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Forcasting Natural Gas Consumption in Nigeria using the Modified Grey Model (MGM(1,1,$\otimes_b$))

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Abstract

Accurate prediction of the natural gas consumption in Nigeria is crucial to Gas management. This study utilizes the improved Grey model (MGM(1,1,$\otimes_b$)), which is an improvement of the modified Grey model (MGM(1,1)), to forecast the natural gas consumption of Nigeria for the year 2021 to 2025. A secondary data retrieved from the NNPC 2019 annual statistics bulletin was used to build a model for this prediction. Noting that MGM(1,1) model uses the Grey action quantity as a unique real number which do not reflect the uncertainty nature of Grey systems. A model MGM(1,1,$\otimes_b$) was developed such that it extends the MGM(1,1) model to retain the uncertainty nature of Grey systems. The new modified Grey model(MGM(1,1,$\otimes_b$)) was used to make prediction of the natural gas consumption of Nigeria and the results shows that the MGM(1,1,$\otimes_b$) model gives a prediction interval which the actual value is bracketed. This implies that natural gas consumption of Nigeria for 2021 to 2025 lies within the MGM(1,1,$\otimes_b$) model prediction values for the same year.

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1. Introduction

Prediction of commercial gas consumption in a country is very important to the energy management sector of the country. Nigeria, having the largest gas reserve in Africa (Nigeria's Natural gas reserves are over 5 trillion m^3 which is several times bigger than the crude oil reserve) [1], produces more oil and pay little attention to gas which is

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consumed in residential, commercial and transportation sectors. natural gas has the potentials of changing the face of Nigerian economy if adequate attention is paid to it. Nigeria is in a phase of energy transition and natural gas has the potentials to dominate the energy outlook. Hence, investors are advice to look toward enhancing efficient utilization of gas within Nigeria [2].

The theory of Grey systems was initiated by Deng Julon (A famous Chinese Mathematician) in 1982, as a predictive tool use for making qualitative decisions. He refers to systems that lack complete information as Grey system. For example the human body, the economy, agriculture etc., are all Grey systems. A combinative interpolation optimization technique is adopted to improve the Grey conventional prediction model [3]. The improved Grey model was applied to predict the amount of malignant tumor patients of Jingzhon city in China for the year 2012 to 2016. Also, the GM(1,1) model has gradually evolve to many novel grey models [4] and has found application stock price prediction. Three important points were considered when considering Grey theory on stock market. These points are as follows: i. the unchanging plan ii. the up and bottom price limit of day trade and iii. the increment in stock price is dis-continuous.

Grey GM(1,1) prediction model was applied China's Beef consumption Forecasting [5]. They improved the conventional model by introducing an optional order of the Grey accumulated generator X^r. Where $X^r(k)$ is given as: $X^{(r)}(k) = \Sigma_{i=1}^{k} \frac{\Gamma(r+k+i)}{\Gamma(r+k+i)\Gamma(r)} X^{(0)}(i)$, where $k = 1, 2, 3, ...n$
and $Z^{(r)}$ is the mean generated sequence of consecutive neighbors of $x^{(r)}$.

The GM(1,1) model was improved to a hybrid carbon emissions forecast model based on multi-factor identification to offer reliable forecasting results. Empirical result indicates that the proposed hybrid model had the best performance compared to other methods. If no effective measures are taken, it is difficult for China to realize its goal for carbon emissions reduction in 2020 [6].

The study of the effect of sample length on forecasting validity of FGM(1,1) was carried out [7]. The results of the study indicates that the forecasting of 4 to 6 sample lengths is the most appropriate. The MAPE of 5 sample length is better than sample lengths 4 or 6. Also, this paper examine a suitable sample size for GM(1,1) model because the more samples selected the more errors [8]. It puts forward the mathematic proofs that the small sample usually has more accuracy than the large sample when establishing GM(1,1) model in theory.

The GM(1,1) model was review and modeled to reflect the uncertainty nature of grey systems by introducing the grey action parameter $\otimes_b$ and is applied to predict china natural gas consumption [9]. Also, a new exponential grey prediction model, which is called as EXGM (1,1), is proposed [10]. By using this model, new cases, deaths and recovered cases of COVID-19 in Turkey is forecast. Numerical results show that EXGM (1,1) is a model that performs more accurately than the comparison models. Again, the idea of Grey-box modelling is adopted in order to take most advantage of known information represented by deterministic structure, and then the neural grey system model is developed[11]. Levenberg-Marquardt algorithm is used to train the proposed model, and the Bayesian regularization is used to tune the regularized parameter automatically.

Here, several novel online Grey systems are proposed [12]. To evaluate the performance of the proposed models, they are compared against a set of benchmark models: GM(1,1) model, Grey Verhulst models with and without Fourier error corrections, linear time series model, and nonlinear time series model. The Grey(1,1) forecasting model was applied to the energy supply in China. They improved on the Grey(1,1) conventional model using the principle of quadric interpolation optimization technique. They used the Chinese energy data to forecast using the improved Grey model (MGM(1,1)). This MGM(1,1) model does not reflect the uncertainty nature of Grey model because is uses the Grey action parameter (b) as a unique real number. Hence, the MGM(1,1) model is extended to reflect the interval Grey action parameter ($\otimes_b$) and then apply the new model (MGM(1,1,$\otimes_b$)) to forecasting natural gas consumption in Nigeria.

2. Grey conventional model (GM(1,1))

The Grey model algorithm according to [13] and [14] is given as:
Grey differential equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{1}$$

which assumes a solution $x^{(0)}(R + 1) + aZ = b$

With mean value generating operator (MGO) $Z = \frac{1}{2}(x(R+1) + x(R))$
The optimized solution of the parameters a, b is derived by the least square method as:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y$$

$$B = \begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(0)}(n) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ . \\ . \\ . \\ x^{(0)}(n) \end{bmatrix} \tag{2}$$

Solution of the GM(1,1) differential equation (discrete time response series) gives

$$x^{(1)}(R+1) = (x^{(0)}(1) - \frac{b}{a})e^{-aR} + \frac{b}{a} \tag{3}$$

To restore original series by IAGO (inverse AGO)

$$x^{(0)}(R) = x^{(1)}(R) - x^{(1)}(R-1), \quad R = 1, 2, 3, ..., n \tag{4}$$

Grey forecasting by extrapolating the model series

$$x^{(1)}(n+1), x^{(1)}(n+2), ..., x^{(1)}(n+m) \tag{5}$$

Theorem 1. *The first order Grey differential equation is given as (1a). Then the recursive function is given by*

$$x^{(1)}(R+1) = (x^{(0)}(1) - \frac{b}{a})e^{-aR} + \frac{b}{a} \tag{6}$$

Proof: *solving using integration factor*

$$I.F. = e^{\int pdt} = e^{\int adt} = e^{\int at} \tag{7}$$

multiply equation(1a) by I.F. in (7)

$$e^{at}(\frac{dx^{(0)}}{dt} + ax^{(0)}) = e^{at}(b) \tag{8}$$

$$\frac{d}{dt}x^{(0)}e^{at} = e^{at}(b) \tag{9}$$

integrating with respect to t, we have

$$\begin{aligned} x^{(0)}e^{at} &= \frac{b}{a}e^{at} + c \\ x^{(0)} &= \frac{b}{a} + ce^{-at} \end{aligned} \tag{10}$$

$let \quad t = R$

$when \quad R = 0,$

$$c = (x^{(0)} - \frac{b}{a}) \tag{11}$$

substituting for c into equation (2.10), we have

$$x^{(1)}(R+1) = \frac{b}{a} + (x^{(0)}(1) - \frac{b}{a})e^{-aR}. \tag{12}$$

3. Model modification MGM(1,1)

$Z^{(1)}(R) = \frac{1}{2}(x^{(1)}(R)+x^{(1)}(R+1))$ is the neighbors mean (background value) of GM(1,1) model. According to [3] "A trapezoidal area with exponential curve side of $x^{(1)}(t)$ is replaced by straight-edge trapezoidal area. the shortcoming is that, as the exponential increase, data sequence varies greatly and the prediction result will have big error $(\triangle s)$". This will affect the applicability of the model to a certain extend.
To ever-come the short coming, a method constructed on the neighbors mean value using combinative interpolation optimization technique, which utilize the idea of numerical approximation combine with interpolation algorithm for problem solving and error reduction [15].

we start by making some numerical treatment on the background value $Z^{(1)}(R+1)$. The algorithm steps are as follows Step 1: Divide the interval $[R,\ R+1]$ into three equal intervals. They are

$$R+\frac{1}{3},\ x^{(1)}(R+\frac{1}{3})),\ (R+\frac{2}{3},\ x^{(1)}(R+\frac{2}{3})),\ (R+1,\ x^{(1)}(R+1)); \tag{13}$$

Step 2: solve $x^{(1)}(R+\frac{i}{3}),\quad i=1,\ 2$
Establish a quadric lagrange interpolation polynomial $P_2(t)$ which is subject to

$$P_2 \approx x^{(1)}(t), \quad [t \in [R, R+1]]$$

then

$$x^{(1)}(R+\tfrac{i}{3}) \approx P_2(R+\tfrac{i}{3}), \quad i=1,2$$

$P_2(t) =$

$$x^{(0)}(R)\frac{(t-R-1)(t-R-2)}{(-1)(-2)} + x^{(0)}(R+1)\frac{(t-R)(t-R-2)}{(-1)(1)} + x^{(0)}(R+2)\frac{(t-R)(t-R-1)}{(1)(2)} \tag{14}$$

we get

$$\begin{aligned} x^{(1)}(R+\frac{1}{3}) &\approx P_2(R+\frac{1}{3}) = \frac{5}{9}x^{(1)}(R) + \frac{5}{9}x^{(1)}(R+1) - \frac{1}{9}x^{(1)}(R+2) \\ x^{(1)}(R+\frac{2}{3}) &\approx P_2(R+\frac{2}{3}) = \frac{2}{9}x^{(1)}(R) + \frac{8}{9}x^{(1)}(R+1) - \frac{1}{9}x^{(1)}(R+2) \end{aligned} \tag{15}$$

Step 3: obtain the piece-wise interpolation function in the interval $[R, R+1]$

$$S_R(t) = \begin{cases} 3[x^{(1)}(R+\frac{1}{3}) - x^{(1)}(R)]t + x^{(1)}(R) & \\ -3R[x^{(1)}(R+\frac{1}{3}) - x^{(1)}(R)], & R \le t \le R+\frac{1}{3} \\ 3[x^{(1)}(R+\frac{2}{3}) - x^{(1)}(R+\frac{1}{3})]t + x^{(1)}(R+\frac{1}{3}) & \\ -3(R+\frac{1}{3})[x^{(1)}(R+\frac{2}{3}) - x^{(1)}(R+\frac{1}{3})], & R+\frac{1}{3} \le t \le R+\frac{2}{3} \\ 3[x^{(1)}(R+1) - x^{(1)}(R+\frac{2}{3})]t + x^{(1)}(R+\frac{2}{3}) & \\ -3(R+\frac{2}{3})[x^{(1)}(R+1) - x^{(1)}(R+\frac{2}{3})], & R+\frac{2}{3} \le t \le R+1 \end{cases}$$

Step 4: Calculate the numerical integration of the background value

Table 1. Comparing the GM(1,1) with MGM(1,1) model (in billion cubic feet)

Year	actual value	GM(1,1) Value	Error	MGM(1,1) value	Error
2010	1811.27	1811.27	-	1811.27	-
2011	1781.37	1880.3	0.5554	1864.3	0.4655
2012	1991.50	1969.5	0.1105	1964.7	0.1346
2013	1916.50	2063.0	0.7642	2070.5	0.8034
2014	2199.88	2160.8	0.1776	2182.1	0.0808
2015	2588.48	2263.3	1.2563	2499.6	0.1160
2016	2465.32	2370.7	0.3838	2423.4	0.1700
2017	2543.93	2483.2	0.2387	2554.0	0.0394
2018	2554.47	2601.8	0.1853	2691.5	0.5364
2019	2620.58	2724.3	0.3958	2836.5	0.8239
		Total error	4.0676		3.17

$z^{(1)}(R+1) = \int_R^{R+1} x^{(1)}(t)dt$

$$\begin{aligned} \int_R^{R+1} x^{(1)}(t)dt &= \int_R^{R+1} S_R(t)dt \\ &= \int_R^{R+\frac{1}{3}} S_R(t)dt + \int_{R+\frac{1}{3}}^{R+\frac{2}{3}} S_R(t)dt + \int_{R+\frac{2}{3}}^{R+1} S_R(t)dt \\ &= \frac{1}{6}x^{(1)}(R+1) + \frac{1}{3}x^{(1)}(R+\frac{2}{3}) + \frac{1}{3}x^{(1)}(R=\frac{1}{3}) + \frac{1}{6}x^{(1)}(R+2) \end{aligned} \tag{16}$$

put the last Step of step (2) into Equation (16) and obtain the optimized background value

$$z^{(1)}(R+1) = \frac{23}{54}x^{(1)}(R) + \frac{35}{54}x^{(1)}(R+1) - \frac{2}{27}x^{(1)}(R+2) \tag{17}$$

This $z^{(1)}(R+1)$ value in (17) gives the new adjacent neighbor(background vale) value which is then substituted into the conventional Grey Model($GM(1,1)$) to optimize the performance of the model.

3.1. Comparing the GM(1,1) with MGM(1,1) model

Using the natural gas consumption data of Nigeria from 2010 - 2019 as obtained from the NNPC 2019 annual statistical bulleting. We analyze their error using mean absolute percentage error (MAPE). The models and results presented in this work are simulated using Mat-lab.
From Table 1, the GM(1,1) model gave an error of 4.0676% while the MGM(1,1) model gave 3.17%. This shows that the MGM(1,1) model has higher precision and accuracy than the GM(1,1).

4. Implementation of the New MGM(1,1,$\otimes_b$) model

From equation (1), the Grey model equation can be rewritten as follows:

$$x^{(0)}(k) + aZ(k) = b(k), \quad where \;\; k = 1, 2, ..., n \tag{18}$$

The natural gas consumption of Nigeria given (in billion cubic squares) as:
$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5), x^{(0)}(6), x^{(0)}(7), x^{(0)}(8), x^{(0)}(9), x^{(0)}(10))$ = (1811.27, 1781.37, 1991.50, 1916.53, 2199
Use the new background value as obtain in equation (17), we implement the MGM(1,1,$\otimes_b$) as follows
$Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), z^{(1)}(4), z^{(1)}(5), z^{(1)}(6), z^{(1)}(7), z^{(1)}(8))$ =
(2686.38, 4593.94, 6521.41, 8571.82, 11003.91, 13515.86, 16025.53, 18570.61)

Table 2. Prediction result using the New MGM(1,1,$\otimes_b$) model(in billion cubic feet)

$\hat{X}^{(1)}(n)_{max}$ **value**	$\hat{X}^{(1)}(n)_{min}$ **value**
2027.15	1603.55
2136.34	1603.55
2251.40	1780.95
2372.66	1876.87
2500.46	1977.96
2635.14	2084.50
2777.07	2196.77
2926.65	2315.09
3084.28	2439.79
3250.41	2571.20

Therefore, given

$$B = \begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ -z^{(0)}(5) & 1 \\ -z^{(0)}(5) & 1 \\ -z^{(0)}(6) & 1 \\ -z^{(0)}(8) & 1 \\ -z^{(0)}(8) & 1 \end{bmatrix} = \begin{bmatrix} -2686.38 & 1 \\ -4593.86 & 1 \\ -6521.41 & 1 \\ -8571.82 & 1 \\ -11003.91 & 1 \\ -13515.86 & 1 \\ -16025.53 & 1 \\ -18570.61 & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ x^{(0)}(5) \\ x^{(0)}(6) \\ x^{(0)}(7) \\ x^{(0)}(8) \end{bmatrix} = \begin{bmatrix} 1781.37 \\ 1991.50 \\ 1916.53 \\ 2199.88 \\ 2588.48 \\ 2465.32 \\ 2543.93 \\ 2554.47 \end{bmatrix}$$

We obtain the parameter *a and b* using least square estimation technique as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y = \begin{bmatrix} -0.05246 \\ 1720.80859807192 \end{bmatrix} \tag{19}$$

Therefore from equation (1), (18) and using a as obtained in (19), we have

$b_{(2)} = 1670.3396261482$
$b_{(3)} = 1540.36768981767$
$b_{(4)} = 1649.38077161638$
$b_{(5)} = 1466.84452655084$
$b_{(6)} = 1622.60504193143$
$b_{(7)} = 1879.42572055166$
$b_{(8)} = 1624.60636943259$

Hence, $\otimes_b = [b_{max}, b_{min}] = [1879.42572055166, 1466.84452655084]$
where b in equation (19) is equivalent to $b_{mid} = 1720.80859807192$
from equation (3) and (4), we obtain the forecast values using the interval Grey action quantity($\otimes_b$) as given below

These values of $\hat{X}^{(1)}$ as presented in Table 2 are the prediction for 2010 to 2019 using the MGM(1,1,$\otimes_b$)

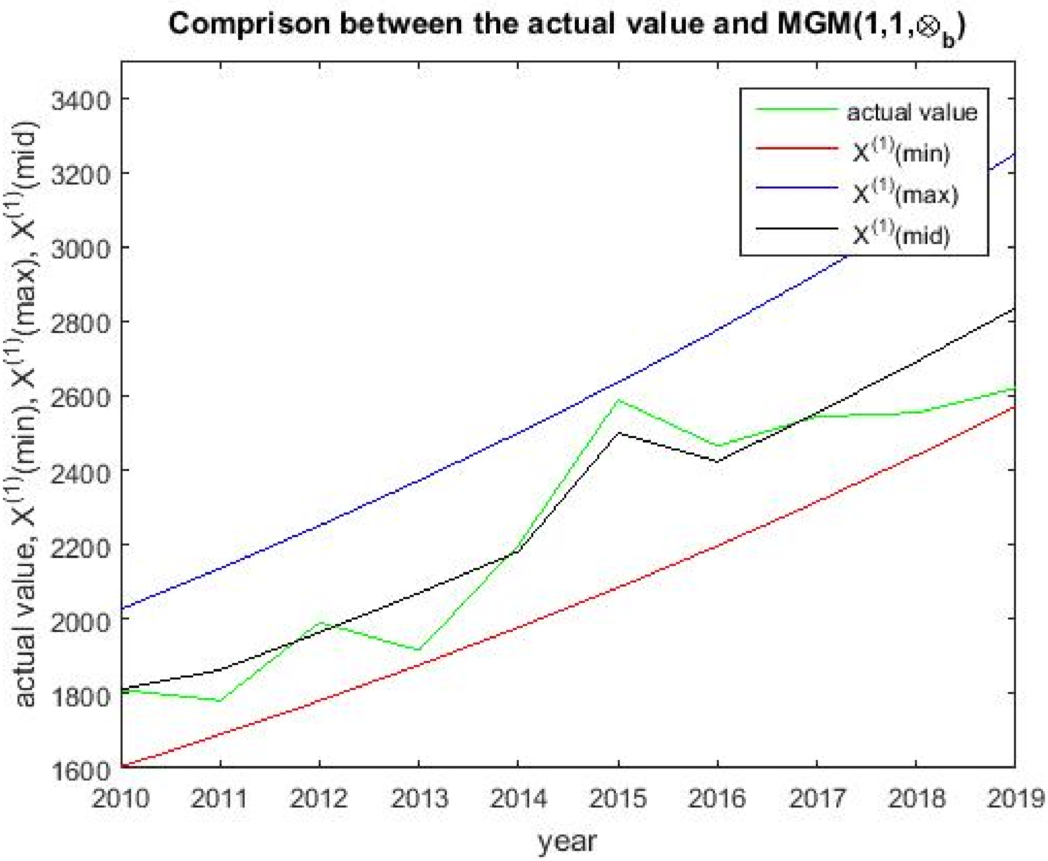

Figure 1. **A graph showing the forecast value using MGM(1,1,$\otimes_b$) model, MGM(1,1) model and the actual value**

4.1. Forecasting the Natural gas consumption of Nigeria using the modified Grey model (MGM(1,1,$\otimes_b$))

Energy is the foundation of economic development, and gas is one of the major energy sources. A nation's energy policy is thus of crucial importance, as it will not only guide the development of that country, but will also affect the operating environment of various industries. Then, the consumption and production of energy, especially natural gas, are still international hot issues, especially in today's social trends of green economy and sustainable development. A good demand forecast is a prerequisite for the effective development of energy and Natural Gas policies, as it can reduce the possibility of errors during consumption and production planning. Thus, producing accurate gas consumption forecasts is very important[16]

Table 3 presents the forecast value using the modified Grey Model MGM(1,1) model for the Natural gas consumption of Nigeria from 2021 to 2025. This result gives the prediction of natural gas consumption in Nigeria where the actual value will surely fall between the prediction interval of MGM(1,1$\otimes_b$).

5. Summary

The Grey modified model (MGM(1,1)) gives a more accurate and better forecast as compared with the conventional Grey Model (GM(1,1)) using MAPE But, the MGM(1,1) model does not reflect the uncertainty nature of the Grey system. Hence, the MGM(1,1,$\otimes_b$) is most suitable for the prediction of natural gas consumption of Nigeria.
Grey MGM(1,1) prediction values conforms with the mean value of MGM(1,1,$\otimes_b$). Hence, $\hat{X}^{(1)}mid$ is the same as MGM(1,1) model prediction values.

Table 3. Forecasting the Natural gas consumption of Nigeria from 2021 to 2025 (in billion cubic feet)

Year	$\hat{X}^{(1)}_{max}$ **Value**	$\hat{X}^{(1)}_{mid}$ **Value**	$\hat{X}^{(1)}_{min}$ **Value**
2021	3425.48	2989.3	2709.69
2022	3609.98	3150.3	2855.63
2023	3804.42	3320.0	3009.44
2024	4009.33	3498.8	3171.54
2025	4225.28	3687.2	3342.36

6. Conclusion

This paper provides mathematical evidence of a corresponding increase in domestic gas consumption in Nigeria and the need for government to put policies in place to increase production thereby impacting on the overall economy of the country. From our findings, it is safe to conclude that:
(i) Comparing the two models (the Grey conventional model (GM(1,1) and the modified Grey model (MGM(1,1))) using an error test criteria (MAPE) shows that the MGM(1,1) model has higher accuracy and better precision than the GM(1,1). Therefore we extend the MGM(1,1) model to reflect the uncertainty nature of Grey system and use the MGM(1,1,$\otimes_b$) to predict the Natural gas consumption of Nigeria from 2021 - 2025;
(ii) As can be seen from the forecast values in Table 3. The corresponding increase (Average Percentage growth is 5.38%) in the Natural gas consumption presents the national energy management sector of Nigeria (which have pay attention to fuel) an opportunity to exploit the gas section and implement policies to burst production in order to meet the rise in consumption.

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References

[1] A. NNPC, *Annual statistical bulletin*, 1st edition (2019).
[2] A. Adamu, "Independence of domestic gas consumption on nigerian economy", Business Science and Management **28** (2014).
[3] L. Xiaofei & H. Renfang, "Grey forecasting model based on interpolation optimization in malignant tumor", in SHS Web of Conferences **6**. EDP Sciences (2014) 03003.
[4] P. H. Hsin & C. I. Chen, "The evolution of grey forecasting and its application on stock price prediction", Stock & Forex Trading **3** (2014).
[5] B. Zeng, S. Li, W. Meng & D. Zhang, "An improved gray prediction model for china's beef consumption forecasting", PloS one **14** (2019).
[6] X. Ma, P. Jiang & Q. Jiang, "Research and application of association rule algorithm and an optimized grey model in carbon emissions forecasting", Technological Forecasting and Social Change **158** (2020) 120159.
[7] X. Zhicun, D. Meng & W. Lifeng, "Evaluating the effect of sample length on forecasting validity of fgm (1, 1)", Alexandria Engineering Journal **59** (2020) 4687.
[8] T. Yao, S. Liu & N. Xie, "On the properties of small sample of gm (1, 1) model", Applied Mathematical Modelling **33** (2009) 1894.
[9] B. Zeng, X. Ma, and J. Shi, "Modeling method of the grey gm (1, 1) model with interval grey action quantity and its application," Complexity **2020** (2020).
[10] H. Bilgil, "New grey forecasting model with its application and computer code", AIMS Mathematics **6** (2021) 1497.
[11] X. Ma, M. Xie & J. A. Suykens, "A novel neural grey system model with bayesian regularization and its applications", Neurocomputing **456** (2021) 61.
[12] G. Comert, N. Begashaw & N. Huynh, "Improved grey system models for predicting traffic parameters", Expert Systems with Applications **177** (2021) 114972.
[13] D. Ju-Long, "Control problems of grey systems", Systems & control letters **1** (1982) 288.
[14] N. Beck, *Ols in matrix form* (2001).
[15] X. Chen, Zhiqiang & Wang, "Applying the grey forecasting model to the energy supply management engineering", Systems Engineering Procedia **5** (2012) 179.
[16] J.-M. Zhou, Wei & He, "Generalized gm (1, 1) model and its application in forecasting of fuel production", Applied Mathematical Modelling **37** (2013) 6234.