

A stochastic multi-range robust approach for low carbon technology participation in electricity markets

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ABSTRACT

Ambitious emission reduction targets require fostering more low-carbon technologies (LCTs) in distribution networks. Projections for future energy use predict a significant implementation of these technologies in residential areas. Despite this, individually they cannot effectively participate in electricity markets. This study examines the potential participation of residential LCTs (RLCTs) in multiple electricity markets, including wholesale day-ahead, real-time, and local energy markets (LEM), through the aggregators. We propose a stochastic weighted multi-range robust model to provide a strategy for RLCT aggregators to function as both sellers and buyers in these markets, as price-makers in LEM and price-takers in wholesale markets. The proposed model accounts for the uncertainty associated with the effect of offers/bids on the market clearing price of LEM and the availability patterns of aggregated LCTs. Results of a case study using realistic data reveal that the proposed approach results in higher overall profits compared to both risk-neutral and risk-averse robust methods. Furthermore, the introduced model is resilient to forecast errors, as evidenced by a 12% decrease in profits with the proposed approach compared to a 26% decrease with a risk-neutral strategy when the forecast error was increased by 20%.

1. Introduction

New decarbonization targets are making conventional power grids undergo radical changes to satisfy these requirements [1]. For instance, the UK recently revised its target to reach zero emissions in the electricity sector for 2035. A similar target has been set for other countries in different parts of the world, including Germany, France, Japan, Canada, and Chile [2,3].

One sector with significant untapped potential to accelerate the process of reaching these targets is residential. The depicted future energy scenarios by distribution companies in the UK demonstrate noticeable uptake of low carbon technologies (LCTs) such as electric vehicles (EVs) and heat pumps (HPs) in the residential part of distribution systems [4]. For instance, in one of these scenarios called the 'leading-the-way scenario', the number of EVs in Northern Powergrid's network, a UK distribution network operator, which is geographically responsible for approximately twenty percent of the UK, will be more than three million in ten years. These small-scale LCTs if considered individually cannot provide noticeable flexibility for the grid operator. However, the coordinated operation of such assets for instance, through an aggregator can offer significant amounts of flexibility that can benefit both the

asset owners by participating in the available electricity markets and the grid operators/market facilitators by helping in relieving network problems.

1.1. Literature review

Several references anticipate the co-existence of multiple markets present in or originating from distribution networks such as local energy markets (LEM), and wholesale day-ahead markets [5–10]. It is therefore reasonable to assume that in such a case aggregators of residential LCTs in distribution networks would gravitate toward participation in multiple markets in order to maximize their profits and benefits to their customers.

The review of existing literature in this regard reveals that there is a gap in comprehensive models that can account for participation in multiple markets while addressing the uncertainties related to the effects of offers/bids on the market clearing price (MCP) in the different electricity markets as pointed out in Table 1. Ref. [5] provides some insights regarding the participation of large-scale distributed energy

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Table 1
Taxonomy of LCTs integration in multiple electricity markets.

Ref. no	Asset type	Market type			Market-Role		Market impact	
		RLCT	DAWEM	RTWEM	LEM	Seller	Buyer	Model
[5]	x	✓	✓	✓	✓	x	BL ^a	x
[6]	✓	x	✓	✓	x	✓	x	x
[7]	x	✓	x	x	✓	✓	x	x
[8]	x	✓	✓	x	x	✓	x	x
[9]	x	✓	✓	x	✓	x	x	x
[10]	x	✓	x	✓	✓	x	BL ^a	x
This study	✓	✓	✓	✓	✓	✓	PQC ^b	✓

^a Bi-level optimization.

^b Price quota curve.

resources (DERs) in multiple markets considering only generation assets. However, such a study cannot be applied to residential LCTs (RLCTs) as both buying and selling roles have to be incorporated in the market-participation model (i.e. some assets like photovoltaic (PV) are only generating power, and some such as HPs will be only consuming energy. EVs can have both roles thus overall at a specific hour of the day and depending on the net flow of the scheduled aggregated assets, the RLCT aggregator could sell or buy energy).

It should be noted that the role of RLCT aggregators in these markets is going to be different. Concerning the size of aggregated assets, the RLCT aggregator will be a price-taker in the wholesale markets therefore not impacting the MCP. However, in the LEM, the RLCT aggregator is a price-maker. These different roles need to be addressed while designing a participation strategy for these markets. For the price-taker role, a prediction of the market price can be incorporated into the offering model. In the case of price uncertainty, probability density function, and scenario-based stochastic approaches are well studied in the literature [11,12]. While a price-taker role is more straightforward, the price-maker needs an economic model to incorporate the impact of offers/bids on the market price as well. There are several approaches used in this regard [13]. First, game-theory-based methods aimed to find an equilibrium of a repetitive game as demonstrated in [14,15]. The second group of methods is the bi-level optimization [5,16]. The first level of the optimization contains a profit maximization/cost minimization from the viewpoint of the main participant (the price-maker) and the second level of the optimization runs the market clearing process. For instance, in [5] the first level contains maximization for an aggregator that participates in the electricity markets. The second level outputs the market clearing price. To solve the bi-level optimization the duality theory and the Karush–Kuhn–Tucker optimality conditions are utilized to convert optimization to a single optimization solvable by the existing commercial solvers. Authors in [16] present a model for the participation of storage facilities in the energy and reserve market formulating the problem as a mathematical programming with equilibrium constraints. Another group of approaches is the agent-based methods [17]. In this group, several agents are considered to recreate market conditions and simulate electricity market operations and interactions.

It is important to note that despite an existing rich literature on different LEMs focusing on the clearance methodologies and market price determination important issues such as strategic bidding and bid/offer creation are not adequately addressed [18]. The aforementioned methodologies based on game theory and agent-based approaches are most suited to provide the analysis of strategic behavior rather than developing a tool for offering/bidding strategies. In addition, bi-level-based approaches have been demonstrated to lead to complex mathematical formulation and high computational burden [19]. Most importantly these approaches rely on full information of all participants of the market which seems to be unrealistic for establishing a market participation strategy.

An alternative approach is to utilize price quota curves (PQCs) to understand the impact of additional generation (GPQC) or demand (DPQC) on the market clearing price [20–22]. Ref. [20] applies GPQC

for the operation planning of a hydro producer which is a price maker in the wholesale electricity market. In [21] GPQCs are used for the self-scheduling problem of a generating company. Ref. [22] utilizes both types of GPQC and DPQC in the economic assessment of a price-maker storage unit. PQC-based approaches have simpler formulations and thus do not carry the computational complexity of bi-level optimizations. However, the construction of an exact forecast of a PQC is difficult due to the inevitable uncertainties of the market and its participants. Refs. [23–25] use scenario-based stochastic method and [13] try to combine the probability density function (PDF) with PQC in order to capture the uncertainty of PQCs. However, these statistical approaches are highly dependent on historical data and require an accurate PDF for PQC. In [26] a model based on the robust optimization is proposed which tends to be conservative as it does not consider conservativeness limitation tools (i.e. budget of uncertainty) and at each time the worst cases of price predictions are incorporated into the model. In addition, their proposed approach is not capable to consider uncertainties of the aggregated RLCTs' power output and availability.

In summary, the existing literature lacks in terms of investigating the potential of participation of RLCTs aggregator in multiple markets (e.g. day-ahead (DA) wholesale market (WEM) and LEM), simultaneous integration of generation and consumption of RLCTs, and consideration of RLCT aggregators as strategic players in the related electrical markets with the associated uncertainties. In this paper, we aim to cover these gaps by developing an extensive model for the simultaneous offering and bidding strategy of RLCT aggregators to three types of electricity markets namely DA WEM, real-time (RT) WEM, and LEM. We aim to provide RLCT aggregators with a tool to handle the prevailing uncertainties while maximizing profit by employing the flexibility of their assets and opportunities in multiple electricity markets.

Although we have deliberately limited the scope of this paper to the commercial aspects of aggregator operation, we recognize the critical importance of addressing grid-related issues associated with aggregated assets. The conventional approach of clearing markets with locational marginal prices, as commonly practiced in wholesale or LEM, may encounter obstacles due to the specific limitations of distribution networks. These networks, often operating at their capacity limits due to widespread electrification, are predominantly radial in structure. This conventional approach could result in a notable portion of transactions being rejected based on infrastructural limitations, despite their economic merit for market players. To advance equitable, economic, and sustainable energy systems, future research efforts must consider various coordination mechanisms. Coordination strategies encompass aligning LEM clearing with contracted flexibility services, leveraging network flexibility through methods such as reconfiguration, battery storage, and capacitor banks, all of which necessitate complete visibility of the LEM by DSO [27]. Recent innovative works, such as [28], propose flexibility mechanisms enabling the DSO to access LEM flexibility and coordinate it with the aforementioned strategies. Additionally, these studies highlight the effectiveness of flexible network technologies, such as soft open points, in supporting local markets while ensuring network security.

We would like to note that there are several business models for the aggregator-customer relationship [29,30], including trading flexibility in wholesale and local electricity markets to minimize the cost of energy or maximize reward and providing ancillary services to the TSO and DSO. In addition, the literature provides various combinations of models to maximize the benefit of distributed flexibility providing stacked applications [31]. In the present paper, as explained above we are considering that aggregators participate in the DA and RT wholesale markets, as well as the LEM. Aggregators act on behalf of their customers (i.e., prosumers) to buy and sell electrical energy to maximize the reward for producers and minimize the cost of energy for consumers.

1.2. Problem statement and paper contributions

We utilize the concept of PQCs and propose a stochastic weighted multi-range robust optimization model to account for the uncertainty of the other market participants' behavior and aggregated assets' output and availability patterns. In the proposed approach, the RLCT aggregator is a price-taker in the wholesale market. However, participates in LEM as a price-maker. Thereby, we model the effect of the offers and bids of the RLCT aggregator on LEM clearing price with PQCs, namely GPQC for the aggregated generation and DPQC for the aggregated demand. As mentioned earlier PQCs are subject to uncertainty. In approaches solely based on robust optimization, the worst-case decisions under the predefined uncertainty set could prove to be overly conservative. Thus, in this paper, considering the market's multi-step offering/bidding capabilities a weighted multi-range robust optimization framework is proposed to account for the uncertainties associated with PQCs. The uncertainty set is divided into several ranges where for each range, a corresponding robust optimization is solved. Note that, unlike the case of a single generation or a single consumption unit, for the case of an RLCT aggregator, the worst-case scenario of the GPQCs and DPQCs dependent on the state of assets requiring proper modeling. The resulting model is nested in a stochastic scenario-based model to deal with the complete set of uncertain parameters in the decision-making of the aggregator. The final model enables the aggregator to be less sensitive to price variation while exploiting fitting price fluctuations by incorporating less conservative actions. In conclusion, the contributions are as follows:

- (1) A novel stochastic weighted multi-range robust optimization is proposed. The proposed approach is a novel solution that models multiple sources of uncertainty faced by the aggregator in a comprehensive manner, namely the effects on LEM clearing price, PV generation and EVs availability. Most studies that consider participation in both local and wholesale electricity markets only provide offering (i.e. supply curves) strategies (Table 1). The proposed methodology provides a tool for the aggregator to produce supply and demand curves simultaneously from one round of optimization while accounting for price-maker/taker roles and the associated impacts on the market clearing price.
- (2) Comparative studies are conducted for the proposed approach and two conventional methods, risk-neutral and classical (risk-averse one-range) robust methods. The results demonstrate that the proposed solution delivers higher profits while maintaining resiliency in the presence of forecast errors. This superior performance is due to the proposed approach's ability to balance risk and reward more effectively than other methods, making it a more advantageous solution for the RLCT aggregator.

2. Mathematical formulation

RLCTs could have different forms like only generation units (e.g. PV), controllable loads (e.g. HPs), and storages (e.g. EVs) which can play both roles. In this study, we assume an aggregator that coordinates RLCTs to take part in three electricity markets including DA WEM, LEM, and RT WEM. The goal is to participate in these markets to maximize the profit of selling the excess energy in the markets while providing the required consumption. There are two timelines that separate decision variables into two different stages.

The first stage includes the here-and-now decision variables which are related to the participation of the aggregator in DA WEM and LEM. In the dynamic landscape of electricity markets as assumed in this paper, the RLCT aggregator assumes distinct roles as a price-taker in the wholesale market and a price-maker in the LEM, dictated by the intricate interplay of market dynamics. Acting as a price-taker in the wholesale market implies the RLCT aggregator's acceptance of market-determined electricity prices without wielding substantial influence. In this vast and competitive market, characterized by numerous

participants and significant power generators, the RLCT aggregator's individual capacity is modest compared to the overall market, rendering it a price-taker. Conversely, within the LEM, the RLCT aggregator transforms into a price-maker, exerting more control over local electricity pricing dynamics. LEMs, often smaller and more localized, provide an environment where RLCT aggregators can wield a more influential role. As the aggregator is a price-taker in the WEM, the only information that is required to be submitted to the DA WEM market is a single power quantity that the aggregator wants to sell or buy for each hour through the course of a day. After the market is cleared by the independent system operator, the MCP is announced to the aggregator. As this price is not known a priori, the aggregator needs to predict the values of price to derive optimal participation in the DA WEM energy market. At the same stage, it is required to establish an offering/bidding strategy for the LEM. For LEM, we presume the aggregator to act as a price-maker that could yield a multi-step offering/bidding curve. The problem dealt with in LEM is an economic offering/bidding therefore the information submitted to the local market operator will be in form of multi-price-power quantities at all time intervals for the next day. In addition, buyer or seller positions can be taken as well in the market depending on the available assets at each hour. Thus, both economic bidding and offering curves should be decided simultaneously in the decision-making problem while accounting for the uncertainty of the aggregated assets.

The aggregator participates in the real-time market considering different possible outcomes of uncertain parameters such as PVs output and EVs availability. The decisions in the second stage are the wait-and-see type, meaning that the previous stage variables are fixed (i.e. power sold to/bought from DA WEM and LEM). The variables of this stage are the adjustment power of controllable units and EVs. We assume that it is possible to change the consumption of controllable units (e.g. HPs) provided the temperature remains within the desired household temperature ranges. Also, EVs can be re-scheduled considering the fact that their availability may differ in different outcome realizations.

2.1. Non-linear deterministic model

The deterministic formulation for the participation of an RLCT aggregator in multi-markets can be formulated as given in (1). This model is a mixed-integer non-linear model which is then linearized as presented in (2).

$$\max \sum_{t \in \mathcal{T}} \left(\lambda_t^{DA} p_t^{Sel,DA} - \lambda_t^{DA} p_t^{Buy,DA} + p_t^{Sel,LEM} \lambda_t^{E,LEM} (p_t^{Sel,LEM}) - p_t^{Buy,LEM} \lambda_t^{E,LEM} (p_t^{Buy,LEM}) + \lambda_t^{RT} p_t^{Sel,RT} - \lambda_t^{RT} p_t^{Buy,RT} \right) \quad (1a)$$

$$\text{s.t. } p_t^{Sel,DA} + p_t^{Sel,LEM} + p_t^{Sel,RT} = p_t^{Sel,tot} \quad (1b)$$

$$p_t^{Buy,DA} + p_t^{Buy,LEM} + p_t^{Buy,RT} = p_t^{Buy,tot} \quad (1c)$$

$$0 \leq p_t^{Sel,tot} \leq P_t^{Sel,max} v_t^{Sel} \quad (1d)$$

$$0 \leq p_t^{Buy,tot} \leq P_t^{Buy,max} v_t^{Buy} \quad (1e)$$

$$v_t^{Sel} + v_t^{Buy} \leq 1 \quad (1f)$$

$$p_t^{Sel,tot} - p_t^{Buy,tot} = p_t^{PV} + p_t^{EV,dis} \eta_d - p_t^{EV,ch} - p_t^{HP} \quad (1g)$$

$$p_t^{PV} = \sum_m p_{t,m}^{PV} \quad (1h)$$

$$p_t^{dis} = \sum_i p_{t,i}^{dis} \quad (1i)$$

$$p_t^{ch} = \sum_i p_{t,i}^{ch} \quad (1j)$$

$$0 \leq p_{t,i}^{dis} \leq \bar{P}_i^{dis} \mu_{t,i} \quad \forall t \in [t_i^b, t_i^e] \quad (1k)$$

$$0 \leq p_{t,i}^{ch} \leq \bar{P}_i^{ch} (1 - \mu_{t,i}) \quad \forall t \in [t_i^b, t_i^e] \quad (1l)$$

$$SoC_{t,i} = \frac{E_i^0}{\bar{E}_i} \quad t = t_i^b \quad (1m)$$

$$SoC_{t,i} = \frac{E_i^0 + \sum_{\tau=t_i^b}^t (p_{\tau,i}^{ch} \eta_c - p_{\tau,i}^{dis})}{\bar{E}_i} \quad \forall t \in [t_i^b, t_i^e] \quad (1n)$$

$$\underline{SoC}_i \leq SoC_{t,i} \leq \overline{SoC}_i, \quad (1o)$$

$$\overline{SoC}_i = \frac{\min\{E_i^0 + (t_i^e - t_i^b) \bar{P}_i^{ch} \eta_c, \bar{E}_i\}}{\bar{E}_i} \quad (1p)$$

$$SoC_{t,i} = \overline{SoC}_i \quad t = t_i^e \quad (1q)$$

$$p_t^{HP} = \sum_j p_{t,j}^{HP} \quad (1r)$$

$$0 \leq p_{t,j}^{HP} \leq \bar{P}_j^{HP} \quad (1s)$$

$$p_{t,j}^{HP} = (C_j + \frac{\Delta t}{R_j}) \theta_{t,j}^r - C_j \theta_{t-1}^r - \frac{\theta_t^a \Delta t}{R_j} \quad (1t)$$

$$\theta_j^r \leq \theta_{t,j}^r \leq \bar{\theta}_j^r \quad (1u)$$

$$p_{t,i}^{ch} = p_{t,i}^{dis} = 0 \quad \forall t \notin [t_i^b, t_i^e] \quad (1v)$$

The first equation, (1a), in the above deterministic formulation depicts the objective function which is maximizing the acquired net profit (i.e. revenue - cost) from all markets. The positive terms are the revenue acquired by selling energy to and the negative ones are costs related to the energy bought from these markets. Here, λ_t^{DA} and λ_t^{RT} are the DA and RT market price at time t respectively. Also, p_t denotes the power that is scheduled to be sold to (*Sel*) or bought (*Buy*) from a specific market distinguished by the related superscript (DA, LEM, and RT). Note that the above objective function is non-linear due to the price-maker role of the aggregator in LEM. Values of power quantities affect the local energy market clearing prices meaning that $\lambda^{E,LEM}$ is a function of power and thus there are two non-linear terms in the above formulation namely $p_t^{Sel,LEM} \lambda_t^{E,LEM}(p_t^{Sel,LEM})$ and $\lambda_t^{E,LEM}(p_t^{Buy,LEM})$. In the rest of this section, we also present the required step to linearize the objective function.

The constraints addressing the market participation part are given in (1b)–(1f). The first two constraints define the total scheduled power to be sold to/bought from markets. The superscript *tot* denotes the total value. The next two constraints limit the maximum power which can be sold to, $P_t^{Sel,max}$, or purchased, $P_t^{Buy,max}$, from all markets. Here, v_t^{Sel} and v_t^{Buy} are binary variables associated with the role of RLCT aggregator in the markets. In (1f) it is ensured that only one of these binaries equals to one: when RLCT aggregator is selling energy to the electricity markets $v_t^{Sel} = 1$, and when buying $v_t^{Buy} = 1$. Constraint (1g) is the power balance of the aggregator at each time where η_d is discharging efficiency. Accordingly, the total energy sold to or bought from the market is equal to the produced power from the PV units, total discharged power from EV batteries minus the consumed power to charge EVs, and supply the total demand. The total power of PVs and EVs that are used in these constraints are defined in Eqs. (1h)–(1j). Here, i is for an EV and m represents a PV unit. The next set of constraints describes EV day-ahead scheduling. Charge/discharge limitations are given in (1k) and (1l). Here, $\mu_{t,i}$ is an auxiliary binary variable to avoid simultaneous charging and discharging of EVs in a specific time period. Note that $\mu_{t,i}$ is one if EV is discharging. The state of charge (SoC) at the start of the charging/discharging period, t_i^b , is provided in (1m) where E_i^0 is the initial energy of the battery and \bar{E}_i is the capacity. Constraint (1n) models the SoC for the charging/discharging period, $[t_i^b, t_i^e]$, having η_c as charging efficiency. The SoC is limited within a specific range provided in (1o). Note that depending on the availability period of an EV and the initial energy, having a fully charged battery at the end of each period may not be possible. Thus, in (1p) the maximum possible value of SoC, \overline{SoC}_i , for each EV is calculated then it is assumed at the end of the charging period, t_i^e the SoC of each battery should

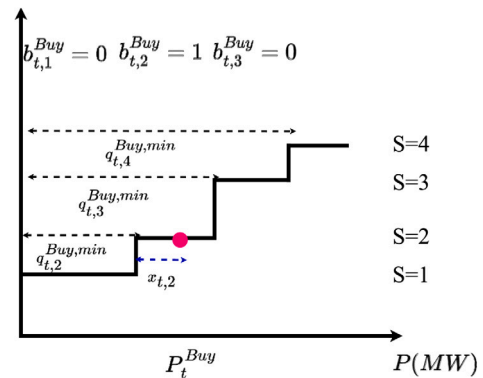


Fig. 1. A multi-step DPQC example utilized for the linearization of the optimization objective function.

match this value as demonstrated in (1q). Constraint (1r) shows the demand coming from HP units, $\sum_j p_{t,j}^{HP}$, j denoting a HP. The power of HPs is adjustable through the day at each hour (1s), however, it is related to the building thermal model as given in (1t). Here, C and R are thermal parameters of a building, θ^r is inside the building and θ^a is ambient temperature. The temperature may vary in the desired (i.e. comfort) zone determined by (1u). The last constraint, (1v) states the charging/discharging powers of EVs outside the availability period.

2.2. Linear deterministic model

The objective function of problem (1) is nonconvex, which makes it difficult to guarantee a global optimum. This subsection provides a method to convexify it, by linearizing it using the step-wise form of PQCs as depicted in Fig. 1 for a DPQC. The demonstrated DPQC includes four steps that depict the potential effect of bids on market price.

For each step, the price is fixed and the value of power can be determined with respect to the accumulated length of previous steps and a variable that varies within the length of the current step (s). Accordingly, power quantities can be rewritten as the summation of two linear terms as shown in (2b) for sold power to LEM and (2e) for the bought power. In other words, the power values are composed of fixed and variable terms. The fixed term, $b_{t,s}^{Sel} q_{t,s}^{Sel,min}$ and $b_{t,s'}^{Buy} q_{t,s'}^{Buy,min}$, depend on the PQC step, b is a binary that denotes a selected step. At each time interval, only one step can be selected (i.e. (2d) and (2g)), o is a binary representing the selling/buying status LEM) which gives a minimum power plus a variable term, x , whose value is limited to a certain range as provided in (2c) and (2f) for the GPQC and DPQC, respectively. Only one type of offer/bid is submitted at each time ensured in (2h). Finally, constraint (2i) depending on the selling/buying status at t determines the price, λ_t^E which is no longer a function of power.

$$\max \quad \sum_{t \in T} \left(\lambda_t^{DA} p_t^{Sel,DA} - \lambda_t^{DA} p_t^{Buy,DA} + \sum_{s \in S^{Sel}} \lambda_{t,s}^{Sel,LEM} (x_{t,s}^{Sel} + b_{t,s}^{Sel} q_{t,s}^{Sel,min}) - \sum_{s' \in S^{Buy}} \lambda_{t,s'}^{Buy,LEM} (x_{t,s'}^{Buy} + b_{t,s'}^{Buy} q_{t,s'}^{Buy,min}) - \lambda_t^{RT} p_t^{Sel,RT} - \lambda_t^{RT} p_t^{Buy,RT} \right) \quad (2a)$$

s.t. (1b)–(1v)

$$p_t^{Sel,LEM} = \sum_{s \in S^{Sel}} x_{t,s}^{Sel} + b_{t,s}^{Sel} q_{t,s}^{Sel,min} \quad (2b)$$

$$0 \leq x_{t,s}^{Sel} \leq b_{t,s}^{Sel} q_{t,s}^{Sel,max} \quad (2c)$$

$$\sum_{s \in S^{Sel}} b_{t,s}^{Sel} = o_t^{Sel} \quad (2d)$$

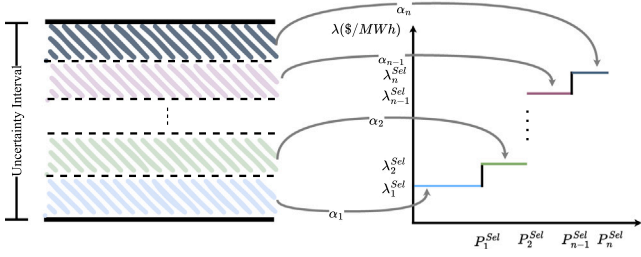


Fig. 2. Mapping example of a multi-range uncertainty set to a multi-step offer curve.

$$p_t^{Buy,LEM} = \sum_{s' \in S^{Buy}} x_{t,s'}^{Buy} + b_{t,s'}^{Buy} q_{t,s'}^{Buy,min} \quad (2e)$$

$$0 \leq x_{t,s'}^{Buy} \leq b_{t,s'}^{Buy} q_{t,s'}^{Buy,max} \quad (2f)$$

$$\sum_{s' \in S^{Buy}} b_{t,s'}^{Buy} = o_t^{Buy} \quad (2g)$$

$$o_t^{Sel} + o_t^{Buy} \leq 1 \quad (2h)$$

$$\lambda_t^E = \left(\sum_{s \in S^{Sel}} b_{t,s}^{Sel} \lambda_{t,s}^{Sel,LEM} + \sum_{s' \in S^{Buy}} \lambda_{t,s'}^{Buy,LEM} b_{t,s'}^{Buy} \right) \quad (2i)$$

2.3. Price-maker uncertainty characterization

In this section, we propose a model for the optimal offering strategy of a price-maker in the electricity markets under uncertainty based on PQC. We consider the participation of the RLCT aggregator in LEM as a strategic player. We develop a simultaneous multi-step offering and bidding curve construction methodology while addressing the uncertainties associated with both GPQCs and DPQCs. In the next section, we incorporate this model into the overall participation strategy of the RLCT aggregator for all markets. Suppose that the RLCT aggregator is participating in the local market as a price-maker.

It is difficult to make an exact prediction and have accurate PQC because knowing the realized accepted values of offers/bids a-priori is complicated. Consequently, the RLCT aggregator needs to consider the uncertainty of PQC while deriving the offering/bidding strategies. We utilize the concept of robust optimization and consider an interval around the forecasted PQC. Choosing a robust approach has the advantage of not requiring the distribution information for price and power quantities while delivering more computational tractability. However, the typical risk-averse robust methods yield over-conservative outputs due to two main factors namely: large forecast ranges (i.e. interval length) and always the consideration of worst cases. This over-conservativeness is not desirable for an agent like an RLCT aggregator as it also likes to explore future realizations that could deliver more revenues. Therefore, we divide the uncertainty interval into multiple smaller ranges spanning realizations from conservative to optimistic ones. We propose a robust approach considering a multi-range set as depicted in Fig. 2 alongside with polyhedral uncertainty model with a budget of uncertainty for each range. The uncertainty interval (i.e. the interval around the predicted PQC) is divided into several ranges. As can be seen, each range is mapped into one step of the offering curve for which the robust optimization (3) with the budget of uncertainty is solved. The provided figure specifically describes how offers can be constructed from a multi-range uncertainty set. A similar procedure can be considered for building the bids. For bids, the first top range of the uncertainty set is mapped to the first step of the bidding curve and this process is repeated for the remaining steps. In the objective function of model (3), it is assumed that the values of price coming from PQC are

uncertain varying in intervals provided in (3a) and (3b). Here, $\hat{\lambda}_{t,s}$ is the uncertain price of step s, s' at time t .

$$\max \sum_{t \in T} \left(\sum_{s \in S^{Sel}} \hat{\lambda}_{t,s}^{Sel,LEM} (x_{t,s}^{Sel} + b_{t,s}^{Sel} q_{t,s}^{Sel,min}) - \sum_{s' \in S^{Buy}} \hat{\lambda}_{t,s'}^{Buy,LEM} (x_{t,s'}^{Buy} + b_{t,s'}^{Buy} q_{t,s'}^{Buy,min}) \right) \quad (3a)$$

$$\text{s.t. (1b)–(1v), (2b)–(2i)}$$

$$\hat{\lambda}_{t,s}^{Sel,LEM} \in [\lambda_{t,s}^{Sel,LEM}, \bar{\lambda}_{t,s}^{Sel,LEM}] \quad (3b)$$

$$\hat{\lambda}_{t,s}^{Buy,LEM} \in [\lambda_{t,s}^{Buy,LEM}, \bar{\lambda}_{t,s}^{Buy,LEM}] \quad (3c)$$

The model described in (3) gives the maximization of the profit of RLCT aggregator in the LEM market. Here, the objective function is the difference between revenue acquired from selling energy to the market and the cost of buying energy depending on a specific period. The market price is no longer deterministic and as displayed in constraints (3b) and (3c) is assumed that could take values within a predetermined interval. With consideration of the uncertainty budget [32], Γ , the robust counterpart of the above problem is presented in (4).

$$\max z \quad (4a)$$

$$\text{s.t. (1b)–(1v), (2b)–(2i)}$$

$$z \leq \sum_{t \in T} \left(\sum_{s \in S^{Sel}} \lambda_{t,s}^{Sel,LEM} (x_{t,s}^{Sel} + b_{t,s}^{Sel} q_{t,s}^{Sel,min}) - \sum_{s' \in S^{Buy}} \lambda_{t,s'}^{Buy,LEM} (x_{t,s'}^{Buy} + b_{t,s'}^{Buy} q_{t,s'}^{Buy,min}) - \left(\sum_{s \in S^{Sel}} \sigma_s^{Sel} \Gamma^{Sel} + \sum_{s' \in S^{Buy}} \sigma_{s'}^{Buy} \Gamma^{Buy} + \sum_{t \in T} \left(\sum_{s \in S^{Sel}} \phi_{t,s}^{Sel} + \sum_{s' \in S^{Buy}} \phi_{t,s'}^{Buy} \right) \right) \right) \quad (4b)$$

$$\sigma_s^{Sel} + \phi_{t,s}^{Sel} \geq l_{t,s}^{Sel} y_{t,s}^{Sel} \quad (4c)$$

$$\sigma_{s'}^{Buy} + \phi_{t,s'}^{Buy} \geq l_{t,s'}^{Buy} y_{t,s'}^{Buy} \quad (4d)$$

$$-y_{t,s}^{Sel} \leq p_{t,s}^{Sel} \leq y_{t,s}^{Sel} \quad (4e)$$

$$-y_{t,s'}^{Buy} \leq p_{t,s'}^{Buy} \leq y_{t,s'}^{Buy} \quad (4f)$$

$$y_{t,s}^{Sel}, \sigma_s^{Sel}, \sigma_{s'}^{Buy}, \phi_{t,s}^{Sel}, \phi_{t,s'}^{Buy} \geq 0 \quad (4g)$$

This mathematical representation is derived by utilizing the duality properties and exact linear equivalences. Note that z is the new objective function and the new variables σ and ϕ are the dual variables related to the price bound and uncertainty set characterization by the budget of uncertainty for both selling and buying prices. In addition, y is an auxiliary variable and parameters $l_{t,s}^{Sel}$ and $l_{t,s'}^{Buy}$ represent the maximum deviation from the predicted values (the uncertainty interval is considered to be symmetric: $\lambda_{t,s}^{Sel,LEM} \pm l_{t,s}^{Sel}$, $\lambda_{t,s'}^{Buy,LEM} \pm l_{t,s'}^{Buy}$) for each step and time. Predetermined Γ^{Sel} and Γ^{Buy} are the budget parameters that can take value in $[0, T]$ where $\Gamma = T$ represents maximum conservativeness and $\Gamma = 0$ is equal to the deterministic model.

The above problem gives the robust optimization associated with one range. We expand this formulation to include all ranges by utilizing the weighted sum approach and introducing the parameter α_k and k ranges. This parameter can be tuned by the decision-maker considering confidence in a price forecast range. Thus:

$$\max \sum_k \alpha_k z_k \quad (5a)$$

$$\text{s.t. (1b)–(1v), (2b)–(2i), (4b)–(4g) \quad \forall k}$$

$$\sum_k \alpha_k = 1 \quad (5b)$$

$$\hat{\lambda}_{t,k}^{Sel,LEM} \geq \hat{\lambda}_{t,k-1}^{Sel,LEM} \quad (5c)$$

$$\hat{\lambda}_{t,k}^{Buy,LEM} \leq \hat{\lambda}_{t,k-1}^{Buy,LEM} \quad (5d)$$

$$p_{t,k}^{Sel,LEM} \geq p_{t,k-1}^{Sel,LEM} \quad (5e)$$

$$p_{t,k}^{Buy,LEM} \geq p_{t,k-1}^{Buy,LEM} \quad (5f)$$

By solving the above optimization the offering and bidding curve at each time period can be extracted. The Eqs. (5a) depicts the objective function. Constraint (5b) ensures that the summation of all given α_k s equals unity. It is important to mention that the RLCT aggregator as a price-maker market participant is only allowed to submit step-wise non-decreasing offering curves and step-wise non-increasing bidding curves to the market. Thus, extra conditions (5c)–(5f) are added to the optimization to guarantee the explained feature for both of the submitted curves. Note that the values of price for each offering/bidding curve step can be recovered from (5) as demonstrated in (6).

$$\hat{\lambda}_{t,k}^{Sel,LEM} = \sum_{s \in S^{Sel}} \lambda_{t,s}^{Sel,LEM} b_{t,s,k}^{Sel} - l_{t,s,k}^{Sel} b_{t,s,k}^{\phi} \quad (6a)$$

$$\hat{\lambda}_{t,k}^{Buy,LEM} = \sum_{st \in S^{Buy}} \lambda_{t,st}^{Buy,LEM} b_{t,st,k}^{Buy} + l_{t,st,k}^{Buy} b_{t,st,k}^{\phi} \quad (6b)$$

$$\hat{\lambda}_{t,k}^{E,LEM} = \hat{\lambda}_{t,k}^{Sel,LEM} + \hat{\lambda}_{t,k}^{Buy,LEM} \quad (6c)$$

Where $b_{t,s,k}^{Sel}$ and $b_{t,st,k}^{\phi}$ are binary variables that become one when ϕ is non-zero meaning that the maximum deviation in a specific step is selected. Note that, the worst case for selling action happens in the lowest value of the price and for buying in the highest value. Thus, Eqs. (6a) and (6b) give the price of selling/buying, and Eq. (6c) provides the final price. The proposed weighted multi-range robust approach provides a decision-making tool for RLCT aggregator as a price-maker to schedule its assets with consideration of the possible outcome of its offering/bidding on the market clearing price. In addition, the extra parameter of α gives the option to signify a specific range. This could be especially useful in the day-to-day offering strategy. For example, the historical data while building PQCs could suggest that the forecast error may be up to twenty percent at some times of the day. However, this is not always the case thus the operator can tune the offering by having a higher weight for the ranges closer to the forecasted ones.

2.4. Stochastic weighted multi-range robust approach

In this section, we incorporate the price-maker model explained in the previous section into the overall multi-market participation model of the RLCT aggregator. The aggregator first decides the DA-related variables which include the amount of power offered to or bought from DA WEM and the offering/bidding curve to be submitted in the LEM. These decisions are here-and-now types which are then fixed while making the next stage (i.e. wait and see) decisions including the power traded with the RT electricity market, EVs rescheduling, and HP consumption adjustment. This structure of decisions is in line with real-world applications. RLCT aggregator decides its strategy in DA markets and HP scheduling. This takes place before the gate closure time and in advance for all of the following time periods. Only, after these decisions are taken, the values of PV production along with EVs availability are realized at each time. Thus, the RLCT aggregator decides how to manage its action optimally (i.e. selling/purchasing additional/deficient power from the market and/or utilizing the HP flexibility) in order to maximize its overall profit. The proposed formulation is given in (7). The RLCT aggregator aims at maximizing the overall profit while participating in all markets. The first term is the profit from the DA WEM as explained previously. The second term represents the objective of the defined model in the previous subsection related to the offering/bidding curve construction of the LEM. The last term is the expected profit from the real-time operation of the RLCT aggregator through the real-time market. We assume that it is possible to modify the HP schedule but with a cost ($cost^{RT}$). The temperature should remain within the defined range. As mentioned earlier the trading happens in two stages wherein the first power traded with DA WEM and LEM is decided and only then when scenarios of the real-time are realized power traded in the real-time market with EVs and HP schedules are decided. In this regard, constraints (7b)

and (7c) separate the first stage power decisions into sold and bought respectively. The difference between these two values should be in balance with the schedule of the assets of the aggregator as enforced in (7d). The power balance equation is given in (7e) where R^{HP} is the change in consumption of HPs in each scenario, ω , and time. The power traded in the RT market equals the difference between the realized value of PV generation and the forecasted one, the difference between the power scheduled for EVs in other markets with the RT market, and the changes of HP consumption at each scenario and time. The cost related to the reduction in power of HP is defined linearly in constraint (7f) as a multiplication of a fixed price, λ^{ξ} , and the accumulated power curtailed from the scheduled HPs. Constraints (7g)–(7h) demonstrate the rescheduling of EVs per each scenario and time which were explained for the deterministic model. Finally, constraints (7i)–(7l) address the utilization of the flexibility of HP and limits of temperature in the real-time market. Note that $R_{t,j,k,\omega}^{HP}$ is a non-negative variable as imposed in (7k).

$$\max \sum_{t \in T} \left(\lambda_t^{DA} p_t^{Sel,DA} - \lambda_t^{DA} p_t^{Buy,DA} + \sum_{k \in K} (\alpha_k z_k + \sum_{\omega} \pi_{\omega} \lambda_{t,\omega}^{RT} (p_{t,k,\omega}^{Sel,RT} - \lambda_{t,\omega}^{RT} p_{t,k,\omega}^{Buy,RT} - cost_{t,k,\omega}^{RT})) \right) \quad (7a)$$

$$\text{s.t. (1k)–(1q) } \forall \omega, k, (5b)–(5b), (6a)–(6c)$$

$$p_t^{Sel,DA} + p_t^{Sel,LEM} = p_{t,k}^{Sel,tot} \quad (7b)$$

$$p_t^{Buy,DA} + p_t^{Buy,LEM} = p_{t,k}^{Buy,tot} \quad (7c)$$

$$p_{t,k}^{Sel,tot} - p_{t,k}^{Buy,tot} =$$

$$p_t^{PV} + p_{t,k}^{EV,dis} \eta_d - p_{t,k}^{EV,ch} - \sum_j p_{t,j,k}^{HP} \quad (7d)$$

$$p_{t,k,\omega}^{Sel,RT} - p_{t,k,\omega}^{Buy,RT} = (p_{t,\omega}^{PV,RT} - p_t^{PV}) + (p_{t,k,\omega}^{EV,dis,RT} - p_{t,k}^{EV,dis}) \eta_d - (p_{t,k,\omega}^{EV,ch,RT} - p_{t,k}^{EV,ch}) + \sum_j R_{t,j,k,\omega}^{HP} \quad (7e)$$

$$cost_{t,k,\omega}^{RT} = \lambda^{\xi} \sum_j R_{t,j,k,\omega}^{HP} \quad (7f)$$

$$p_{t,k,\omega}^{dis,RT} = \sum_i p_{t,i,k,\omega}^{dis,RT} \quad (7g)$$

$$p_{t,k,\omega}^{ch,RT} = \sum_i p_{t,i,k,\omega}^{ch,RT} \quad (7h)$$

$$p_{t,j,k}^{HP} - R_{t,j,k,\omega}^{HP} = (C_j + \frac{\Delta t}{R_j}) \theta_{t,j,\omega}^{r,RT} - C_j \theta_{t-1,j,\omega}^{r,RT} - \frac{\theta_t^a \Delta t}{R_j} \quad (7i)$$

$$p_{t,j,k}^{HP} - R_{t,j,k,\omega}^{HP} \geq 0 \quad (7j)$$

$$R_{t,j,k,\omega}^{HP} \geq 0 \quad (7k)$$

$$\theta_{-j}^{r,RT} \leq \theta_{t,j,\omega}^{r,RT} \leq \theta_j^{r,RT} \quad (7l)$$

$$p_{t,i,k,\omega}^{ch,RT} = p_{t,i,k,\omega}^{dis,RT} = 0 \quad t \notin [t_{i,\omega}^{b,RT}, t_{i,\omega}^{e,RT}] \quad (7m)$$

3. Numerical studies

In this section, numerical simulations are provided for the participation of an RLCT aggregator in the local and wholesale electricity markets in two separate case studies. In case I, we consider participating only in LEM to better investigate the impact of the aggregator offering/bidding in the market and associated uncertainties. In case II, all markets are considered. We analyze how adding the capability to participate in extra markets could impact the performance of the aggregator. The optimization problem is solved using the CPLEX solver in GAMS software and the reported average time consumption is three

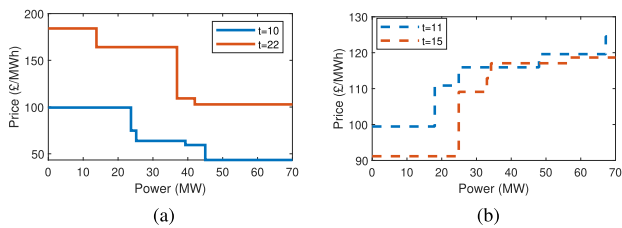


Fig. 3. Example of PQCs for different hours (a) GPQC (b) DPQC.

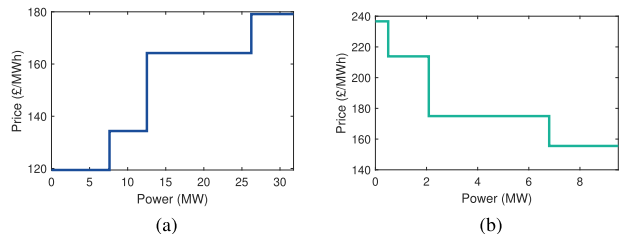


Fig. 4. Offering and bidding curve in LEM (a) Offering curve (Hour 17) (b) Bidding curve (Hour 20).

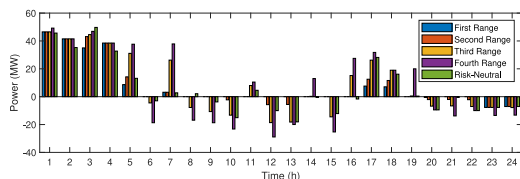


Fig. 5. Scheduled exchanged power with the LEM for different hours of the day for different ranges of the proposed and risk-neutral strategy. Here, the positive values are the supply and the negative ones are the demand.

and nineteen minutes for cases I and II, respectively. Prior to going into details of each case, the input data and LEM setup are explained as follows.

3.1. Data and LEM setup

To build up the LEM environment that the RLCT aggregator participates in, we consider the leading-the-way scenario (the year 2035) from the distribution future energy scenarios and the county Durham located at the northeast of England as the geographical area [4]. Considering this scenario, there will be a high uptake of LCTs in the grid having the potential to directly or indirectly (through an aggregator) participate in the LEM. The considered information in the aforementioned scenario includes the number of EVs, number of HPs, domestic installed PV capacity, large solar generation installed capacity, wind generation installed capacity, non-renewable generation installed capacity, domestic underlying consumption, and industrial and commercial energy consumption all provided in Table 2. We assume that twenty-five percent of the residential assets including EVs, HPs, and fifty percent of domestic PV capacity are aggregated by the RLCT aggregator to participate in the market with the goal of maximizing the overall profit as explained in the mathematical formulation section. The data for wind speed and solar irradiance comes from [33] for county Durham. The days under study that use this data start from the first of March for Case I and from the first of April for Case II. The EV and HP types/models are from [11] assuming ten different types. Also, for the second case, five scenarios with different PV irradiance and EV availability are built according to the model explained in [11]. For LEM, it is assumed that ten generation units participate in the market including seven renewable and three non-renewable, along with four demand units representing the I&C consumers. The depicted capacities from the scenario given in Table 2

are divided among these units. The offers/bids are built according to the approach proposed in [34]. Finally, after acquiring offers/bids PQCs for both generation and demand are constructed some samples of which are provided in Fig. 3.

3.2. Case I: Participation only in LEM

In this case, the aggregator participates in LEM while only dealing with the uncertainty of offers/bids impacts on the market price. We first investigate the performance for a day (first of March) and then the overall performance is analyzed for longer periods. We use the data explained above to investigate and analyze the performance of the RLCT aggregator in the LEM. Four ranges are considered within the twenty percent confidence interval around the predicted PQCs ultimately resulting in offers/bids with a maximum of four steps. The model (5) is used to derive the offers/bids in LEM. For the first day, examples of the acquired curves are demonstrated in Fig. 4. For the offering curve first, the power of 7.6 MWh is offered with the price of £119.2/MWh. The next submitted offers are (12.5 MWh, £143.3/MWh), (26.2 MWh, £164.2/MWh), and (31.4 MWh, £179.2/MWh). For each range, the scheduled power to be exchanged in the LEM is given in Fig. 5. The first range corresponds to the lowest range (e.g. first step in Fig. 2) when selling and the highest range for buying electricity for the market hence the worst case of the price realization. Thus, the activity, in this case, is the minimum as in this situation model tries to refrain from fully participating in the market and impacting market price unfavorably (i.e. lowering/increasing price when selling/buying) at different hours. Thus, as can be seen from this figure, for this range RLCT aggregator has lower activity in the market where in several hours of the day no power is exchanged with the market meaning that it is more beneficial to provide the necessary consumption and avoid impacting markets by extra offering/bidding in the market. This behavior changes in other ranges. As can be seen from Fig. 5, the most supply and demand come from the fourth range which exploits a range where the price is high when offering and low when bidding. In other words, the price in this range varies in the highest bounds of GPQCs and the lowest of DPQCs, depicting a situation that despite having a high sensitivity to the market clearing price, more participation in the market will still be profitable. The obtained price profiles for each range are gathered in Fig. 6. As already explained lower ranges correspond to more conservative prices. This translates to lower selling prices in the market and higher prices required for buying electricity in the market. On the other hand, two conditions are enforced on the offering/bidding curves as explained in the mathematical formulation section, namely non-decreasing for the offering curves and non-increasing for the bidding curves. The average expected profit for this day is £22189 which is calculated via averaging the expected profit acquired from all ranges as the considered α is the same for all ranges. The corresponding scheduled power and price profiles are depicted in Figs. 5 and 6. For the sake of comparison, two results of two more strategies are considered, namely a risk-neutral and a classical robust approach (risk-averse with one range). Toward investigating the actual performance of each strategy a validation after-the-fact analysis is carried out using real data as explained in the following. First, the offering/bidding quantities are derived for the whole period of the optimization. Then, considering these hourly offers/bids, the market is cleared, realizing the power exchanged with the market and the resulting price. With this information, the overall profit of each strategy per day can be calculated. For better demonstration, the obtained profits for eleven consecutive days starting from the first of March are given in Fig. 4. As can be seen from 4, except for one day for the risk-neutral strategy, the proposed approach outperforms other strategies. The reason for the day with lower achieved profit than risk-neutral is due to the fact that on this day the predicted PQCs were realized very close to the real ones. However, on other days where this was not the case, the difference is much more noticeable between the proposed and the risk-neutral strategy. Also, the classical

Table 2
The input data of the future distribution energy scenarios (* Industrial and commercial).

Data	Value	Data	Value
No. of EVs	109364	No. of HPs	66652
Domestic PV	98.88 MW	Large solar	427.255 MW
Wind	148.94 MW	Non-Renewable	47.813 MW
Domestic load	514112 MWh/yr	I&C* load	951117 MWh/yr

Table 3
The total resulting profit (£) of each strategy for a period of one month with respect to different forecasting errors.

Strategy	Forecasting error		
	10%	20%	30%
Risk-Neutral	0.786	0.690	0.576
Classical robust	0.686	0.622	0.557
Proposed	1.271	1.189	1.121

Table 4
Potential profit with the proposed approach (% of Ideal Case).

Error percentage	Proposed (%)
10%	92.1
20%	86.06
30%	80.14

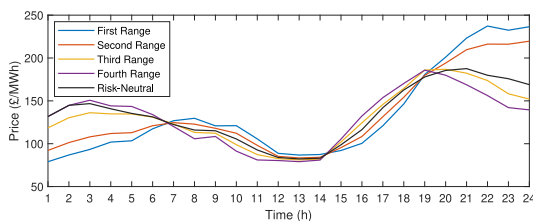


Fig. 6. Acquired price profiles corresponding to different ranges of uncertainty set of the proposed and risk-neutral strategy.

robust approach seems to perform better than risk-neutral on some of the days with high price fluctuations from the predicted values, however, its achieved profit is lower than that of the proposed strategy for all days. The same process is repeated for the period of a whole month and the total acquired profit by each strategy is provided in Table 3 with respect to different forecasting errors. As can be seen, the increase in prediction error results in lower overall profits. However, this has different impacts on the considered strategies in terms of profit achieved. For the proposed strategy the total profit is decreased by 12 percent for the 30 percent forecasting error compared to the 10 percent forecasting error. This is 26 percent for the risk-neutral strategy and 19 percent for the classical robust approach. The risk-neutral is an optimistic approach and is thus sensitive to the prediction error. The more the prediction error the more impact on the acquired profits. The classical robust approach, however, despite reaching a lower total performance is moderately sensitive to prediction errors.

We conducted a comprehensive comparison of our approach, specifically evaluating the potential acquired profit in contrast to an ideal case, which serves as a benchmark with perfect hindsight demonstrated in Table 4. In the previous table findings demonstrated that the proposed method consistently achieves noticeably superior performance in terms of both acquired profit and resilience to prediction errors. Importantly in Table 4, comparable results were obtained even when compared to a scenario with no uncertainty. These results underscore the efficacy of our proposed method as a valuable tool for decision-makers operating in similar market settings, highlighting its robustness and effectiveness in addressing real-world uncertainties (see Fig. 7).

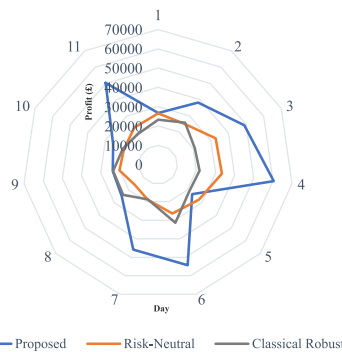


Fig. 7. The obtained profit of the after-the-fact analysis on different days for the proposed, risk-neutral, and classical robust strategies.

3.3. Case II: Participation in multiple markets

In the previous case study, the RLCT aggregator participation in LEM was demonstrated while investigating the different aspects of the proposed uncertainty modeling approach. In this case, we consider all markets. Five scenarios are constructed for the RT realization of uncertain parameters according to the procedure provided in [11]. Fig. 8 summarizes the expected power traded in LEM, power traded in DA WEM, and power traded in RT WEM. As can be seen, the proposed strategy utilizes the price of DA WEM and LEM as a signal to schedule the assets and trade accordingly in these markets. When there is a noticeable difference in market prices the trading is shifted completely toward the market with a favorable price. To keep the figure more tractable, for three scenarios, the power adjustment in RT WEM is provided in this figure. For the first two scenarios, the variation in realized PV generation and the EVs availability necessitates buying some of the power back or selling the excess scheduled power in the RT market. The third scenario depicts less contribution to this market coming from the realized values being close to the prediction of assets' behavior. Table 5 provides a comparison of profit acquired from LEM between two cases: having only the option of participating in LEM and all markets simultaneously considering the expected profit of each range of uncertainty set. As can be seen, the difference is more considerable for the first and second ranges. This means that in these ranges when there is an option of participating in additional markets other than LEM since price fluctuations through the day are smaller it is preferable to withhold more power from LEM and participate more in the DA WEM market. However, when the difference between the low and high prices of LEM during the day increases, meaning more chance for arbitrage, the activity in LEM increases to benefit from buying at lower prices and then selling at higher prices. Note that in presence of the option of participating in DA WEM the activity in LEM is still lower than LEM only case, in order to avoid impacting the MCP leading to lower profits. Finally, Table 6 summarizes the total profit acquired by the aggregator by participating only in LEM and also in all markets. The results demonstrate that the profit is increased by 28 percent when the RLCT aggregator is participating in all markets. Indeed, if the aggregator schedules all of its assets toward LEM which is naturally a smaller market in terms of the sizes of traded energy compared to DA WEM, it moves MCP lower when offering the generation to market and higher when buying energy from the market. Thus, it is beneficial that the aggregator cut off some of the energy traded in LEM toward DA WEMs. However, more interestingly, as reported in this table, the traded energy in all markets is fifty percent more than energy traded only in LEM which demonstrates that providing the capability to participate in multi-markets simultaneously releases more flexibility from the residential side, hence increasing the potential profit of the aggregator representing customers alongside bringing more resources to the markets when required.

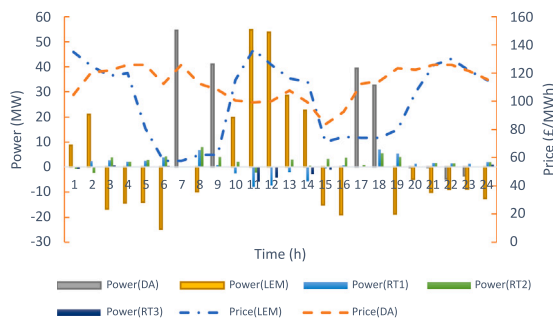


Fig. 8. The expected values of power traded in the markets along with DA and LEM market prices.

Table 5

Comparison of expected profit (£) acquired from LEM for different ranges of uncertainty set: (a) when the aggregator is only participating in LEM and (b) when all markets are considered in the optimization formulation.

	Range			
	First	Second	Third	Fourth
(a)	15 904.24	22 542	45 172.15	61 488.63
(b)	2364.47	6187.75	35 635.02	52 321.71

Table 6

The expected profit and traded energy in case of only participating LEM and both WEM and LEM.

	WEMs and LEM	LEM
Total profit (£)	46 534.29	36 276.76
Total traded energy (MWh)	766.1	508.05

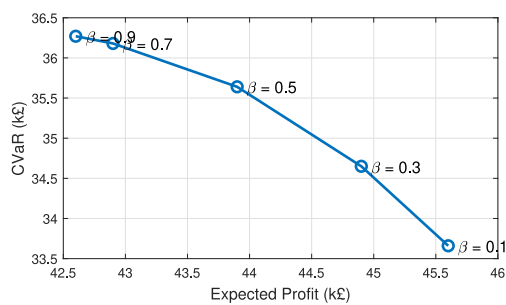


Fig. 9. Profit and CVaR with different values of risk-aversion parameter.

We also calculated conditional value at risk (CVaR) based on the methodology outlined in [35] and depicted the results in Fig. 9. This visual representation unveils the impact of the risk aversion parameter β on the intricate interplay between profit and CVaR within the aggregator’s decision-making framework, considering uncertainties from PV and EVs. Each data point is annotated with the corresponding β values, offering a transparent insight into the aggregator’s risk preferences. The observed trend aligns with theoretical expectations: heightened risk aversion prompts a more conservative approach. On the other hand, lower β values signify a more aggressive strategy, resulting in elevated total profit. This figure succinctly captures the relationship between risk aversion and decision-making, illustrating how aggregators balance profit and risk regarding variable uncertainties coming from the aggregated assets.

4. Conclusion

The future energy scenarios in several countries around the world depict a noticeable uptake of low-carbon technologies, especially at

the residential level. This paper addressed several challenges that an aggregator of such technologies could face in energy markets:

- (1) Deriving a suitable strategy to participate in multiple electricity markets at local and wholesale levels.
- (2) Consideration of seller/buyer and price-maker/price-taker roles in the aforementioned markets.
- (3) Accounting for the uncertainty associated with the potential effects of offering/bidding on the MCP when the aggregator could be a price-maker.

The proposed stochastic multi-range robust approach has proven to be a well-suited solution for effectively addressing the intricate challenges inherent in managing uncertainties associated with offer/curves. This model optimally derives offer/curves while accounting for uncertainties in both the price-maker model and the outputs of aggregated assets.

The obtained results underscore the efficacy of our approach, showcasing that an aggregator can attain higher overall profits in comparison to conventional robust and risk-neutral approaches. Importantly, our method ensures that these profits exhibit reduced sensitivity to variations in forecast errors. This heightened resilience to uncertainties enhances the reliability and stability of the aggregator’s decision-making process. Throughout this study, our primary goal centered on profit maximization, a pursuit that translated into increased participation in various markets. This holds particularly true when considering the simultaneous engagement with multiple markets. By achieving superior profitability and mitigating sensitivity to forecast errors, our proposed approach not only aligns with the main objectives of the aggregator but also positions itself as a robust and practical solution for contemporary challenges in energy market participation. The positive outcomes obtained in our research underscore the significant contributions of our proposed model in fostering more effective and profitable participation of aggregators in diverse energy markets.

5. Future work

As for future research direction, we envision three promising avenues. Firstly, in the presence of sufficient data, the exploration of data-driven methods for comparison and potential tuning of uncertainty sets stands out as a valuable direction. This approach could leverage the richness of available data to refine the model’s understanding of uncertainties, further enhancing its adaptability and performance. Secondly, we propose future investigations into incorporating distribution network information into the offering model of the aggregator. Considering the intricacies of the distribution network could provide a more comprehensive representation of the operating environment, potentially leading to more accurate and context-aware decision-making by the aggregator. This avenue opens new possibilities for aligning the model with the evolving dynamics of distribution networks, offering a holistic perspective for future research endeavors. Thirdly, incorporating the dynamics of energy and reserve markets into our modeling framework could provide a more comprehensive understanding of the aggregator’s decision-making landscape. This avenue holds the potential to enhance the adaptability and robustness of our proposed approach, offering valuable insights for a more resilient and efficient integration of low-carbon technologies into the broader energy landscape.

CRedit authorship contribution statement

Arman Alahyari: Conceptualization, Formal analysis, Methodology, Visualization, Writing – original draft, Writing – review & editing, Software. **Charalampos Patsios:** Supervision, Writing – review & editing. **Natalia-Maria Zografou-Barredo:** Validation, Writing – review & editing. **Timur Saifutdinov:** Validation, Writing – review & editing. **Ilias Sarantakos:** Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix

A general optimization form of the problem addressed in the paper can be formulated as shown in (8).

$$\max \mathbf{a}^T \mathbf{x} + \mathbb{E}[\min_{\psi_r \in \Psi} p_r(\mathbf{x}, \psi_r)] + \mathbb{E}[q(\mathbf{x}, \omega)] \quad (8a)$$

$$\text{s.t. } q(\mathbf{x}, \omega) \leq \mathcal{Z} \quad (8b)$$

$$p_r(\mathbf{x}, \psi_r) := \max_{y_r \in F(\mathbf{x}, \psi_r)} \mathbf{c}^T y_r \quad (8c)$$

$$F(\mathbf{x}, \psi_r) \neq \emptyset \quad (8d)$$

The objective is the aggregator profit set to be maximized over three markets of DA wholesale, local market and real-time market as demonstrated respectively in (8a) followed by constraint (8b)–(8d). Note that Ψ is a full uncertainty set containing all ranges, $p_r(\mathbf{x}, \psi_r)$ is the profit over local market with multi-range uncertainty constrained by $F(\mathbf{x}, \psi_r)$ and $q(\mathbf{x}, \omega)$ which stands for the optimal value of the second-stage real-time problem. In this regard, our approach can provide a solution that maximizes the first-stage profit in the DA market and the expected worst profit in the local market, along with the expected profit in the real-time market, considering the collection of all uncertainty sets and scenarios, respectively. Thus, as explained in the paper, the first term does not incorporate uncertainty. For each future turnout, only one value is determined, thereby eliminating the requirement for the expectation. The second term represents the weighted multi-range uncertainty set for modeling bidding in the local market, followed by the expectation of the operation over scenarios of different outcomes in real-time. In this part, we want to provide some insight into the conservatism of the utilized uncertainty set in the paper compared to the robust method, assuming that both methods use the same shape for describing the variation of uncertain variables. For instance, both be a box or an ellipsoidal uncertainty set.

Note that, regarding the defined type of the uncertainty set in the paper for robust optimization, the expectation can be rewritten as $\sum_j \alpha_j (\min_{\psi_j} p_j(\mathbf{x}, \psi_j))$, translating into finding the worst case in multiple ranges with different associated weights. It can be demonstrated that such a set of uncertainty would lead to a less conservative solution compared to a classical robust approach with one range. To prove this, consider an uncertainty set comprising two ranges. The first range of the uncertainty set, ζ_1 , has a more conservative associated optimal solution compared to the second range, ζ_2 . If these two sets were considered separately, meaning weights for both are equal to one ($\alpha_1 = \alpha_2 = 1$), the relation of associated optimal values can be described as $g(\zeta_2) = \min[g(\zeta_1), g(\zeta_1 \setminus \zeta_2)]$. In the proposed method, we consider both ranges with $\alpha_1 + \alpha_2 = 1$. Thus, for the multi-range set, the optimal value can be formulated as $g(\zeta_1, \zeta_2, \alpha_1, \alpha_2) = \alpha_1 g(\zeta_1) + \alpha_2 g(\zeta_2)$, which shows that for two ranges, the conservatism of the proposed method is less than the classical robust method: $g(\zeta_1) \leq g(\zeta_1, \zeta_2, \alpha_1, \alpha_2) \leq g(\zeta_2)$. This can be easily generalized for more than a two-range uncertainty set.

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