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Global dual sourcing: ARMA(1,1) demand and its decomposition

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Abstract

We consider the case of reshoring, where a global firm takes back a portion of its low-cost offshore supply to be produced in nearshore factory in order to establish a dual sourcing supply chain equipped with both low cost and responsiveness. We first establish the performance benchmark of a single (nearshore or offshore) supplier model. In the dual sourcing setting, a firm decomposes the first order auto-regressive moving average, ARMA(1,1), market demand process into two parts: one for the nearshore source and one for the offshore source. Order-up-to policies determine the order quantities for both sources. We show how to reduce inventory costs in the dual-sourcing case to a level identical to the near-shore single-source case. Furthermore, if certain conditions are met, the nearshore manufacturing cost reduces in the offshore lead-time. This suggests low-cost and low-emission transport modes should be utilised (slow steaming vessels which are both low cost and environmentally friendly, but may endure longer offshore lead-times come to mind), breaking the trade-off between economic and environmental performance.

Keywords: dual sourcing, supply chain dynamics, ARMA(1,1) demand, order-up-to policy

1. Introduction

Supply chain management has come of age. Never before has so much public attention been given to our field. The impact of the Covid-19 pandemic and the subsequent geopolitical tensions have led to a continuing trend to rethink global supply chains. Newspapers and 24-hour news channels now regularly discuss factors such as “reshoring”, “de-risking”, “war sanctions”, “piracy”, the “US inflation reduction bill”, “EU emissions tax” on high carbon imports, and “export controls” on dual use technology, high-end chips, rare earth minerals, data even. We should also be careful to remember global warming, high inflation, growing government and personal debt, the unknown threat of AI, cyber-attacks, fake news, election interference, and the arbitrary enforcement of local laws. Never before has supply chain management become so important to us all.

In a small conference paper, we cannot hope to consider all these factors. Let us therefore focus on *reshoring with dual souring*. Reshoring with dual sourcing is the act of bringing back a proportion of total production from a global supplier to be produced in a local factory in a dual sourcing supply chain. Reshoring with dual sourcing allows one to avoid some unexpected disruptions and effectively shorten the geographical distance of the entire supply chain, Hufford (2023).

It has been shown in much of the supply chain management literature that long lead times increase supply chain costs. For example, the bullwhip effect often, but not always, increases in the lead time, Gaalman et al. (2022). However, some closed-loop supply chain (CLSC) studies, which include two sources of supply (one for new products, another for re-manufactured products), have shown that there are economic benefits when one of the two sources has a longer lead time (Hosoda et al., 2015; Hosoda & Disney, 2018). Longer lead times could even be more environmentally friendly, BIMCO (2023).

Herein, we consider a single-item, periodic-review inventory system with two supply sources in a centrally organised supply chain facing an auto-correlated demand process. To preserve analytical tractability, we assume all lead-times are constant, excess demand is backlogged, and no capacity constraints are present. Two different supply sources adopt the same review period to manage their production systems. The objective is to minimise the expected per period sum of capacity, overtime, inventory holding, and backlog costs in the whole system.

2. Literature review

There is much literature on the dual sourcing problem. Fukuda (1964) considers the case where there are two different lead times (for example, due to two sources, or two different transport modes) and the difference between the two deterministic lead times is one period. Fukuda (1964) finds dual-base-stock policies are optimal. Gijsbrechts et al. (2024) uses the same one-time-period lead time difference assumption and shows a modified dual-base-stock policy with three base-stock levels is optimal. Boute & Van Mieghem (2015) proposes a dual-sourcing smoothing (DSS) policy applicable for both single and dual sourcing settings. Their results include a simple but practically useful square-root bound for the strategic sourcing allocations in a dual sourcing setting.

Allon & Van Mieghem (2010) study the Tailored Base-Surge (TBS) sourcing strategy where a global supplier receives a constant order and focuses on cost efficiency and the local supplier's quick response delivers high customer service levels. The TBS strategy is similar to the *standing order* issued to the global supplier in the study by Rosenshine & Obee (1976). The standing order scheme effectively eliminates the impact of the long global lead time.

Boute et al. (2022) consider dual sourcing under i.i.d., AR(1), and IMA(0,1,1) demand with unit local lead time and an arbitrary global lead time. The global orders were a constant in the i.i.d. and AR(1) cases; the non-stationary IMA(0,1,1) global orders were dynamic. The local orders were determined via the proportional order-up-to policy. They found that price parity does not have to be reached to make re-shoring a small proportion of demand back to the local factory economic. The shorter local lead time allowed for tighter control of the local finished good inventory that led to an economic benefit. This benefit increases in the global lead time and also in the demand correlation.

A closely related problem to reshoring is an inventory system with both regular and emergency replenishment orders, for example see Tagaras & Vlachos (2001), Chiang & Gutierrez (1996), and Whittemore & Saunders (1977).

3. Base case: The ARMA(1,1) supply chain

Assume a firm faces a stochastic market demand process, represented by a first-order auto-regressive moving average, ARMA(1,1), model (Box et al., 2008)

$$d_t = \mu_d + \phi(d_{t-1} - \mu_d) + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{1}$$

where d_t is the market demand at time period t, $\mu_d = \mathbb{E}[d]$ is the mean demand, ϕ is an auto-regressive parameter bounded by $|\phi| < 1$, θ is a moving average parameter bounded by $|\theta| < 1$, and ε_t is a random error term realised at time period t with zero mean, constant variance σ_ε^2, and drawn from a normal distribution. The long run variance of d_t, $\mathbb{V}[d]$, is

$$\mathbb{V}[d] = \frac{1+\theta^2 - 2\theta\phi}{1-\phi^2}\sigma_\varepsilon^2, \tag{2}$$

Box et al. (2008). This ARMA type demand assumption is common in the supply chain literature (see, for example Lee et al. (2000), Hosoda & Disney (2006), and Gaalman et al. (2022)). The ARMA(1,1) demand process contains i.i.d., AR(1), and MA(1) processes as special cases.

3.1. The order-up-to replenishment policy for the single source

For brevity, we deal here only with the case of a single *offshore* source. The results here can easily be interpreted for a single nearshore supplier. We assume the firm uses the order-up-to (OUT) policy with minimum mean square error (MMSE) forecasts, Box et al. (2008), and the following inventory balance equation exists,

$$ns_t = ns_{t-1} + o_{t-L-1} - d_t.$$

Here, ns_t is the local net stock level at time t, o_t is the order quantity placed by the firm at time t with the global offshore source, and $L \in \mathbb{N}_0$ is the replenishment lead-time from the global offshore source. It is known some different formulations of the OUT policy exist. The first formulation uses the difference between the desired inventory position, s_t, and the actual inventory position (the work-in-progress plus current net stock level), ns_t, (Li et al., 2014),

$$o_t = s_t - \Big(\underbrace{\textstyle\sum_{i=1}^{L} o_{t-i}}_{\mathrm{WIP}_t} + ns_t\Big); \quad \text{where} \quad s_t = \mu_{ns} + \mathbb{E}\left[\textstyle\sum_{i=1}^{L+1} d_{t+i} | t\right]. \tag{3}$$

Here, $\mu_{ns} = \mathbb{E}[ns]$ represents the time-invariant target net stock (or, safety stock) level. The expected demand i periods ahead, conditional upon the information available at time t, is given by

$$\mathbb{E}[d_{t+i}|t] = \mu_d + \phi^i(d_t - \mu_d) - \phi^{i-1}\theta\varepsilon_t,$$

which can be obtained by recursion and knowing the conditional expectation of future realisations of the noise term has an expectation of zero. This can be used to determine the conditional expected demand (the MMSE forecast) over the lead-time and review period,

$$\mathbb{E}\left[\textstyle\sum_{i=1}^{L+1} d_{t+i}|t\right] = (L+1)\mu_d + \phi\Lambda_{L+1}(d_t - \mu_d) - \theta\Lambda_{L+1}\varepsilon_t : \quad \Lambda_{L+1} = \frac{1-\phi^{L+1}}{1-\phi}. \tag{4}$$

The second formulation of the OUT policy (Lee et al., 2000; Hosoda & Disney, 2006) is

$$o_t = d_t + (s_t - s_{t-1}). \tag{5}$$

This formulation is useful as it contains only feed-forward paths, facilitating its analysis.

3.2. Variances in the case of single offshore source

It is recognised that the variance of the net stock levels maintained by the OUT policy is identical to the variance of the demand forecast errors over the lead-time and review period (Vassian, 1955; Hosoda & Disney, 2006). With the knowledge of (1), the demand forecast errors over the lead-time and review period is given by

$$\textstyle\sum_{i=1}^{L+1} d_{t+i} - \mathbb{E}\left[\sum_{i=1}^{L+1} d_{t+i}|t\right] = \varepsilon_{t+L+1} + \sum_{i=1}^{L}\left(1 + \frac{1-\phi^i}{1-\phi}(\phi-\theta)\right)\varepsilon_{t+L+1-i}.$$

The variance of the net stock levels in the case of the base case can be written as

$$\mathbb{V}[ns] = \mathbb{E}\left[\left(\textstyle\sum_{i=1}^{L+1} d_{t+i} - \mathbb{E}\left[\sum_{i=1}^{L+1} d_{t+i}|t\right]\right)^2\right] = \sigma_\varepsilon^2 + \sum_{i=1}^{L}\left(1 + \frac{1-\phi^i}{1-\phi}(\phi-\theta)\right)^2\sigma_\varepsilon^2. \tag{6}$$

Mathematical simplification yields the following Lemma.

Lemma 1. ***(Variance of the net stock levels)*** *The variance of the net stock levels maintained by the OUT policy is given by*

$$\mathbb{V}[ns] = \sigma_\varepsilon^2\left(1+\frac{L(\theta-1)^2}{(\phi-1)^2}+\frac{\phi(\phi-\theta)\left(\phi^L-1\right)\left(\theta\left(\phi(1-\phi^L)+2\right)+\phi\left(\phi(1+\phi^L)-2\right)-2\right)}{(\phi-1)^3(\phi+1)}\right). \tag{7}$$

Proof of Lemma 1. All proofs in this short conference paper are omitted to save space. □

Remarks on Lemma 1. The net stock variance is always greater than σ_ε^2, a fact most easily seen in (6). Eq. (7) shows the inventory variance is has a component that increases in the lead time L, the second addend. The third addend oscillates depending on the parity of L when a negative ϕ is present. When $L=0$, then $\mathbb{V}[ns]=\sigma_\varepsilon^2$. When $\phi=\theta$, then $\mathbb{V}[ns]=(L+1)\sigma_\varepsilon^2$.

Lemma 2. ***(Variance of the global orders)*** *The variance of the global production orders is*

$$\mathbb{V}[o] = \frac{\sigma_\varepsilon^2}{(\phi-1)^2}\left(\frac{2(\phi-\theta)^2\phi^{2L+2}}{\phi+1}+(\theta-1)\left(\theta+2(\phi-\theta)\phi^{L+1}-1\right)\right). \tag{8}$$

Remarks on Lemma 2. When $\phi=\theta$, the order variance $\mathbb{V}[o]=\sigma_\varepsilon^2$. The order variance is increasing in ϕ and decreasing in θ when $\theta<\phi$. If L was replaced by ℓ in (7) and (8) they would be representative of the dynamic performance of a single local supplier.

4. Dual sourcing case: Market demand process decomposition

Let's assume a firm who currently uses a global offshore supply chain with a lead-time L in order to enjoy lower labour cost, is thinking of re-shoring. The firm intends to take back a portion of its orders placed on low-cost offshore sources to produce at a nearshore factory with a shorter lead-time $\ell\in\mathbb{N}_0(<L)$ and operate a dual sourcing supply chain, Boute et al. (2022). A key issue of dual sourcing is how to coordinate both the offshore and nearshore sources effectively. It is often advocated that the global offshore source operates under smoothed orders (covering the base demand) and the nearshore source is used as a responsive supplier in a TBS arrangement, Allon & Van Mieghem (2010). Thus, the order-smoothing policy is used for global offshore source and a quick-response policy with a volatile order is used for the nearshore source. We explore this idea further here. The significant difference from previous research is that we decompose the market demand process into two processes and two different ordering policies are used to determine the order quantities for each source.

We assume the firm decomposes the ARMA(1,1) market demand process (1) into two demand processes: the demand for nearshore source d_t^n and the demand for the global offshore source d_t^g, in order to reduce the variability in the offshore replenishment orders. The motivation to decompose

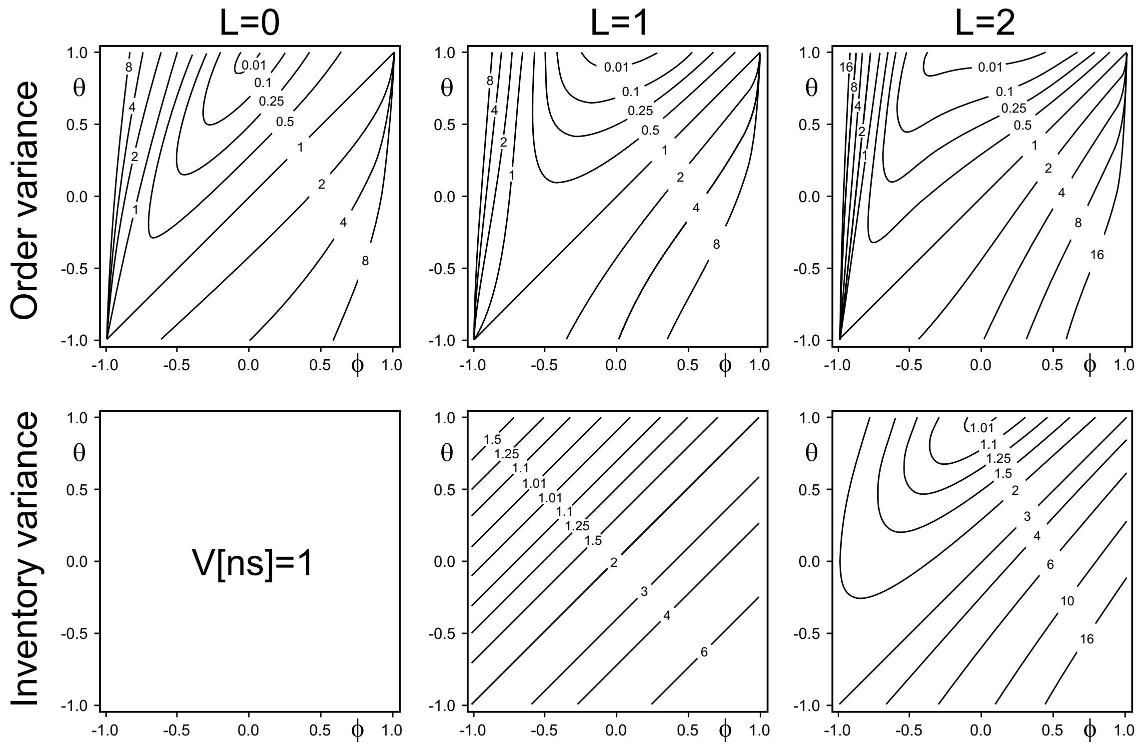

Figure 1: Base case: The order and inventory variances maintained by a single global supplier when $\sigma_\varepsilon^2 = 1$

the demand process comes from a very simple reason. Box et al. (2008) describes the ARMA(1, 1) process as a mix of auto-regressive and moving average processes. Therefore, it is natural to break them back down again into the two original processes. In what follows, we will use $\{n, g\}$ for nearshore and global offshore sources. The two demands, d_t^n and d_t^g are given by

$$d_t^n = \mu_n + \phi(d_{t-1} - \mu_d) + \varepsilon_t, \quad \text{and} \quad d_t^g = \mu_g - \theta\varepsilon_{t-1},$$

where $\mu_n > 0$, $\mu_g > 0$ and $\mu_n + \mu_g = \mu_d$. Note: $d_t = d_t^n + d_t^g$. It is easy to see that $\mathbb{E}[d_t^n] = \mu_n$ and $\mathbb{E}[d_t^g] = \mu_g$. It is assumed that both μ_n and μ_g are decision variables. It is obvious that d_t^g is a scaled white noise process when $\theta \neq 0$. If the market demand process is AR(1) (i.e. $\theta = 0$), then d_t^g becomes constant over time, μ_g. We will show this decomposition creates the convenient situation where the variance of d_t^g is always smaller than the variance of the market demand, $\mathbb{V}[d]$, given in (2). The variance of the two demand processes becomes

$$\mathbb{V}[d^n] = \phi^2\mathbb{V}[d] + \sigma_\varepsilon^2, \quad \text{and} \quad \mathbb{V}[d^g] = \theta^2\sigma_\varepsilon^2. \tag{9}$$

After taking the difference, simplification yields the relationships

$$\mathbb{V}[d^n] > \mathbb{V}[d^g], \quad \text{and} \quad \mathbb{V}[d] > \mathbb{V}[d^g]. \tag{10}$$

This market demand decomposition reduces the variability of the replenishment orders placed on the offshore supplier, helping to reduce the off-shore supplier capacity costs.

5. Dual sourcing case: Ordering policies for decomposed demand processes

Before we investigate the ordering policy for the dual sourcing setting, we assume the following relationship governs the dynamics of the net stock in our dual sourcing supply chain:

$$ns_t = ns_{t-1} + o^n_{t-(\ell+1)} + o^g_{t-(L+1)} - d_t, \tag{11}$$

where ns_t is the local net stock level, o^n_t and o^g_t are the replenishment order quantities placed at time period t to the nearshore and the global offshore sources, respectively. ℓ and L are the replenishment lead-times of the nearshore source and the global offshore sources, respectively. We assume $L > \ell$.

5.1. Ordering policy for global offshore source

First, we will consider the ordering policy for the global offshore supplier. To avoid the bullwhip effect, we use the order-up-to policy with a time-invariant order-up-to level s^g for the global offshore source to determine the order quantity. With the knowledge of (5), the OUT policy with a time-invariant order-up-to level (i.e. $s_t = s_{t-1} = s^g$) is given by

$$o^g_t = d^g_t + (s^g - s^g) = d^g_t, \tag{12}$$

which suggests when the decomposed offshore demand is d^g_t, the replenishment order $o^g_t = d^g_t$, and $\mathbb{V}[o^g] = \mathbb{V}[d^g]$. From (9) and (10), we have $\mathbb{V}[d] > \mathbb{V}[d^g]$, which suggests that $\mathbb{V}[d] > \mathbb{V}[o^g]$ and no bullwhip exists for the global offshore source. In the special case of $\theta = 0$, we have $d^g_t = o^g_t = \mu_g$, and the global supplier's replenishment order is a constant.

5.2. Ordering policy for nearshore source

The OUT policy with an order-up-to level at the nearshore source of s^n_t is

$$o^n_t = s^n_t - \left(\sum_{i=1}^{\ell} o^n_{t-i} + ns_t \right), \tag{13}$$

where $\sum_{i=1}^{\ell} o^n_{t-i}$ is the work-in-progress in the nearshore factory at time period t, WIP^n_t, Li et al. (2014). Note, s^n_t in (13) is fundamentally different to s_t in (3) which only considers the safety stock level and the future demands over the lead time plus one time period. In the dual sourcing setting it is natural to assume the order quantity for the arriving offshore source previously placed should be taken into account when determining the later, local order o^n_t. To obtain s^n_t, we rearrange the modified inventory balance equation (11) to yield,

$$o_t^n = ns_{t+\ell+1} - ns_{t+\ell} - o^g_{t-(L-\ell)} - d_{t+\ell+1}. \tag{14}$$

Using (14), WIP_t^n can be rewritten as

$$\mathrm{WIP}_t^n = \sum_{i=1}^{\ell} o^n_{t-i} = ns_{t+\ell} - ns_t - \sum_{i=1}^{\ell} o^g_{t-(L-\ell)-i} + \sum_{i=1}^{\ell} d_{t+i}. \tag{15}$$

After substituting (14) into the LHS, and (15) into the RHS, of (13), simplification yields

$$ns_{t+\ell+1} = s_t^n + \sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i} - \sum_{i=1}^{\ell+1} d_{t+i}. \tag{16}$$

Using the conditional expected values for future demand, s_t^n can be obtained from (16)

$$s_t^n = \mathbb{E}\left[\sum_{i=1}^{\ell+1} d_{t+i}|t\right] - \sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i} + \mu_{ns}, \tag{17}$$

where

$$\mathbb{E}\left[\sum_{i=1}^{\ell+1} d_{t+i}|t\right] = (\ell+1)\mu_d + \phi\Lambda_{\ell+1}(d_t-\mu_d) - \theta\Lambda_{\ell+1}\varepsilon_t : \quad \Lambda_{\ell+1} = \frac{1-\phi^{\ell+1}}{1-\phi}. \tag{18}$$

The target net stock (or, safety stock) level $\mu_{ns} = \mathbb{E}[ns_{t+\ell+1}]$ is a constant value. Eq. (17) reveals the value of s_t^n should equal the difference between the MMSE forecast of the market demand (not the decomposed demand) over $\ell+1$ time periods and the sum of the offshore orders already placed that will be received by the firm by the end of time period $t+\ell+1$, plus the safety stock μ_{ns}. Note, as $\ell < L$, the value of $\sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i}$ is known at time t. Substituting (17) into (13) yields the following OUT policy for the nearshore source under the dual source setting:

$$\begin{aligned} o_t^n &= \mathbb{E}\left[\sum_{i=1}^{\ell+1} d_{t+i}|t\right] - \sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i} + \mu_{ns} - \left(\sum_{i=1}^{\ell} o^n_{t-i} + ns_t\right) \\ &= \mathbb{E}[d_{t+\ell+1}|t] + \underbrace{\mu_{ns} - ns_t}_{\text{Inventory feedback}} + \underbrace{\mathbb{E}\left[\sum_{i=1}^{\ell} d_{t+i}|t\right] - \left(\sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i} + \sum_{i=1}^{\ell} o^n_{t-i}\right)}_{\text{WIP feedback}}. \end{aligned} \tag{19}$$

Based on the knowledge obtained so far, we can define another form of the OUT policy in the dual source setting. By incorporating (15) into (13) and after some simplification, we have

$$o_t^n = s_t^n - \left(ns_{t+\ell} - \sum_{i=1}^{\ell} o^g_{t-(L-\ell)-i} + \sum_{i=1}^{\ell} d_{t+i}\right). \tag{20}$$

With the knowledge of (16), $ns_{t+\ell}$ can be written as

$$ns_{t+\ell} = s^n_{t-1} + \sum_{i=0}^{\ell} o^g_{t-(L-\ell)-i-1} - \sum_{i=1}^{\ell+1} d_{t+i-1}. \tag{21}$$

After substituting (21) into (20), algebra reveals another formulation of the OUT policy for the nearshore source under the dual sourcing setting:

$$o^n_t = d_t - o^g_{t-(L+1)} + (s^n_t - s^n_{t-1}), \tag{22}$$

where $d_t - o^g_{t-(L+1)}$ can be interpreted as the net demand for the nearshore source, which is the portion of the market demand to be fulfilled by the nearshore source. This net demand drives the value of o^n_t, not the market demand. The policy in (22) is useful as it contains only feed-forward paths, facilitating analysis of the policy. In summary, the firm that dual sources after re-shoring will order using (12) from the offshore source and (13), (19), or (22) (they are all equivalent) from the nearshore source.

5.3. Variances in the case of dual sourcing

For the economic analysis, we require expressions of both the net stock level variance at the local firm and the nearshore production order variance under the dual sourcing setting (we already know the offshore supplier's order variance). After substituting (17) into (16), simplification yields

$$ns_{t+\ell+1} - \mu_{ns} = \mathbb{E}\left[\sum_{i=1}^{\ell+1} d_{t+i}|t\right] - \sum_{i=1}^{\ell+1} d_{t+i}.$$

Therefore, the variance of the net stock levels at the local firm in the case of the dual sourcing is

$$\begin{aligned}\mathbb{V}[ns] &= \mathbb{E}\left[(ns_{t+\ell+1} - \mu_{ns})^2\right] = \mathbb{E}\left[\left(\mathbb{E}\left[\sum_{i=1}^{\ell+1} d_{t+i}|t\right] - \sum_{i=1}^{\ell+1} d_{t+i}\right)^2\right] \\ &= \sigma^2_\varepsilon\left(1 + \frac{\ell(\theta-1)^2}{(\phi-1)^2} + \frac{\phi(\phi-\theta)\left(\phi^\ell - 1\right)\left(\theta\left(\phi(1-\phi^\ell)+2\right) + \phi\left(\phi(1+\phi^\ell)-2\right)-2\right)}{(\phi-1)^3(\phi+1)}\right).\end{aligned} \tag{23}$$

Comparing (23) with (7) provides following Lemma.

Lemma 3 (Variance of local net stock levels in the dual sourcing setting). *When the firm decomposes the* ARMA$(1,1)$ *demand process into d^n_t and d^g_t and places orders o^n_t and o^g_t using the OUT policies, the variance of the local firm's net stock levels becomes identical to the variance of the net stock maintained by a single near-shore supplier, and the offshore lead-time (L) has no impact on the local net stock variance.*

This finding is attractive for the companies who are considering re-shoring. The offshore lead-time (L) does not affect the value of the net stock variance, $\mathbb{V}[ns]$. It suggests that inventory costs, which are a linear function of $\sqrt{\mathbb{V}[ns]}$, can be reduced to the same level maintained by the single nearshore source, while still enjoying lower manufacturing costs offered by the offshore source. A similar finding can be seen in Boute & Van Mieghem (2015). Note, Lemma 3 holds regardless of how the demand process is decomposed as long as the firm uses the OUT policies shown above.

The variance of the nearshore production orders o_t^n can be obtained from (22), (17), and (18);

$$o_t^n = d_t + \phi\Lambda_{\ell+1}(d_t - d_{t-1}) - \theta\Lambda_{\ell+1}(\varepsilon_t - \varepsilon_{t-1}) - o_{t-(L-\ell)}^g, \tag{24}$$

where $\Lambda_{\ell+1} = (1-\phi^{\ell+1})(1-\phi)^{-1}$. Substituting the RHS of (1) into (24) yields

$$\begin{aligned} o_t^n =& \mu_d + (\phi + \phi\Lambda_{\ell+1}(\phi-1))(d_{t-1}-\mu_d) + (1+\Lambda_{\ell+1}(\phi-\theta))\varepsilon_t - \\ & \theta(1+\Lambda_{\ell+1}(\phi-1))\varepsilon_{t-1} - o_{t-(L-\ell)}^g. \end{aligned}$$

Using $\mathbb{V}[o^n] = \mathbb{E}\left[(o_t^n - \mu_d)^2\right]$, the variance of o_t^n is

$$\begin{aligned} \mathbb{V}[o^n] =& (\phi + \phi\Lambda_{\ell+1}(\phi-1))^2\mathbb{V}[d] + (1+\Lambda_{\ell+1}(\phi-\theta))^2\sigma_\varepsilon^2 + (\theta+\theta\Lambda_{\ell+1}(\phi-1))^2\sigma_\varepsilon^2 + \theta^2\sigma_\varepsilon^2 \\ & - 2(\phi+\phi\Lambda_{\ell+1}(\phi-1))(\theta+\theta\Lambda_{\ell+1}(\phi-1))Cov\left[(d_{t-1}-\mu_d)\cdot(\varepsilon_{t-1})\right] \\ & - 2(\phi+\phi\Lambda_{\ell+1}(\phi-1))Cov\left[(d_{t-1}-\mu_d)\cdot\left(o_{t-(L-\ell)}^g\right)\right], \end{aligned} \tag{25}$$

where $Cov[X \cdot Y]$ is the covariance between X and Y. To obtain (25), we used the following: $Cov\left[(d_{t-1}-\mu_d)\cdot\varepsilon_t\right] = 0$, $Cov\left[\varepsilon_t \cdot \varepsilon_{t-1}\right] = 0$, $Cov\left[\varepsilon_t \cdot o_{t-(L-\ell)}^g\right] = 0$, and $Cov\left[\varepsilon_{t-1} \cdot o_{t-(L-\ell)}^g\right] = 0$. As $Cov\left[(d_{t-1}-\mu_d)\cdot(\varepsilon_{t-1})\right] = \sigma_\varepsilon^2$ and $Cov\left[(d_{t-1}-\mu_d)\cdot\left(o_{t-(L-\ell)}^g\right)\right] = \left(\theta^2\phi^{L-\ell-1} - \theta\phi^{L-\ell}\right)\sigma_\varepsilon^2$, the variance of o_t^n can be rewritten as[1]

$$\begin{aligned} \mathbb{V}[o^n] = \sigma_\varepsilon^2 \Bigg(& \theta\left(\theta + \phi^{2\ell+2}(\theta-2\phi) + 2(\phi-\theta)\phi^{L+1}\right) + \\ & \frac{(\theta(\theta-2\phi)+1)\phi^{2\ell+4}}{1-\phi^2} + \frac{\left(\theta+\phi^{\ell+1}(\phi-\theta)-1\right)^2}{(\phi-1)^2} \Bigg). \end{aligned}$$

Note, although $\mathbb{V}[o^n]$ represents the variance of orders to the nearshore source, the offshore lead-time (L) appears in this formula, which leads to the following Lemma.

Lemma 4 (Impact of offshore lead-time on the nearshore order variance). *If an ARMA(1,1) demand process is decomposed into d_t^n and d_t^g and orders o_t^n and o_t^g are placed using OUT policies, when $\theta(\phi-\theta) > 0$, the variance of nearshore orders ($\mathbb{V}[o^n]$) reduces in the offshore lead time L.*

[1] This variance expression only holds when $\ell \leq L$.

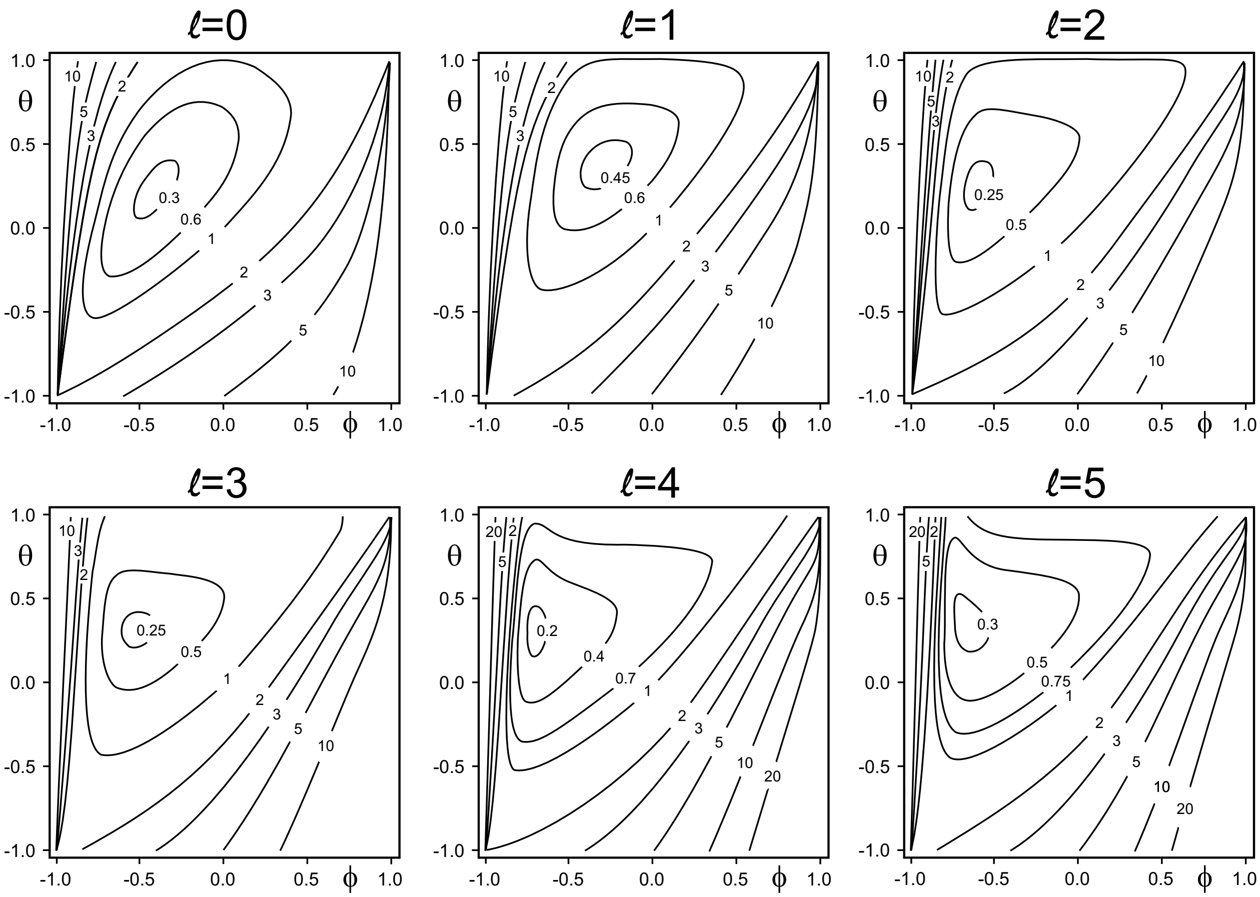

Figure 2: The local order variance when the global supplier has a lead-time of $L = 5$ and $\sigma_\varepsilon^2 = 1$

This is a counter-intuitive finding. Lemma 4 implies that, when certain conditions are met, nearshore order variance, $\mathbb{V}[o^n]$, (a proxy for the nearshore capacity cost) can be reduced by deliberately lengthening the offshore lead-time, L. Remember, Lemma 3 shows long offshore lead-times do not impact the local inventory variance. Lemma 4 shows that longer offshore lead-time could have a positive impact on the nearshore capacity cost and this is favourable from an ESG perspective. For example, Lemma 4 implies it makes economic sense to utilise vessels that are more environmentally friendly with low operational costs despite the longer transportation lead-times from offshore sources that may result.

6. Economic analysis

We now analyse the effectiveness of our proposed OUT policies for the dual sourcing setting by using the following two cost functions: The first one is for the base cases, single global offshore and nearshore source supply chains:

$$C_j = \underbrace{(h\mathbb{E}[[ns_t]^+] + p\mathbb{E}[[-ns_t]^+])}_{\text{Local inventory costs}} + \underbrace{(u^j k^j + u^j m^j \mathbb{E}[[o_t^j - k^j]^+])}_{\text{Offshore/nearshore capacity costs}},$$

where $j \in \{n, g\}$. Here h is the per period per unit inventory holding costs, p the per period per unit backlog penalty costs, u^j is the per unit production (labour) cost within the per period nominal capacity of k^j and $u^j m^j$, (where $m^j \geq 1$ is the over-time multiplier for production above the nominal capacity of normal working hours) is the per unit production (labour) cost in overtime above the nominal capacity of k^j. Notice, labour are guaranteed their nominal per period (perhaps weekly) wage of $u^j k^j$, but overtime has volume flexibility. The inventory costs are minimised by setting the safety stock target, μ_{ns}, to the newsvendor critical factile,

$$\mu_{ns}^{\star} = \sqrt{\mathbb{V}[ns]}\, \Phi^{-1}\left[\frac{p}{p+h}\right].$$

When $\mu_{ns}^{\star}$ is present, the minimised inventory costs are given by

$$h\mathbb{E}[[ns_t]^+] + p\mathbb{E}[[-ns_t]^+] = (h+p)\sqrt{\mathbb{V}[ns]}\, \varphi\left[\Phi^{-1}\left[\frac{p}{p+h}\right]\right],$$

Churchman et al. (1957). Here, $\varphi[\cdot]$ and $\Phi^{-1}[\cdot]$ is the pdf and inverse cdf of the standard normal distribution. Lemma 3 revealed the inventory costs in C_n and C_g are identical. Using the same newsvendor techniques, the capacity costs for the source $j \in \{n, g\}$ are minimised by setting the nominal capacity k^j to

$$k^{j\star} = \mu_j + \sqrt{\mathbb{V}[o^j]}\Phi^{-1}\left[\frac{m^j - 1}{m^j}\right].$$

When $k^{j\star}$ is present, the minimised capacity costs are

$$u^j k^{j\star} + u^j m^j \mathbb{E}[(o_t - k)^+] = u^j \mu_j + u^j m^j \sqrt{\mathbb{V}[o^j]}\varphi\left[\Phi^{-1}\left[\frac{m^j - 1}{m^j}\right]\right],$$

Boute et al. (2022).

The second cost function is for the case of the dual sourcing supply chain can be written as

$$C_d = \underbrace{(h\mathbb{E}[[ns_t]^+]) + p\mathbb{E}[[-ns_t]^+])}_{\text{Local inventory costs}} + \sum_{j\in\{g,n\}} \underbrace{(u^j k^j + u^j m^j \mathbb{E}[[o_t^j - k^j]^+])}_{\text{Offshore and nearshore capacity costs}}.$$

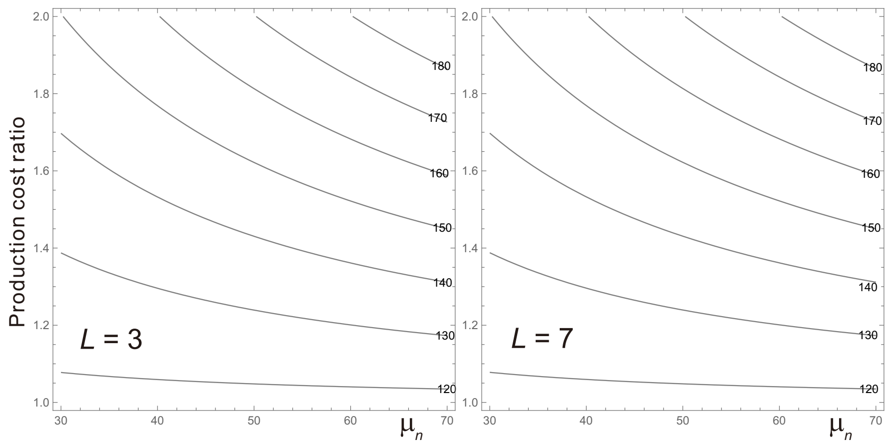

Figure 3: Impact of μ_n, u^n and L on the total cost of dual sourcing supply chain, C_d. Values of parameters used herein are: $\phi = 0.6$, $\theta = 0.3$, $\mu_d = 100$, $31 \leq \mu_n \leq 69$, $\mu_g = \mu_d - \mu_n$, $\sigma = 5$, $L = \{3,7\}$, $\ell = 1$, $h = 1$, $p = 10$, $m = m^n = 1.2$, $u = 1$, and $u^n = u \times r$, where r is production cost ratio ($1 \leq r \leq 2$).

6.1. Numerical analysis

To illustrate the economic behaviour of our model, a numerical study is conducted. There are three goals of this analysis: i) To understand the impact of the decision variables, μ_n and μ_g, that determine the allocation between the offshore and the nearshore sources, on the total supply chain costs, ii) To understand the impact of the offshore lead time, L, on the total supply chain costs and iii) To understand how the demand parameters ϕ and θ affect the economics of dual sourcing.

Fig. 3 shows the impact of μ_n on the total cost of dual sourcing supply chain, C_d. When we conduct the analysis, to avoid negative values in o_t^g and o_t^n, we constrain the demand process such that $\mu_j - 4\sqrt{\mathbb{V}[o^j]} > 0$, where $j \in \{n,g\}$. To meet this constraint, the value of μ_j is restricted between $31 \leq \mu_j \leq 69$, in addition to the total volume constraint that $\sum_j \mu_j = \mu_d = 100$. It is clearly shown that if the supply chain wants to allocate larger volume to the nearshore source, the production cost ratio ($r = u^n/u$) should be as low as possible to enjoy lower total supply chain costs, especially when the offshore lead time is short. In terms of the second goal, it is quite difficult to perceive the difference between two graphs in Fig. 3 despite L changes from 3 to 7.

Fig. 4 and 5 illustrate the impact of ϕ and θ on the total cost of each supply chain, when $L \in \{3,7\}$. Area I represents the area where a single global supplier dominates dual sourcing which in turn dominates a single local supplier, $C_g^* \leq C_d^* \leq C_n^*$. Area II is where dual sourcing dominates a single global supplier which in turn dominates a single local supplier, $C_d^* \leq C_g^* \leq C_n^*$. Area III is the case where dual sourcing dominates a single local supplier which in turn dominates a single global supplier, $C_d^* \leq C_n^* \leq C_g^*$. The parameter values used herein are: $\mu_d = 100$, $\mu_n = 31$, $\sigma = 5$, $\ell = 1$, $h = 1$, $p = 10$, $m = m^n = 1.2$, $u = 1$, and $u^n = 1.5$. A comparison with Fig. 2 shows that the greater the variance of nearshore orders, the lower the total costs tend to be when dual sourcing is used. In addition, the longer the offshore lead time, the more likely dual sourcing becomes the

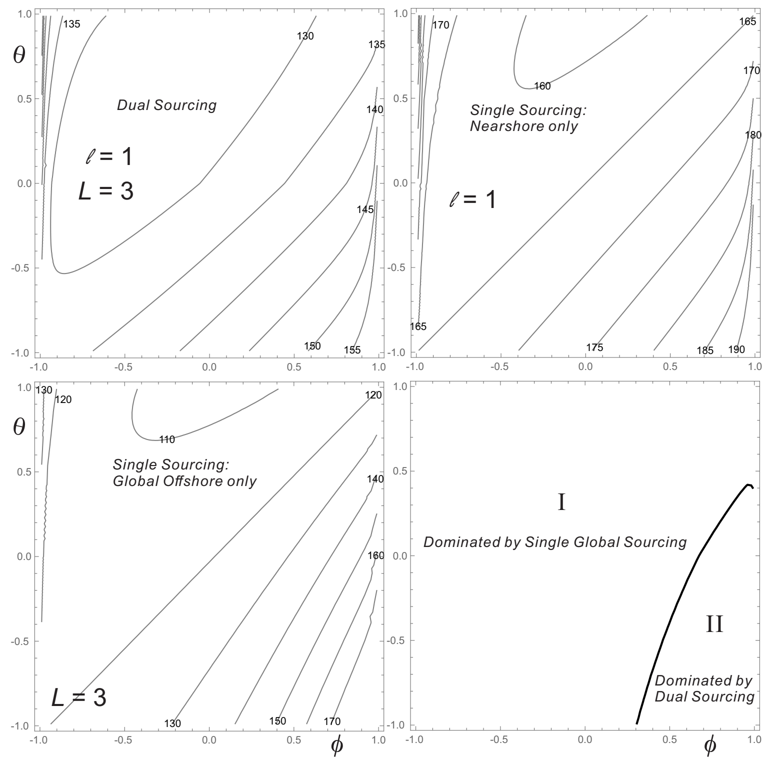

Figure 4: Total costs contours and strategy selection in the case of $L = 3$.

lowest cost strategy.

7. Conclusions

Using an analytical approach coupled with a numerical analysis, we have investigated the dynamics and economics of a reshoring setting with dual sourcing. We assumed demand was an ARMA(1,1) process. We decomposed the ARMA(1,1) process into two demand streams. The demand stream for the global supplier was based on the MA component of the ARMA(1,1) demand; the demand stream for the local supplier comprised of the AR component of the ARMA(1,1) demand. We obtained expressions for the variance of the local and global orders, as will as the variance of the finished goods inventory. Uniquely, we quantified the cost of an installed capacity and overtime at both production sources and the inventory holding and backlog costs. We identified areas in the demand parameter space where the local, global, and dual sourcing strategies dominate for different global lead times L.

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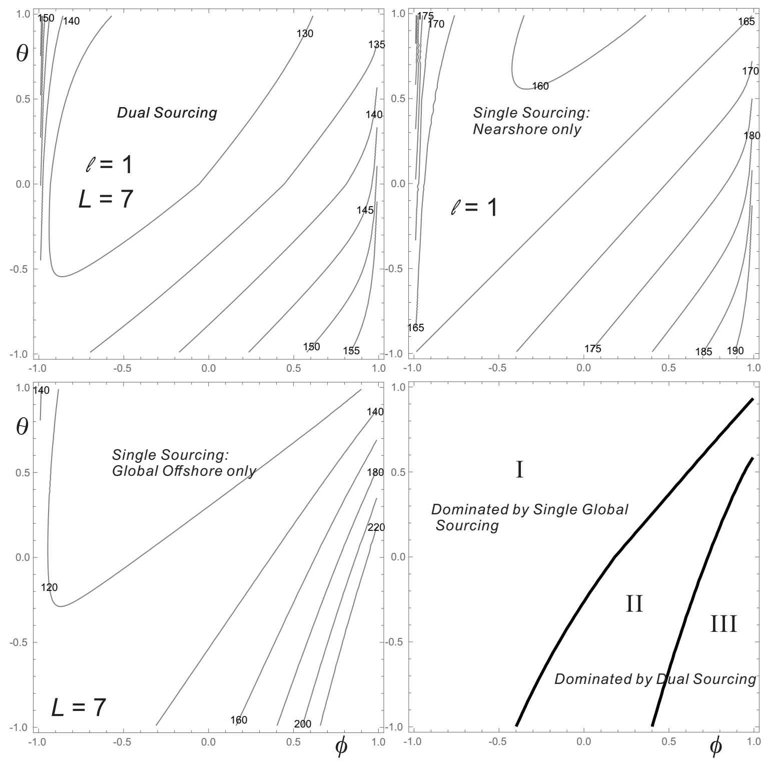

Figure 5: Total cost contours and strategy selection in the case of $L = 7$.

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