Wave Propagation In A Hygrothermoelastic Half-space Along With Non-local Variable

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Received: Dec. 16, 2022; Accepted: Feb. 13, 2023

This particular work is an effort to investigate the propagation of wave in hygrothermoelastic medium in the light of Eringen's nonlocal theory of elasticity. The coupled wave equations in terms of displacement, temperature and moisture concentration are solved in an analytical way. The phase velocities of longitudinal displacement wave, transverse displacement wave, diffusion wave and thermal wave, under the influence of nonlocal variable, moisture concentration, and diffusion coefficient, in the medium are obtained analytically and presented graphically to show the effect of these parameters on the wave velocities.

Keywords: Wave propagation, Hygrothermoelastic, Nonlocal, normal mode analysis, Phase velocity, Diffusivity

http://dx.doi.org/10.6180/jase.202405_27(5).0001

1. Introduction

Eringen [1] was one of many developers of nonlocal theory. The theory explained the dependency of stress at a point in a continuum body on strain at specific point and on its surrounding points. In problems related to waves nature of material is influenced by the extrinsic (e.g., wavelength) and intrinsic (e.g., atomic size) characteristic length. The relevance of this theory is prominent in case of proportionate external and internal characteristic lengths. The micropolar elastic setup is appropriate in light of the nonlocal theory due to equivalent characteristic lengths in the micropolar solids.

The dispersion relation for the nonlocal micropolar elastic solids representing the dipolar materials was derived by Eringen [2]. Further, the reflection of plane longitudinal waves from the stress-free boundary surface of a nonlocal micropolar solid half-space was examined by Khurana and Tomar [3]. They revealed the existence of two waves, which are dilatational waves and transverse waves. The dilatational waves are uncoupled in scalar potentials and the transverse waves are coupled in vector potential. In light of Eringen's nonlocal theory of elasticity, Sarkar and Tomar [4] explained time-harmonic plane waves in an infinite thermoelastic solid with voids. dilatational waves and thermal properties but nonlocality affect all the types waves present in the medium. In recent times, Singh [5] studied the Rayleigh waves propagation in an isotropic and homogeneous nonlocal generalized thermoelastic solid half-space with voids. Sarkar et al. [6], Biswas [7], Mondal et al.[8], Das et al. [9] are other contributors in this area of research. Lata [10] investigated the effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium. Lata and Kaur [11] discussed plane wave propagation in transversely isotropic magneto-thermoelastic rotating medium. Recently Lata and Singh [12] studied plane wave propagation in a homogeneous isotropic nonlocal magneto-thermoelastic medium under the effect of Hall current.

Diffusion is the process in which of atoms and molecules moves region lower concentration from that of higher concentration till the condition of equilibrium is achieved in solids. This movement of atom and molecules causes migration to occupy a definite position in this situation. The migration in result produce disturbance.

Further, non-uniform moisture distribution will develop the concentration gradient which will lead to the movement of moisture. Movement of moisture will alter temperature and moisture present in the material with reference to time and position. Hence, theories of heat transfer and moisture transfer can be considered equivalent. The temperature and moisture distribution may be changed to a considerable amount when mechanical stresses are applied. The interdependency of mechanical deformation and diffusion due to this change provides a thrust for investigation. Many engineering problems with practical interest show this relationship of moisture, heat, and deformation. Any problem involving the study of solids under the effects of moisture and heat falls within the domain of hygrothermoelasticity. Szekeres [13, 14] investigated problems on the coupling of heat transfer and moisture. Gasch et al. [15] compared the damages caused by temperature and moisture variation with mechanical loadings in which he concluded that temperature and moisture causes more damage than the later. Szekeres and Engel Brecht [16] developed fundamental equations governing coupled hygrothermoelasticity by setting fundamental analogy between heat and moisture. Gigliotti et al. [17] explored the cyclical and transient hygrothermoelastic stress in laminated composite plates. A mathematical model was set up by Gawain et al. [18] for analysing the behaviour of concrete in hygrothermal medium. Raja et al. [19] carried out piezohygrothermoelastic analysis by developing a formulation for this problem via finite element method. A micro-macro mechanical method was given by Aboudi and Williams [20] to record the response of hygrothermoelastic composites. Rao and Sinha [21] carried out analyses on how moisture and temperature influences free vibrations for multidirectional composites in three dimensions. Vlase et al. [22] presented a method to simplify the calculus of the eigenmodes of a mechanical system with bars concerning the deformations and the loads in the elements of the system.

In this work, we have developed the governing equations for hygrothermoelastic medium within the scope of nonlocal theory. The chances of propagation of plane waves are studied along with its various characteristics features. The effect of various parameters on the phase velocities of components of waves is displayed graphically. Numerical calculations are carried out on MATLAB software for a given material and the derived outcomes discussed with the help of graphical results. This article is an effort to through light on effect of nonlocal variable on hygrothermoelastic problems.

2. Basic equations

The constitutive equations, heat equation, field equations and moisture diffusion for homogeneous, isotropic hygrothermoelastic solid, in absence of incremental body forces and heat sources given by Hosseini et al. [23] and Montanro [24] are:

$$\sigma_{ji,i} = \rho(1-\epsilon^2\nabla^2)\ddot{u}_i \tag{1}$$

$$D_\tau T_{,ii} + D_T^m m_{,ji} - \dot{T} - \frac{\beta_{ij^T} T_0}{\rho c}\dot{u}_{j,j} = 0 \tag{2}$$

$$D_m m_{,ji} + D_m^T T_{,ii} - \dot{m} - \frac{\beta_{ij}^m T_0}{k_m}\dot{u}_{j,j} = 0 \tag{3}$$

where

$$\beta_{ij}^T = \beta_T\delta_{ij}, \quad \beta_T = (3\lambda+2\mu)\alpha_T, \tag{4}$$

$$\beta_{ij}^m = \beta_m\delta_{ij}, \quad \beta_m = (3\lambda+2\mu)\alpha_m, \tag{5}$$

$$\sigma_{ij} = C_{ijkl}\eth_{kl} - \beta_{ij}^m - \beta_{ij}^T, \tag{6}$$

$$C_{ijkl} = \frac{2Gv}{1-2v}\delta_{ij}\delta_{kl} + G\delta_{ik}\delta_{jl} + G\delta_{il}\delta_{jk}, \tag{7}$$

$$\varepsilon_{ij} = \frac{u_{j.i}+u_{i,j}}{2}, \tag{8}$$

Here, σ_{ij} denotes components of stress, ε_{ij} strain and $\bar{\omega}_{ij}$ displacement, respectively. Further, ρ is density, K_m is moisture diffusivity, D_m is diffusion coefficient of moisture, D_T is temperature diffusivity, T is temperature, m is moisture concentration, D_T^m, D_T^m are coupled diffusivities, T_0 is the reference temperature, P is the initial pressure, c denotes heat capacity, m_0 is initial moisture, β_{ij}^T is material coefficients due to coupling between stresses and temperature, β_{ij}^m is material coefficients present due to coupling between stresses moisture concentration, respectively, α_T refers to coefficient of linear thermal expansion, coefficient of moisture expansion is α_m, ϵ is non-local parameter, and Lame's constants are λ, μ.

3. Formulation

The Cartesian coordinate system (x, y, and z) with the z-axis pointing perpendicularly down is taken under consideration. To simplify, we have considered that the waves are moving in the plane x-z.

Thus, in 2-D x–z plane the displacement vector in hygrothermoelastic medium is reduces to, $\vec{u} = (u, 0, w)$ where $u = u(x, z, t)$, $w = w(x, z, t)$ which changes the equations of motion and coupled generalized equations of heat conduction and moisture diffusion Eqs. (1) to (3) along with other relations Eq. (6) in 2-D and in the absence of body forces to following form:

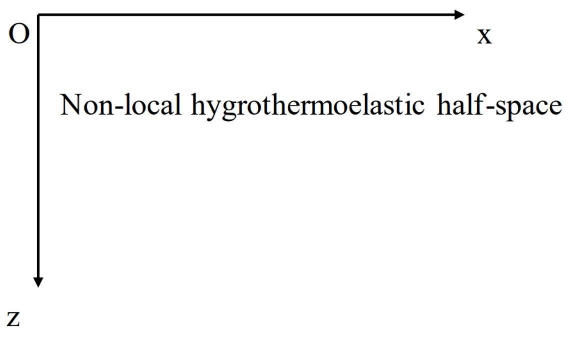

Fig. 1. Geometry of the problem.

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z}+\mu\frac{\partial^2 u}{\partial z^2}-\beta_m\frac{\partial m}{\partial x}-\beta_T\frac{\partial T}{\partial x}=\rho\left(1-\epsilon^2\nabla^2\right)\frac{\partial^2 u}{\partial t^2},\tag{9}$$

$$\mu\frac{\partial^2 w}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial z}+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\beta_m\frac{\partial m}{\partial z}-\beta_T\frac{\partial T}{\partial z}=\rho\left(1-\epsilon^2\nabla^2\right)\frac{\partial^2 w}{\partial t^2}\tag{10}$$

$$D_T\nabla^2 T+D_T^m\nabla^2 m-\frac{\partial T}{\partial t}-\frac{\beta_T T_0}{\rho c}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0\tag{11}$$

$$D_m\nabla^2 m+D_m^T\nabla^2 T-\frac{\partial m}{\partial t}-\frac{\beta_m m_0 D_m}{k_m}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0\tag{12}$$

$$\left(1-\epsilon^2\nabla^2\right)\sigma_{xx}=\lambda\frac{\partial w}{\partial z}+(\lambda+2\mu)\frac{\partial u}{\partial x}-\beta_m m-\beta_T T\tag{13}$$

$$\left(1-\epsilon^2\nabla^2\right)\sigma_{xz}=\mu\frac{\partial u}{\partial z}+\mu\frac{\partial w}{\partial x},\tag{14}$$

$$\left(1-\epsilon^2\nabla^2\right)\sigma_{xx}=\mu\frac{\partial u}{\partial z}+\mu\frac{\partial w}{\partial x},\tag{15}$$

$$\left(1-\epsilon^2\nabla^2\right)\sigma_{zz}=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu)\frac{\partial w}{\partial z}-\beta_m m-\beta_T T\tag{16}$$

To simplify numerical calculations, few dimensionless quantities are introduced as given below:

$$x'=\frac{x}{l},z'=\frac{z}{l},u'=\frac{u}{l},w'=\frac{w}{l},\epsilon'=\frac{\epsilon}{l},t'=\frac{D_m}{l^2}t,\quad m'=m,T'=\frac{T}{T_0},\sigma_{ii}'=\frac{\sigma_{ii}}{\lambda},\tag{17}$$

where the quantity l has the dimension of length.

With the help of quantities in Eq. (17) Eqs. (9) to (12), reduces to the following non-dimensional equations (after dropping the primes):

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z}+\mu\frac{\partial^2 u}{\partial z^2}-\beta_m\frac{\partial m}{\partial x}-\beta_T T_0\frac{\partial T}{\partial x}=\frac{\rho D_m^2}{l^2}\left(1-\epsilon^2\nabla^2\right)\frac{\partial^2 u}{\partial t^2}\tag{18}$$

$$\mu\frac{\partial^2 w}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial z}+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\beta_m\frac{\partial m}{\partial z}-\beta_T T_0\frac{\partial T}{\partial z}=\frac{\rho D_m^2}{l^2}(1-\epsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2},\tag{19}$$

$$D_T T_0\nabla^2 T+D_T^m\nabla^2 m-D_m T_0\frac{\partial T}{\partial t}-\frac{\beta_T T_0 D_m}{\rho c}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0,\tag{20}$$

$$D_m\nabla^2 m+D_m^T T_0\nabla^2 T-D_m\frac{\partial m}{\partial t}-\frac{\beta_m m_0 D_m^2}{k_m}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0.\tag{21}$$

We use potential functions φ and ψ to represent displacement components u and w by following relation (Helmholt'z representation)

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\tag{22}$$

Using Eq. (22) in Eqs. (18) to (21), we get

$$(\lambda+2\mu)\nabla^2\phi-\beta_m m-\beta_T T_0 T=\frac{\rho {D_m}^2}{l^2}\left(1-\epsilon^2\nabla^2\right)\frac{\partial^2\phi}{\partial t^2}\tag{23}$$

$$\mu\nabla^2\psi=\frac{\rho {D_m}^2}{l^2}\left(1-\epsilon^2\nabla^2\right)\frac{\partial^2\psi}{\partial t^2}\tag{24}$$

$$D_T T_0\nabla^2 T+D_T^m\nabla^2 m-D_m T_0\frac{\partial T}{\partial t}-\frac{\beta_T T_0 D_m}{\rho c}\frac{\partial}{\partial t}\nabla^2\phi=0,\tag{25}$$

$$D_m\nabla^2 m+D_m^T T_0\nabla^2 T-D_m\frac{\partial m}{\partial t}-\frac{\beta_m m_0 {D_m}^2}{k_m}\frac{\partial}{\partial t}\nabla^2\phi=0\tag{26}$$

4. Analytic solution

The Method of Normal mode analysis is used to decompose the solution of the physical variables under consideration as $\left(\phi,\psi,T,m,\sigma_{ij}\right)=\left(\phi^*,\psi^*,T^*,m^*,\sigma_{ij}^*\right)e^{ik(x\,\mathrm{Sin}\,\theta+y\,\mathrm{Cos}\,\theta-vt)}$ where v is phase velocity and ϕ^*,ψ^*,T^*,m^* and σ_{ij}^* are the amplitudes of field quantities. Thus from Eqs. (23) to (26), we get

$$\left(a_{11}-a_{12}v^2\right)\phi^*+a_{13}T^*+a_{14}m^*=0,\tag{27}$$

$$\left\{\mu-\frac{\rho D_m^2\left(1+\epsilon^2k^2\right)}{l^2}v^2\right\}\psi^*=0,\tag{28}$$

$$a_{21}\phi^*+\left(a_{22}+a_{23}v^2\right)T^*+a_{24}m^*=0,\tag{29}$$

$$a_{31}\phi^*+a_{32}T^*+\left(a_{33}+a_{34}v^2\right)m^*=0,\tag{30}$$

Where

$$a_{11} = (\lambda + 2\mu), a_{12} = \frac{\rho D_m^2 \left(1+\epsilon^2 k^2\right)}{l^2}, a_{13} = \frac{\beta_T T_0}{k^2}, a_{14} = \frac{\beta_m}{k^2},$$
$$a_{21} = \frac{\beta_T T_0 D_m i\omega}{\rho c}, a_{22} = D_T T_0, a_{23} = \frac{D_m T_0}{i\omega}, a_{24} = D_T^m,$$
$$a_{31} = \frac{\beta_m m_0 {D_{(m)}}^2 i\omega\big)}{k_{(m)}}, a_{32} = D_m^T T_0, a_{33} = D_m, \quad a_{34} = \frac{D_m}{i\omega}. \tag{31}$$

Solving equation Eqs. (27), (29) and (30), we obtain

$$\begin{vmatrix} \left(a_{11} - a_{12}v^2\right) & a_{13} & a_{14} \\ a_{21} & \left(a_{22} + a_{23}v^2\right) & a_{24} \\ a_{31} & a_{32} & \left(a_{33} + a_{34}v^2\right) \end{vmatrix} = 0 \tag{32}$$

On solving the determinant Eq. (32), the following sixth degree equation is obtained,

$$B_1 v^6 + B_2 v^4 + B_3 v^2 + B_4 = 0 \tag{33}$$

where,

$$\begin{aligned} &B_1 = -a_{12}a_{23}a_{34} \\ &B_2 = a_{11}a_{34}a_{23} - a_{12}a_{22}a_{34} - a_{12}a_{33}a_{23}, \\ &B_3 = a_{11}a_{22}a_{34} + a_{11}a_{33}a_{23} - a_{12}a_{22}a_{33} + a_{12}a_{24}a_{32} - \\ &a_{13}a_{21}a_{34} - a_{14}a_{31}a_{23}, \\ &B_4 = a_{11}a_{22}a_{33} - a_{11}a_{24}a_{32} + a_{13}a_{24}a_{31} + a_{14}a_{32}a_{21} - \\ &a_{14}a_{31}a_{22} - a_{13}a_{21}a_{33}. \end{aligned} \tag{34}$$

Rewriting Eq. (28) as

$$C_1 - C_2 v^2 = 0 \tag{35}$$

where, $C_1 = \mu, C_2 = (\rho D_{(m)}^2 (1 + \epsilon^2 k^2))/l^2$.

The Zeros of Eqs. (33) and (35) i.e. $v_j (j = 1, 2, 3, 4)$ represents the phase velocities of the P_1, P_2, P_3, P_4 waves. If $v_j^{-1} = V_j^{-1} + i\omega^{-1} q_j (j = 1, 2, 3, 4)$ where the phase velocity v and wavenumber k are complex value, which can be calculated as $k = \frac{\omega}{V} + iq$ s.t. V and q are real. If the Re(v) $\geq$ 0, then the real parts of roots of Eqs. (33) and (35) indicate propagation speed of P_1, P_2, P_3, P_4, and Img(v) ≤ 0 is the damped wave. Hence, V_j can be called as the propagation speeds and q_j is the attenuation coefficients of the coupled P_1, P_2, P_3, and P_4 waves.

Particular case- Wave propagation in hygrothermoelastic half-space

Neglecting non-local parameter ϵ in Eq. (1), the problem reduces to wave propagation in hygrothermoelastic half-space discussed by Ailawalia et al. [25] as a special case after neglecting hydrostatic initial stress in the medium.

5. Numerical results

Analytical results in the above section are confirmed by showing a numerical example of taking wood slab as a porous material. For this material, various physical constants are given by Chang and Weng [26] and Yang et al.[27] as per Table 1.

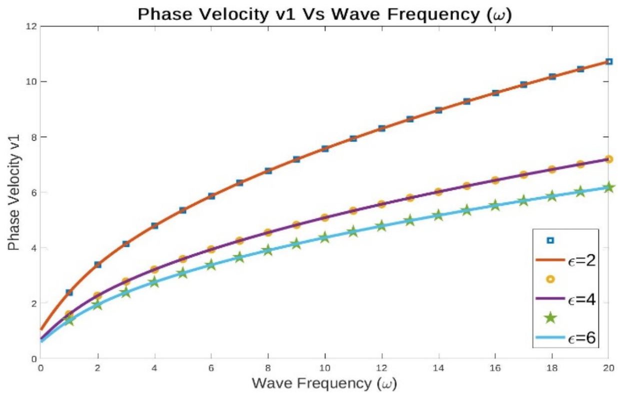

Fig. 2

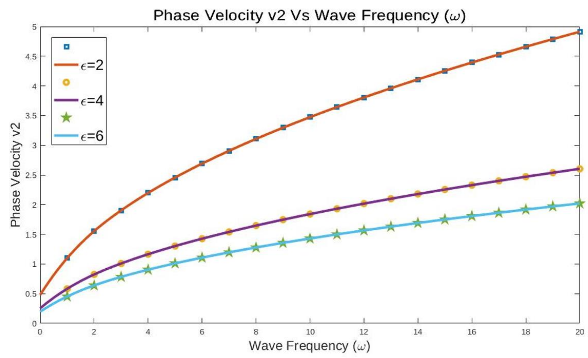

Fig. 3

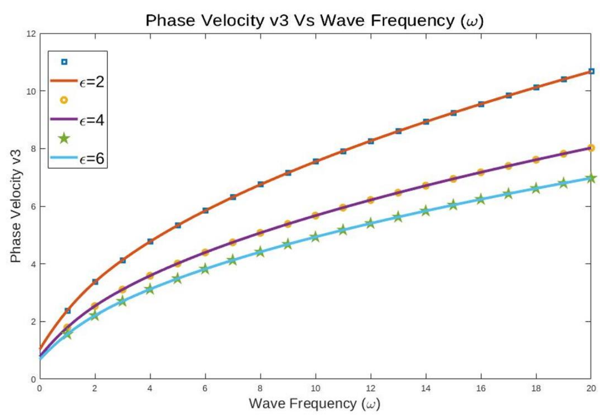

Fig. 4

Table 1. Comparison of time performance and performance with different superpixel methods

Quantity	Symbol	Value with units
Lame's constant	λ	$46.9 \times 109\ \mathrm{N/m^2}$
Lame's constant	μ	$24.17 \times 109\ \mathrm{N/m^2}$
Density	ρ	$370\mathrm{Kg/m^3}$
Poisson's ratio	v	.33
Moisture reference	$\mathrm{m_0}$	10%
Coefficient of Moisture expansion	α^m	$2.68 \times 10^{-3}\ \mathrm{cm/cm\,(H_2O)}$
Coefficient of linear thermal expansion	α^T	$31.3 \times 10^\wedge(-6)\mathrm{cm}/cm(H_2O)$
Temperature	$\mathrm{T_0}$	283 K
Heat Capacity	c	$2500\mathrm{j/Kg}\,(^\wedge 0\ \mathrm{K}),$
Moisture diffusivity	k_m	$2.2 \times 10^{-8}\mathrm{Kg/msM}$
Diffusion coefficient of moisture	D_m	$2.16 \times 10^{-6}\ \mathrm{m^2/sec}$
Coupled diffusivity	D_m^T	$0.648 \times 10^{-6}\ \mathrm{m^2\,(\%H_2O)\,/s\,(^0\ K)}$
Coupled diffusivity	D_T^m	$2.1 \times 10^{-7}\ \mathrm{m^2\,(^0\ K)\,/s\,(\%H_2O)}$
Temperature diffusivity	D_T	$\frac{k}{\rho c}$

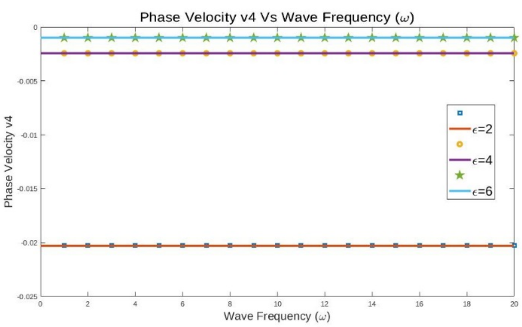

Fig. 5

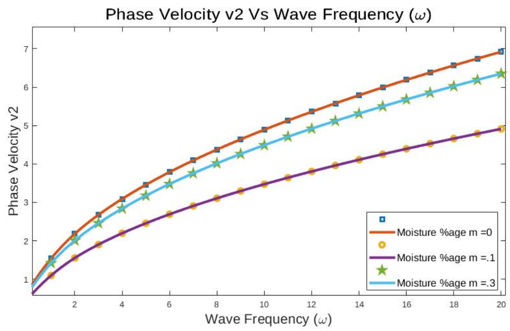

Fig. 7

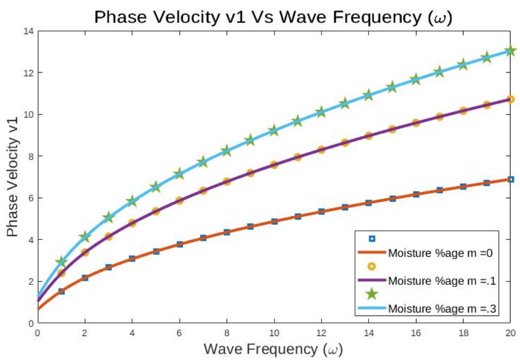

Fig. 6

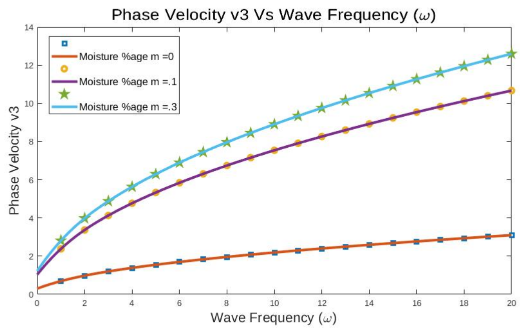

Fig. 8

6. Discussions

a. Effect of the non-local variable on phase velocities for a fixed value of moisture concentration:

The numerical calculations are made at surface z=1.0, for t= 1.0 and dimensionless quantity l = 1.0. The results for penetration depth of waves at different values of wave frequency ω are shown in Figs. 2 to 5 graphically .We have shown this variation for three values of nonlocal variable i.e. ϵ =2,4,6 against frequencyω(0 to20). We can see the increasing trends in phase velocity for different values of ϵ(non-local parameter) for P1, P2, and P3. Whereas, for P4 waves the phase velocity varies for different values ϵ but remains constant for different wave numbers ω.

b. Effect of moisture concentration on phase velocities for a known value of the non-local variable:

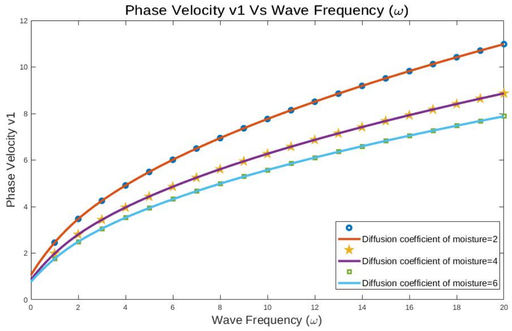

Fig. 9

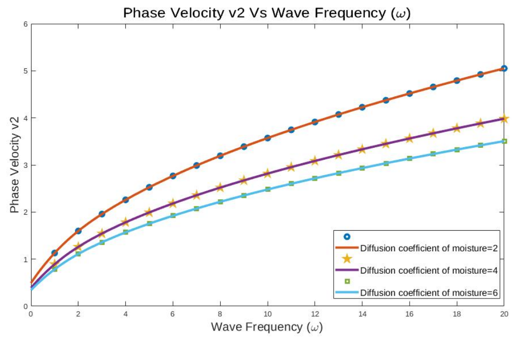

Fig. 10

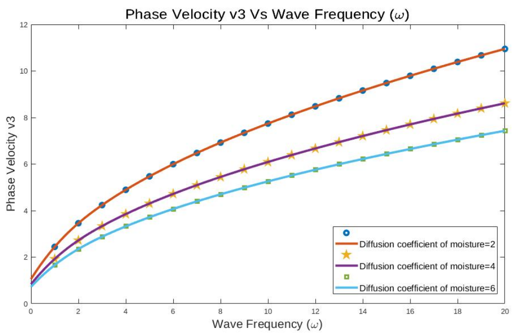

Fig. 11

The graphical results of Phase velocity of waves for distinct values of wave frequency ω are shown in Figs. 6 to 8. We have shown these variations for three values of moisture references i.e. m_o=0.0,0.3,0.5 against frequency ω. We observe that in the absence of moisture, the phase velocities $v1$ and $v2$ keep increasing along with frequency ω. However,in the presence of moisture component, we can see similar trends as in the previous case (i.e. absences of moisture). But, at the same time, we find that the values of phase velocities $v1$ and $v3$ are increases as we hiked the moisture content i.e. values of m_o.

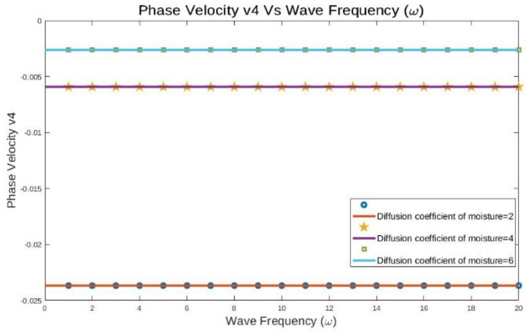

Fig. 12

Surprisingly, the phase velocity $v2$ shows opposite results. The variations are similar with increasing trends both in the presence and absence of moisture but the value of velocity $v3$ for a fixed frequency decrease as moisture content increase. These values of velocity $v4$ increase as the value of moisture is increased.

c. Effect of moisture diffusivity on phase velocities for a known value of the non-local variable and moisture concentration:
   The graphical results of Phase velocity of waves for different values of wave frequency ω are shown in Figs. 9 to 12. The variation for three values of the diffusion coefficient of moisture i.e.D_m=2,4,6 against frequency ω have plotted .Here, we have noticed decrease in all three phase velocities with an increase in value of the coefficient of the diffusion coefficient D_m. The decrease in phase velocity of all components with the value of frequency is similar to previous cases. For the P4 waves, the phase velocity increases for different values D_m diffusion coefficient of moisture but remains constant for different wave frequencies ω.

7. Conclusion

a. The coupled wave equations are derived in terms of displacement, temperature, and moisture concentration. Further, speed waves, namely, P1, P2, P3, and P4, are calculated for a given material. The influence of ϵ the nonlocal Variable is reflected very clearly in various plots for different wave frequencies ω on speeds. It is observed that phase velocity for P1, P2and P3 keeps increasing along with frequency ω, and P4shows similar trends for different values of wave frequencies ω.

b. The influence of moisture concentration on the phase velocities is depicted in the present work. We can see an increase in the phase velocities as moisture concentration is increased.

c. We may conclude that the coefficient of diffusion affects the phase velocities oppositely as moisture concentration.

d. The change in the phase velocities is very less when temperature changes up to 50 K but the values increase if temperature variation is high.

e. Our results aids in the evaluation of the wave propagation phenomenon in a thermoelastic medium containing moisture. The results obtained are very useful in seismology.

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