A Linear Approximation Method For A Stochastic Constraint For A Multi-Objective Nonlinear Eurobond Investment Portfolio Model

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Nonlinear mathematical models are widely used better to reflect the stochastic structure of financial investment problems and to express them numerically. However, in some real-life situations, it is necessary to consider not only one purpose but many purposes simultaneously. Therefore, we have to define these models with multi-objective programming. This study defines a multi-objective nonlinear Eurobond investment portfolio and showcases the normal distribution of purchase and selling prices. The study then proposes a mechanism to convert the stochastic constraint into an equivalent deterministic form and provides near-optimal solutions in reasonable times.

Keywords: financial investment models; chance constraints; nonlinear optimization

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1. Introduction

Financial management enables to generate and implement ideas about where to find financial resources to achieve goals and objectives in an organization. A finance manager's primary task is to measure organizational efficiency through appropriate allocation, purchasing, and management. Guidance in financial planning and the use of several scientific methods are essential. It is also essential to obtain funds from different sources, make quality and efficient production with cheap resources, and deliver these products to the user or consumer the shortest and fastest way.

The 'why' behind financial models is to depict various real business situations to enable the audience to understand the potential or actual financial results and their dependence on various inputs. The goal here is to simulate or model a real business scenario or economic phenomenon and be able to play with various inputs and observe their effect on results. Once such a relationship is established between inputs and outcomes, financial models will likely be used to predict the most likely financial outcomes based on probability distributions of various input parameters. The predicted results are then used as inputs for making business decisions.

Eurobond is lesser known compared to usual investment tools. It is generally a long-term debt instrument that states or companies offer for sale in foreign currencies in international markets in order to obtain funds outside their own countries [1].

In Turkey, Eurobonds are generally issued by the Treasury of the Republic of Turkey in USD or EURO in international markets. However, they can be exported in other well-known foreign currencies as well. They are usually long-term investments that pay back the initially invested amount in 5 to 40 years. They work like a regular interest-based investment. Once the investor makes the payment, it provides a regular cash flow to the investor. It is less known in Turkey because it operates on foreign currency, and awareness among investors is low, limited to 30-50 thousand investors in total. In addition, since the payment structure is similar to a common interest, people do not seek an alternative in foreign currencies. The main difference between Eurobonds is that they make payments either in 6 months or annually, depending on whether they are Euro-based or USD based.

We choose Eurobond for our study because their purchasing and selling prices are stochastic, and it showcases the stochastic structure we need in our mathematical model. In particular, they may change based on inflation, deflation, and other governmental policies, and it is impossible to estimate their prices accurately. Second, it is convenient to choose Eurobond as an investment tool as the Turkish currency lost significant value in the last five years against Euro and US dollar. An investment should have meaning in terms of the inflation rate if an investment is made. Since Eurobond offers a regular payback period in foreign currency, using it in our financial investment model makes sense.

In addition to these factors, we assume that Eurobond portfolio profits are higher than those from local currency interest rates and those from foreign currency exchange. These two conditions provide a basis for our Eurobond model because otherwise, there is no need for Eurobond investment, and investors are better off by regular interest investments. The stochastic behavior of purchasing and selling prices of Eurobond coupons brings us to the central issue of utilizing chance constraints in our model. The main purpose of writing the chance constraints is to express mathematically the situations where some constraints do not need to be met 100%, especially in complex models. Mathematically, a chance constraint is given in the form in Eq. (1):

$$P\{\sum_{j=1}^{n} a_j x_j \leq b\} \geq a \tag{1}$$

Here, the left side inside the probability parenthesis is the original constraint. We convert it into a chance constraint by writing it as in Eq. (1), meaning that the probability that the constraint will be less than or equal to b will be α.

The mathematical formulation of this constraint is inconsistent with the linear programming assumptions. However, assuming that the parameter b, which is the right-hand side of the constraint, is normally distributed such that b N(μ_b,σ_b^2), the new constraint becomes as stated in Eq. (2).

$$\sum_{j=1}^{n} a_j x_j \leq \mu_b + z_\alpha \sigma_b \tag{2}$$

Therefore, this constraint becomes compatible with linear programming assumptions. Here, the value indicated by z_α represents the z score corresponding to the α probability value in the normal distribution table. In summary, if a parameter has a stochastic and normally distributed structure, a mathematical program constraint for this parameter, expressed as a probabilistic equation, can be written linearly. While providing this, the following three assumptions are made:

1. The right-hand side of the constraint, that is, the b values, are independent of each other.
2. Due to the nature of probabilities and chance constraints, the constraint will not be taken as pure equality.
3. These independent b values fit the normal distribution.

Writing the constraint may become more complicated in cases where the values taken by the parameter do not fit the normal distribution, or where the right-hand side of the constraint is constant and the technology coefficients are probabilistic. Therefore, in order to benefit from this definition, it should be tested whether the data comply with the above assumptions.

The rest of this study is as follows: We provide previous studies to see the methods to handle with stochastic and nonlinear chance constraints in Chapter 2. Then we give our chance constrained stochastic investment model and our approximation scheme in Chapter 3. We provide the results in Chapter 4 and then finally discuss and provide further insights in Chapter 5.

2. Previous work

One of the benchmark studies to handle chance constraints are conducted by [2]. The study uses a linear orthonormal transformation and the resulting constraint set can be solved both by Simplex method and Dantzig-Wolfe decomposition.

By comparing five different widely used stochastic optimization algorithms, [3] have done a study to show which algorithm gives better results in which case. These algorithms are random search, simultaneous stochastic perturbation approximation, simulated annealing, evolutionary strategies, and genetic algorithms. The authors showcase these algorithms and provide insights into using them in specific scenarios.

[4]also provides solution methods on nonlinear stochastic models with one or more constraints. Duality is used to create linear algebraic constraints to replace the nonlinear stochastic ones, and many solution procedures are showcased ın the study. The methods solve convex problems to optimality, or they can provide good enough solutions for

relaxed versions of stochastic nonlinear models. This study is essential for our study as it provides many insights at the same time to compare and select the best one.

[5] redefines a chance constraint such that it is feasible for a stochastic problem with a certain probability. They argue that any solution generated by their approximation methods to the problem should satisfy the constraint set with some probability as well. They then investigate these probabilities and conditions for optimality under different scenarios. More detailed studies and methodologies can be found in the book [6].

The reader may refer to [7] for a comprehensive review of many other models and extensions of multi-objective stochastic programming in financial investment. They provide an extensive review on the development and recent methods on how to solve multi-objective stochastic models. Linear approximation of nonlinear functions is one of these methods to handle uncertainty in real life practices.

[8] provides a linear approximation formula for chance constraints. This formula can be used in models with multi-objective as well as single-objective functions. The key to the formula is that it bounds the constraint with the nonlinear value, and ensures that the constraint is met while losing some of the feasible region.

[9] propose a method to learn from the operational trends of power networks and pre-assign lower and upper bounds to chance constraints to solve them in a nonlinear modeling environment. In the constraint set they generate, if something is impossible, it has a probability of zero, so they remove the constraint from the model and iteratively try to find the optimal solution.

[10] define the so-called random-rough (Ra-Ro) variables to handle the uncertainty in financial investment decision-making. They convert the stochastic nonlinear model into an equivalent deterministic but quadratic mathematical model to eliminate the uncertainty.

[11] provide a review of methods used to handle chance constraints. They divide the methods into three categories. They share the studies that use robust optimization, those using scenarios, and those with sample average optimization. They also provide methods that are based on data that do not fit well-known discrete and continuous distributions.

[12] provide a model to solve continuous time chance constraints for calculating risk guarantees for an unstable feedback control policy system.

[13] created a portfolio selection model with an expected return, chance, and cardinality constraints. They produced a data set by making use of mixed distributions and a robust formulation, and they produced solutions to their models with mixed integer programming containing convex functions.

[14] deal with a robust model to solve a quadratic cost function with two-sided chance constraints. They transform the chance constraints into deterministic forms and satisfy them while minimizing their function. They use a data set with Wasserstein distance to account for data perturbation. They ultimately transform the model into a linear program that commercial solvers can easily solve.

[15] provide a simple convex optimization problem by approximating model predictive control with stochastic constraints. They do not have distribution information, so they enforce the chance constraints as robust in their model.

[16] deal with multiple uncertain factors in their model by introducing a mixed integer programming model with robust chance constraints to model uncertain charging demands. They produce a bi-level programming model to solve the mixed integer problem and provide experimental design on three factors affecting real-life situations.

[17] solve a two-stage stochastic program by linear program rules and show that stochastic structure can be reduced to linear structure and chance constraints can be eliminated by Wasserstein metrics.

[18]suggest a purpose-based investment model for personalized and lifelong financial planning. The model assigns priority coefficients for different purposes in each period and shows how chance-based risk constraints are resolved for optimum investment with uncertain situations.

Another Wasserstein ambiguity set based formulation of an energy optimization problem is provided by [19]. They provide a bilinear program out of a stochastic nonlinear model and show that the resulting model is solved in shorter times compared to solutions with other algorithms in the literature. Other models and strategies to deal with energy and electricity flow networks by chance constrained optimization can be found in [20] and [21].

3. The eurobond model and the proposed method for linear approximation

3.1. The Nonlinear Model with Chance Constraints

The Eurobond investment model is given as follows:

Decision Variables:

x_{it}: amount of bond i purchased in period t
y_{it}: amount of bond i sold in period t
oh_{it}: amount of bond i on hand in period t
d_1: slack variable for target income TARG
d_2: excess variable for target income TARG

Parameters:

PB_{it}: sale price for bond i in period t

PA_{it}: purchase price for bond i in period t

A_i: periodical income for bond i

C_1: coefficient for slack variable d_1

CAP: initial capital

TARG: target capital

W: coefficient of the capital in the chance constraint

T: total or the last number of periods (if there are 25 periods, T is 25)

Based on the decision variables and parameters, the mathematical model is given in Eq. (3)-Eq. (7)

$$minz = c_1 d_1 \tag{3}$$

$$I_0 = CAP - \sum_{i=1}^{N} PA_{i0} x_{i0} + \sum_{c=1}^{C} PP_{c0} bp_{c0} \tag{4}$$

$$I_t = \sum_{i=1}^{N} (-PA_{it} x_{it} + PB_{it} y_{it} + A_{it} oh_{it}) + I_{t-1} \qquad \forall t \tag{5}$$

$$I_T + d_1 - d_2 = TARG \tag{6}$$

$$W * CAP - \sum_{i=1}^{N} (-\mu_{iT}^{PA} x_{iT} + \mu_{iT}^{PB} y_{it} + A_{iT} oh_{iT}) + I_{t-1} \geq z_\alpha \sum_{i=1}^{N} \sqrt{(\alpha_{iT}^{PA} x_{iT})^2 + (\alpha_{iT}^{PB} y_{iT})^2} \tag{7}$$

Eq. (3) denotes the objective function and tries to minimize the penalty resulted from failing to reach the target income. Eq. (4) and Eq. (5) are the balance constraints for the money flow in each period. The objective is defined as a goal programming model by making it possible to have an income less or more than a predefined target in Eq. (6). Eq. (7) is the chance constraint stating that the total income at the end should be larger than or equal to W times the target income by a probability that corresponds to z_α in the normal table. The index T is the number of the final period considered. To illustrate the use of this constraint, consider the following probability for the total income:

$$P(I_T \geq 1.10CAP) \geq 0.9 \tag{8}$$

Eq. (8) states that with the probability 0.9, the total income at the end of the last period T is greater than 1.10 times the initial capital. Since we assume normal distribution for Eurobond prices, the stochastic constraint in Eq. (8) conveniently transforms to the constraint in Eq. (7), having the W equal to 1.10 and z_α equal to 1.28.

To explain Eq. (8), the multiplicative value of 1.10 and a probability value of 0.9 are used. The value 1.1 corresponds to around 2-3% interest rate and is a global value offered by many banks in developed countries. On the other hand, the value 0.9 corresponds to a risk of 10% which is taken by many investors especially in high risk investments. The probability value can be made 80%, 95% or even 100% to demonstrate no risk situation. However, since we aim to include risk in our calculations, we showcased values less than 100%.

3.2. Approximation to the Chance Constraint and a Solution Algorithm

The linear approximation of a differentiable function L(x) at some point a is given by

$$L(x) = f(a) + f'(a)(x - a) \tag{9}$$

Since the derivative is needed in Eq. (9), we reduce the constraint in Eq. (7) by subtracting the right side from both sides, and take the derivative. Doing this for all x_{iT} variables, yields Eq. (10):

$$\frac{\partial \left(-\sum_{i=1}^{N} \left(-\mu_{iT}^{PA} x_{iT} + \mu_{iT}^{PB} y_{it} + A_{iT} oh_{iT} \right) + I_{T-1} \right)}{\partial x_{iT}} - \frac{\partial \left(z_\alpha \sum_{i=1}^{N} \sqrt{\left(\sigma_{iT}^{PA} x_{iT}\right)^2 + \left(\sigma_{iT}^{PB} y_{iT}\right)^2} \right)}{\partial x_{iT}} \geq 0 \;\; \forall i \tag{10}$$

Since all other variables are treated as numbers in the partial derivatives, Eq. (10) further reduces to

$$\mu_{iT}^{PA} - z_\alpha \frac{1}{\sqrt{\left(\sigma_{iT}^{PA} x_{iT}\right)^2 + \left(\sigma_{iT}^{PB} y_{iT}\right)^2}} \left(\sigma_{iT}^{PA} x_{iT}\right) \geq 0 \tag{11}$$

By incorporating Eq. (9) into Eq. (11), and applying the approximation scheme at point a yields the following inequality in Eq. (12).

$$\mu_{iT}^{PA} a_{iT} - z_\alpha \sqrt{\left(\sigma_{iT}^{PA} a_{iT}\right)^2} + \left(\mu_{iT}^{PA} - z_\alpha \frac{1}{\sqrt{\left(\sigma_{iT}^{PA} a_{iT}\right)^2 + \left(\sigma_{iT}^{PB} y_{iT}\right)^2}} \left(\sigma_{iT}^{PA} a_{iT}\right) \right) (x - a) \geq 0 \tag{12}$$

Thus, the nonlinear chance constraint in Eq. (7)becomes linear. Here, we used the advantage that our chance constraint is differentiable and is easy to provide an approximate value for the given sets of optimization problem.

4. Designed experiments and findings

We solve the resulting linear model using different values of W, confidence level α, and different numbers of Eurobonds. Then we compare the results with the solutions obtained from solving the original nonlinear problem. SNGP and LGP denote the original nonlinear goal programming model and the linearized goal programming model, respectively. We start with a rate of return of 2% and gradually increase it to 20% to compare the results. The results are summarized in Table Table 1 below.

Based on the table, the solutions worsen as we increase the target income from 1.02 to 1.2. The linearized and non-linear models give worse solutions, and the objective function value no longer becomes zero after the rate of 1.08 for the linearized model.

Both models converge to a total income as we continue to increase the rates of return in our experimental design. The LGP converges to a total income of 1,080,086 at the rate of 1.1, and increasing the rate of return further only worsens the objective function value as the model cannot find any solution that reaches the target value and is still feasible. For the SNGP, convergence exits at the value of 1,096,104 when we increase the rate of return to 1.12. In other words, the LGP converges to a value sooner than the SNGP with a return rate of 8% and does not provide any better solution, while the SNGP converges to a final total income at the value of 1.12 with a rate of 9.6%. Thus, the LGP gives values 1.6% worse than the original solution. The total income values of LGP and SNGP are provided in for the different significance values of 0.9, 0.95, and 0.99.

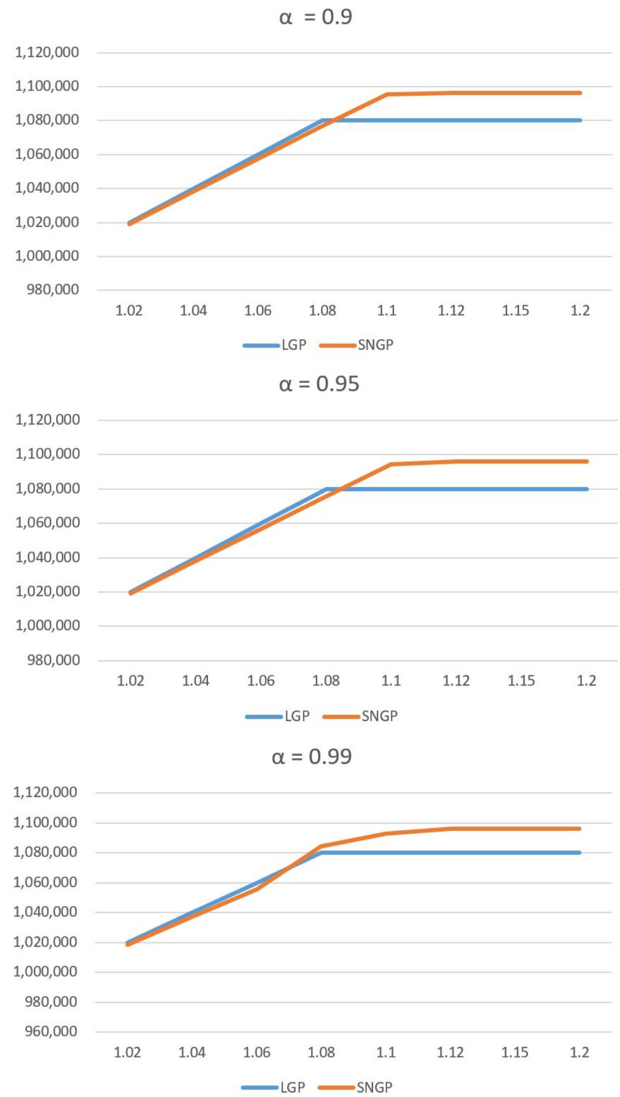

Fig. 1. Objective Function Values for LGP and SNGP for Three Different Confidence Levels

5. Result and further discussion

Regardless of whether they are borrowing or have the funds in possession, if a company or individual investor is preparing to allocate capital for various projects, it is necessary to measure and compare the financial results of various projects using financial models as a decision maker. Through these financial models, decision-makers can better prioritize the allocation of capital that will help the company or themselves achieve their financial goals.

As mentioned above, since the steps to be taken regarding money and finance in many areas require financial analysis, the ability to create financial models and the presence of financial modeling in daily life are inevitable.

In this paper, we first provided a Eurobond investment model that incorporates uncertainty and multiple purposes at the same time. Further, we showcased the situation where the purchase and selling prices of Eurobonds are normally distributed. Then, we introduced a mechanism to convert the nonlinear stochastic goal programming model into a linear goal programming equivalent. Since the constraint structure allowed for using linear solvers, we reached a solution quickly. At the same time, we observed that the linearized constraint first showed better total income results at lower values of rates of return. However, as we increased the rates of return higher and higher, the original problem started to show better results as the linear constraint made the feasible region tighter compared to the chance constraint. Considering the structure of the feasible region of the linear equivalent, this is entirely logical because the feasible region becomes restricted and does not allow taking any risks in investments. Furthermore, even though the linear equivalent can represent the whole

Table 1. Designed Experiment Results

W	α	LGP	Z	SNGP	Z
1.02	0.9	$1,020,000$	0	$1,019,178$	821.52
1.04		$1,040,000$	0	$1,038,357$	1643.08
1.06		$1,060,000$	0	$1,057,622$	2377.69
1.08		$1,080,000$	0	$1,076,829$	3170.26
1.1		$1,080,086$	19913.54	$1,095,473$	4526.28
1.12		$1,080,086$	39913.42	$1,096,104$	23895.9
1.15		$1,080,086$	69913.42	$1,096,104$	53895.7
1.2		$1,080,086$	119913.4	$1,096,104$	103897.3
1.02	0.95	$1,020,000$	0	$1,018,989$	1011.12
1.04		$1,040,000$	0	$1,037,978$	2022.45
1.06		$1,060,000$	0	$1,056,966$	3033.87
1.08		$1,080,000$	0	$1,075,954$	4045.33
1.1		$1,080,086$	19913.54	$1,094,507$	5492.63
1.12		$1,080,086$	39913.42	$1,096,104$	23896.1
1.15		$1,080,086$	69913.42	$1,096,104$	53897.2
1.2		$1,080,086$	119913.4	$1,096,104$	103897.3
1.02	0.99	$1,020,000$	0	$1,018,553$	1446.64
1.04		$1,040,000$	0	$1,037,107$	2893.4
1.06		$1,060,000$	0	$1,055,659$	4340.11
1.08		$1,080,000$	0	$1,084,213$	5786.82
1.1		$1,080,086$	19913.54	$1,092,756$	7244.1
1.12		$1,080,086$	39913.42	$1,096,104$	23896.1
1.15		$1,080,086$	69913.42	$1,096,104$	53897.2
1.2		$1,080,086$	119913.4	$1,096,104$	103897.3

financial model, conversion from a nonlinear model always needs assumptions. These assumptions may also affect model accuracy if not addressed correctly. Even though mathematically correct, the linear model may propose other investment opportunities than those proposed by the stochastic model for the same set of available investment data.

In addition, our chance-constrained problem was solved at three different confidence levels. As we increased the probability of having a total net income larger than the target income, the linearized model yielded better results, but after reaching a certain rate of return (10% in our case), the chance-constrained model started yielding better solutions than the linearized model. Thus, our model showed a better performance until we reached a threshold value of the rate of return. After the threshold value, our linearized model yielded results within 1.5% to 12% of the optimal solution yielded by the chance-constrained model, thus still performing very well and taking less time to compute and find the set of solutions. This result is significant because it shows that our model can be a helpful tool in deciding which financial investment opportunities to select, especially when time is limited and investor objectives are urging many restrictions. Further, the model provides a risk aversion mechanism to choose the threshold risk value after which an investor is unwilling to take further investment risks. Moreover, these risk values can be combined with investors' past data, and a forecasting tool can be implemented to incorporate more risk values into investment policies.

Another advantage of using our linearized model is the earlier detection of a threshold value that shows the maximum rate of return given the set of purchase and selling prices of Eurobond stocks. This provides a forecasting mechanism in cases where Eurobond prices assume normal or any other probability distribution. However, they have a mean and standard deviation for expected value and forecasting calculations. It is essential in financial planning to know and estimate how the market conditions will exist in the planning horizon, and an accurate model will play a crucial role in two things. First, it will help provide a robust plan that will not be heavily affected by sudden price changes. Second, total risk and total income value deviation will be minimized. In this aspect, our model can achieve both purposes.

An extension of the model would be to include more financial instruments and not depend solely on Eurobond stocks. This will require information on multiple probability distributions, and they may not fit into reputable distributions, but defining extra binary constraints for discrete distributions or approximation methods for continuous distributions will assist in defining multi-commodity financial models.

One limitation of the study is that our research is conducted and designed on the Eurobonds provided and regulated by the Turkish Treasury. The Eurobonds we considered are also issued by many other countries, such as Lithuania or Netherlands, and they are global. This is why they can be used to generalize an investment policy. However, they are treasury bonds, and assumptions on them are related to the safety guarantee of the government. In other words, not only one but many countries protect these Eurobonds, and their risk is due to market structure.

However, some instruments may fail to be secure, which may include higher uncertainty ultimately. Many other mechanisms exist, such as cryptocurrency, free Bonds, and leverage accounts. They possess a very high uncertainty, and only specific investor profiles are authorized to deal with them as it includes a high risk of loss at the same time. In future models, this uncertainty may be dealt with extended models, and more insights can be gained in the case that other countries regulate financial instruments or there is less control of the government on the instruments.

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