf platforms. Furthermore, machine learning has emerged as a promising tool to effectively address the challenges within the mechanics community.

Summary A wide range of kinematic and dynamic numerical models is available for simulating the behaviors of soft robots, offering exceptional adaptability to different geometries and structures based on existing modeling theories and numerical solution algorithms. However, the trade-off between computational complexity and simulation accuracy remains a challenge in achieving fast, accurate, and robust control of soft robots in complex environments.

Keywords Soft robotics · Nonlinear dynamics · Numerical simulation · Model order reduction · Discrete model

Introduction

Inspiration from the diverse creatures in nature has sparked the emergence of numerous soft robots in recent years. These robots, such as soft robotic octopuses, snakes, caterpillars, birds and click beetles [1–5], exhibit remarkable flexibility in locomotion, exceptional dexterity in manipulation, and advanced adaptability to complicated environments. In contrast to conventional piece-wise rigid machines, soft robots possess an infinite number of Degrees of Freedom (DOFs) and inherent softness, which enables them to overcome the limitations of confined space, offers enhanced dexterity, and ensures safer interactions with humans.

Designing and fabricating a soft robot with particular function requires meticulous consideration of numerous factors and issues. However, the current methods often involve an iterative experimental trial and error processes, resulting in significant time costs, tedious assembly processes, and substantial interference from the environment. In contrast, numerical simulation has served as an efficient tool to guide the design, analysis, fabrication, and control of soft robots [6].

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During the design phase, simulation performs as a valuable tool for visualizing the overall structure and analyzing kinematics, including singularity, feasibility, and reachable space range. This allows for adjustments, improvements, and optimizations before the fabrication of real robots. Following the fabrication of the robot, the numerical tool continues to be utilized as a complementary verification since experiments cannot cover all working conditions within a short period of time, and, more importantly, subjecting the robot to extreme environments may lead to damage or even complete failure.

Accurate simulation models that capture the dynamic behavior of soft robots promote the realization of precise and robust model-based controls [6]. They also enable the application of deep learning based control methods, such as deep reinforcement learning, by constructing virtual training environments. Therefore, the precise and rapid modeling of soft robotic dynamics holds great significance within the soft robotics community.

Various mechanical theories and numerical solvers offer the flexibility to develop customized kinematic and dynamic models for soft robots. In the case of cable or tendon-driven soft robots, geometric structures are often modeled in a reduced-order fashion, such as the one-dimensional (1D) slender rod model or the two-dimensional (2D) thin shell framework, wherein the main components are retained while trivial details are disregarded to enhance computational efficiency. However, for simulating complex structures and exploring intricate mechanical details, three-dimensional (3D) models need to be established and analyzed using Finite Element Method (FEM) with readily available platforms. The simulation process should also be incorporated with the contact dynamics and external actuation. Moreover, Machine Learning (ML) techniques show promise as modeling approaches that can complement or even replace traditional mathematical models. Additionally, model-based control has gained significant attention as an attractive topic, given its robustness and high precision in controlling soft robots.

Compared to theoretical modeling, numerical approaches are widely embraced due to their advantages in terms of computational cost, numerical robustness, and overall applicability. Hence, this concise review primarily focuses on the numerical simulation of soft robots. The review begins with introducing various structural mechanics models, with particular attention given to reduced-order models (ROMs) in "One-Dimensional Reduced-Order Models." Subsequently, a review of 3D models is provided in detail in "Three-Dimensional Models." The review also explores the role of ML techniques in soft robot simulation and delves into the realm of model-based control, which is encompassed in the "Machine Learning-Based Models." The interaction challenges are then addressed through discussions on frictional contact, fluid-structure interaction, and interactions with external fields or multi-physics phenomena in the "Interaction Models." Finally, the review concludes with a summarizing "Conclusion."

One-Dimensional Reduced-Order Models

The reduced-order models (ROMs) are a type of physical model that reduces the computational complexity of a system while maintaining acceptable prediction errors. This reduction is achieved by simplifying the system's structure, boundary conditions, nonlinear mapping relations, and other factors. It is important to note that ROMs should not be confused with Model Order Reduction (MOR), which will be discussed in the context of "Three-Dimensional Models."

ROMs have gained widespread adoption due to their ability to simulate key motions of simplified geometries. Designers can conveniently develop their own ROMs using existing mechanics theories. In the realm of soft robotics, the 1D mechanics model explores the highly nonlinear dynamics of elastic bodies such as rods or beams. Typically, the modeling framework begins with discretizing the slender structure into finite discrete segments. The equations of motion are then established based on relevant balance laws, such as energy conservation. Finally, the solution is obtained using numerical treatment methods.

Planar Beam Theory

The Euler-Bernoulli beam theory is a simple yet highly useful theory that assumes the cross-section of the beam is infinitely rigid within its own plane and remains plane and normal to the deformed beam axis during deformation [10]. When a distributed load q is applied to the beam with elastic modulus E and second moment of beam's cross-sectional area I, the relationship between the load and the induced deflection, $\omega(x)$, can be expressed as $\frac{d^2}{dx^2}(EI\frac{d^2\omega}{dx^2}) - q = 0$ along with a specified boundary condition [11].

The Euler-Bernoulli beam method is commonly used to capture continuum bending deformations. In [12], a 3D Euler-Bernoulli beam-based inverse dynamic model was developed for a fluidic elastomeric actuator finger, which served as the basis for deriving a modular dynamic model for the Cable-Driven Soft Robot (CDSR) [13]. Figure 1(a1) and (b1) depict a classic soft robot that utilized linear beam theory, Coulomb's friction law, and simplified energy analysis to establish the relationship between bending curvature, flexural rigidity, and air pressure [7]. By incorporating parametric kinematic Pythagorean Hodograph curves and considering external forces, static shape reconstruction and real-time control were achieved based on actuator inputs [12]. In control problems, accounting for interactions with external forces is crucial. For example, a simplified solid mechanics model was employed to capture the soft body deformation and investigate the peeling-and-loading mechanism of an untethered soft robot capable of climbing 3D surfaces, by controlling external magnetic fields [14]. Although the discretized model with absolute or relative states used in this approach achieved lower simulation accuracy compared to other general ROMs approaches, it offered improved computational efficiency and time savings [15]. When higher accuracy is desired for precise control, the Euler-Bernoulli beam model can be further extended, as demonstrated by the combined Piecewise Constant Curvature (PCC) model used in the simulation of a flexible link [16].

Piecewise Constant Curvature Model

PCC models are extensively applied to simulate continuum robots, based on the assumption that the major structure is approximately represented by a series of connected tangent arcs with constant curvatures. This simplification significantly reduces the complexity of calculating the bending angles, making kinematic modeling and real-time control more convenient to implement [17, 18]. PCC models can be classified into different types based on various perspectives. For instance, they have been categorized as robot-independent mapping models and robot-specific mapping models in [19] and as kinematics-based models and mechanics-based models in [20••].

In PCC models, three key parameters are used to characterize the arc of the robot: curvature κ, the angle of the plane containing the arc ϕ, and arc length l, as shown in Fig. 2(a). In the context of soft robots, which have infinite dimensionality, the homogeneous transformation matrix $T(\kappa, \phi, l)$, can be determined for any point along the arc from the arc base. This parameterization combined with PCC assumption, allows for more convenient establishment of the kinematic models. Moreover, many modeling methods and control strategies developed for rigid robots can be transferred to soft robots, as demonstrated by the example of the six-segment soft robot shown in Fig. 1(a1) and (b1) [8]. The classical Denavit-Hartenberg parameter method is a widely used approach in constructing kinematic models [21].

The PCC model is effective in modeling structures that consists of bending actuators. However, its accuracy diminishes when applied to closed kinematic chains or predicting the position of the tip in tendon-driven continuum robot. Extensive efforts have been dedicated to developing extended PCC models to improve modeling accuracy. These efforts include adopting independent curvature instead of constant curvature [22], incorporating energy minimization techniques [23], and introducing tension propagation models [24].

In the case of a CDSR, it has been validated that multiple cable actuation tended to introduce undesirable axial compression and coupling. To mitigate this, a variable stiffness formulation corresponding to the axial compression can be utilized to reduce tip positioning errors [25]. The drawbacks of the PCC model, such as singularities, non-linear function, non-direct reversibility, and discontinuities, are considered byproducts of the commonly employed direction/angle of bending parameterization. These problems can be addressed

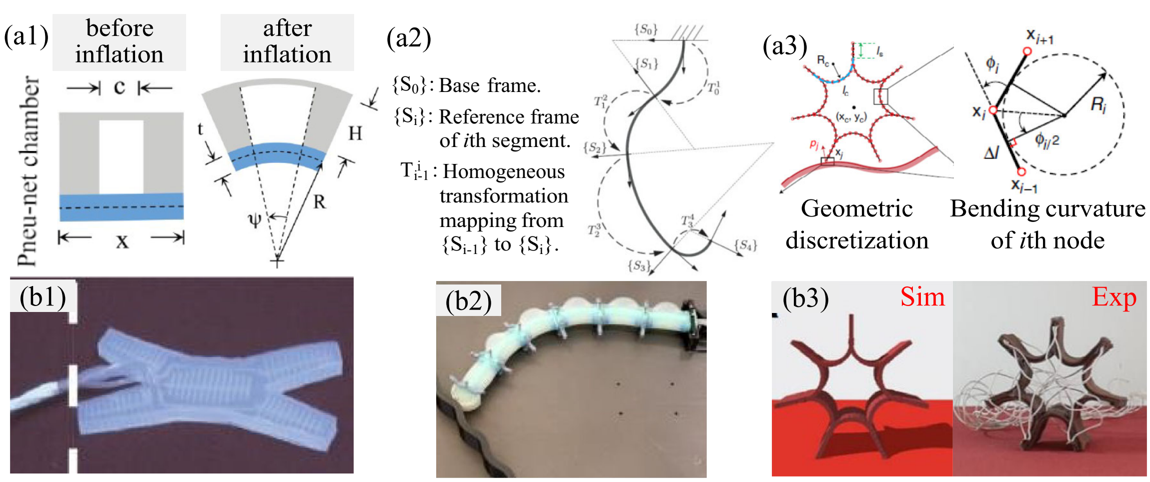

Fig. 1 Three types of soft robots modeled with 1D ROMs. **a1** The bending geometries of a pneu-net chamber before and after inflation of a soft quadruped robot in (**b1**) [7]. c, H: the width and height of the air chamber. x, t: the arclength and thickness of its neutral bending plane. R, ψ: the bending radius and the angle of arclength c, $\psi = c/R$. **a2** The constant curvature-based kinematics model of a four-segment soft robot in (**b2**) [8]. **a3** The geometric discretization and bending curvature of a soft rolling robot in (**b3**) simulated with discrete elastic rod model [9]. (x_c, y_c): position of the central point. l_c, l_s, R_c: length of the curved and straight part, radius of the curved actuator. x_j, P_j: the jth contact point and the contact force. x_{i-1}, x_i, x_{i+1}: node point. R_i, Δl, ϕ_i: radius, turning angle and edge length of node i

by constructing a new state representation [26]. Alternatively, linearisation of the relationship between cable length and individual segment angle can also solve the issue [27].

Cosserat Rods

Kirchhoff's rod theory is applicable to slender solid bodies that satisfy the condition of 1D geometry, where the length (L) is significantly larger than the radius (r), i.e., $L \gg r$). This theory models the bending and torsion of such slender structures. In the early 20th century, the Cosserat brothers reformulated Kirchhoff's rod theory by introducing directors, leading to the development of Cosserat rod theory [28, 29]. The Cosserat rod theory is a generalization of Kirchhoff rods, as it additionally considers stretching and shearing, encompassing all possible deformation modes of an elastic rod system. As shown in Fig. 2(b), a Cosserat rod is described by its centerline $\boldsymbol{r}(s, t)$ and a local reference frame $\{\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3\}$, where $s \in [0, L]$ represents the arc length of the rod, t denotes time, and d_i represent the directors. The strain vectors can be obtained by calculating the rate of change of a body-fixed frame with respect to the arc length, s. The balance of forces and moments can be established by describing their evolution over time. Linear constitutive equations for bending and torsion are used to relate the geometric deformations, material properties, and the corresponding forces. Building upon the Cosserat rod theory, numerous mechanics models have been developed to further understand and analyze the behavior of slender structures.

Compared to the PCC method that approximates constant-curvature arcs, Cosserat rod theory is considered a geometrically exact modeling approach. With the Cosserat rod model, real-time forward dynamics simulations can be achieved by discretizing the time derivatives of partial differential equations and solving the resulting ordinary differential equation boundary value problem along arc length at each timestep. Computational efficiency is attained by exploring the stability of the implicit methods at large timesteps [30]. In the kinematic modeling of tendon-actuated continuum robots, a comparison between beam mechanics and Cosserat rod methods revealed that the former approach consumed significantly less computation time while the latter provided slightly higher accuracy [31]. The computational complexity can be reduced by simplifying the models through certain assumptions, such as employing structure-induced assumptions to generate a compact and computationally efficient formulation [32], designing novel numerical solver [33], or modifying the shooting method [34].

When applying Cosserat rod models to cooperative or parallel soft robots, factors such as the coupling effect, compensation of interaction forces and moments, large deformations of manipulated objects, and terminal constraints should be considered [35, 36]. Conventional Cosserat models may lead to exponentially increased computation complexity in workspace estimation. In such cases, optimization involving solving the inverse model and mapping the workspace boundary has been effective in improving efficiency [37]. Establishing an exact model requires consideration of both internal forces and external constraints from loads or the environment [38].

To improve prediction accuracy, Cosserat rod models can be extended by combining them with other methods, such as the minimum potential energy principle [39], or the Newton-Euler law [40]. Based on Cosserat rods, a kinetostatic model can be constructed for parallel continuum robots [41]. A combination of screw theory, Lie groups and Lie algebras, Cosserat rod models, and the finite element method has been employed to accurately and computationally efficiently model nonlinear arms [42]. Additionally, an open-source environment called Elastica has been developed based on Cosserat rods to model the 3D dynamics of soft slender rods, accounting for bending, twisting, shearing, and stretching. This environment significantly reduces computation time and enables dynamic modeling of multiple active or passive

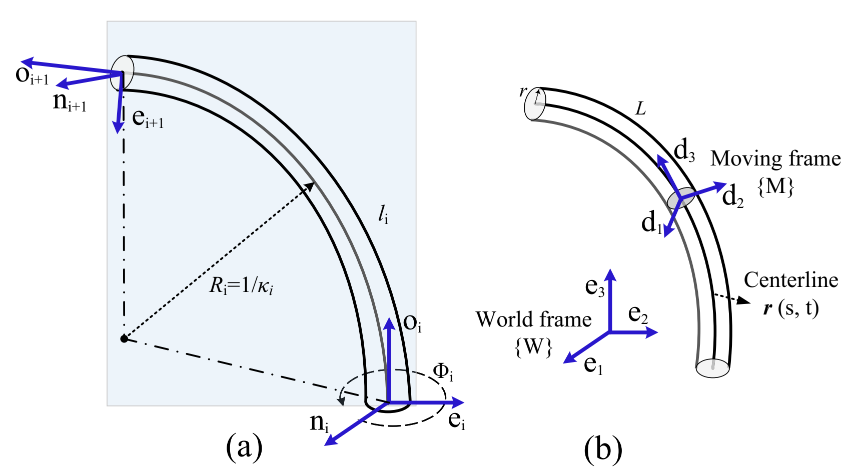

Fig. 2 Geometric representation of **a** PCC model and **b** Cosserate rod model. $\{e_i, n_i, o_i\}$ and $\{e_{i+1}, n_{i+1}, o_{i+1}\}$: two local frames at the two ends of the segment. l_i, Φ_i, R_i, κ_i: length of the segment, orientation of the plane containing the arc, radius and curvature of the segment. $\{e_1, e_2, e_3\}$ and $\{d_1, d_2, d_3\}$: The basic vectors of the world frame $\{W\}$ and moving local frame (or reference frame) $\{M\}$. L: arclength of the centerline $\boldsymbol{r}(s, t)$. r: radius of the Cosserat rod

Cosserat rods interacting with each other and their environments [43].

In recent years, a novel modeling method called Discrete Elastic Rods (DER) has emerged based on the Kirchhoff theory of elastic rods. DER has been proven to be effective in simulating slender rods [44–46], e.g., knots [47], flagella [48], tendril [49], and gridshells [50]. In DER, the material frame is represented by its angular deviation from the natural Bishop frame, which constitutes the kinematic description. The normalized discrete curvature between two consecutive edges is defined as $\kappa_i = 2\tan(\phi_i/2)$, where ϕ_i represents the turning angle, as shown in Fig. 1(a3). By treating the centerline and constraints quasi-statically, combined with parallel transport and timestep updates, the discrete equations of motion can be established, incorporating the total elastic energy $E(\Gamma)$ which consists of discrete stretching, bending, and twisting energies: $E(\Gamma) = E_{\text{stretching}}(\Gamma) + E_{\text{bending}}(\Gamma) + E_{\text{twisting}}(\Gamma)$. DER has been validated as a highly efficient simulation tool in the discrete differential geometry community.

An example of DER application can be seen in Fig. 1(a3) and (b3), where a star-shaped rolling robot composed of Shape Memory Alloy (SMA) limbs was modeled using DER. The elastic energy resulting from the strains was represented by a linear sum of stretching and bending energy [9]. DER has also demonstrated excellent performance in establishing state-space models for a polychaete worm-inspired soft body [51] and an untethered sea star-inspired soft robot [52]. Thanks to its high computation efficiency and modeling accuracy, DER has been implemented to guide the design of an untethered frog-inspired soft robot to achieve faster locomotion speed [6]. A trajectory library capturing the dynamics of the frog robot based on DER simulation is generated and utilized to achieve real-time online path planning [6].

Three-Dimensional Models

Despite their convenience, ROMs often rely on simplification assumptions and may fail to accurately capture the detailed mechanical behaviors of soft robots. They can suffer from limited accuracy and a lack of precise distribution of internal strain and stress. In the case of 2D structures, such as plates or shells, which deform in 3D space but have a much smaller thickness compared to their planar dimensions, we will not delve into them further as most of these robot structures can be effectively simulated using higher computational efficiency through reduced-order 1D models, such as the shell-like legs of a soft rolling robot described in [9]. It is worth noting that these 2D models differ from existing planar or 2D mechanical models, which essentially assume motion within a single plane [8, 53, 54]. In this section, our focus will primarily be on discussing the 3D modeling frameworks and related issues.

3D Framework

While 1D ROMs are commonly used to simulate most 3D soft robots, their prediction accuracy tends to decrease, and the complete mechanical behaviors cannot be fully captured due to simplifications made in geometry, structure, boundary constraints, and other aspects. In scenarios where precise control or manipulation is required, the utilization of 3D modeling frameworks becomes necessary to effectively address various mechanics problems, especially those involving complex structures. In the development of ROMs, 3D frameworks serve as valuable reference tools to validate proposed models. Moreover, 3D frameworks find widespread application in assisting and optimizing the design of sensors, actuators, and control observers, as depicted in Fig. 3(a) and (b), since they can offer visual representations of the distribution of internal strain and stress, interactions with the surrounding environment, and the impact of materials, geometries, and other factors [55–57].

Currently, 3D models are commonly established using FEM, which involves dividing a large system into smaller finite elements through space discretization. The typical workflow of FEM includes steps such as geometrical structure modeling, configuration and constraint definition, meshing, analysis settings, solving, and post-processing. By employing FEM-based static models, the precise visualization and estimation of the exterior workspace boundary can be achieved, facilitating optimization-based control [62]. For instance, unlike the 1D ROM approach used in [63], the soft bellow actuators in [56] were modeled using FEM to explore their mechanical characteristics along different latitudes, enabling the realization of precise control based on pressure and action curves. In many cases, the presence of factors such as robot gravity, loadings on actuators or motors, and contact forces with internal components or external environments introduces significant errors when using kinematic models with ROMs. FEM models have been found effective in addressing these challenges [64]. Furthermore, for accurate simulation of tensegrity topologies, both bending degrees of freedom and regional elongation need to be taken into account in order to capture bending and contraction motion patterns. Tensegrity structures can be decomposed into multiple components (including struts, springs, and cables) and nodes, where the generalized coordinates of each element are chosen as the sum set of the position vectors of two nodes, forming a positional formulation within FEM [65].

Model Order Reduction

The computational complexity associated with high-dimensional models is a significant drawback that limits the widespread application of FEM, particularly in real-time control scenarios. To address this challenge, one feasible solution

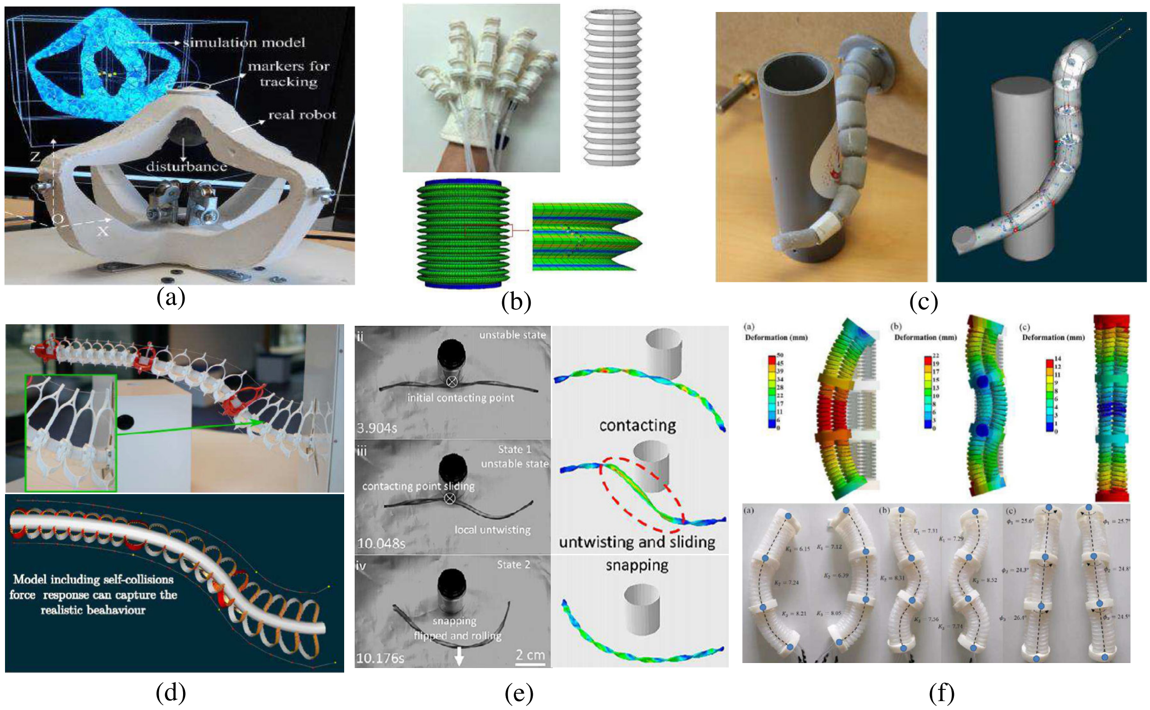

Fig. 3 3D models for simulation of soft robots. **a** A parallel soft robot and its simulation model based on real-time FEM [55]. **b** FEM-based simulation for the optimal design of a pneumatic bellow actuator used in soft-robotic glove [56]. **c** Simulation of a tentacle robot using ROM techniques [58]. **d** Simulation of a flexible manipulator robot with MOR technique applied to tackle the self-collisions [59]. **e** Simulation of soft body-obstacle interactions with Abaqus [60]. **f** Simulation of a bionic spine motions with Ansys [61]

is to replace highly nonlinear mappings, complex evaluation methods, or time-consuming computation matrices with approximate counterparts. For instance, the real-time FEM developed in [55, 66] adopts the linear elasticity hypothesis, enabling local pre-computations that accelerate online execution. Another popular approach is the use of Model Order Reduction (MOR) techniques, which aim to establish a model with a minimal number of DOF while minimizing the loss in accuracy (Fig. 3(c)) [58, 67]. In most MOR methods, a learning phase is employed, during which snapshots of the full-order model for different parameters are computed.

Among a variety of MOR methods, proper orthogonal decomposition is widely adopted. As a data-driven method, proper orthogonal decomposition utilizes two orthogonal projectors to decompose the large-scale vector into a reduced order state and a neglected state [68]. To preserve the stability and mechanical structure properties, the proper orthogonal decomposition projectors could be modified [69]. However, when intermittent contacts are present, where contact areas or locations vary, data-driven MOR approaches like proper orthogonal decomposition are not applicable due to the unavailability of intermediate mechanical variables. In such cases, Balanced Model Reduction has been effective by allowing switching among the defined reduced-order models [70]. In the treatment of self-collisions, MOR could be utilized to reduce the contact force space by projecting the force onto a reduced positive space, as shown in Fig. 3(d) [59]. A data-driven hybrid MOR strategy was proposed which applied proper symplectic decomposition and proper orthogonal decomposition to derive the smooth force and nonsmooth force respectively, resulting in a 65.89% [71]. Furthermore, a real-time dynamic model of a cable-driven continuum robot was established based on the principle of virtual power, considering cable constraints and friction effects [72].

Off-the-Shelf Platform

Although there are numerous off-the-shelf simulation tools available, most of them lack support for the specific actuation methods used in soft robots, such as cable-driven, tendon-driven, and pneumatic actuators, making them unsuitable for modeling soft robots. However, several conventional FEM platforms are well-suited for soft robot simulation and are widely utilized, including Abaqus, Ansys, and COMSOL. For instance, Abaqus is extensively adopted for establishing FEM models and analyzing the spatial distribution and variation of mechanical quantities (Fig. 3(e)) [60, 73, 74]. Ansys is powerful in simulating soft structures as well as interactions with surrounding environments (Fig. 3(f))

[61, 75]. COMSOL offers a broad range of material properties for users to select from and is particularly effective in handling multi-physical fields [76–78]. SOFA (Simulation Open Framework Architecture), a popular open-source simulator, has also been widely used by researchers for developing soft robots and simulation software [58, 79, 80]. In addition, several other simulators are available for soft robots, e.g., Bullet Physics Library [81], ChainQueen [82], Gym [83], SOMO [84], and ANCF [85]. Refer to [86•] for more details about off-the-shelf simulators.

Machine Learning-Based Models

Conventionally, the modeling of soft robots involves analytical or numerical techniques, which require solving a series of ODEs or PDEs, resulting in time-consuming and cumbersome derivations. In contrast, ML-based models, including artificial neural networks, offer a data-driven or surrogate approach to address complex and intractable issues. For simpler mapping relationships, shallow artificial neural networks like multi-layer perceptron networks, feedforward neural networks, and radial basis function networks can be directly employed to estimate unknown or difficult-to-measure intermediate variables. These variables may include unknown functions arising from model order reduction, robot configuration, or Jacobian matrices for forward and inverse kinematics [87–89]. For capturing complex functions or intricate mechanical relationships using ML, deep learning methods are often preferred, with Long Short-Term Memory being a typical representative [89, 90]. Deep learning-based controllers have found numerous applications, among which Deep Reinforcement Learning stands out as a successful control model. Deep reinforcement learning has been applied to teach underwater soft robots how to swim or to help robots master manipulation skills [79, 91]. The combination of RL and the Cosserat rod-based simulator, Elastica, has been validated through a series of applications [43]. The above modeling methods are summarized in Table 1.

Interaction Models

Couplings involved in the soft robot, and the interactions with surrounding environments, such as frictional contact, fluid-structure interaction, and multi-physics fields, are of significance in establishing a precise mechanical model.

Table 1 Summary of the modeling methods and their characteristics

Modeling method		Theory/assumptions	Applicable type of soft robot	Pros	Cons
1D ROM	Planar Beam Theory	Euler-Bernoulli beam method, rigid cross-section of the beam remains normal to axis	CDSR, continuum robot, etc	Simplification of linear theory of elasticity. High computational efficiency	Transverse shear strain is not considered. Inapplicable to thick-beam structures
	PCC model	Major structure is approximated by connected tangent arcs with constant curvatures	CDSR, bending actuators, tendon-driven continuum robot etc	Reduced computational complexity. Advantageous in simulating curve-like structures	Ideal PCC assumption is difficult to satisfy. Numerical instability
	Cosserat Rods	Kirchhoff's rod theory. Length is significantly larger than the radius	Slender-rod structure, cooperative or parallel robot, SMA-driven soft actuator	Easy implementation, High computational efficiency	Only valid for slender structures
3D model	FEM	Large system is divided into finite elements. Procedures: modeling, meshing, settings, solving etc	Arbitrary robot structure	Broad applicability. High accuracy. General and local details available	Tedious computation process. Expensive time cost
ML-based model	ANN, RBF, LSTM, RL etc	Intermediate variables or parameters are approximated with data-driven regression models	Arbitrary robot structure	Able to capture mappings or predict parameters that are difficult to handle with traditional mechanics	Requirement of large-size dataset. Affected by the network architecture, parameters and training data

Frictional Contact

To simplify the calculation of friction models, the static and kinetic coefficients of friction were often assumed to be equal [40]. At times, the friction forces are assumed to be equal to the maximum static friction force, which is proportional to the normal force [24].When considering the contact and sliding between driving cables and guiding channels of CDSR, both continuous saturated viscous friction models and Coulomb friction models can be employed. The former offers higher accuracy, while the latter is simpler to implement (Fig. 4(a1) and (b1)) [22, 93]. Anisotropic characteristics of snakes on heterogeneous terrains were explored using the Coulomb friction model to capture the frictional effects [94].

For ground-traveling soft robots, friction plays a vital role in facilitating motions such as crawling, burrowing, and locomotion of earthworm-like soft robots. When soft robots move on steep inclines or vertical walls, precise modeling of the friction force becomes particularly crucial, and the stick-slip effect deserves special attention [95, 96]. Additionally, estimating the contact status, location, and force in varying or complex contact situations has been the focus of intensive research [16, 40, 97]. Incremental potential contact method and maximum dissipation principle, originally developed in the computer graphics community [98] have also been employed for modeling the frictional contact dynamics of soft robots due to their computational efficiency [99].

Fluid-Structure Interaction

In the design of underwater soft robots or soft bodies confined in fluid-filled spaces, it is essential to consider fluid-structure interaction. Typically, the fluid is assumed to be stagnant or flowing in the low Reynolds number regime. In [100], a robotic fish was modeled as a 2D swimming elastic beam, with contractive strains imposed on two sides periodically. To simulate the locomotion of a magnetic soft millirobot in a fluid-filled environment, the computational fluid dynamics (CFD) model was established to mesh the midplane of the robot with shell elements [101]. Fluid flow generated by robot motion can be modeled using Stokes equations and computed by boundary element method [102]. In scenarios where soft robots move in incompressible and irrotational flow, the potential-flow theory and unsteady vortex-shedding method can be employed [103].

To capture the dynamic locomotion of bacteria-inspired soft robots, the DER model was combined with both Lighthill Slender Body theory and Regularized Stokeslet Segments formulation for single/multiple flagellar propulsion [52, 104]. Furthermore, the Fluid-Structure Interaction interface can be applied to FEM models, enabling the incorporation of multiple material and flow properties [76]. In addition to conventional simulators, newly emerged simulators, such as a differentiable soft-body simulator DiffPD, can be utilized

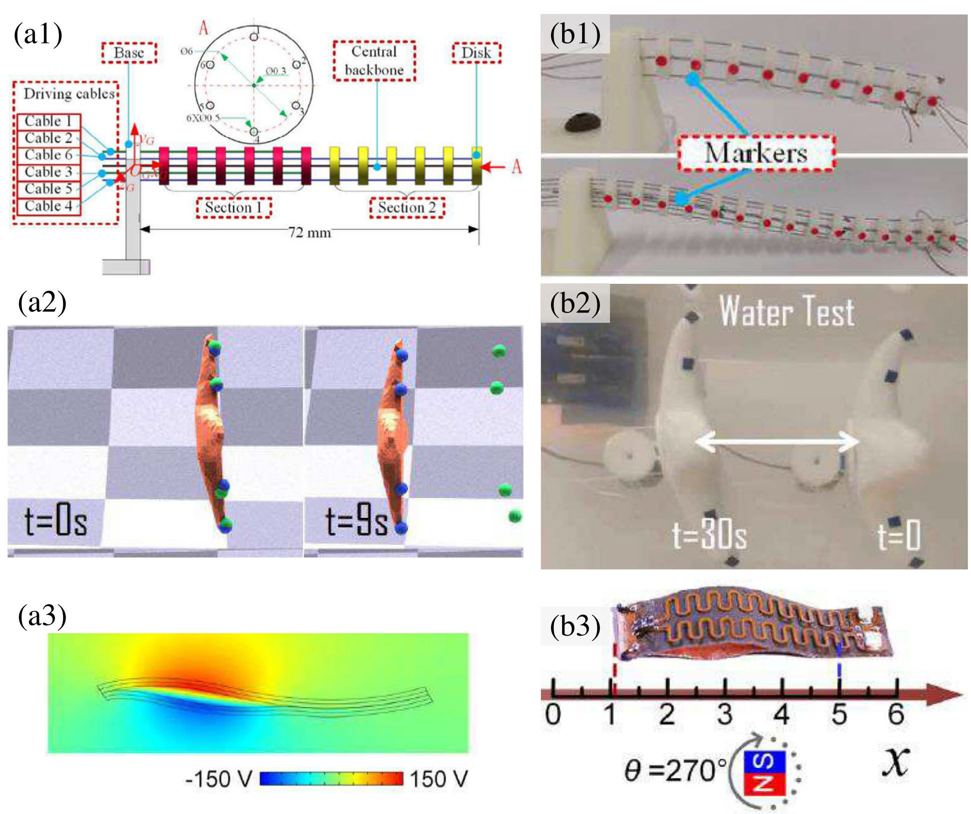

Fig. 4 Three interaction examples of soft robots. **a1** The schematic of multi-section planar continuum robot in (**b1**) [22]. **a2** The simulated motion of a Starfish robot in (**b2**) using DiffPD [92]. **a3** Simulated potential distribution under open circuit condition from FEM for a slug-inspired magnetic soft millirobot in (**b3**) 4

for simulating an underwater starfish robot [92] (Fig. 4(a2) and (b2)).

Multi-Physical External Fields

The incorporation of multi-physical external fields opens up possibilities for designing soft robots with novel mechanisms and control methods. One area that has received significant attention recently is magnetically actuated soft robots, which offer flexible remote control and promising applications in biomedicine [102, 105]. Simulating static magnetic fields is relatively straightforward, often achieved by using constant magnetic flux. Distributed magnetic torques can be computed and combined to generate opposing and tangential surface forces [106]. The Lorentz force and torque acting on the body magnet can also be integrated to simulate magnetic interactions [107].

For complex structural robots or intricate interactions, FEM tools are commonly employed. COMSOL Multiphysics software, for instance, has been widely used to simulate various physical environments. For example, it has been used to analyze the potential distribution under open-circuit conditions for a slug-inspired magnetic soft millirobot [105] (Fig. 4(a3) and (b3)). Additionally, a 3D Helmholtz coil model was developed to generate a uniform magnetic field[108], and a magnetoelastic rod model was utilized for dynamic analysis of cilia carpet robots [109]. Heat-actuated robots have emerged based on an electrical-thermal-mechanical mechanism. Analyzing radiative heat transfer and modeling the heater actuator have been investigated in this context [77, 110]. These advancements enable the design and control of soft robots with heat-responsive behaviors.

Conclusion

In conclusion, the development of kinematic and dynamic models for soft robots has gained significant attention in recent years. These models offer cost-effective simulation environments for optimizing robot designs and expediting the overall development process while validating their performance. Among various modeling approaches, ROMs have emerged as a preferred choice in the mechanics community, which can be easily tailored to suit specific soft robot configurations and boundary constraints. They offer advantages in terms of running speed and flexibility in adjusting model parameters. Most of these models are established for relatively simple geometries or structures that can be simplified into 1D or 2D models. For complex structures, FEM is a better option, with numerous off-the-shelf platforms available. To construct accurate models, careful consideration should be given to internal and external forces, including contact forces, friction, and other interaction forces.

While cable- or tendon-driven robots currently dominate the field of soft robots, various actuators based on novel materials or actuation mechanisms are emerging. Bio-inspiration remains a prevalent source for robot design. ML-based methods, particularly deep learning, offer the possibility to estimate unknown intermediate variables and approximate complex mapping relationships. Combining deep learning with conventional mechanics theories leads to more precise models. The integration of deep learning and model-based control holds promise for intelligent and robust control of soft robots. In a long run, the computational efficiency and modeling accuracy are always pursued and it dominates the development of robot simulators. In order to improve the computation speed and realize real-time simulation, how to establish a light-weight model without performance degradation is a promising trend, which is expected to motivate the surge of various model order reduction techniques. Meanwhile, a great deal of attentions will be paid to the extension of typical modeling theories and the fusion of different simulators, thus increasing the model accuracy. Along with the prosperity of deep learning in many fields recently, the demands for kinematic and dynamic modeling of soft robots vastly increase, which will alleviate the rigorous requirement of data amount, provide precise virtual training environment and guide their optimization as physical engines.

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Data Availability No datasets were generated or analyzed during the current study.

Declarations

Human and Animal Rights and Informed Consent This article does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest The authors declare no competing interests.

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Papers of particular interest, published recently, have been highlighted as: • Of importance •• Of major importance

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