# A Study of the Behavior of Shell Footings using Finite Element Analysis

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## ABSTRACT

In this research, the conical shell foundation is investigated. The two components of the interacting system; the soil and the shell foundation, are modelled using the finite element method. In this study, 15-node isoparametric triangular axisymmetric elements with two degrees of freedom at each node are used to model the shell and soil. The soil-structure interaction between the footing and the supporting medium are modelled using interface elements. Comparison between the results obtained by the present analysis and those obtained by other investigations are made. The present analysis shows satisfactory results when compared with those obtained by other studies with largest percentage difference of 14% in the value of the ultimate load. Parametric studies have been carried out to investigate the effect of some important parameters on the behaviour of shell foundations. Three parameters are considered which are: semi-vertical angle, footing embedment and edge beam.

**keywords:** Analysis, Conical, Finite Element, Foundation, Shells, Soil, Interface Element.

## دراسة سلوك الأسس القشرية باستخدام طريقة العناصر المحددة

**الخلاصة**

في هذا البحث تم دراسة تصرف الأسس القشرية. تتكون أنظمة الأسس القشرية من جزئين وهما التربة والأسس والتي تم تمثيلهما باستخدام العناصر المثلثة المتناظرة محوريا وبخمسة عشر عقدة ولكل عقدة درجتين للحرية. تم تمثيل مناطق الاتصال بين التربة والأساس باستخدام عناصر التداخل. تم عمل مقارنة بين النتائج النظرية باستخدام طريقة العناصر المحددة والنتائج العملية لباحثين أخريين ولقد وجد توافق جيد بين تلك النتائج وكان أعلى نسبة فرق في الحمل الأقصى بينهما هو 14%. تم عمل دراسة مقارنة لبعض العوامل ودراسة تأثيرها على قابلية تحمل التربة ومنها الزاوية الراسية للأساس المخروطي ومنسوب الدفن للأساس ووجود عوارض حلقية في نهاية الأساس.

## INTRODUCTION

S hells are structures that derive their strength from form rather than mass. This form enables shells to put a minimum of material to maximum structural advantages.

While a plain element like a roof slab undergoes bending when subjected to

vertical loads including self-weight, a shell which is non-planar or a spatial system sustains the applied loads primarily by direct in-plane or membrane forces (compression or tension). Bending forces, even when present, it normally assumes only a place of secondary importance. Among the shells, which have come into wider use in foundations is the conical shell. The frustum of a cone is probably the simplest form in which a shell can be put to use in foundations. While smaller shells of this type can be used as footings for columns Figure (1). Because of its circular plan, the axisymmetric conical shell is, however, limited to individual units [1].

## PREVIOUS STUDIES

The early attempt at the finite element analysis of a shell foundation available in literature seems to be by Jain et al. (1977) [2]. In that study, the behaviour of a conical shell foundation under vertical loads has been studied. A linear solid axisymmetric finite element analysis was used to determine the contact soil pressure and stress distributions in shells. Based on contact pressure distribution a membrane stress analysis of conical shell foundation was developed for comparison. The effect of variation of several parameters such as soil modulus of elasticity, half vertex angle of the conical shell and the provision of edge beam at the end have been studied. The general conclusion that emerged from their investigations on conical foundation was that, the bending effects are small and that the load carrying capacity of the shell is mainly due to the membrane behavior.

Kurian (1993) [3] investigated the performance of shell foundations on soft soils. Two types of shell foundations have been used, which were the hypar and the conical shell foundations. The soil was represented by Winkler springs, in which the subgrade reaction $(k_n)$ of the soil is varied to simulate a wide range of soil conditions. The behaviour of these shell foundations has been studied in terms of the variation of vertical deformation (w), membrane stress (N) and bending stress resultants (M) in the shell and the edge beams.  It was indicated, from the results, that with the increase of $(k_n)$-value the membrane stress resultants decrease, whereas the bending stress
resultants increase. However the absolute vertical displacement is found to be more sensitive with $(k_n)$-value and decreases fast with the increase in $(k_n)$-value.

Kurian (1995) [4] presented a parametric study on the behavior of conical shell foundations. The finite element method was also used with simulating the soil as a Winkler medium. The study was conducted to investigate the influence of the rise and thickness of the shell and the existence of ring beams at top and bottom. The general conclusions that emerged from the study were that it is not advantageous to adopt unduly high values of the rise of the shell. Half cone angle (or vertex angle) in the range of 60° to 45° represents a desirable compromise between reduction in stresses and facility of

construction. For the same material input, a shell with its thickness tapering down is more advantageous than a shell of uniform thickness.

Al-Juboory (1998) [5] studied a conical footing under a vertical load. A finite element method was used. The main aim of that was to investigate the effect of the variation of the semi-vertical angel (α) of the cone on the bearing capacity of the soil. During that study, the concrete footings were treated as rigid rough bodies on cohesive soils. From the results that have been reviewed, it can be concluded that the bearing capacity value increases with increasing the vertex angle. The bearing capacity of a conical shell foundation is less than that of a circular footing of the same base diameter. A conical footing has in general the same behavior of circular footing regarding the stress distribution and the development of failure zone in the soil under the footing, excepting a more concentration near the edges of the conical footing.

Al-Azzawi (2000) [6] studied the nonlinearity in material and geometry by using the finite element method for the analysis of reinforced concrete shell foundations under static loading. The response of reinforced concrete shell foundation was traced through its elastic, inelastic and ultimate load ranges. He also used a layered approach in his analysis by dividing the concrete into eight layers. Also, a number of steel reinforcement layers were smeared into the concrete layers at appropriate position. The shell was modeled by using nine-node curved shell element. The soil-structure interaction between the shell elements and the supporting medium was modeled using two approaches. In the first approach, the three-dimensional computational model for soil, interface and infinite domain elements were adopted. The eight and twenty-node isoparametric brick elements were used to model the soil. Also, the eight and twelve-node infinite domain elements were used to model the far field behavior of the soil. The elastoplastic constitutive relations with a double yield surface were employed for modeling the soil. In the second approach, the soil medium was represented by a Winkler model with normal compressional and tangential frictional resistances. A parametric study was made by Al-Azzawi, (2000) [6] to examine the influence of some selected parameters of the reinforced concrete shell foundation system on their ultimate strength with those parameters. Comparison between the finite element and available experimental results was made and found it to be satisfactory.

Hassan (2002) [7] investigated the behavior of hypar and conical shells on Winkler foundations. The two components of the interaction system; the soil and the

foundation, were modeled using the finite element method. Four-node elements with six and five degrees of freedom per node were used in the analysis. Parametric studies were made to examine the effect of some selected parameters on the footings behavior. Comparisons between the results obtained and those from other investigations found it to be satisfactory with largest percentage difference of 8 percent in the value of the vertical displacement.

Maharaj (2003) [8] analyzed an axisymmetric conical shell foundation on clay by nonlinear finite element method. The shell foundation and soil were divided into four-node isoparametric finite elements. Load-settlement curves have been produced based on the nonlinear finite element analysis. The effect of soil modulus and taper angle on the

load settlement behavior of conical shell foundation has been studied. A comparison between the load carrying capacity of the conical shell foundation and that of the circular foundation of the same base area was made. The load carrying capacity of the conical shell foundation was found to increase with increase in soil modulus. The effect of increase in taper angle was to increase the load carrying capacity of the conical shell foundation. This effect was seen significant in soil with low modulus than in soil with high modulus due to the soil-structure-interaction. The conical shell foundation was found to have greater bearing capacity than that of the flat circular foundation of the same base area. Also, it was found to undergo lesser settlement than that of the flat circular foundation. Maharaj, (2003) [8] proved that a significant improvement in bearing capacity and settlement would be by providing conical shell foundation instead of flat circular foundation.

Maharaj (2004) [9] analyzed an axisymmetric shell raft foundation by nonlinear finite element method. The shell raft and soil has been discretize into four-node isoparametric finite elements. The soil has been modeled as Drucker-Prager elastoplastic medium. The effect of soil modulus, pile and pile stiffness on the load- settlement behaviour of shell raft has been studied. The load settlement curves of shell raft, shell raft with pile and the regular raft have been compared. The effect of increase in soil modulus has been found to improve the load carrying capacity of the shell raft foundation. The addition of even a single pile below a shell raft increases considerably its load carrying capacity. The effect of increase in pile modulus has been found to increase the load carrying capacity and reduce the settlement of the shell raft. The improvement in load carrying capacity due to increase in stiffness is only up to a limiting value of pile stiffness. Maharaj, (2004) [9] proved that a significant improvement occurs in the load carrying capacity by providing a raft shell foundation instead of the regular raft foundation.

Fernando et. al. (2011) [10] studied the bearing capacities of conical and pyramidal shell foundations on dry sand. The obtained experimental results were compared with those of circular and square flat foundations, respectively. Four foundation models on dry sand were tested in which the influence of the shell configuration on the bearing capacity and settlement were investigated. It was concluded that the ultimate capacities of shell foundations are higher than that of their flat counterparts with the same plan dimensions. And that the failure mechanism under the shell foundation is similar to its conventional flat counterpart.

## FINITE ELEMENT ANALYSIS

The shell footings and the soils were modeled and analyzed using the finite element software PLAXIS. The program uses the incremental tangent stiffness approach in the analysis, in which the load is divided into a number of small increments, which are applied simultaneously and carrying out iteration for each load step. During each load increment, the stiffness properties appropriate for the current stress level are employed in the numerical analysis. Experimental results from earlier work of Fernando et al (2011) [10] were used to validate the finite element modeling of the present study. Two types of footing models; flat circular footing and conical shell footing are selected for the analysis

and compared with the experimental results obtained earlier by Fernando et al. (2011) [10]. The cross sections of the modeled footings are shown in Figure (2).

The material properties given in Table 1 are used as input for modeling the footings in finite elements. The soil is modeled using the Mohr- Coulomb model with properties given in Table (2).

The finite element mesh geometry for the axisymmetric condition is symmetrical about the centerline, therefore only one half of the cross section passing through the axis of symmetry of the footing is considered. Standard fixities are used in the present study in which the nodes along the bottom of the section are considered as pinned supports, i.e., no movement is allowed in both vertical and horizontal directions and roller support at both sides of mesh. The soil and the footing were modeled using 15-noded triangular elements with quadratic variations for the displacement along the sides of the element. Also interface elements are used between the footing and soil with a coefficient of friction (R=0.1). Figure (3) shows the typical generated mesh used in the present study.

## ULTIMATE LOADS

The ultimate load is not always well defined. Load-settlement curve (a) in Figure (4) shows a well defined ultimate load while curve (b) does not. To obtain the ultimate load from curve (b), a simple method is to find the intersection of the tangents of the two parts of the curve. The value at the ordinate of the intersection C in Figure (4) is the ultimate bearing capacity.

Tangent method is used in this study for determination of ultimate bearing capacity and working load from the load-settlement relations of shell foundations.

## NUMERICAL VERFICATION

In this paper, the experimental results obtained by Fernando ET. al. (2011) [10]are compared with the finite element results obtained from PLAXIS program.

Figure (5) shows the load–settlement curves of the finite element and the laboratory experiment models. In general there behaviors are in good agreement. However, the

Results of the finite elements are slightly higher than those of the laboratory experiments. The difference is about 10 and 14% for the circular and conical footings, respectively. This is due to the simple finite element analysis carried out in this study in which two dimensions are used. Knowing that, the experimental study was carried out for the three dimensional model. However both the experimental and finite element models clearly show that load carrying capacity of the conical shell footing, with almost similar cross sectional area, is higher than the flat circular footing. Shell footing ensures better covering of the soil inside the space of the footing by preventing the soil from flowing outward. This can be very significant, particularly when the soil is poor.

## PARAMETRIC STUDY

In order to study the influence of variation of selected parameters on the behavior of the conical shell foundations, three parameters are considered which are: semi-vertical angle ($\alpha$), embedment ratio (D/B) and the effect of edge beam the influence of variation

of the semi-vertical angle (or half vertex angle) on the conical shell foundation behavior is now considered. Three values for $\alpha$ were studied, 90° (flat), 60° and 30°. Figure (6) shows the load settlement curves for different semi vertical angles. It is seen that as the semi vertical angle increased the ultimate load decreased due to decreasing the membrane stresses in the shell and contact area between shell and soil as shown in Figure (7).

The presence of an edge beam at the toe of the shell would reduce the soil pressure and increase the bearing capacity. To examine this, a finite element analysis is done to study the effect of adding edge beam. Also, the effect of embedment ratio (D/B) on the behavior is considered. The footing dimensions used in the parametric study of edge beam and embedment ratio are given in Figure (8).

The load-settlement behavior of shell footing model with edge beams is given in Figure (9). The initial portion of the two curves overlaps each other up to load of about 1 kN. After this load, the load carried by shell footings with edge beam is significantly higher than the footing without the edge beam. This shows that there is a significant improvement in settlement-load carrying capacity of the footing when there is an edge beam.

Embedment ratio (D/B) is taken 0.5 for fully embedded footing and zero for footing with no embedment. From Figure (10), it is obvious that as the embedment ratio increased, the ultimate load carried by shell footing increased. This shows the benefit of fully embedment of footing on the load carrying capacity of the shell footing.

## CONCLUSIONS

A non-linear finite element analysis is carried out to study the behavior of shell footings. It was found that the shell footing had a better load carrying capacity compared with the flat footing for a similar cross sectional area. The finite elements showed a reasonably good agreement with the laboratory experimental results; with a discrepancy of within 10 to 14%. The effect of semi-vertical angle has been studied Using finite elements and found that as the semi-vertical increased the load carrying capacity decreased. The effect of adding edge beams at the bottom of the shell footings has been studied also and found to be beneficial in increasing the load carrying capacity of the footing. Fully embedded shell footing (D/B=0.5) is shown to have a better load carrying capacity compared with the footing with no embedment.

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**Table (1) Timber Footing Properties (Fernando et. al. in ( 2011))[10].**

| Properties | Value |
|---|---|
| Unit weight (kN/m$^3$) | 12 |
| Young modulus (MN/m$^2$) | 3200 |
| Poisson's ratio $n$ | 0.3 |

**Table (2) Sand Properties (Mohr-Coulomb model)**
**(Fernando et. al. in ( 2011))[10].**

| Properties | Value |
|---|---|
| Unsaturated unit weight (kN/m$^3$) | 16.3 |
| Saturated unit weight (kN/m$^3$) | 17.3 |
| Young modulus (kN/m$^2$) | 30000 |
| Poisson's ratio $n$ | 0.3 |
| Cohesion coefficient C  (kN/m$^2$) | 0.0002 |
| Angle of internal friction  $f$ | 43$^o$ |

**Figure (1) conical shell foundation.**
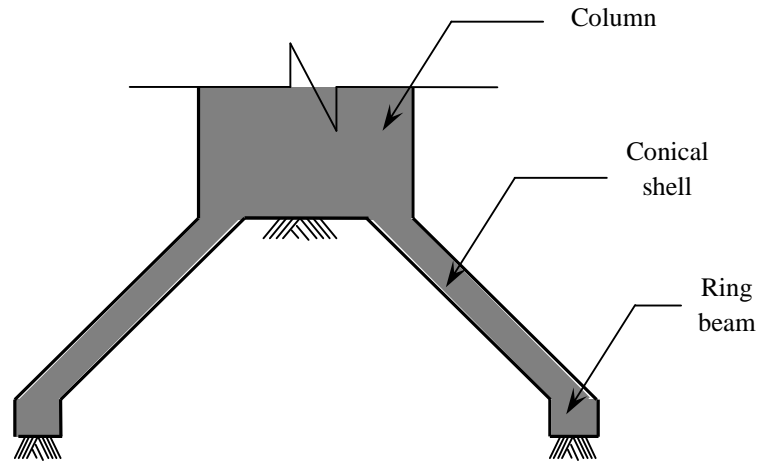


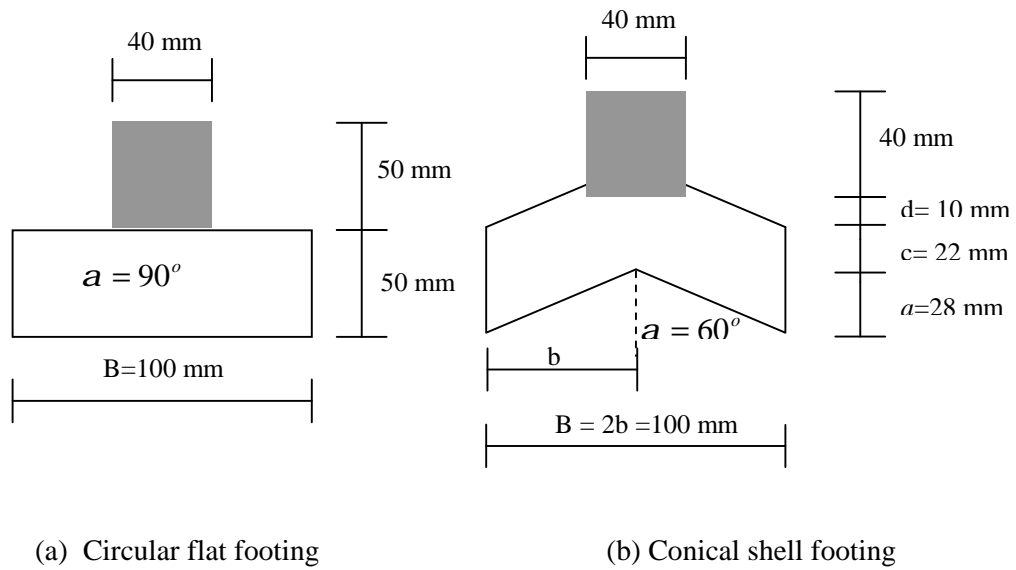(a)  Circular flat footing                    (b) Conical shell footing

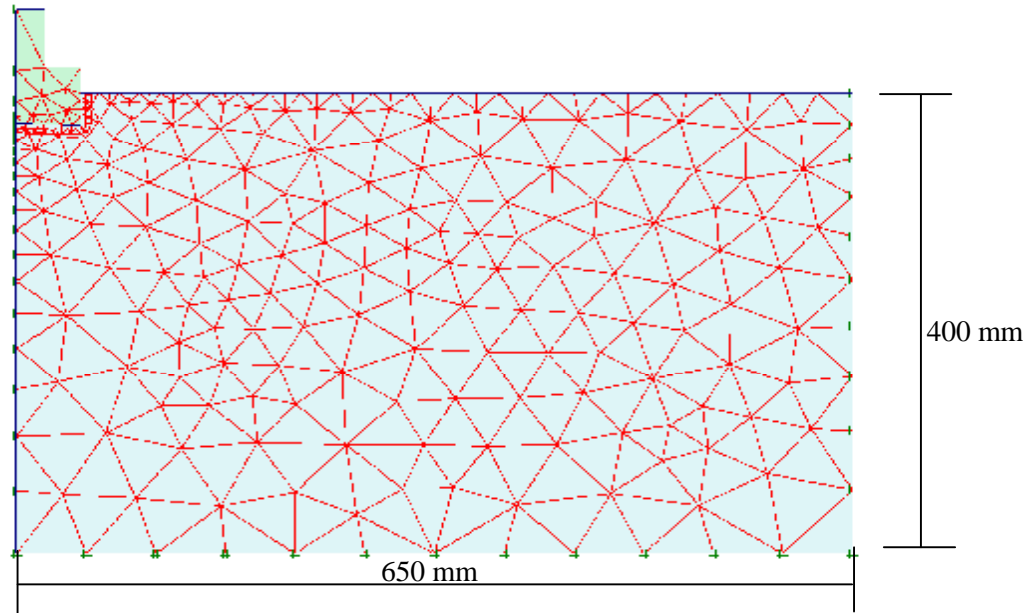**Figure (2) Sketch of the modeled Foundations.**

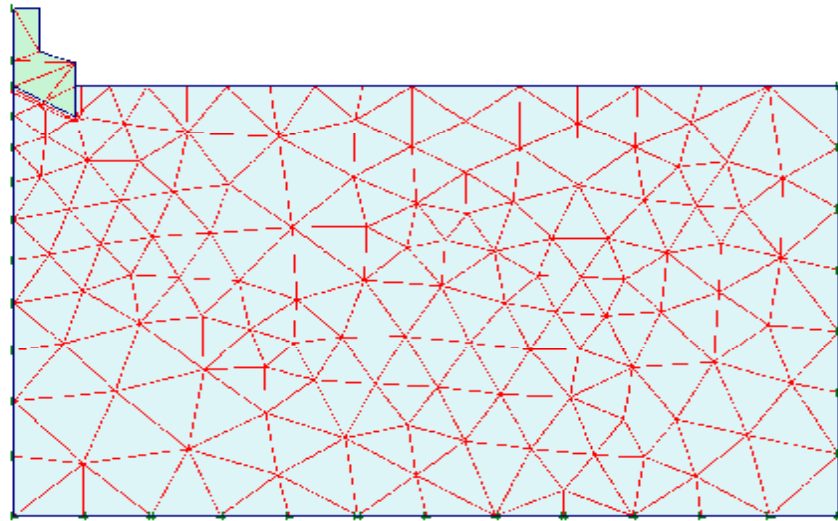**Figure (3 –a) Typical finite element mesh for circular footing.**



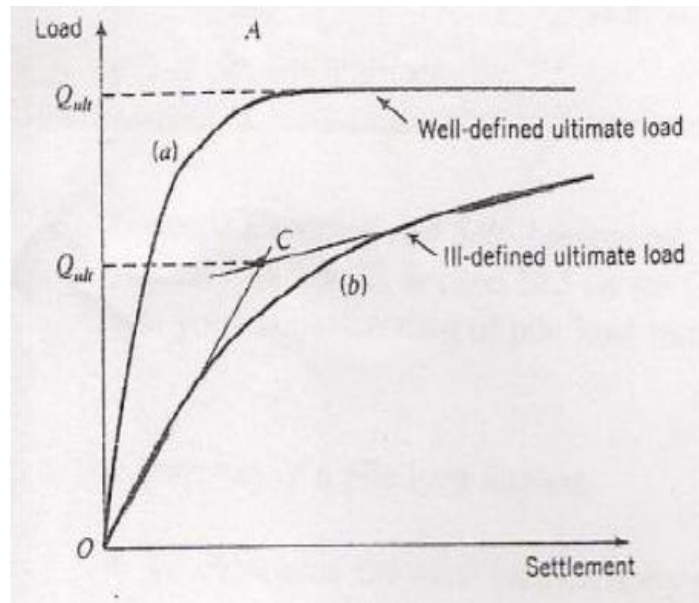**Figure (3 –b) Typical finite element mesh for conical footing.**

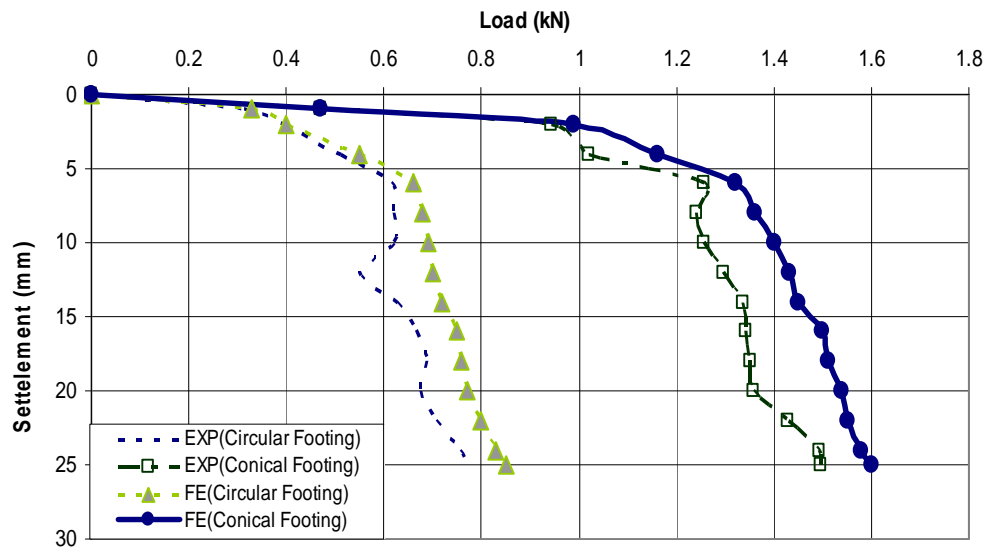**Figure (4) Determination the ultimate bearing capacity by the Tangents method.**



**Figure (5) Load-settlement curves of the finite elements and experimental modelof the circular and conical footings.**
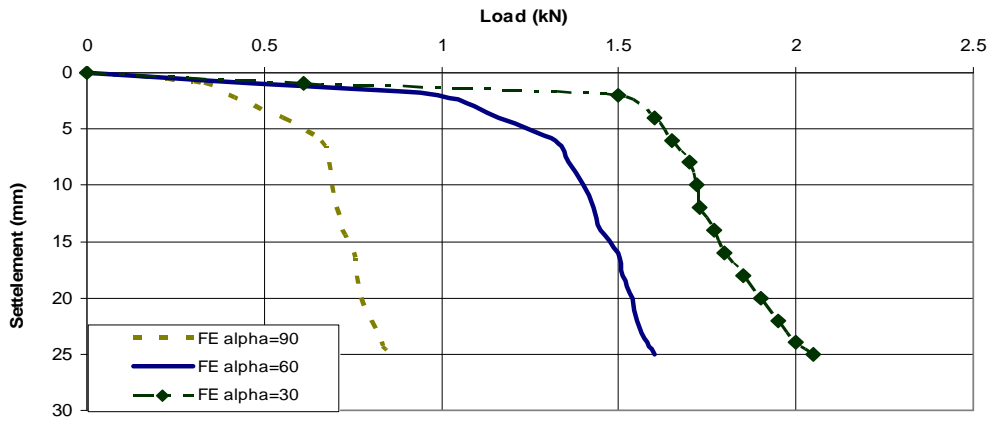
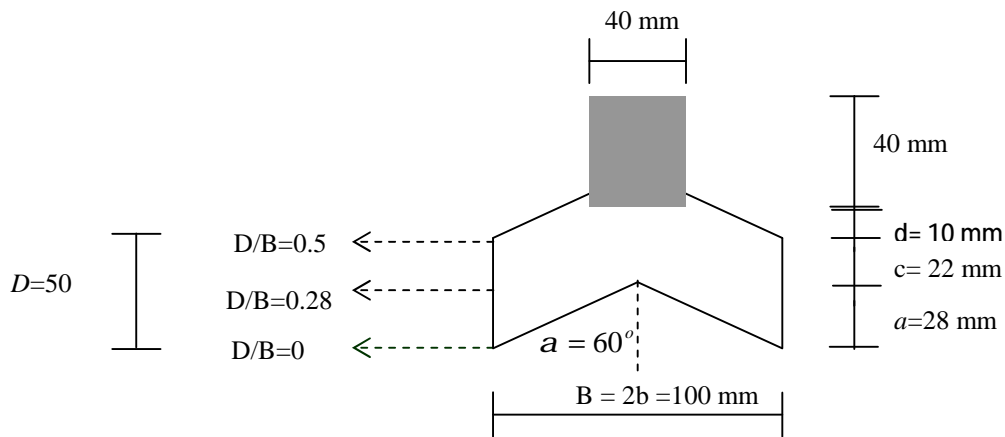**Figure (6) Load settlement curves for different semi vertical angles.**
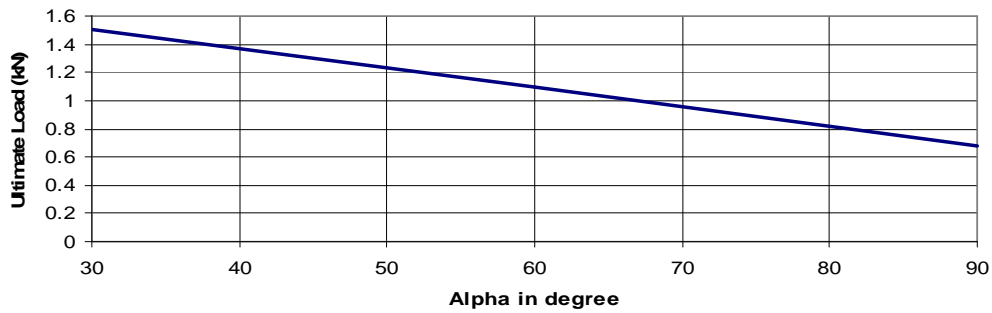


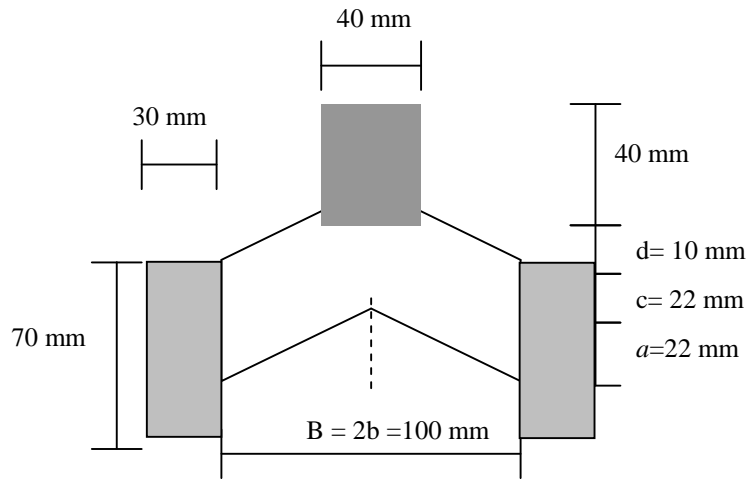**Figure (7) Effect of semi vertical angle on the ultimate load.**



(a) Embedment ratio D/B

40 mm

30 mm

40 mm

d= 10 mm

c= 22 mm

70 mm

*a*=22 mm

B = 2b =100 mm

(b) Edge beam

**Figure (8) Effect of embedment ratio and edge beam.**
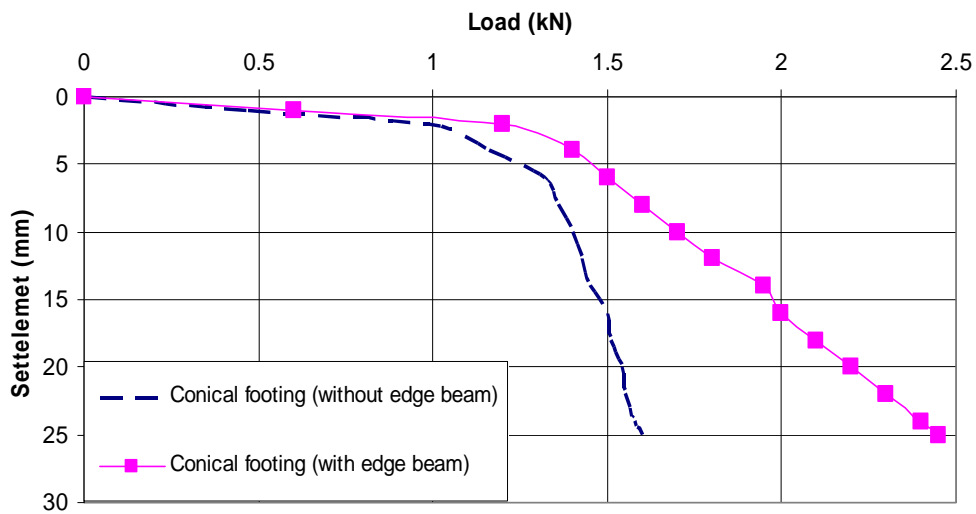
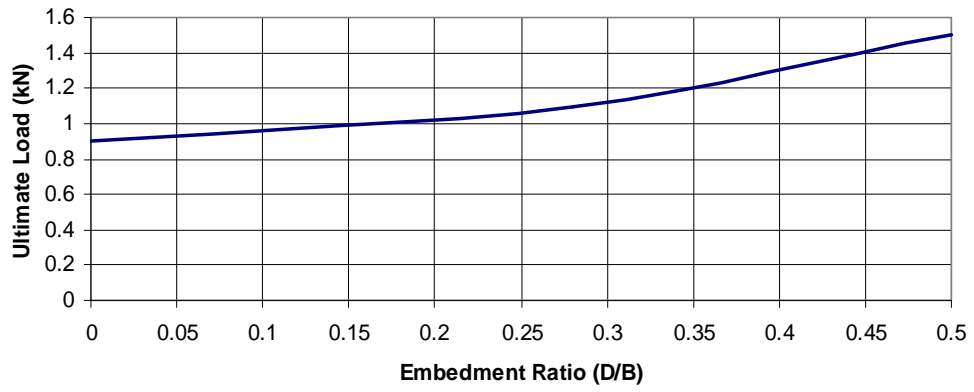**Figure (9) Load-settlement curves for conical footing with
and without edge beam.**

**Figure (10) Effect of embedment ratio on the load carrying capacity.**