Seepage Analysis Through and under Hydraulic Structures Applying Finite Volume Method

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Received on:7/5/2009 & Accepted on: 10/1/2013

ABSTRACT

In this paper, the seepage analysis through and underneath the hydraulic structures is studied at the same time without dividing the structure into parts, and then analyze each part individually. The analysis has been done using the finite volume method using rectangular elements. This method implemented on several types of structures and the comparison of the results is made with the one solved using finite element method. The comparison showed close results. The finite volume method has been implemented on non-rectangular structures. The present work studied the effect of heterogeneous foundations on the uplift pressure and exit gradients at the downstream and comparison with homogenous foundations. Also it studied the evaluation of effect of position and inclination of cut-offs at upstream or downstream of structures on uplift pressure and exit gradients at downstream. In addition, it studied the effect of impervious body inside the structure or foundation on uplift pressure and exit gradients at downstream.

تحليل التسرب خلال وتحت المنشآت المائية باستخدام طريقة العناصر الحجمية

الخلاصة

تم في هذا البحث تحليل التسرب خلال وتحت المنشآت المائية في آن واحد دون اللجوء الى تقسيم المنشأ الى اقسام وتحليل كل جزء على حدة، تم التحليل باستخدام طريقة العناصر الحجمية وباستخدام عناصر مستطيلة الشكل. طبقت الطريقة على عدة انواع من المنشآت ومقارنة النتائج مع نتائج الحل بطريقة العناصر المحددة فكانت نتائج المقارنة متقاربة. تم تطبيق الطريقة على المنشآت ذات الاشكال غير المستطيلة. وايضا دراسة تأثير الاسس غير المتجانسة على ضغط الرفع والانحدار الهيدرولي عند المؤخر مقارنة مع الاسس المتجانسة، بالاضافة الى تقييم تأثير موقع ودرجة ميلان الحواجز الرأسية عند مقدم او مؤخر المنشأ على ضغط الرفع والانحدار الهيدرولي عند المؤخر. كما تم دراسة تأثير وجود جسم غير نفاذ للماء في المنشأ او اساسه على ضغط الرفع والانحدار الهيدرولي عند المؤخر.

https://doi.org/10.30684/etj.2013.82185
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INTRODUCTION

Hydraulic structures are a specific type of engineering structures designed and executed in such a way in order to utilize it to control natural water or save industrial sources to ensure optimum use of water. Hydraulic structures of such a field normally consist of two parts: (1) the superstructure part which comprises the piers, abutments, retaining walls, arches, and all the upper components of the structure, and (2) The substructure part, which generally consists of a continuous masonry or concrete foundation called the floor, and of step walls or sheet piles.

The design of a complete hydraulic structure then is divided into two parts, the hydraulic design and structural design.

One of the most important problems that cause damage to hydraulic structures is seepage through and/or under dams, which occurs due to the difference in water level between the upstream and downstream sides of hydraulic structure.

Seepage is inevitable in all earth dams and ordinarily does not harm. Uncontrolled seepage may, however, cause erosion within the embankment or in the foundation, which may lead to piping.

Seeping water may prove harmful to the stability of the dam by causing softening and sloughing of slopes due to the development of pore pressures and thereby leading to the weakling of the mass and even failure by shear [2].

Therefore, the study of seepage through earth dams is one of the important analyses in dam design to calculate the quantity of losses from reservoir, estimating the pore pressure distribution, and locating the position of the free surface, which are used in analysis of the dam stability against the shear failure. In addition, studying of the hydraulic gradient gives a general idea about potential piping.

Seepage flow below the foundation of hydraulic structures founded on permeable soils exerts upward pressure on the structure (floor) and tends to reduce the frictional resistance between the structure and its foundation, and increase the tendency to slide. This thrust force is called the uplift pressure. In addition, it may tend to wash away soil under the hydraulic structure, leading to piping. Excessive uplift pressure and piping are often the cause of damage of the stability of the structure and may cause its failure.

In general, the problems to be considered due to seepage flow through and beneath structures can then be grouped into two categories, those due to (1) excess leakage and (2) excess pressure or gradients.

Excess quantity of seepage is caused by high permeability, short seepage paths, and defects such as cracks, fissures, and uneven settlement. The seepage discharge can be reduced by using soils of low permeability, placing cores (in earth fill structures), cut-offs in the foundations, and by increasing the seepage path by extending the floor using auxiliary revetments. Excessive uplift pressures, particularly at point where there is little weight of structure materials to resist them, lead to boiling and piping. Control of these pressures and gradients by using step walls or sheet piles on upstream or/and downstream, internal drains, filter trench on downstream, pressure relief wells on downstream side.

AIM OF THE RESEARCH

The aim of the present work is to verify the finite volume method adopted in this study to obtain solutions for seepage flow through porous media comprising irregular zones and compare the results obtained with finite element method. In addition, an investigation is to be done to show the effect of anisotropy of foundation soil on the uplift pressure and exit gradient and compare it with the isotropic soil foundation. Also, to evaluate the effect of location and inclination of the cut-offs on the uplift pressure and exit gradient. As well, to evaluate the effect of impermeable solid rock located in the foundation.

FINITE VOLUME METHOD

The finite volume method (FVM) is one of the numerical methods used to solve the partial differential equations of such applications: fluid flow, heat transfer, combustion …etc. The FVM was originally developed as a special finite difference formulation and then thoroughly validated general purpose computational fluid dynamics (CFD) technique. The numerical algorithm consists of the following steps [4][5]:

- Formal integration of the governing equations of fluid flow over all the (finite) control volumes of the solution domain.
- Discretisation involves the substitution of a variety of finite-difference-type approximations for the terms in the integrated equation representing flow processes such as groundwater flow and sources. This converts the integral equations into a system of algebraic equations.
- Solution of the algebraic equations.

FVM FOR TWO-DIMENSIONAL STEADY STATE SEEPAGE

Consider steady state seepage (without external source) in a two dimensional domain defined in Figures (1) through (3). The process is governed by Laplace equation.

$$\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)=0 \qquad \ldots(1)$$

Where k is the coefficient of permeability. Boundary values of ϕ are prescribed. The numerical algorithm steps are as follows:

Step 1: Grid generation

The first step in the finite volume method is to divide the domain into discrete control volumes. Let us place a number of nodal points in the domain. The boundaries (or faces) of control volumes are positioned mid-way between adjacent nodes. Thus, each node is surrounded by a control volume or cell. It is common practice to set up control volumes near the edge of the domain in such a way that the physical boundaries coincide with the control volume boundaries as shown in Figure (1).

A general nodal point is identified by P and its neighbours in a two-dimensional geometry, the nodes to the west, east, south, and north are identified by W, E, S, and N respectively. The west side face of the control volume is referred to by '*w*' and the east side of the control volume face by '*e*' and the same for rest faces south '*s*' and north '*n*'. The distances between the nodes W and P, P and E, P and S, and P and N are identified by δx_w , δx_e , δy_s , and δy_n respectively. Similarly, the distances between face w and point P and between P and face e are denoted by δx_{wP} , δx_{Pe} , δy_{sP} , and δy_{Pn} respectively. Figure (3) shows that the control volume width is $\Delta x = \delta x_{we}$ and height is $\Delta y = \delta y_{sn}$.

Step 2: Discretisation

The key step of the finite volume method is the integration of the governing equation (or equations) over a control volume to yield a discretised equation at its nodal point P. For the control volume defined above and for a case of steady state seepage with a source gives Poisson's equation.

$$\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)+S_\phi=0 \qquad \ldots(2)$$

Where S_ϕ is the source term. Integrating the above equation gives:

$$\int_{\Delta V}\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)dx.dy+\int_{\Delta V}\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)dx.dy+\int_{\Delta V}S_\phi dV=0 \qquad \ldots(3)$$

$$\int_{y_s}^{y_n}\int_{x_w}^{x_e}\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)dx.dy+\int_{x_w}^{x_e}\int_{y_s}^{y_n}\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)dx.dy+\bar{S}\Delta V=0 \qquad \ldots(4)$$

exact evaluation of the inner integral yields the following:

$$\int_{y_s}^{y_n}\left[\left(k\frac{\partial \phi}{\partial x}\right)_e-\left(k\frac{\partial \phi}{\partial x}\right)_w\right]dy+\int_{x_w}^{x_e}\left[\left(k\frac{\partial \phi}{\partial y}\right)_n-\left(k\frac{\partial \phi}{\partial y}\right)_s\right]dx+\bar{S}\Delta V=0 \qquad \ldots(5)$$

For the first term in the above equation, will be assumed that the quantities between square brackets do not vary with y, and for the second term the quantities between square brackets do not vary with x, which yields the following expression:

$$\left[\left(k\frac{\partial \phi}{\partial x}\right)_e-\left(k\frac{\partial \phi}{\partial x}\right)_w\right]\Delta y+\left[\left(k\frac{\partial \phi}{\partial y}\right)_n-\left(k\frac{\partial \phi}{\partial y}\right)_s\right]\Delta x+\bar{S}\Delta V=0 \qquad \ldots(6)$$

By considering a control volume of unit thickness, then face areas of the control volume are $A_e=A_w=\Delta y$ and $A_n=A_s=\Delta x$, this tends to:

$$\left[k_e A_e\left(\frac{\partial\phi}{\partial x}\right)_e - k_w A_w\left(k\frac{\partial\phi}{\partial x}\right)_w\right] + \left[k_n A_n\left(\frac{\partial\phi}{\partial y}\right)_n - k_s A_s\left(\frac{\partial\phi}{\partial y}\right)_s\right] + \overline{S}\Delta V = 0 \cdots (7)$$

The above equation expresses the continuity equation across the control volume.

Here ΔV is the volume and $\overline{S}$ is the average value of the source S_ϕ over the control volume. The source term S_ϕ may be a function of the dependent variable. In such cases the FVM approximates the source term by means of a linear form:

$$\overline{S}\Delta V = S_u + S_P\phi_P \qquad \ldots(8)$$

To calculate gradients (and hence fluxes) at the control volume faces an approximate distribution of properties between nodal points is used. Linear approximations seem to be the obvious and simplest way of calculating interface values and the gradients. This practice is called central differencing. Thus, we can re-write equation (7) as follows[5] [4]:

$$k_e A_e\frac{(\phi_E-\phi_P)}{\delta x_{PE}} - k_w A_w\frac{(\phi_P-\phi_W)}{\delta x_{WP}} + k_n A_n\frac{(\phi_N-\phi_P)}{\delta y_{PN}} - k_s A_s\frac{(\phi_P-\phi_S)}{\delta y_{SP}} + S_u + S_P\phi_P = 0 \cdots (9)$$

This equation can be re-arranged as:

$$\left(\frac{k_w A_w}{\delta x_{WP}} + \frac{k_e A_e}{\delta x_{PE}} + \frac{k_s A_s}{\delta y_{SP}} + \frac{k_n A_n}{\delta y_{PN}} - S_P\right)\phi_P = \left(\frac{k_w A_w}{\delta x_{WP}}\right)\phi_W + \left(\frac{k_e A_e}{\delta x_{PE}}\right)\phi_E + \left(\frac{k_s A_s}{\delta y_{SP}}\right)\phi_S + \left(\frac{k_n A_n}{\delta y_{PN}}\right)\phi_N + S_u \ldots (10)$$

Identifying the coefficients of ϕ_E , ϕ_W , ϕ_S , ϕ_N ,and ϕ_P in the above equation as a_W , a_E , a_S , a_N , and a_P . Hence, the above equation can be written as:

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N + S_u \qquad \ldots(11)$$

The above equation represents the discretised form of equation (1) and it applies to each internal node in the domain shown in Figure (1). This defines a linear system of m equations in the m unknown internal values of ϕ .

Step 3: Solution of equations[4][5]

Discretised equations of the form (11) must be set up at each of the nodal points in order to solve a problem. For control volumes that are adjacent to the domain boundaries, the general discretised equation (11) should be modified to incorporate boundary conditions. The resulting system of linear algebraic equations is then solved to obtain the distribution of the property ϕ at nodal points. There are two families of solution techniques for linear algebraic equations: direct methods and indirect or iterative methods.

The finite volume method usually yields systems of equations each of which has a vast majority of zero entries. In this proposed work, the matrix of coefficients will be a penta-diagonal system that has five non-zero coefficients. Since the systems are often very large - up to 100000 or 1 million equations - one find that iterative methods are generally much more economical than direct methods.

Jacobi and Gauss-Seidel iterative methods are easy to implement in simple computer programs, but they can be slow to converge when the system of equations is large. Hence, they are not considered suitable for general CFD procedures. Thomas (1949)[5] developed a technique for rapidly solving tri-diagonal systems that is now called the Thomas algorithm or the tri-diagonal matrix algorithm (TDMA). The TDMA is actually a direct method for one-dimensional situations, but it can be applied iteratively, in a line-by-line fashion, to solve multi-dimensional problems and is widely used in CFD programs. It is computationally inexpensive and has the advantage that it requires a minimum amount of storage.

VERIFICATION OF FINITE VOLUME METHOD

The capacity and effectiveness of the finite volume method have been examined by applying it to solve several typical and practical seepage examples. The examples contained both isotropic and anisotropic body and foundation material.

Having determined the values of potentials, the uplift pressure distribution beneath the structure, the seepage discharge, the exit gradients …etc. can be determined accordingly.

The method has been used to solve the following examples:

a. Seepage flow beneath a solid rigid impermeable base to a water impounding structure supported by a uniform isotropic permeable foundation, Figure (4). It can be shown analytically that, for this case, the equipotential lines are symmetrical. Figure (4) shows the equipotential lines produced by the numerical solution of both finite element and the finite volume; it is seen that the flow pattern is symmetrical as logical would expected. It shows that the result of the finite volume is similar to the Jumaily[1] finite difference/element results.

b. The same previous example, but now the permeable foundation is anisotropic (kx = nky). Different values of n were taken. Figure (5) shows the comparison between the response of the finite difference/element and finite volume methods for these various conditions that show the similarity in results, Jumaily[1].

MORE ILLUSTRATIVE EXAMPLES

Figures (6 to 8) show the comparison between the response of the finite element and finite volume methods for various boundary conditions and seepage control devices that show the similarity in results, Khasaf[3], except for Figure (8) which shows a different equipotential lines. A thorough check done using finite volume method to reach the result that the figure showed by Khasaf is for ($\theta = 60$) and not ($\theta = 150$) as printed in his thesis. Figures (9 to 10) show other cases of dams. Figure (10) shows a case which is difficult to be solved using finite difference as there is a rounded rock under the dam.

OTHER CHARACTERISTICS OF FVM

The finite volume method is not limited to Cartesian grids but can be used with a grid in any orthogonal coordinate system. i.e. one can use rectangular elements in Cartesian coordinate, sector elements in polar coordinates, and cuboids elements in three dimensional Cartesian coordinates.

The finite volume method is applied to different types of material, structures, boundary conditions and it shows good results in comparison to ones done by finite element method.

The finite volume method shows a good response easy to implement in anisotropy, non-homogeneous material, and discontinuous boundary.

The finite volume method is easy as finite difference and powerful as finite element and it is easy to program and implement.

CONCLUSIONS

In this study the finite volume method was used to analyze the seepage flow under hydraulic structures founded on isotropic, anisotropic, homogeneous, and non-homogeneous material. Rectangular elements proved their efficiency in computing the potential head. Comparison between the results of finite volume method and finite element method is obtained and the solutions show good agreement.

A special code written to program the finite volume method solutions, so as potential head and exit gradient can be obtained at any point within the flow domain. The present finite volume model is general and can be applied to a wide range of practical problems.

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LIST OF SYMBOLS

Symbol	Definition	Dimensions
A_w, A_e, A_s, A_n	Face areas of the control volume at *w, e, s, n*	[L^2]
a_W , a_E , a_S , a_N , a_P	Coefficients	[-]
k	Coefficient of permeability	[L/T]
k_a	Coefficient of permeability of material a	[L/T]
k_b	Coefficient of permeability of material b	[L/T]
k_w, k_e , k_s , k_n	Coefficient of permeability at control volume faces	[L/T]
P	General nodal point	[-]
S_u	source term	[unit/L^3]
S_ϕ	source term	[unit/L^3]
x,y	coordinates	[-]
W, E, S, N	Node position at west, east, south, and north	[-]
w, e, s, n	west, east, south, and north side face of the control volume	[-]
θ	Orientation of the direction of major principal hydraulic conductivity ellipse with x-axis	[$^\circ$]
ΔV	Volume of the control volume	[L^3]
Δx	Control volume width	[L]
Δy	Control volume Height	[L]
δx_w , δx_e , δy_s , δy_n	The distances between the nodes W and P, P and E, P and S, and P and N	[L]
δx_{wP} , δx_{Pe} , δy_{sP} , δy_{Pn}	The distances between face w and point P, P and face e, *s* and face P, P and face *n*,	[L]
δx_{we}	Control volume width in respect with east-west directions	[L]
δy_{sn}	Control volume Height in respect with north-south directions	[L]
ϕ	Flow potential	[L]
ϕ_E , ϕ_W , ϕ_S , ϕ_N , ϕ_P	Flow potential at nodes E, W, S, N, and P	[L]

Symbol	Definition	Dimensions
ϕ_j	Potential at extremity (j)	[L]
ϕ_P	Potential at node under consideration (P)	[L]

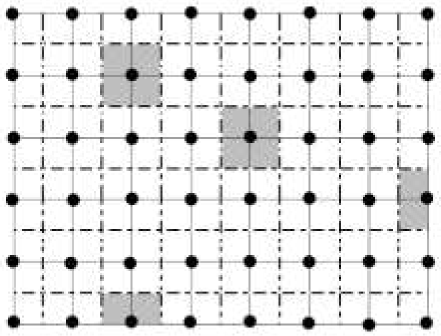

Figure (1) Control Volume surrounding grid nodes and covering

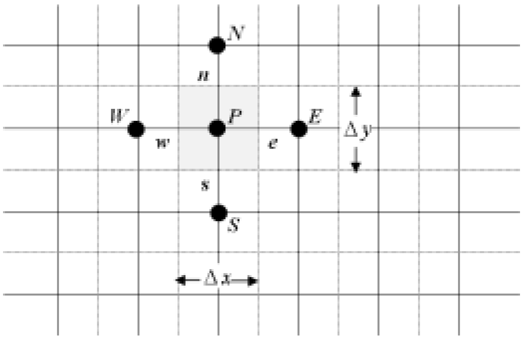

Figure (2) A part of the two dimensional grid

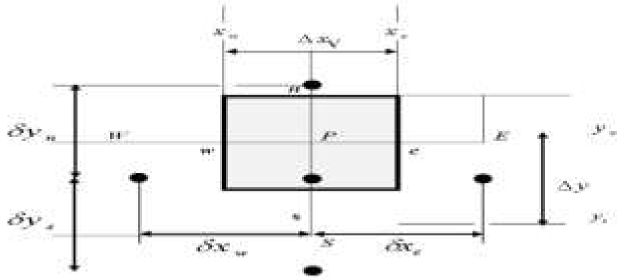

Figure (3) Geometric variables for a typical control volume.

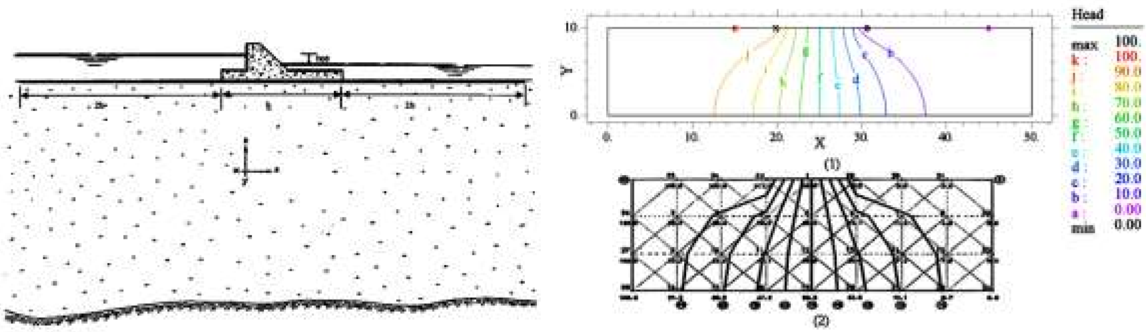

Figure (4) Homogeneous uniformly isotropic soil example, comparison results between (1) finite volume & (2) finite difference, Jumaily[1]. (kx=ky).

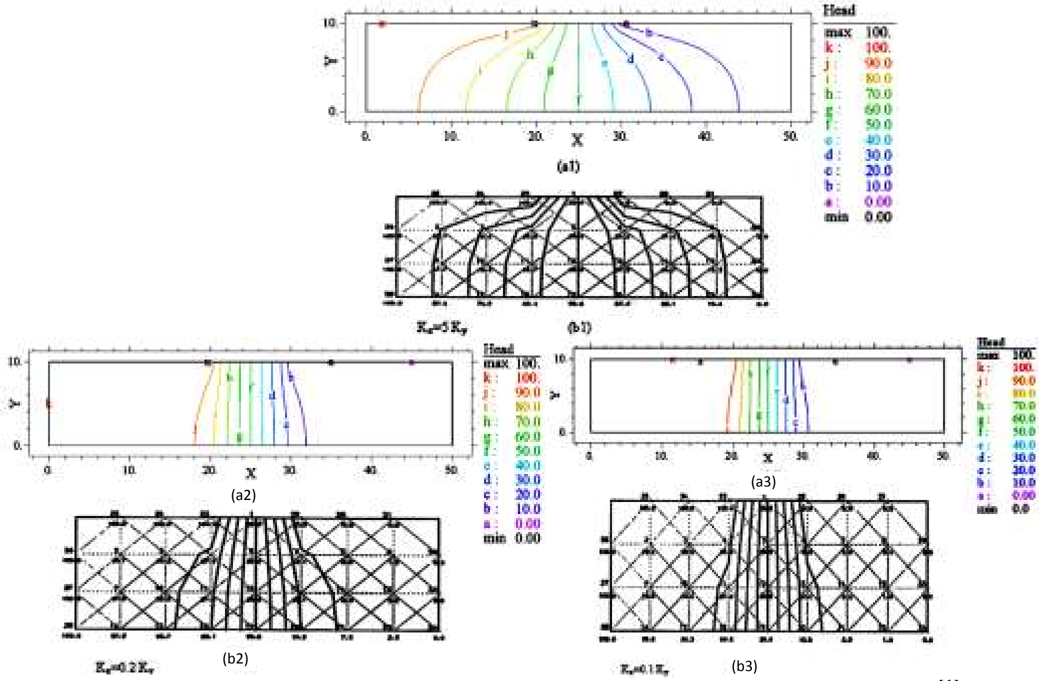

Figure (5) Comparison between finite volume & finite difference, Jumaily[1]. (a1) finite volume & (b1) finite difference (kx=5 ky), (a2) finite volume & (b2) finite difference (kx=0.2 ky), (a3) finite volume & (b3) finite difference (kx=0.1 ky).

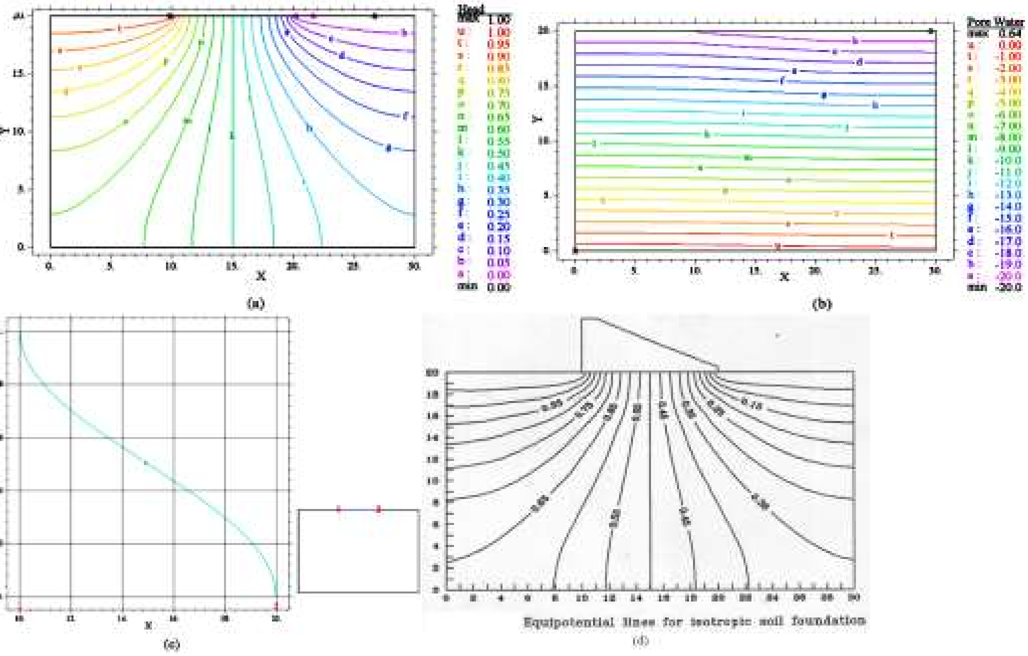

Figure (6) A dam supported by isotropic foundation impounding a depth of water (H=1) (a) Equipotential lines, (b) Pore water pressure, (c) Uplift Pressure distribution under the dam base, (d) FEM, Khasaf [3].

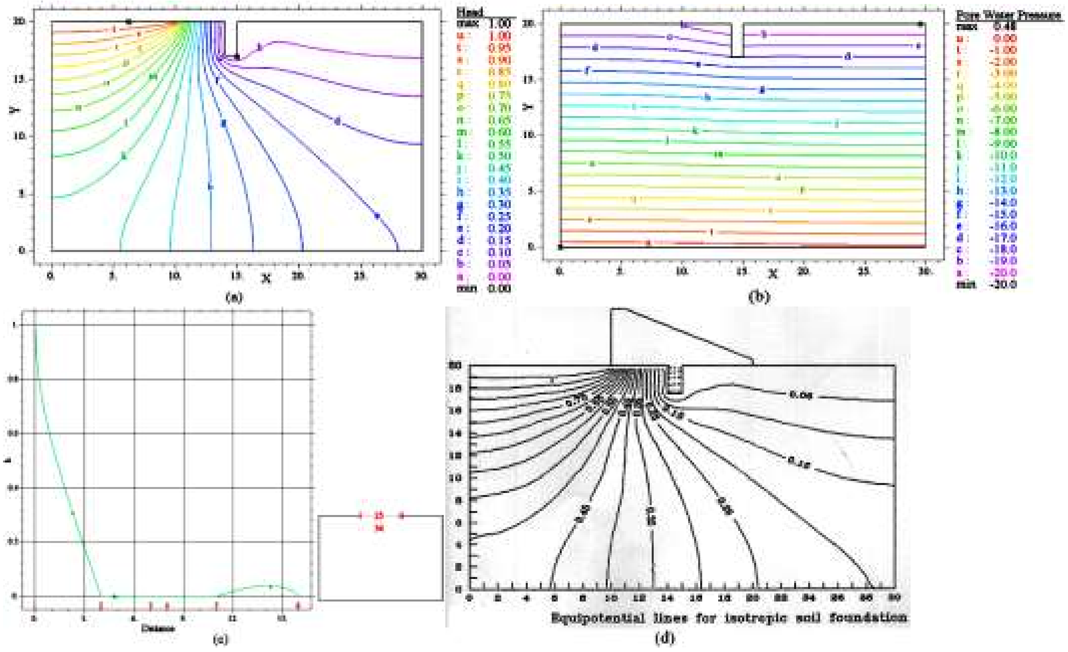

Figure (7) A dam supported by isotropic foundation impounding a depth of water (H=1) and filter trench at x=14 width=1 (a) Equipotential lines, (b) Pore Water Pressure, (c) Uplift Pressure distribution under the dam base, (d) FEM, Khasaf [3].

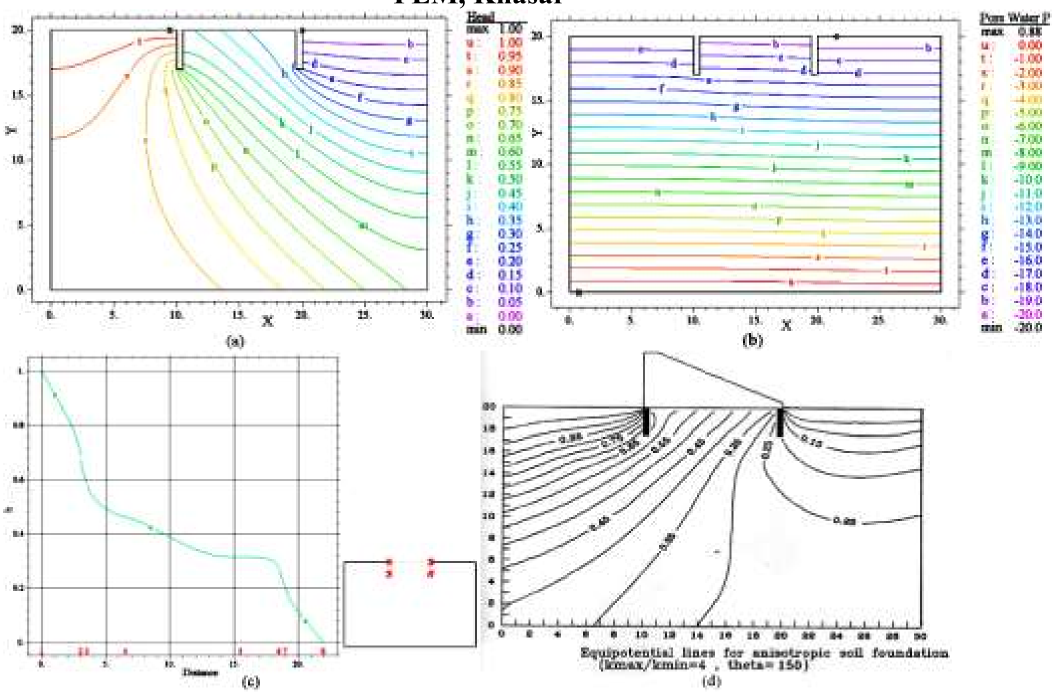

Figure (8) A dam based on anisotropic foundation (kx =4ky with $\theta = 150$) impounding a depth of water (H=1) with two cut-offs at far ends (a)

Equipotential lines, (b) Pore water pressure, (c) Uplift Pressure distribution under the dam base, (d) FEM, Khasaf[3].

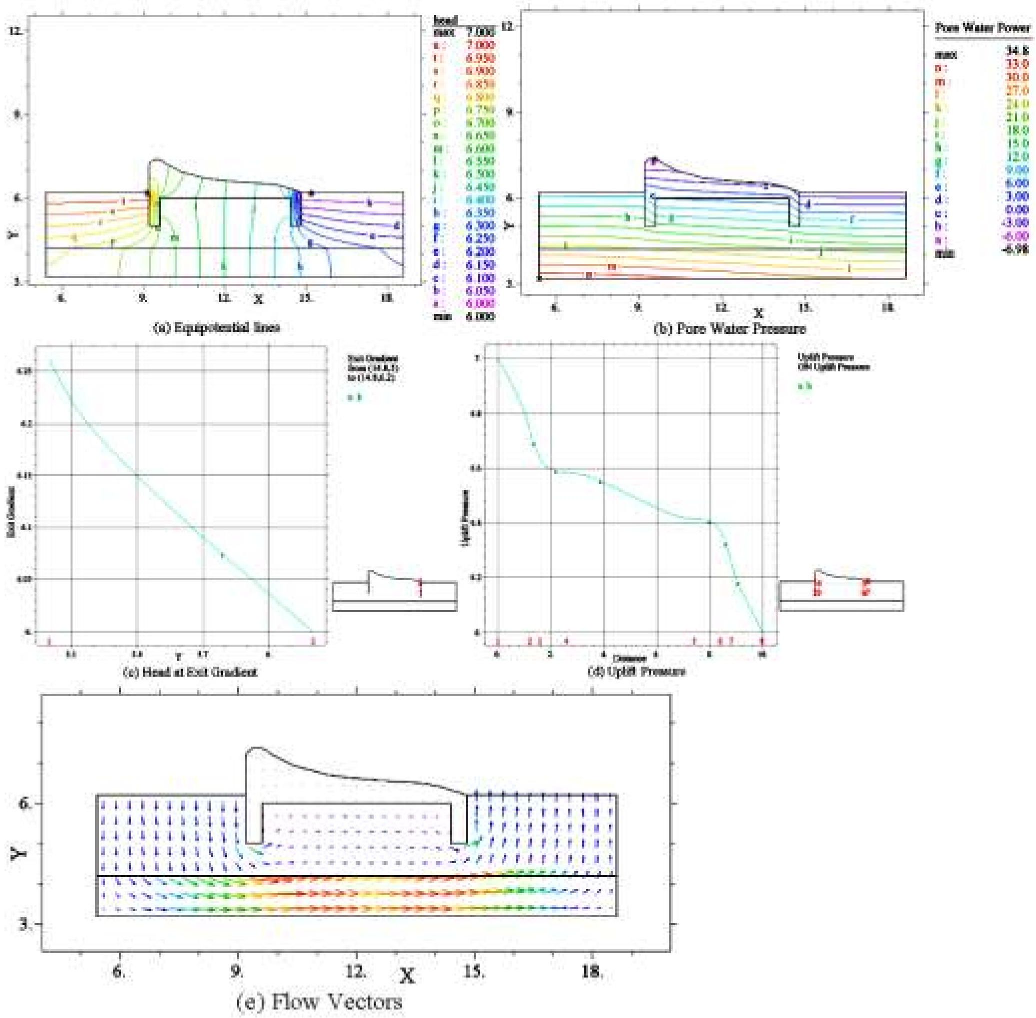

Figure (9) A dam its k=1E-12 and Layer1 k=1E-10 and Layer2 k=1E-8 impounding a depth of water (H=7) and D/S H=6 with two cut-offs at far ends (a) Equipotential lines, (b) Pore water pressure, (c) Head at Exit Gradient, (d) Uplift Pressure distribution under the dam base, (e) Flow Vectors.

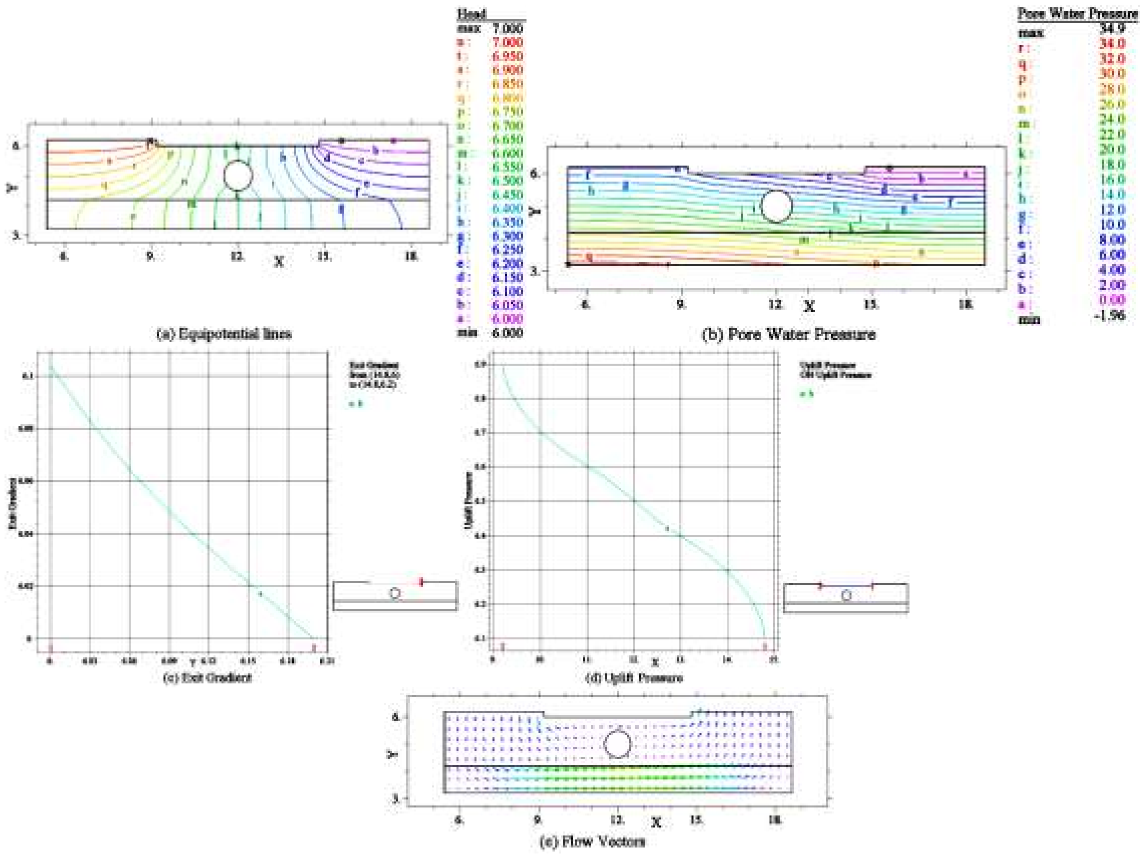

Figure (10) A dam its k=1E-12 and Layer1 k=1E-10 and Layer2 k=1E-8 impounding a depth of water (H=7) and D/S H=6 with spherical rock under the middle of the dam(a) Equipotential lines, (b) Pore water pressure, (c) Head at Exit Gradient, (d) Uplift Pressure distribution under the dam base, (e) Flow Vectors, (f) Head around the Rock.