Wave Energy Dissipation of Waves Breaking on a Reef with a Steep Front Slope

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WAVE ENERGY DISSIPATION OF WAVES BREAKING ON A REEF WITH A STEEP FRONT SLOPE

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Abstract: The transformation of waves propagating over a steep bottom slope is of great importance regarding the coastal processes in the near-shore area. This study will contribute with tools to predict the dissipated wave energy for irregular waves passing a steep submerged slope. An extensive number of tests with regular and irregular waves breaking over a steep bottom slope have been performed in the Hydraulics & Coastal Engineering Laboratory, Aalborg University. Based on these experimental data formulae have been developed capable of predicting the transmitted wave energy over steep slopes.

In addition to the experimental work a numerical wave model has been developed, which can simulate irregular waves propagating over steep slopes and the related wave energy dissipation due to breaking waves and bed friction. The propagation of waves in the model is based on the extended refraction and diffraction equation, often referred to as the Extended Mild Slope Equation.

The results from this study will be applicable in the design of coastal structures as submerged breakwaters or artificial and natural reefs.

INTRODUCTION

The objective of the study is to establish tools to predict the wave energy dissipation for waves breaking on a steep submerged slope. Sets of formulae have been developed based on experimental tests performed in the hydraulic laboratory. Furthermore, a comprehensive numerical wave model has been developed, which can simulate irregular waves propagating over steep slopes and the related wave energy dissipation.

The study was carried out at Aalborg University, Hydraulics & Coastal Engineering Laboratory as a part of a Ph.D. degree awarded February 2003. In Jensen (2003) the study is described in detail.

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TESTS IN HYDRAULIC LABORATORY

In 2000 Jensen M.S performed a large series of tests at the hydraulic laboratories of Aalborg University. A 23 m long and 1.5 m wide wave flume was used for the simulation of waves breaking over a simulated steep reef-slope. The water depth range in front of the reef-slope was between 0.56-0.71 m. The height of the slope was 0.38 m with a water depth of 0.18-0.33 m on the horizontal plateau. The length of the reef-plateau was 14 m to provide enough length for the waves to reform after breaking. The waves were dissipated after the reef-plateau by a mild sloping gravel beach. The energy dissipation due to wave breaking was of interest and the bottom friction was therefore minimised by applying a layer of thick plastic on top of the plateau. The layout of the wave flume and the location of 14 wave gauges are shown in Fig.1.

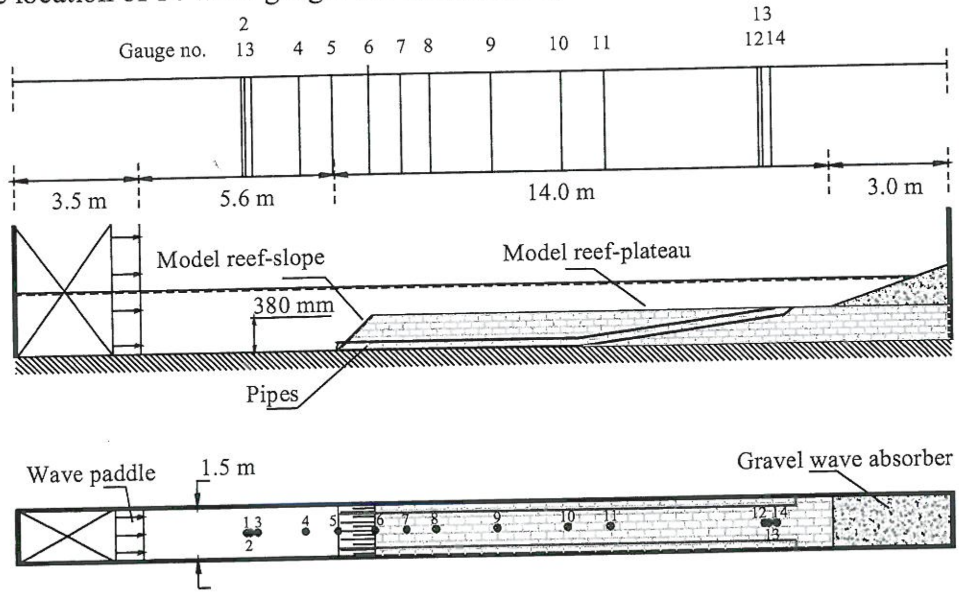


Fig. 1. Experimental layout in the hydraulic laboratory

A return flow system was provided to avoid the effect of water being pumped over the reef-plateau. Due to the waves being reflected from the steep slope an active wave absorption system was used. The absorption system was developed at Aalborg University and described in details by Frigaard et al. [1994]. The incident waves were controlled with good accuracy. The layout of the model, with the relative small width of the wave flume being 1.5 m, ensures that minimum resonant cross oscillations were present. Four different layouts of the reef-slope were used with the inclinations 1:0.5, 1:1, 1:2 and an additional S-formed slope. The waves were measured along the flume with capacitance gauges. Regular waves were generated in order to establish threshold values for wave breaking and irregular waves were generated in order to evaluate the wave energy dissipation. In total 301 tests with regular waves and 110 tests with irregular waves were performed. The irregular wave trains consisted of 650 waves.

Ratios between important wave parameters are given in Table 1. Ratios are derived from measurements of irregular waves at gauge No.11. The ratio H_{max}/H_s=1.37 is lower compared with a ratio of 1.80 assuming that wave heights are Rayleigh distributed. The ratio $H_{max}/h_t \approx 0.5$ is smaller than the often used breaking criterion ratio of 0.78 in water of constant depth (McCowan, 1894). The relation between H_{mo} and H_s is in good compliance with other studies showing that H_{mo} is approximately 10% larger than H_s. It is also shown that a modified Swart and Loubser's (Swart and Loubser, 1978) type parameter F_{c0} is useful in connection with waves breaking. Gourlay (1994) also argued that $F_{co} = (H_o/h_t)^{1/2}\left(T\sqrt{g/h_t}\right)^{5/2}$ based on the deep-water wave height, H_0, and a representative depth over the plateau, h_t, is a suitable parameter classifying wave transformation on a steep bottom slope. For irregular waves the incident wave height H_s can be used instead of H_0 and T_p instead of T. F_{c0} less than 10 indicate deep-water waves and values in the range 10-500 characterise transitional waves. Values above 500 indicate shallow water waves. In Jensen (2003) it is shown that some relations, as indicated in Table 1, are correlated with F_{c0}.

Table 1. Important ratios between often-referred wave parameters

Important ratios	Range	Comment
H_{max} / h_t	0.4-0.6	Correlated with F_{co}
H_{rms} / H_{max}	0.55	Constant ratio
H_{max} / H_s	1.37	Constant ratio
H_{mo} / H_s	1.0 - 0.85	Correlated with F_{co}

Based on the data obtained in the experimental tests it can be concluded that the transmitted wave energy depends on the seaward incident wave height, the wave period and the water depth on the plateau. The transmitted wave energy depends only marginal on the steepness of the slope within the range of slopes tested, i.e. 1:2–1:0.5. This is considered to be an important observation.

PREDICTIVE FORMULAE FOR WAVE HEIGHT TRANSMISSION

Both formulae for the transmission of wave height and wave energy flux, respectively, have been derived in Jensen (2003). In the following the formula describing the wave height transmission is presented. An expression of the wave height transmission is developed using the concept of gradient of the energy flux on the reef plateau. Massel and Brinkman (2001) used this approach based on a model proposed by Dally et al. (1985). The model predicts the transformation of energy flux along the reef plateau. Applying this model to the experimental layout an equation for wave height on the reef plateau can be obtained as

$$\frac{H_t(x)}{h_t} = \left\{\left[\left(\frac{H_t}{h_t}\right)_i^2 - \gamma^2\right]\exp\left[-k_a\frac{(x-x_i)}{h_t}\right] + \gamma^2\right\}^{1/2} \qquad (1)$$

where $H_t(x)$ is the transmitted wave height along the reef plateau. Lower index i indicates the value of non-dimensional wave height (H_t/h_t) at the i^{th} gauge. Location of the gauge is denoted x_i. The breaker index γ is the ratio of the water depth yielding the maximum allowable wave height and k_a is the attenuation coefficient. Wave heights at i^{th} gauge are measured in the experimental tests.

Transmission of irregular waves is of practical interest and therefore a formula will be based on the significant wave height. By rearranging Eq.1 and using wave gauge No.6 as a reference point the significant wave height transmission coefficient $K_{t,H}(x) = H_t(x)/H_s$ becomes (H_s is the incident significant wave height)

$$K_{t,H}(x) = \sqrt{\left(\frac{\gamma h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_t}{h_t}\right)_6^2 - \gamma^2\right] \exp\left[-k_a \frac{(x - x_6)}{h_t}\right]\right\}} \tag{2}$$

At $x = x_6$ and for $x \to \infty$ the wave height transmission coefficient becomes $K_{t,H}(x_6) = H_6/H_s$ and $K_{t,H}(x \to \infty) = \gamma h_t / H_s$. Such results indicate that the limits of the formula are sound. Based on the experimental data a least square method is used to obtain a calibration of γ and k_a. The calibration yields γ =0.35 and k_a= 0.20. The length along the reef plateau is given by the parameter X, with starting point at X= 0 at x_6. In Eq.(3) the calibrated constants are inserted into Eq.(2)

$$K_{t,H}(x) = \sqrt{\left(\frac{0.35 h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_{t,X=0}}{h_t}\right)^2 - 0.35^2\right] \exp\left[-0.2 \frac{X}{h_t}\right]\right\}} \tag{3}$$

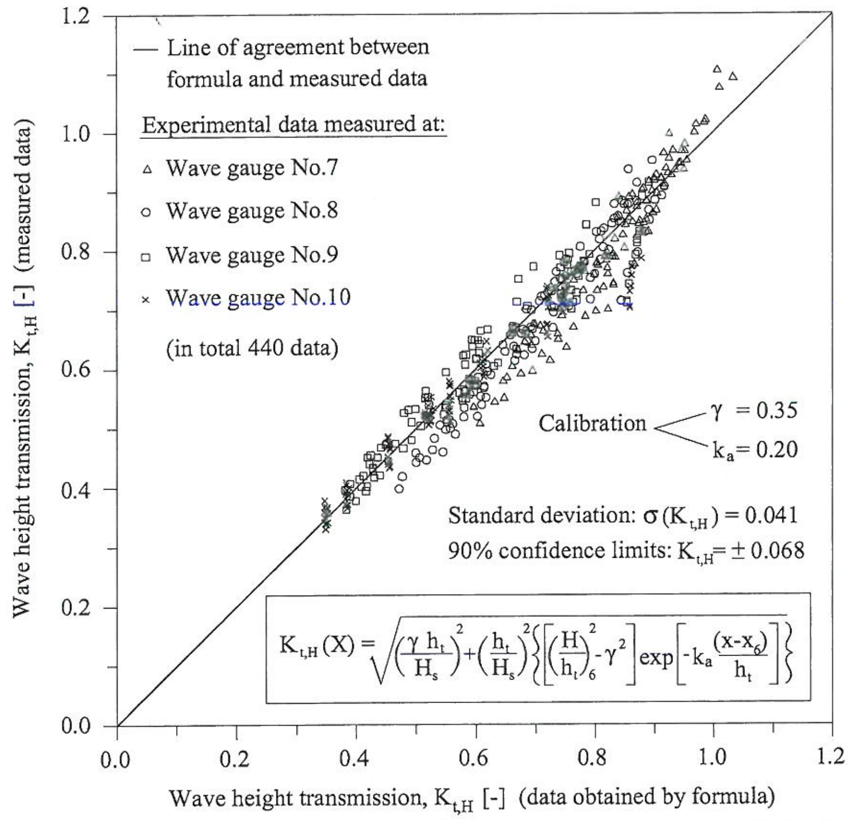


Fig. 2. Predicted wave height transmission versus measured data, Eq.(3)

Fig.2 shows a graph comparing predicted transmitted wave heights based on the expression given in Eq.(3) with measured data obtained in the experiments. There are plotted data that seem to follow another trend than the rest of the depicted data (around $K_{t,H}$= 0.8). These data are all obtained in the experiments involving the smaller incident significant wave height combined with the deepest water depth at the reef plateau. In these wave trains the individual waves are only "mildly" breaking, i.e many of the waves pass the reef slope without breaking. To ensure wave breaking over the reef slope, the requirement using formulae Eq.(3) is $H_s/h_t \geq 0.4$. Furthermore, the transmitted significant wave height on the reef plateau, $H_{t,X=0}$ shall not be smaller than the reformed wave height on the reef plateau. This constraint results in the requirement $H_{t,X=0}/h_t \geq \lambda \Rightarrow \; H_{t,X=0}/h_t \geq 0.35$.

Eq.(3) provides a tool for prediction of the significant wave height transformation along the reef. The incident wave height, H_s, can be omitted in Eq.(3). The equation is not useful as a design tool when the wave heights are not known on the reef or an artificial reef is to be designed. It is experienced from the experimental data that there exists a close relationship between the incident wave height and the wave height measured at the reef edge. Instead of the reference wave height, H_6, on the reef plateau, the incident significant wave height is used. This results in Eq.(4).

$$K_{t,H}(X) = \sqrt{\left(\frac{\gamma h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_s}{h_t}\right)^2 - \gamma^2\right] \exp\left[-k_a \frac{X}{h_t}\right]\right\}} \tag{4}$$

The calibration constants γ and k_a remains unchanged. Eq.(5) is the formula predicting the significant wave height on the reef plateau based on the incident significant wave height (measured before the slope). The top graph shown at Fig.3 is based on Eq.(5).

$$K_{t,H}(X) = \sqrt{\left(\frac{0.35 h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_s}{h_t}\right)^2 - 0.35^2\right] \exp\left[-0.2 \frac{X}{h_t}\right]\right\}} \tag{5}$$

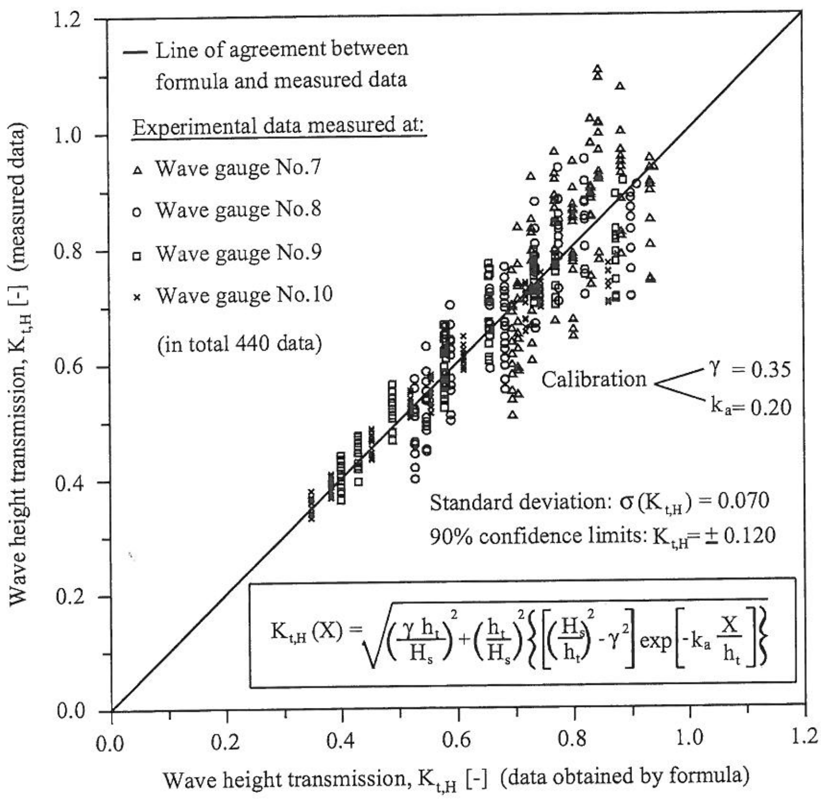
By comparing Fig.2 and the top graph in Fig.3 it is clear that the scatter is increased when the reference significant wave height is replaced with the incident significant wave height. The wave period is not represented in Eq.(5) and it is observed that the amount of waves, in the wave train, breaking over the steep slope is correlated with the steepness of the waves. A best fit is obtained by relating the wave steepness to the wave attenuation parameter. The wave steepness is given by $S_i = H_s/L_P$. The wave length is calculated using the incident spectral peak period and the finite water depth seaward the reef as $L_P = (g/2\pi)T_P^{\,2} \tanh(2\pi\, h/L_P)$. Eq.(6) is the modified formula included the wave steepness and calibration factor m.

$$K_{t,H}(X) = \sqrt{\left(\frac{\lambda h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_s}{h_t}\right)^2 - \lambda^2\right] \exp\left[-m S_i k_a \frac{X}{h_t}\right]\right\}} \tag{6}$$

The values of γ and k_a are unchanged. A best fit results in m =22. Inserting the calibrated parameters Eq.(7) can be written.

$$K_{t,H}(X) = \sqrt{\left(\frac{0.35 h_t}{H_s}\right)^2 + \left(\frac{h_t}{H_s}\right)^2 \left\{\left[\left(\frac{H_s}{h_t}\right)^2 - 0.35^2\right] \exp\left[-4.4 S_i \frac{X}{h_t}\right]\right\}} \tag{7}$$

A requirement using Eq.(7) is $H_s/h_t \geq 0.4$ to ensure wave breaking on the plateau. The bottom graph shown at Fig.3 is based on Eq.(7). It is seen that the scatter has decreased.



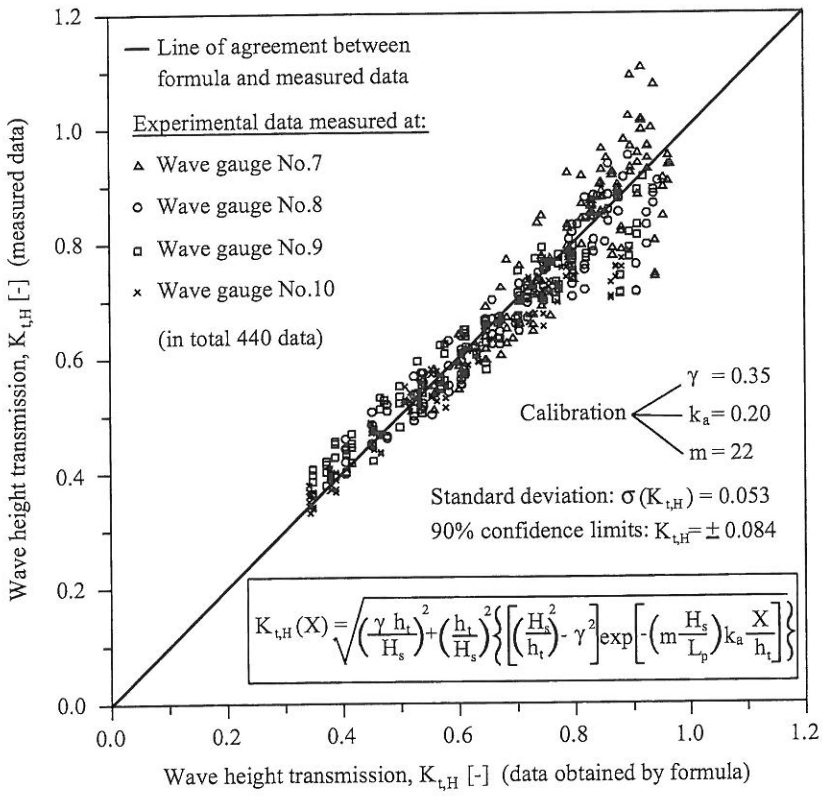


Fig. 3. Predicted wave height transmission versus measured data. Eq.(5) is depicted in top graph and Eq.(7) is depicted in bottom graph

NUMERICAL WAVE MODEL

A numerical wave model has been developed, which can simulate irregular waves propagating over steep slopes including wave energy dissipation. The propagation of waves is based on the extended refraction and diffraction equation, often referred to as the Extended Mild Slope equation, provided by Massel [1993] and Massel and Gourlay [2000]. This equation takes into account steep and rapidly varying bottom topography. Wave energy dissipation due to bottom friction is modelled by an expression for the average rate of energy dissipation in waves given by Gerritsen [1981]. The breaking process has been parameterised using a modification of the Battjes and Janssen [1978] periodic bore approach. An empirical parameter related to the bore model is calibrated against the experimental data. The present model deals with one-dimensional irregular, dissipating propagating waves but is also capable of generating two-dimensional monochromatic waves. Other researchers have demonstrated the possibility of generating directional waves. Waves are generated internally in the numerical model with sponge layers placed at the outside boundaries for absorption of the reflected wave energy. A modified formulation by Radder and Dingemans (1985) is used in order to make the equations suitable to be solved in a numerical approach. A numerical solver of the equations is based on the Adams-Bashforth-Moulton scheme.

The numerical model is verified in several cases when compared to the theoretical solution. The extended refraction and diffraction equation is an extension of the classical Mild-Slope equation. The classical Mild-Slope equation is limited to milder slopes than applied in this study. Results to be noted and obtained in several test examples are the importance of the higher-order bottom terms. A test regarding the reflection of waves from a plane slope improves the solution significantly. This is also the case for even milder slopes than 1:3, which is a commonly referenced limit for the application of the classical Mild-Slope equation.

The transient Extended Mild Slope equation governing the propagation of irregular waves over a rapidly varying slope including energy dissipation is seen in Eq.8. The top expression is the classical Mild Slope equation. The middle expression is the higher order bottom terms. The bottom expression is the wave energy dissipation due to wave breaking and the energy dissipation due to bed friction.

$$
\begin{aligned}
&(\textit{classical Mild Slope eq.}) && \varphi_{tt} - \nabla\cdot(\overline{C}\,\overline{C}_g \nabla\varphi) + (\varpi^2 - \overline{k}^2\overline{C}\,\overline{C}_g)\varphi + \\
&(\textit{higher order terms}) && \varpi^2\left\{R_1(\nabla h)^2 + R_2\nabla^2 h\right\}\varphi + \\
&(\textit{wave energy dissipation}) && \left\{\frac{\alpha_0\,\varpi}{\pi\,h}\left(\frac{H_{\max}^3}{H_{rms}^2}\right)\left(\frac{\sqrt{gh}}{\overline{C}}\right)Q_b + \frac{16 f_r}{3\pi}\frac{|U|^3}{gH_{mean}^2}\right\}\varphi_t = 0
\end{aligned}
\qquad (8)
$$

φ	The velocity potential
φ_{tt}	Second derivative of the velocity potential $\varphi_{tt} = \partial^2\varphi/\partial t^2$
$\overline{C}$	Wave celerity associated with the wave carrier frequency
$\overline{C}_g$	Group velocity associated with the wave carrier frequency
$\overline{\omega}, \overline{k}$	Angular wave frequency and wave number associated with the wave carrier frequency
R_1, R_2	Parameters determining higher-order bottom effects. Can be found in Jensen (2003)

α_0	Coefficient describing the intensity of wave breaking
H	Water depth
Q_b	Fraction of wave breaking
f_r	Wave energy dissipation factor
U	Amplitude of the bed velocity variations
H_{mean}	Mean wave height in a wave train

Energy dissipation due to wave breaking can be introduced in the numerical model as Eq.9. All parameters are in general known except the parameter α_0.

$$W_f = \frac{\alpha_0\,\varpi}{\pi\,h}\left(\frac{H_{\max}^3}{H_{rms}^2}\right)\left(\frac{\sqrt{gh}}{C}\right) Q_b \qquad (9)$$

According to Massel and Gourlay [2000] α_0 is correlated with F_{c0}. The experimental data have been used to calibrate α_0 and Eq.10 suggests an expression for α_0

$$\alpha_0 = \frac{F_{c0}^{0.77}}{100}\frac{h_t}{H_s} \qquad (10)$$

The wave model is verified in the range of F_{c0} = [100-1000]. F_{c0} = 100 is the threshold of waves breaking and F_{c0} = 1000 is the limit of the wave model, where the extended refraction-diffraction model provides realistic results.

It is suggested that the periodic bore model approach for description of the energy dissipated, Eq.(9), is modified compared to the original proposed model by Battjes and Janssen [1978]. The maximum wave height should be determined as H_{max}/h_t= 0.5 instead of a ratio being 0.8 as used in the original model. Furthermore, when the ratio between the root-mean-square wave height obtained in the numerical model and the maximum wave height exceeds 0.55, i.e. when H_{rms}/H_{max} > 0.55, then the fraction of waves breaking, Q_b, is set to 1.0 and $H_{\max}^3/H_{rms}^2$ is set to H_{rms} as written in Eq.11.

$$\begin{aligned} &H_{rms} \le 0.55\,H_{\max} \Rightarrow W_f = \frac{\alpha_0\,\varpi}{\pi\,h}\left(\frac{H_{\max}^3}{H_{rms}^2}\right)\left(\frac{\sqrt{gh}}{C}\right) Q_b \qquad else \\ &H_{rms} > 0.55\,H_{\max} \Rightarrow W_f = \frac{\alpha_0\,\varpi}{\pi\,h}\left(H_{rms}\right)\left(\frac{\sqrt{gh}}{C}\right) \{Q_b = 1.0\} \end{aligned} \qquad (11)$$

Wave model to simulate experimental tests by Thorkilsen et al.

In the following are shown results obtained from the wave model. The model set-up and resulting experimental data used are obtained from Thorkilsen et al. [1991]. The original wave breaking model by Battjes and Janssen [1978] is applied in the wave model. The maximum allowed wave height used to establish the probability of wave breaking, Q_b, is 0.8 times the local water depth. The root-mean-square wave height used to calculate Q_b, is estimated from a wave train of 100 wave periods (T_p). The wave carrier period is chosen similar to the wave peak period, with a minimum cut-off frequency of 0.74 times the peak frequency and a maximum cut-off frequency of 1.74 times the peak frequency. The reduction of the spectrum yields also a reduction of the generated energy in the wave model. In order to compensate for this reduction, the spectrum generated is $H_s = 0.146$ m, which corresponds to an incident wave height of $H_{m0} = 0.140$ m. The experimental set-up by Thorkilsen et al. [1991] is shown in Fig.4.

The significant wave height is calculated along the slope for α_0 in the range 0.5-2.0. Furthermore, wave energy dissipation due to bed friction is introduced in one test. It is seen from Fig.4 that the numerical results and the experimental data compares well when α_0 is chosen in the range 1.0-1.5. By introducing the bed friction $f_r = 0.1$ in combination with $\alpha_0 = 1.0$, the numerical results come even closer to the experimental data. It is concluded that the numerical model can predict the wave energy dissipation for irregular waves breaking over a mild slope well. The value of α_0 is in the order of 1 for waves breaking at mild slopes. It is noted that the higher-order bottom terms, which are included in the wave model, have no influence on the results because the bar is mildly sloping and the refraction coefficient is small.

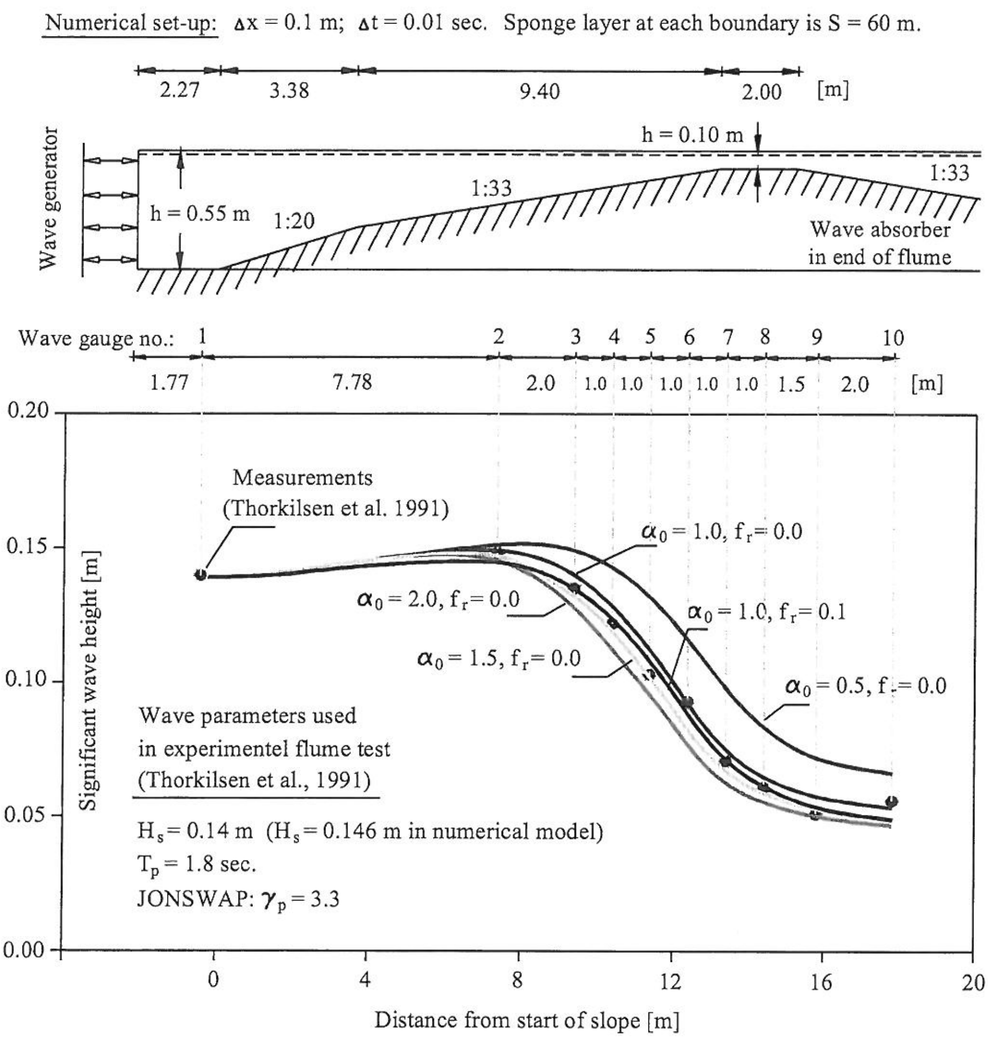


Fig. 4. Wave model results compared with experimental tests by Thorkilsen

Wave model used to simulate the experimental tests

The tests performed by Jensen M.S are simulated by the wave model. The breaking process is described using a modification of the Battjes and Janssen [1978] bore model. Estimation of the root-mean-square wave height, H_{rms}, is important, because it is controlling the rate of wave breaking. It has been chosen to estimate the root-mean-square wave height on basis of the preceding 100 waves obtained in the wave model. Battjes and Janssen [1978] used a maximum non-dimensional wave height of $H_{max}/h_t=$ 0.80. A more realistic value being H_{max}/h_t=0.5 is used. In the experimental tests the dissipation on the reef-plateau due to bottom friction is believed to be of significantly less influence. In the numerical model the bottom friction is simulated using a friction dissipation coefficient of f_r = 0.01.

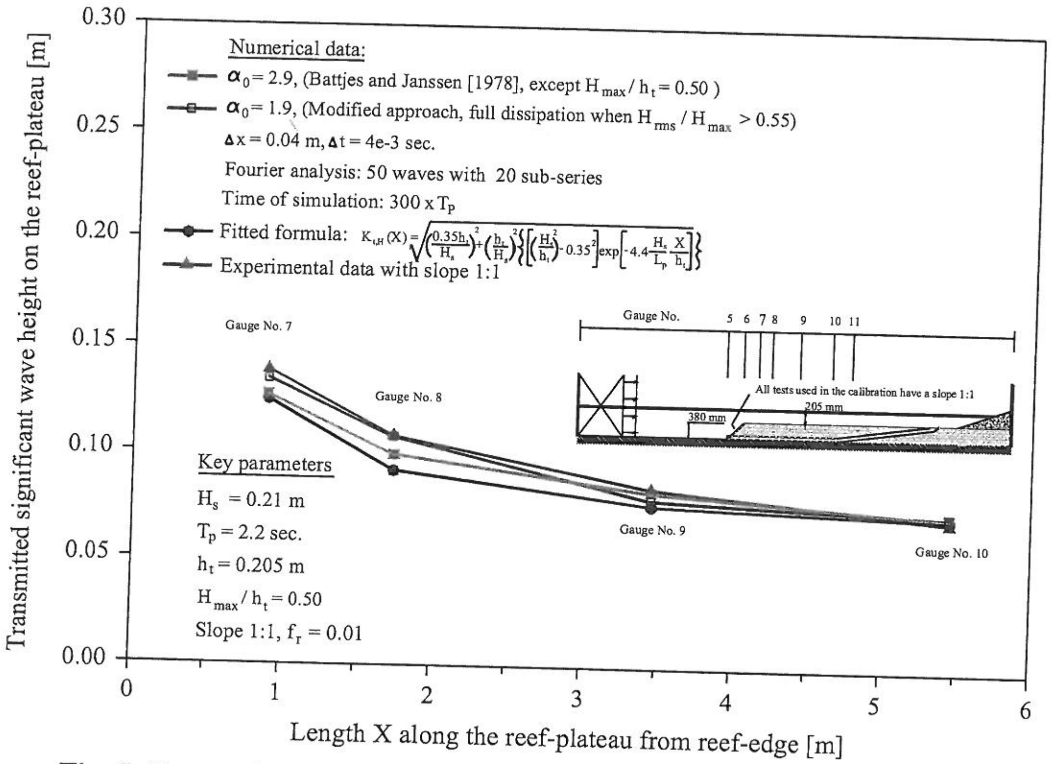


Fig. 5. Transmitted significant wave height over a steep bottom slope

Fig.5 shows the results of a calibration applied to a test with significant wave breaking. The incident significant wave height is H_s = 0.21 m and the water depth on the reef-plateau is h_t = 0.205 m, which yields the highest value of F_{c0} = 914 among all the irregular tests performed. The results are obtained as the transmitted wave heights and are compared with the measured wave heights along the reef-plateau and furthermore the wave height estimated by the formulae obtained for transmitted wave heights, Eq.7. Two dashed lines are depicted as results obtained with the numerical model. One dashed line is reflecting the results strictly following the approach proposed by Battjes and Janssen with the slight modification that the maximum allowable ratio of maximum wave height over water depth at the reef-plateau is taken as 0.5 instead of 0.8. The second dashed line reflects a modified approach as described by Eq.11. It is seen that during significant wave breaking using Battjes and Janssen [1998] approach with a calibrated value of $\alpha_0 = 2.9$ does not predict the transmitted wave height as well as the modified approach where $\alpha_0 = 1.9$. It can be concluded that if the root-mean-square wave height controls the rate of wave breaking, when the ratio of the root-mean-square wave height and the maximum wave height exceeds 0.55, the predictions improve in the case of high values of F_{c0} associated with waves breaking over a steep bottom slope. It is noted that the predicted transmitted wave heights obtained from the formulae given in Eq.7 compare well with the experimental data.

CONCLUSIONS

The transmitted wave energy depends on the incident wave height, the wave period and the water depth on the reef-plateau. The transmitted wave energy depends only slightly on the steepness of the slope within the range of slopes tested. Important relations between often referred wave parameters are given and it is found that a modified Swart and Loubser's type parameter is useful in connection with describing waves breaking on a steep bottom slope.

Non-dimensional formulae based on a theoretical approach predicting the transmission of wave heights are provided.

A numerical model simulating the wave energy dissipation of irregular waves propagating over a rapidly varying seabed is developed. The model is verified in several cases when compared to the theoretical solution. Furthermore, the experimental tests are used to calibrate and thereby predict the parameter α_0, which controls the intensity of the energy dissipation due to wave breaking. The numerical model developed is able to predict the experimental data very well.

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