



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## On some serious methodological problems concerning the analysis of the Danish Welfare Commission and the use of the DREAM model

Zambelli, Stefano

*Publication date:*  
2006

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Zambelli, S. (2006). On some serious methodological problems concerning the analysis of the Danish Welfare Commission and the use of the DREAM model. Aalborg: Centre for Comparative Welfare Studies, Institut for Økonomi, Politik og Forvaltning, Aalborg Universitet.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

**Working Papers from Department of Economic, Politics and  
Public Administration  
Aalborg University**

---

**On some serious methodological  
problems concerning the analysis of the  
Danish Welfare Commission and the use  
of the DREAM model**

**Stefano Zambelli  
ISSN: 1396:3503  
ISBN: 87-90789-76-8  
2006:1**

*On some serious methodological problems concerning the analysis of the Danish Welfare Commission and the use of the DREAM model*

*Stefano Zambelli*

*Copyright: The author*

*Working Paper Department of Economic, Politics and Public Administration  
Aalborg University  
Fibigerstraede 1  
DK-9220 Aalborg*

*ISSN: 1396:3503*

*ISBN: 87-90789-76-8*

*2006:1*

*Aalborg 2006*

*Print: UNI.Print*

**On some serious methodological problems concerning the analysis  
of the Danish Welfare Commission and the use of the DREAM  
model**

**Stefano Zambelli**

Department of Economics, Politics and Public Administration  
Aalborg University  
Fibigerstraede, 3  
9220 Aalborg  
Denmark

e-mail: [stefano@socsci.aau.dk](mailto:stefano@socsci.aau.dk)



## **Abstract**

The Danish Welfare Commission has expressed a great concern for the consequences that the ageing of the Danish population will have for the state finances and the future welfare state.

In this article it is claimed that this fiscal problem can be solved in many different ways. Each of these possible ways has important and different implications for the welfare of the individuals. The Welfare Commission (2005b, Ch.3, p.2) uses for its evaluations the model DREAM (Danish Rational Economic Agent Model). The computation of the 'welfare values' of the current and future generations is one of the results of the model. The DREAM model builders and the Welfare Commission, however, **do not** publish and/or **do not** use these values to draw their conclusions.

This is a very serious methodological problem. Without the publication of these values the Welfare Commission cannot claim that its policy proposal is better than any other financially sustainable policy. Therefore one is justified in concluding that the reform proposed by the Welfare Commission is highly arbitrary.



## **Table of contents**

<b>On Fiscal Sustainability and the Ministry of Finance Problem</b> .....	1
<b>The Kotlikoff-Auerbach General Equilibrium Overlapping Generations Model</b> .....	7
The ‘Theoretical’ DREAM .....	7
The ‘actual’ DREAM .....	8
<b>Conclusion</b> .....	11
<b>Appendix. Some simple computations: productivity gains and their intergenerational distribution</b> .....	13
<b>References</b> .....	19





## **Note to Readers**

This article can be approached from different levels. Some parts of the article are easy to read for both experts and non experts, while others might require further reflections and some technical notions.

Some of my colleagues have advised me to make two articles out of it: one for the non economists and one for the economists.

For the time being, I have chosen not to follow their advice. It is my hope that the interested reader will get the essence of the argument and will excuse me for the unusual mixture of technical remarks (mostly relegated to the footnotes) and discursive ones.

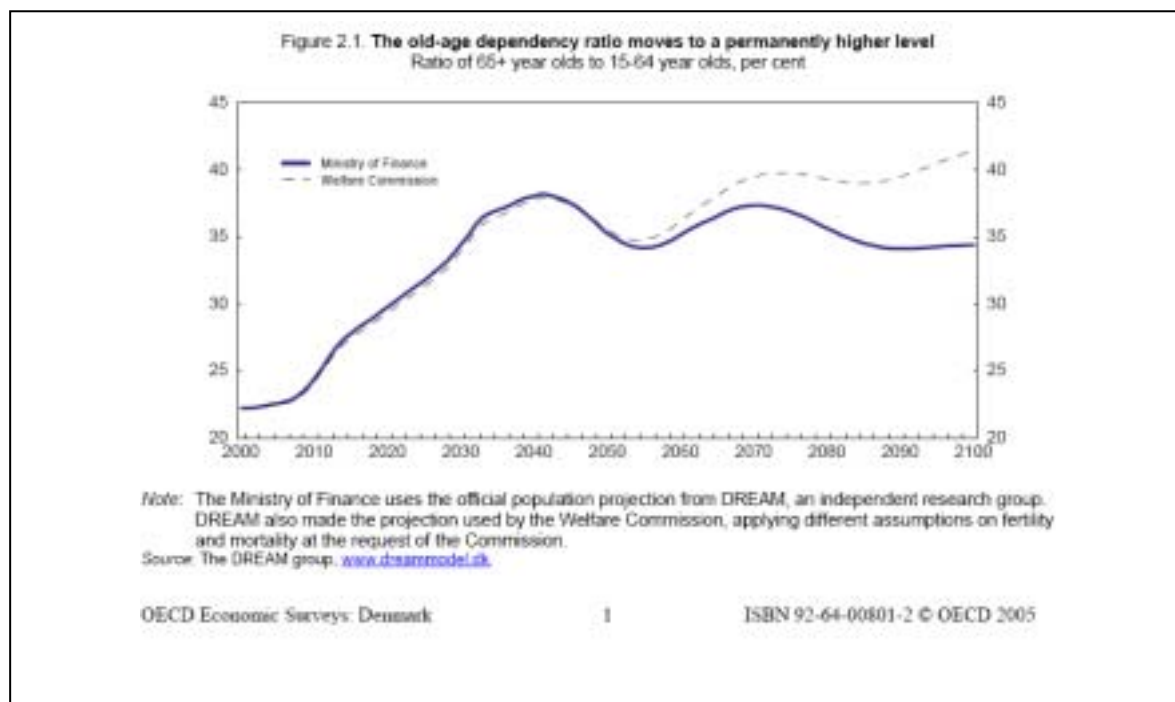
Thank you.



## On fiscal sustainability and the Ministry of Finance problem

The Danish Welfare Commission has primarily focussed on the problems that the welfare state will have to deal with in light of the specific demographic development which implies a future reduction of the amount of the working population with respect to the non-working population (Figure 1. See OECD, 2005, p.1).

**Figure 1**



(OECD, 2005, p.1)

The major challenge that the welfare state will have to face is to prepare the society to the change in the demand of goods and services that the new ageing structure will inevitably require.

Among all the problems that the welfare state has and will have to deal with, the Welfare Commission has focused on the state fiscal problem determined by the expected increase of future state expenditures coupled with a decrease of future expected state revenues. This is a budgetary problem for the Ministry of Finance, but not necessarily a problem for the society as a whole. It is important to stress that **a state financial unbalance is, almost always, a loss for some citizens and a gain to others**. Fiscal financial unbalances determine a secure change in the distribution of resources among citizens, but not necessarily a change of the total availability of them.

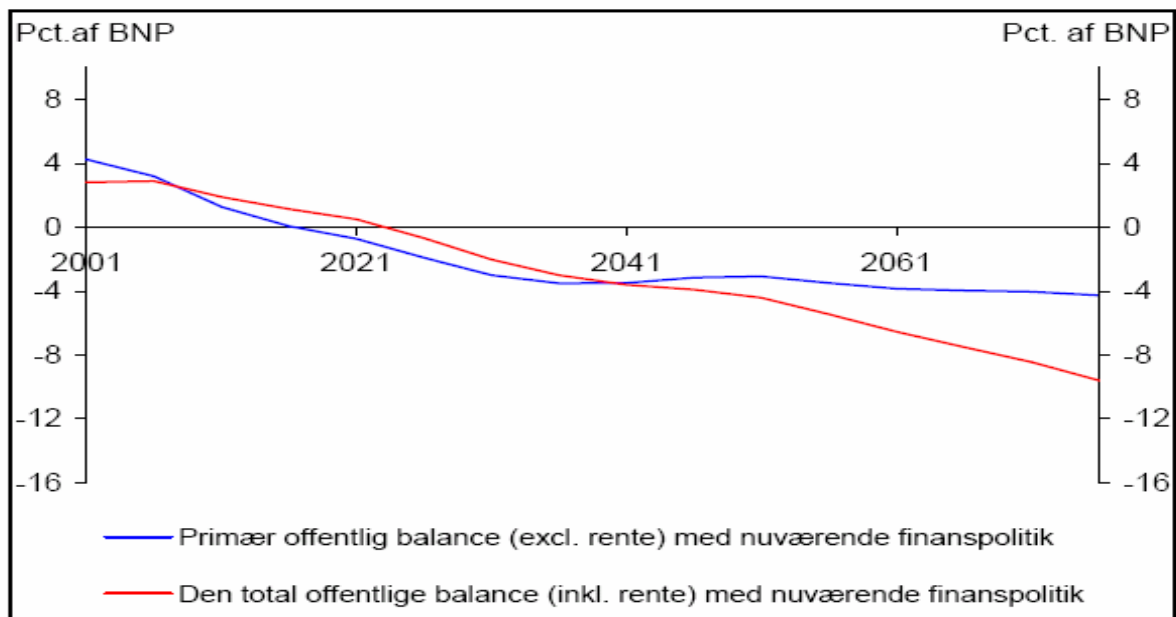
It is well known that a given Government expenditure can be financed by tax revenues or by issuing government bonds (or a combination of the two). Let us recall that **government expenditure is deliveries of services to the population and long term investments in infrastructures or transfers from some groups of individuals to others**. In order for these services and investments to be delivered labour supplied by some citizens has to be employed (that is wages have to be paid), and goods supplied by other citizens have to be bought. This means that at least for those employed by the public sector and for those selling goods to it, **there are big advantages to be associated with the existence of these government expenditures**. But these advantages have to be measured

also in terms of the provided services and transfers to the citizens. Obviously one draw back is that in order to finance these expenditures the state has to tax citizens or has to borrow from them. If the state runs a deficit this often means that government bonds would be issued and that the government debt would increase. **But this increase in government debt is NOT a problem for the whole population, it is a problem for some and an advantage for others<sup>1</sup>, or it simply does not matter<sup>2</sup>.** Some in some generations would pay fewer taxes, being consequently able to consume more or, alternatively, to save more and others would lend to the state, and by so doing would substitute, for example, current consumption in favour of future consumption. It has to be pointed out that these behaviours would be different with the different ages of the individuals (in accordance with individual life cycles). **Hence, as long as the citizens are willing to bear the problems regarding the redistribution associated with its financing, the emergence or increase of government debt is not necessarily a threat for any state.**

For example, the figure below shows, under the **very specific but unknown assumptions** made by the DREAM modellers and the Welfare Commission, a situation in which the state would have to run a deficit.

**Figure 3. Evolution of the Primary and Total State Budget**

**Figur 4.1: Udviklingen i den primære og den samlede budgetsaldo**



Kilde: Egne beregninger på DREAM-modellen.

<sup>1</sup> It is a common tendency, supported by the media coverage of this phenomenon, to view a government debt as a big problem like that experienced by a household holding a big debt. But there is a great difference between a government debt and the debt of a household. The government debt is financed by the citizens themselves, while the household debt is financed by the surplus of another household. If we make the analogy of the government debt to the economy of a household, the emergence of a government debt means the emergence of a debt between, let us say, the son versus the father, or vice versa. While this might be unpleasant for the members of the household, it cannot be seen in itself as a threat for the economic survival of the household.

<sup>2</sup> Under certain stringent conditions some economists have shown that the way in which the state finances the current expenditures, either through taxation or through the issuing of government bonds, has no effect on the individuals' welfare on the ground that the individuals would foresee that non paid current taxation would be transformed into future taxation and hence the individuals would transform non paid current taxes into accumulation of savings to be used to pay future taxes. This phenomenon is known as the Barro-Ricardian equivalence.

**Is this deficit going to be a problem for the citizens? Not necessarily – the reason is that citizens are interested, one can presume, in having a decent consumption level and a decent quantity of services through their whole life.** An increase in government debt means that some generations would have an increase of the amount of government bonds owned which would give to the same citizens a higher purchasing capacity, relative to the other living generations, when old (i.e. transferring purchasing power to the future). **Some citizens might be willing to substitute current consumption in favour of future consumption and consider this a good opportunity, and by so doing could increase their welfare.** At some point young generations would buy government bonds from older generations, this effect would imply that older generations can buy more of the consumption goods and that young generations would buy less (because they have to buy government bonds from the old). But the same would in turn happen when the once young generation becomes old. Clearly, this buying and selling of bonds between different generations does not imply a loss in the welfare of the individuals. Moreover it should also be mentioned that for some members of the young generations it would not be necessary to buy these government bonds simply because they will inherit them.

These effects could be quite different in accordance with different expected productivity improvements (see the Appendix). In these cases young generations could reduce current consumption in favour of older generations in exchange for higher future consumption. Following the above simple reasoning we can see that the emergence of government debt is not necessarily to be avoided and under certain conditions it is also something to be desired for the welfare of the individuals.

The builders of DREAM and the Welfare Commission postulate that the existence of government debt is a big threat for the survival of the Welfare State. Consequently they analyze different scenarios where the aim is that of having (at a future point of time) the disappearance (or consolidation) of government debt<sup>3</sup>. This situation is defined as Fiscal Sustainability. But if some of the arguments that I have made above hold, it is not clear why the Fiscal Sustainability should be the citizens' major concern. To repeat, the existence of government debt means that some generations would for example hold government bonds when young and sell them to the incoming young generations when old. This is a 'Sustainable' situation that might also have the advantage of increasing the welfare of some citizens and eventually to most of them.

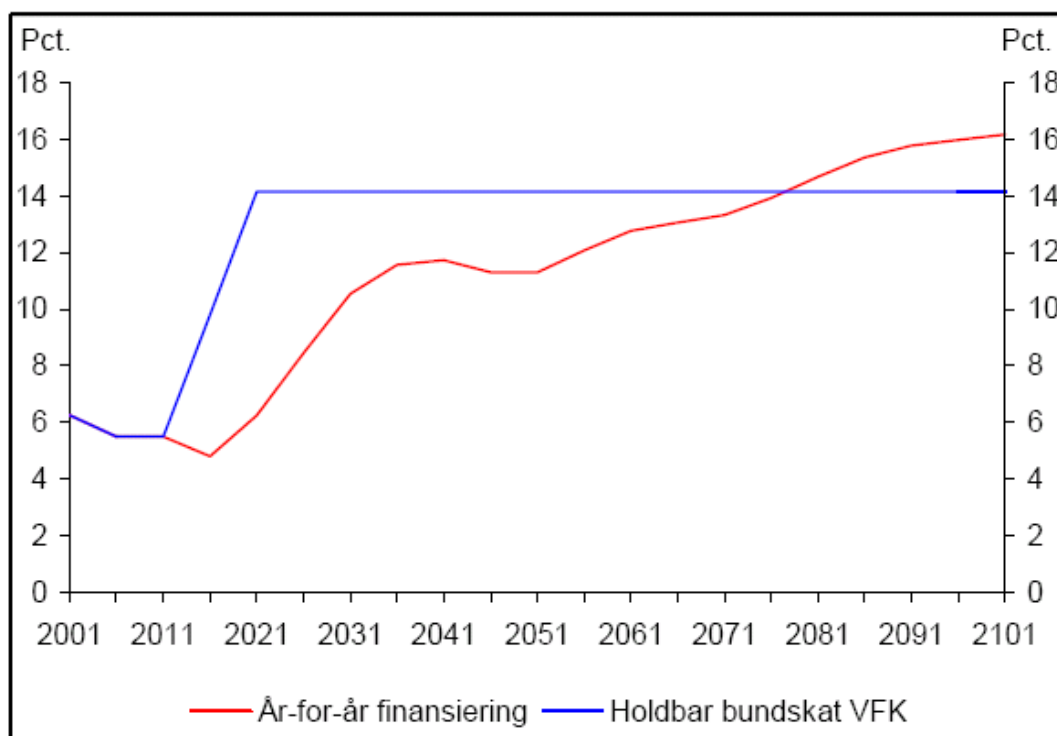
The Welfare Commission publishes, given some assumptions on government expenditures, some possible paths for the evolutions of Government Finances (Welfare Commission, 2005a and also OECD, 2005), where the disappearance of the debt is expected to occur at the latest 2080, (and for some cases earlier). There exist an infinite number of paths where this can be achieved. But it should be noticed that each path implies a reallocation of resources between the different generations (and also inside the same generation). **When comparing two alternative paths, only for very special cases it would happen that some generations would improve without having others to loose.**

---

<sup>3</sup> Technically the objective is that of Fiscal Sustainability where a policy is implemented so that the present value of the future surpluses and deficits are assumed to be equal to zero (See also OECD, 2005). Note that the Fiscal Sustainability may be analyzed also under very high increases or reductions of government expenditure and under an infinite number of different dynamic taxation rules. The Welfare question is to determine which of them would be the optimal one for the citizens' welfare and not for state finances. Moreover the final date in which the balanced budget condition is going to take place has in itself big redistribution implications. That is, whether this balance is expected to be reached in 2080 or 2090 will have important welfare effects for the individuals.

**Figure 4. Tax rate relative to a sustainable pay-as-you-go financing and a sustainable fiscal policy finance with a permanent tax increase**

**Figur 8.1: Bundskattesatsen ved år-for-år finansiering og ved holdbar finanspolitik finansieret ved permanent skattestigning**

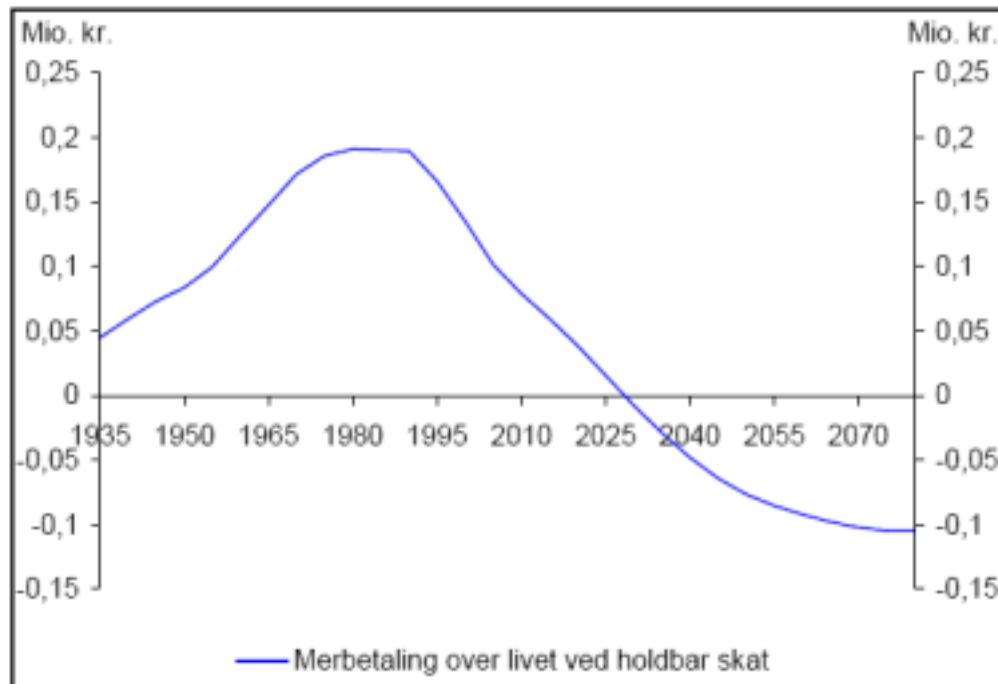


Kilde: Egne beregninger på DREAM-modellen

The above are two alternative cases where ‘Fiscal Sustainability’ is obtained. The technical report published by the Welfare Commission (2005a, Ch.8 and subsequent chapters) seem to prefer the ‘Sustainable Tax-Rate’ (‘Holdbar bundskat’) in stead of the pay-as-you-go (År-for-år finansiering) solution, both of them fulfil the requirement of Fiscal Sustainability. **In evaluating the two schemes the problem is to figure out the effects that they have for the different citizens.** Both schemes, as well as any other alternative scheme, imply different decisions of the agents. **From the point of view of each member of the society the two schemes are not equivalent.** One could say that this is obvious. In fact the Welfare Commission publishes (Figure 5) what the difference between the two schemes would imply in terms of tax payments for the representative generations.

Figure 5. Change in the life tax payments for the different generations. Sustainable tax financing relative to the pay-as-you-go financing (millions of Danish kroner per person)

Figur 8.2: Merbetaling til den offentlige sektor over livet fordelt på fødselsårgange. Finansiering med holdbar skat målt i forhold til år-for-år finansiering (mio. kr. pr. person)



Kilde: Egne beregninger på DREAM-modellen

(Danish Welfare Commission, 2005a, p. 33)

**But the content of such a computation in terms of the welfare of the individual citizens is totally unclear.** These two alternative cases imply that the different generations would change their consumption decisions and working efforts during their life span. The two taxation schemes imply two completely different life distributions of the taxation burdens. In practice **whether the substitution of consumption when young with consumption when old as well as a different distribution of working efforts might be welfare improving or not depends on the propensities of the individuals (utility function) and on the effects that the decisions of the whole population would have for the overall economic conditions.**

Another point where the Welfare Commission unexplained concern for Fiscal Sustainability leads towards questionable positions is **the denial that future productivity increases would make substantial differences both for state finances and in the inter and intra-generational relative individual welfares.** The argument of the Welfare Commission is that in the future an increase in productivity would determine a concurrent increase of state expenditures and revenues. Here I will not deal with the technical details, but different productivity levels could have important effects on the decisions of the agents so as to change their life cycle expenditures and labour supply plans. Moreover, **so long as a government debt exists a different expectation of future productivity evolutions would determine substantial differences on the value of the state debt and hence the emergence of different demands and supplies of the financial holdings of the different agents.** And this would have important re-distributional effects.



All of the above comments and remarks lead to an important question. **How are we going to evaluate the welfare effects of different alternative policies?** This is obviously a very difficult question. Nevertheless a model like DREAM should help in providing some tentative answers, because it was build with this precise scope.

The methodological critic that I am addressing here is that **the modellers of DREAM do not publish results that are consistent with the aim of evaluating the welfare effects of alternative policies.** This critique is based not on the specific details of the model, which are not known to the public, but on the published results, which are most of the time not fit to answer the above relevant question.

In the sequel I will first make a description of the class of models that inspire DREAM, the Kotlikoff-Auerbach General Equilibrium Overlapping Generations Model, and subsequently I will sketch the questions that the model should ask (and answer). Finally I will claim that the developers of DREAM and the Welfare Commission are not following this tradition and misuse the model. This raises serious doubts on the tenability of their analysis and conclusions<sup>4</sup>.

---

<sup>4</sup> Please note the type of critic I will make is from 'inside' that is: I question the use of the model, but I do not deal with the question of its usefulness in general. Which could certainly be addressed, but I do not.

# The Kotlikoff-Auerbach General Equilibrium Overlapping Generations Model

## *The 'Theoretical' DREAM*

The core ideas of the model originally developed by Kotlikoff and Auerbach (1987) can be explained in few sentences.

The model assumes that there are **many co-existing generations and that each generation** (from young to older ones) is represented by family cohorts who are identical, but differ in the date of birth and expected date of death and in the wealth owned<sup>5</sup>. **Each family cohort maximizes its remaining-life well-behaved inter-temporal utility function subject to its remaining-life budget constraint by taking decisions with respect to present and future consumption, present and future working efforts** (that is labour supply and leisure time) and hence accumulation or de-accumulation of savings. Moreover the family cohorts are assumed to take decisions according with the **rational expectation hypothesis**. That is: the agents are assumed to know the model of the economy they operate in and are assumed to take full accounts of the effects that changes in policy would have for the present and future condition of the economy.

Moreover, the **national production is determined through the implementation of an aggregate neoclassical well-behaved production function** where physical capital and the families' supplied labour are the production factors. Families are paid wages and interests. In other words the individual family cohorts take the best decisions that the economic circumstances allow them to do also with respect to the best forecasts of future conditions (and the **markets are full competition markets so as to allow these decisions to be implemented**).

Once the details of the model are specified and the parameters are 'calibrated' so as to somehow fit the historical data, one can study the properties of the model and have an idea of the welfare level of the different family cohorts measured in terms of the inter-temporal utility function.

And here comes the crucial point of how the model is and should be used. Given that the model allows for the measurement of the welfare of the different family cohorts, policy experiments can be implemented. **A policy experiment is, in the literature following the Kotlikoff and Auerbach (1987) tradition, a change in an important policy parameter<sup>6</sup>** (or of strategic changes of government expenditure through time). **What has to be done is to run the model under policy parameters that differ from the existing ones and subsequently compare the utility of the different family cohorts<sup>7</sup>.**

---

<sup>5</sup> Families own labour force, physical and financial capital and production takes place by using labour force and capital supplied by these families. The assumption of perfect competition assures that all the markets are in equilibrium (**there is no undesired unemployment of resources**, either labour or capital). Moreover the assumption of perfect competition does in this context imply that the families make their decisions as price-takers, i.e., they have no influence in the determination of the prices, typically the wages and the interest rates. Furthermore the underlining assumption of Walrasian General Equilibrium assures that the wage rate is equal to the marginal productivity of labour and the profit rate is equal to the marginal productivity of capital and that markets clear at all times, i.e. there is only voluntary unemployment.

<sup>6</sup> Such as taxation level, pension age, central bank interest rates, government expenditure and so forth and so on.

<sup>7</sup> In the case in which policy parameter changes that improve the utility of at least one family cohort without making some others worse do not exist, the initial situation is said to be Pareto Optimal.

In this literature a **better policy is the one that allows for an increase in the utility/welfare of at least one generation without reducing the utility/welfare of other generations** (Principle known as Pareto improvement). A particular interesting feature of this kind of models is that, **regardless of the standard neoclassical properties attached to the model, it is still possible that the dynamics of the model and the emerging equilibrium(s) may be sub-optimal**. Technically one says that the system may turn out to be **dynamically inefficient**<sup>8</sup>. A system is dynamically inefficient when it can be shown that a policy exists that might improve the well being of some generations without worsening the well-being of other generations<sup>9</sup>.

Now, there is much that can be said about this class of models, but the core is described by the above few lines.

### ***The 'actual' DREAM***

To find out about the actual functioning of the DREAM model is not an easy task and it is not a task I will take up here.

The details of the model change from publication to publication and the Danish citizens have no access to the actual codes. Nevertheless one has to trust **that the general framework is like the one just described above** (Knudsen et al., 1998) (plus unpublished assumptions)<sup>10</sup>.

---

<sup>8</sup> Dynamically inefficiency is a well known situation where, regardless of the perfect competition assumption and the assumption of utility maximizing agents, it is still possible that the welfare of the system can be improved through policy (see for example the Advanced Macroeconomic textbook by Romer, 2001, p.85-89).

<sup>9</sup> For a textbook example where a case in which a pay-as-you-go system is Pareto improving see Romer, p. 94, exercise 2.16.

<sup>10</sup> The DREAM model is constructed so as to follow in the spirit the Auerbach and Kotlikoff tradition. This means that demographic studies on the future evolution of the population are considered. The population is divided into family cohorts and for each cohort there is a representative agent that maximizes the utility function subject to budget constraints. The functional forms and the values of the parameters are obviously *ad hoc* chosen by the model builders.

Subsequently the model is 'calibrated' in the following way (the numbers after the letter indicate possible alternatives of the same step):

a1) The model is calibrated so as to assume that during a past base year the model is in a stationary equilibrium (steady state), this means that without any change in the parameters for the future all the new or young family cohorts would take the same optimal inter-temporal consumption and labour supply decisions.

a2) The model is calibrated so as to assume that, for example, the year 2005 represents the decision of the existing generations, but not necessarily a steady state, which implies that the decisions of future generations may be quite different from those of the 2005 generations.

NOTE that in both cases a1) and a2) it is assumed that also the decisions of future generations are computed simply because the decision of, let's say, the 2005 generation is also depended on the expected decision of future generations (let's say, 2040 generation, but the decision of the 2040 generation would be a function of the 2075 generation and so forth and so on with an infinite reiteration towards the future).

b) Here the utility values of the different existing generations and future generations would be computed (and this can be done for both cases a1) and a2)). Given the decisions of the generations the implicit evolution of the state balances is computed.

c) Given the model and the results produced in a1), a2) it is now possible to work with **Policy Experiments** and new utility values are computed. The Welfare Commission seems to base its suggestions on these experiments, where one aims at solving the '2040 state financial crisis' with for example a change in the taxation rules, like a change in the different tax rates as compared with a pay-as-you-go system.

Many results of the DREAM model have been and are used for the analysis of economic problems. In order not to distract attention to the main problem addressed in this note, I will not indulge on a discussion on how and if this use is appropriate in general, but confine it to **what the model should be used for**.

Given a certain change of some policy parameters or policy rules the DREAM model is used to generate virtual dynamic paths so as to show the evolution of state finances.

**But this is NOT the way in which it should be used and it is here that my major critic lies.**

Each family cohort is assumed to maximize its inter-temporal utility function and because it is equipped with rational expectations the individual would discount the impact that a change in policy would have for the present and future economic conditions. The logic of the model requires that the individuals have the same information as the model builders have and hence they will take full account of the effects that their decisions have on the economy as a whole.

Therefore an expected change in the tax basic rate or the implementation of the ‘pay-as-you-go’ strategy might have different impacts on the decisions of the family cohorts. In the presence of a policy change the different generations would modify their inter-temporal consumption decisions, labour efforts and hence their income and saving plans so as to maximize their whole life welfare. The DREAM model is used to compute these and the welfare values. **This is important information that should be given to the public**, because it would allow estimating the welfare gain or welfare losses consistently with the model assumptions. As claimed above, the increase of government debt associated with the different policies (e.g. no policy, increase in the taxation basis, adoption of the pay-as-you-go system) is not necessarily a problem when seen individually, or it would be a problem to some, but not for others. In particular for the case of a model like DREAM, the optimal decisions of the family cohorts are computed assuming that the individuals would maximize their utility taking into account the future effects of the announced policies (rational expectations) and hence the individual would reallocate their working efforts and consumption and saving decisions accordingly. In other words the agents described in DREAM would always maximize their utility both in the case of pay-as-you-go as well as in the case of the so-called sustainable taxation. But these utilities would not be the same and information on these utilities is **essential for the evaluation of policies**.



## Conclusion

Here I have addressed a methodological critique to the DREAM model and its use by the Welfare Commission: namely that the results of the model are not properly published. Therefore **the model does not provide information of the estimated welfare changes that are associated to the different policy experiments.**

Without this information, and without a precise description on how the model functions, the results of the model and its interpretations are of dubious relevance.

As I have claimed above, a balanced budget policy, following the same logic underpinning the model DREAM, is not necessarily a desirable condition. It is in fact possible that an increase in government debt is welfare improving, the whole depends on how the agents reallocate their consumption and working decisions and how the markets re-equilibrate (which would also have an impact on the value and ownership of government bonds and hence on redistribution). This is particularly important in cases of transitions from one steady state to the other (or when population readjustments occur, as it is for the demographic transition from here to year 2040).

Nevertheless, the Welfare Commission postulates, i.e. without showing why, that fiscal sustainability is a necessity. But even in this case it is not clear which policy, again following the same methodological approach implicit with the overlapping generations models, is from the point of view of welfare optimal (and if not why and how). **Lacking information on the welfare of the individual generations renders the whole exercise a useless one.**

For example **the Welfare Commission does not provide methodological consistent criteria to tell whether the so called ‘Sustainable Tax-Rate’ (‘Holdbar bundskat’) is a better policy than the ‘Pay-As-You-go’ (År-for-år finansiering) financing method<sup>11</sup>.**

The choice of not publishing this data, that must certainly be available to the model builders and to the members of the welfare commission, **reduces the credibility of their policy suggestions.** One could, although this is not the belief of the present author, easily suspect that what is published is what serves the rhetoric of the arguments, and that non published data is what could be used by the opponents.

Let me conclude with more general remarks. This inappropriate use of the DREAM model has led to divert attention away from other important welfare improving policies. For example, current research in economics places a great deal of attention on the effects that current allocation of resources on human capital, research and development and social capital

---

<sup>11</sup> The point of the present note is to stress the methodological problem discussed above. Nevertheless if the objective is that of fiscal sustainability, let me point out that there are very good arguments, lacking of other information, to prefer a slightly modified version of the ‘Pay-As-You-go’ system instead of a once and for all change in the tax rate. Such arguments might be sustained by using the Time Inconsistency literature and the Rules versus Discretion literature, that is very neoclassical literature. The ‘Pay-As-You-go’ system is a very clear rule that guarantees the fiscal sustainability and reduces uncertainty (and it can also be welfare improving, see Romer, 2001, p. 94, exercise 2.16). Moreover, if the agents are assumed to be equipped with rational expectations any future shock disturbing the system could be absorbed by the agents with lower unpleasant consequences than other taxation schemes would imply.

has for the economic growth of the future. The Welfare Commission has not considered these possibilities if not in terms of the fiscal contribution or costs.

But current investment in infrastructure, education as well as current investment in research and development has long lasting implications for the future of the welfare of the individuals. Paradoxically an increase of current government expenditure so as to increase the current and future supply of public goods is not something that the Welfare Commission has worked with.

**Fiscal sustainability is only a budget constraint and cannot alone be considered an objective of the welfare state; it is at best a mean.** This is a very agreeable and almost trivial point.

It is too bad that the way in which the debate has been conducted has posed attention almost exclusively on the fiscal problem. And this is not a good starting point for a serious reform of the welfare state.

## Appendix

### Some simple computations: productivity gains and their intergenerational distribution<sup>12</sup>

The Welfare Commission assumes a productivity increase of 2% per year, which implies that by year 2040 the real national income would increase by circa 80% and by year 2050 by more than 100% (see Figure. 1A).

This productivity gain is something that will inevitably be distributed among the different citizens in different ways. Nevertheless we can have an idea of the possible distributions between those belonging to the working age population and those belonging to the non working age population<sup>13</sup>.

For example a social contract could be made that implies an equal redistribution of the productivity gains between those that belong to the working age population and those that belong to the non working age one. In this case, if we make equal to 100 the average income of somebody belonging to the working age group during 2005 and 100 the average income of somebody belonging to the non working age during 2005, those belonging to the working age and those belonging to the non working age would have an increase of income that would follow precisely the increase of aggregate income plotted in Fig. 1A<sup>14</sup>.

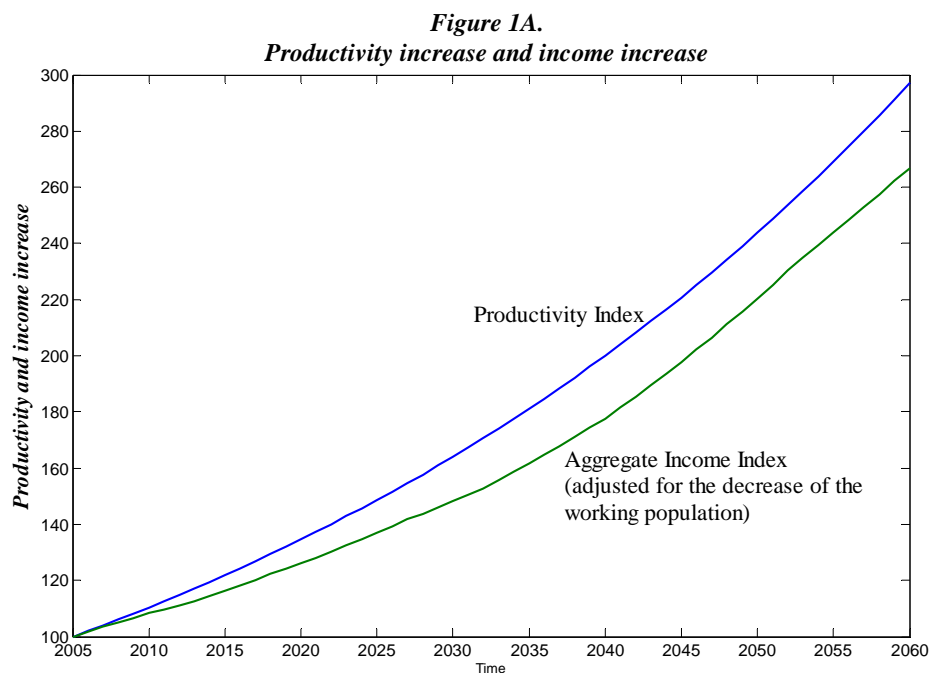
---

<sup>12</sup> Similar but more general and specific computations can be found in Brink (2001).

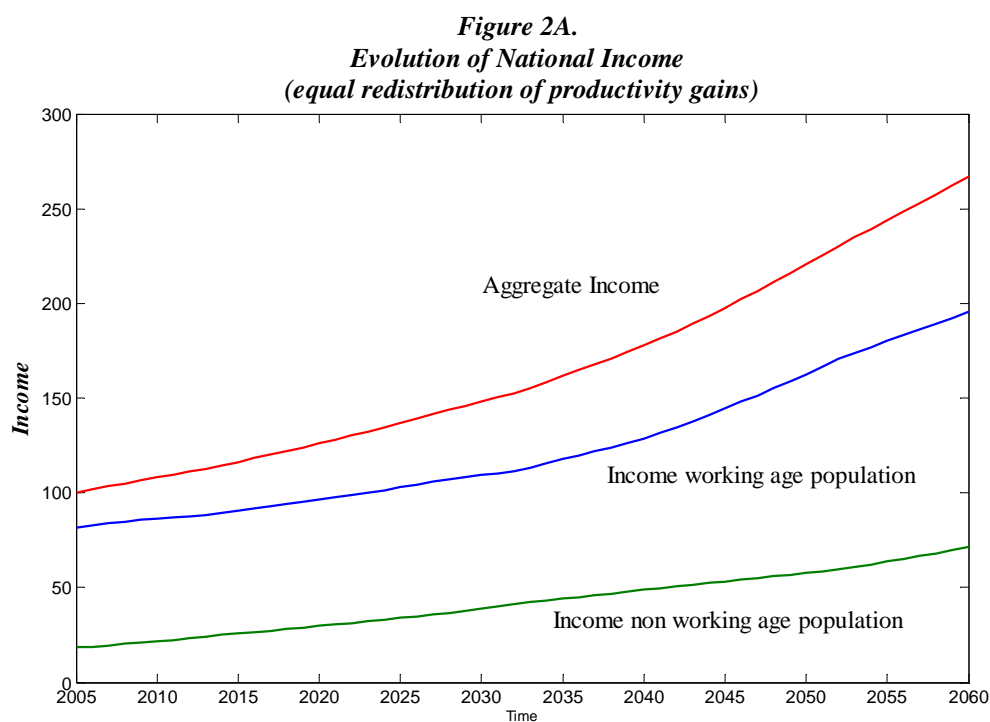
<sup>13</sup> These computations are made by using the information implicit in the figure 1 reported in the main text. Here the population is separated between the working population and non working population. For the sake of simplicity only 60% of the working age population is assumed to work (which approximates from below the actual ratio of the Danish active population with respect to the working age population). It is assumed an annual productivity increase of 2% and that during the year 2005 the income of the working individual and non working one is the same (but changes in this last assumption would not influence the results substantially). The evolution of the aggregate income index is taking under consideration the above assumptions and is computed with respect to the 2005 level.

<sup>14</sup> Please note that here it is not necessary to deal with the problem of fiscal sustainability. As explained in the main text there is an endless number of ways in which to finance state expenditures (let's assume that the state finances any eventual deficit with a pay-as-you-go procedure that by construction guarantees the balancing of the state budget).



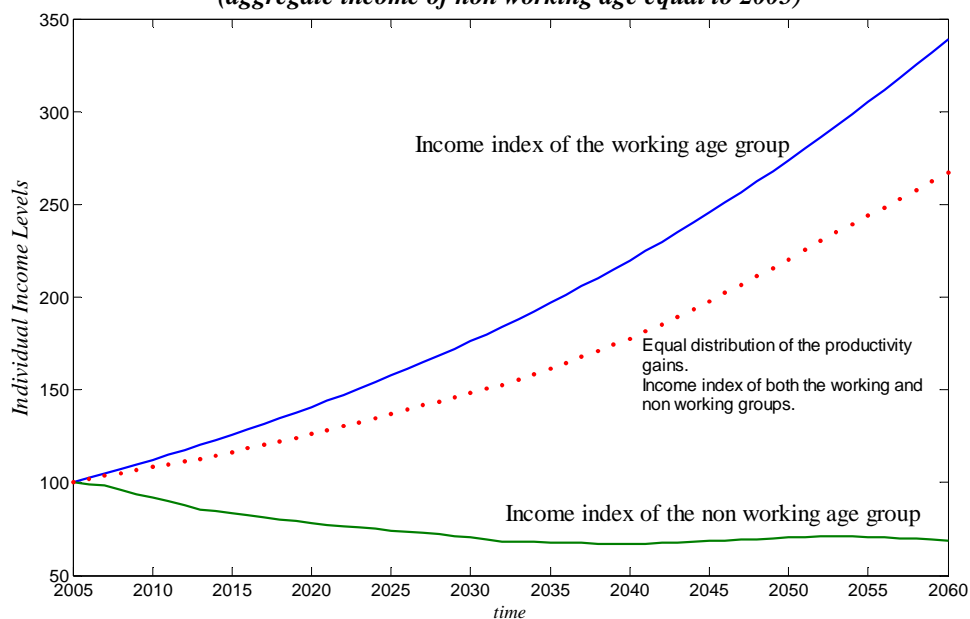


The national income distribution implied by the above is shown in Figure 2A.



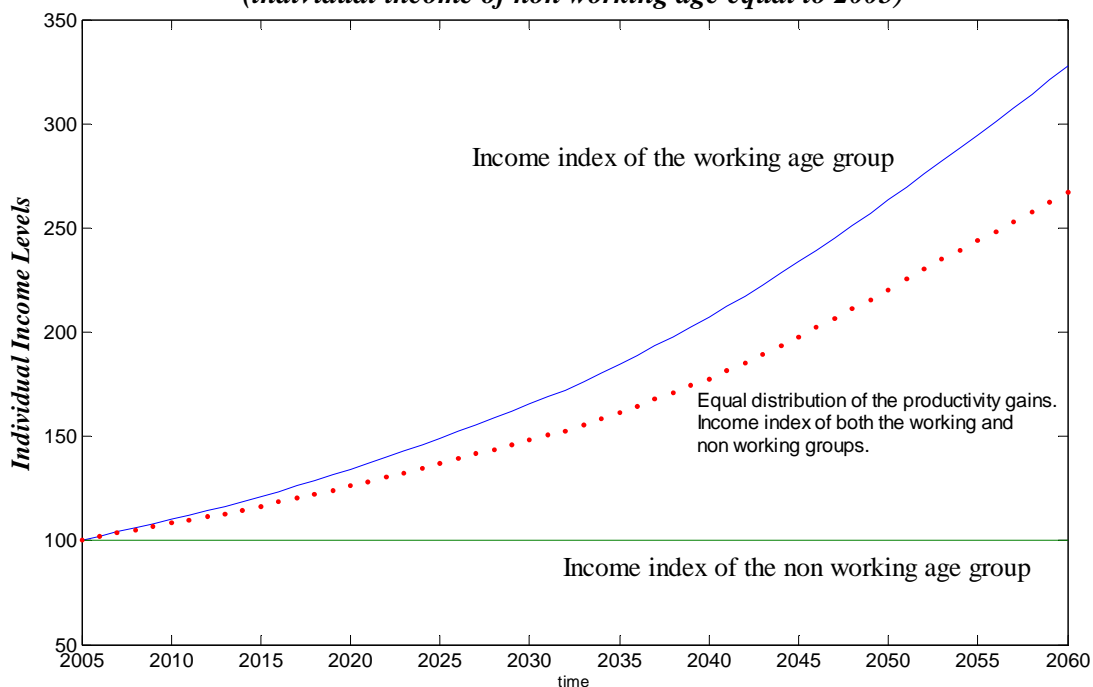
A different from the above social contract would be the one in which the working age population supports the non working age population so that in the aggregate they should get the same as in year 2005. The implications for the individuals are shown in figure 3A. In this case it is clear that the elderly will see their individual situation deteriorate. But this implies a total insensitiveness of those belonging to the working age group.

**Figure 3A.**  
**Income gains of the working age and non working age population**  
**(aggregate income of non working age equal to 2005)**



Another social contract would be the one in which a member of the non working age group would earn on the average and individually the same as a member of the non working age group earns in 2005. The implications for the individuals are shown in figure 4A. Here the situation of the members of the non working age group is by construction not deteriorating but the income improvements of the working age group are quite considerable.

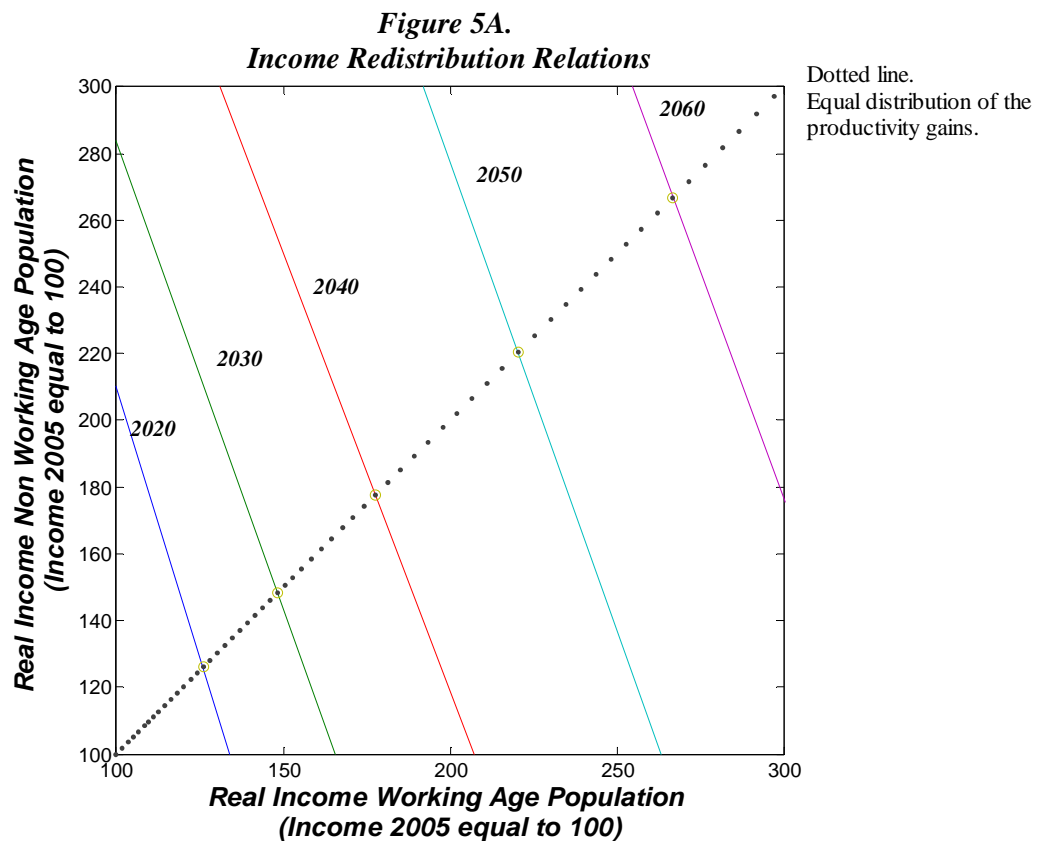
**Fig. 4A.**  
**Income gains of the working age and non working age population**  
**(individual income of non working age equal to 2005)**



Finally Figure 5A reports the different income substitution possibilities relative to the 2005 base levels for the years 2020, 2030, 2040, 2050 and 2060. The dotted line is relative to the equal allocation of the productivity gains as it is reported in Figure 1A. Moving along the different lines indicates the implications that the allocation of income in favour of one group would have for the other. Obviously they all represent cases of improvements from the 2005 income.

Clearly, according to the different welfare reforms there are different redistributions associated with them. The simple computations reported above should indicate that, with a 2% productivity increase, the ageing problem is not such a big problem. I am 47 years old. I will retire at 67. Personally I would hope to have, in 2025, the same real income level of my colleagues that are retiring today. If this would be the social contract, Figure 4A indicates that my daughters, that in the year 2025 would be respectively 25 and 30 years old, would have a real income which will be 50% higher of what I earn today. I expect to die, in line with the Welfare Commission forecasts, at 82 years, that is the year 2040 is a critical year for me, and at that time my daughters should earn in real terms double as much of what I earn today.

Well, as I said above, in year 2025 I would be satisfied with the today's standards for the elderly, but I cannot imagine that my daughters will not be willing to transfer some of their 'productivity gains' to me and to my peers. In the case that they will not be willing to do it; the reason for the collapse of the welfare state would be their selfishness and certainly NOT the so called ageing problem.



The different lines represent possible individual income redistributions between the working age population and the non working age population. Different policies would imply the determination of different points on the lines (but just one point per policy).

The dotted line represents one possible 'policy' where through time the productivity gains are evenly redistributed.



## References

Auerbach, A. and L., Kotlikoff, 1987, *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.

Brink, H., 2001, *Makroøkonomi*, Jurist- og Økonomforbundets Forlag, pp.161-87.

Knudsen, M., L. Pedersen, T. Pedersen, P. Stephensen, and P. Trier, 1998, *Danish Rational Economic Agent Model – DREAM Ver. 1.2*, Working Paper, [www.dreammodel.dk](http://www.dreammodel.dk), DREAM, København.

OECD, 2005, *Is the Welfare System Sustainable?*, OECD, Economic Survey for Denmark, 2005.

Romer, D., 2001, *Advanced Macroeconomics*, second edition, McGraw-Hill, New York.

Velfærdkommissionen, (Danish Welfare Commission), 2005a, *Befolkningsudvikling, velstands dilemma og makroøkonomiske strategier*, Teknisk analyserapport, [www.velfaerd.dk](http://www.velfaerd.dk), November.

Velfærdkommissionen, (Danish Welfare Commission), 2005b, *Fremtidens velfærd - vores valg*, [www.velfaerd.dk](http://www.velfaerd.dk), December.