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Approximate Solution of Fractional Integro-Differential Equations by Using Bernstein Polynomials

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ABSTRACT

In this paper, Bernstein piecewise polynomial is used to approximate the solution of the fractional integro-differential equations, in which the fractional derivative is described in the (Caputo) sense. Examples are considered to verify the effectiveness of the proposed derivation, and the approximate solutions guarantee the desired accuracy.

Keywords: - Fractional integro-differential equations, Bernstein polynomials.

الحل التقريبي للمعادلات التفاضلية – التكاملية ذات الرتب الكسورية باستخدام متعددات حدود برنشتاين

الخلاصة

في هذاالبحث، يتم أستخدام متعددات حدود برنشتاين، لتقريب حل المعادلات التفاضلية - التكاملية ذات الرتب الكسورية، والتي اعتمد تعريف (كابوتو) فيها بالنسبة للمشتقة الكسورية،وتم تقديم امثلة للتحقق من فعالية الاشتقاق المقترح، والحلول التقريبية تضمن الدقة المطلوبة.

INTRODUCTION

The concept of fractional or non-integer order derivation and integration can be traced back to the genesis of integer order calculus itself [1]. Almost every mathematical theory applicable to the study of non-integer order calculus was developed through the end of 19^{th} century.

However it is in the past hundred years that the most intriguing leaps in engineering and scientific application have been found. The calculation technique has to change in order to meet the requirement of physical realityin some cases.

The use of fractional differentiation for the mathematical modeling of real world physical problems has been wide spread in recent years, e.g. the modeling of earthquake, the fluid dynamic traffic model with fractional derivatives, measurement of viscoelastic material properties, etc, [2].

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There are several approaches to the generalization of the notion of differentiation of fractional orders, e.g. Riemann-Liouville, Grünwald–Letnikov, Caputo and generalized Functions approach [3].

Riemann-Liouville fractional derivative is the mostly used by mathematicians but this approach is not suitable for real world physical problems since it requires the definition of fractional order initial conditions, which have no physically meaningful explanation yet. Caputo introduced an alternative definition, which has the advantage of defining integer order initial conditions for fractional order differential equations [4].

Unlike the Riemann-Liouville approach which derives its definition from repeated integration, the Grünwald–Letnikov formulation approaches the problem from the derivative side; this approach is used in numerical algorithms.

A fractional Integro-Differential equation arises in modeling processes in applied sciences (Physics, Engineering, Finance, Biology …).Many problems in Viscoelasticity, Acoustics, Electromagnetics, Hydrology and other areas of application can be modeled by fractional Integro-Differential equations.

In recent years, fractional Integro-differential equations have been investigated by many authors. Momani [5] obtained local and global existence and uniqueness solution of fractional Integro-Differential equation. Rawashdeh [6] used the Collocation Spline method to approximate the solution of fractional Integro-differential equations. Mittal [7] used Adomian decomposition method to solve the fractional Integro-differential equation. Arikoglu [2] solved the fractional Integro-differential equation by using the differential transform method.
Mohammed [8] used the Homotopy Analysis method to solve the Fractional Integro-Differential equations.

In this paper we shall approximate the solution of the fractional order Integro-differential equation of the type:

$$D_*^{\alpha} y(t) = f(t) + \int_a^b k(t,s) y(s)\, ds, \quad y(a) = y_a, 0 < \alpha \leq 1 \qquad (1)$$

where D_*^{α} is Caputo fractional derivative, α is a parameter describing the order of the fractional derivative by using the Bernstein Polynomials. Bernstein Polynomials have been used recently to solve some linear as well as non-linear differential equations, ordinary and partial, approximately by Bhatta and Bhatti [9], and some Integral equations, by Mandal and Bhattackarya [10]. These polynomials defined over an interval forms a complete basis there, and each of them are positive and there sum is unity. Description of the properties of Bernstein Polynomials can be found in the paper of Bhatta and Bhatti [9]. Using these polynomials; we have obtained an approximate numerical solution of (1) which is acceptable when compared with the exact solution. It is important to notice that, all the computations needed in this paper are coded in Mathcad 14 software and the results are presented in tabulated form.

FRACTIONAL CALCULUS

Many definitions of fractional calculus have been proposed [3]. Most frequently occurring are Caputo and Riemann-Liouville [4]. In this section, we shall discuss definitions and some basic properties of these two types of fractional derivatives.

Abel-Reimannn fractional integral and derivatives [7]

The Abel-Riemann $A-R$ fractional integral of any order$\alpha > 0$for a function$\psi(t)$ with $t \in \mathbb{R}^{+}$is defined as:

$$J^{\alpha}\psi(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\psi(\tau)d\tau, t > 0, \alpha > 0 \qquad (2)$$

The $A-R$ integrals possess the semigroup property:

$$J^{\alpha}J^{\beta} = J^{\alpha+\beta}, \text{for all } \alpha, \beta \geq 0 \qquad (3)$$

The $A-R$ fractional derivative (of order $\alpha > 0$) is defined as the left inverse of the corresponding $A-R$ fractional integral, i.e.

$$D^{\alpha}J^{\alpha} = I \qquad (4)$$

For positive integer m such that $m-1 < \alpha \leq m$, $(D^m J^{m-\alpha})J^{\alpha} = D^m(J^{m-\alpha}J^{\alpha}) = D^m J^m = I$, so that $D^{\alpha} = D^m J^{m-\alpha}$, i.e.

$$D^{\alpha}\psi(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_0^t \frac{\psi(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau & m-1 < \alpha < m \\ \frac{d^m}{dt^m}\psi(\tau) & \alpha = m \end{cases} \qquad (5)$$

Properties of the operators $J^{\alpha}$and D^{α}can be found in [3], for $t > 0, \alpha \geq 0$, and $\gamma > -1$, we mention the following:

$$J^{\alpha}t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+\alpha)}t^{\gamma+\alpha}, \qquad D^{\alpha}t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)}t^{\gamma-\alpha}$$

Caputo fractional derivatives [7]

In the late sixtieth decade an alternative definition of fractional derivative was introduced by Caputo. Caputo and Mirandi used this definition in their work on the theory of viscoelasticity. According to Caputo's definition $D_*^{\alpha} = J^{m-\alpha}D^m$ for $m-1 < \alpha \leq m$, i.e.

$$D_*^{\alpha}\psi(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{\psi^m(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau & m-1 < \alpha < m \\ \frac{d^m}{dt^m}\psi(\tau) & \alpha = m \end{cases} \qquad (6)$$

One of the basic properties of the Caputo fractional derivative is:

$$J^{\alpha}D_*^{\alpha}\psi(t) = \psi(t) - \sum_{k=0}^{m-1}\psi^{(k)}(0^+)\frac{t^k}{k!} \qquad (7)$$

Caputo's fractional differentiation is a linear operation, similar to integer order differentiation, $D_*^{\alpha}[\lambda\, f(t) + \mu\, g(t)] = \lambda D_*^{\alpha} f(t) + \mu D_*^{\alpha} g(t)$, where λ and μ are constants.

BERNSTEIN POLYNOMIALS

The general form of the Bernstein polynomials of n^{th} degree over the interval is defined by

$$B_{i,n}(t) = \binom{n}{i}\frac{(t-a)^i(b-t)^{n-i}}{(b-a)^n}, a \le t \le b, i = 0,1,2,\cdots,n$$

Note that for more properties one can see [12].

FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION APPROXIMATE SOLUTION FORMULATION

In this section, we shall derive the approximate solution of the fractional Integro-differential equation by using the Bernstein polynomials as follows:
Consider the Fractional Integro-Differential equation of the form,

$$D_*^{\alpha} y(x) = f(x) + \int_a^b k(x,t)y(t)dt\,, 0 < \alpha \le 1 \qquad (8)$$

where D_*^{α} is the Caputo fractional derivative of order $\alpha$and $f(x)$, $k(x,t)$ are given functions.
Our approach begins by taking the fractional integration to both sides of equation (8) we get:

$$y(x) = y(0) + I^{\alpha} f(x) + I^{\alpha}\left(\int_a^b k(x,t)y(t)dt\right) \qquad (9)$$

To determine the approximate solution of (8), we use the Bernstein polynomial basis on $[a,b]$ as

$$y(x) = \sum_{i=0}^{n} a_i B_{i,n}(x) \qquad (10)$$

where: a_i $(i = 0,1,\dots,n)$are unknown constants to be determined.
Substituting equation (10) into equation (9), we obtain:

$$\sum_{i=0}^{n} a_i B_{i,n}(x) = y(0) + I^{\alpha} f(x) + I^{\alpha}\left(\int_a^b k(x,t)\sum_{i=0}^{n} a_i B_{i,n}(t)\, dt\right)$$

Hence

$$\sum_{i=0}^{n} a_i B_{i,n}(x) - I^{\alpha}\left(\sum_{i=0}^{n} a_i \phi(x)\right) = y(0) + I^{\alpha} f(x)$$

where$\phi(x) = \int_a^b k(x,t)B_{i,n}(t)dt$

thus, we have

$$\sum_{i=0}^{n} a_i \left[B_{i,n}(x) - I^{\alpha}\phi(x)\right] = y(0) + I^{\alpha} f(x) \qquad (11)$$

Now, we put $x = x_j, j = 0,1,\cdots,n$ into equation (11), $x_j^{'s}$ are being chosen as suitable distinct points in (a,b), putting $x = x_j$ we obtain the linear system:

$$\sum_{i=0}^{n} a_i \alpha_{ij} = f_j \,, j = 0,1,\cdots,n \qquad (12)$$

where $\alpha_{ij} = B_{i,n}(x_j) - I^{\alpha}\phi(x_j)andf_j = y(0) + I^{\alpha} f(x_j)$.
The linear system (12) can be easily solved by standard methods for the unknown constants a'^{s}. These a_i $(i = 0,1,2,\cdots n)$ are then used in equation (10) to obtain the unknown function $y(x)$ approximately.

ILLUSTRATIVE EXAMPLES

In this section, we shall give some illustrative examples in order to clarify our approach,

Example(1)

Consider the fractional Integro-Differential Equation:

$$D_*^{\alpha} y(x) = 1 - \frac{1}{3}x + \int_0^1 xty(t)dt \,, y(0) = 0, 0 < \alpha \le 1 \qquad (13)$$

By taking the fractional integration for both sides of the above equation, we get:

$$y(x) = y(0) + I^{\alpha}\left(1 - \frac{1}{3}x\right) + I^{\alpha}\left(\int_0^1 xty(t)dt\right) \qquad (14)$$

To determine the approximate solution of eq.(13), we set $y(x) = \sum_{i=0}^{3} a_i B_{i,3}(x)$ and after substituting it into eq.(14), we get:

$$\sum_{i=0}^{3} a_i B_{i,3}(x) = y(0) + I^{\alpha}\left(1 - \frac{1}{3}x\right) + I^{\alpha}\left(\int_0^1 xt \sum_{i=0}^{3} a_i B_{i,3}(t)\, dt\right)$$

So,

$$\begin{aligned} &a_0(1-x)^3 + a_1 3x(1-x)^2 + a_2 3x^2(1-x) + a_3 x^3 \\ &- I^{\alpha}\left[a_0 x \int_0^1 t(1-t)^3 dt + a_1 x \int_0^1 3t^2(1-t)^2 dt \right. \\ &\left. + a_2 x \int_0^1 3t^3(1-t)dt + a_3 x \int_0^1 t^4 dt\right] = I^{\alpha}\left(1 - \frac{1}{3}x\right) \end{aligned}$$

Hence,

$$\begin{aligned} &a_0\left[(1-x)^3 - 0.05\frac{\Gamma(2)}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + a_1\left[3x(1-x)^2 - 0.1\frac{\Gamma(2)}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + \\ &a_2\left[3x^2(1-x) - 0.15\frac{\Gamma(2)}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + a_3\left[x^3 - 0.2\frac{\Gamma(2)}{\Gamma(\alpha+2)}x^{\alpha+1}\right] = \\ &\frac{x^{\alpha}}{\Gamma(\alpha+1)} - \frac{\Gamma(2)x^{\alpha+1}}{3\Gamma(\alpha+2)} \end{aligned} \qquad (15)$$

Case (1)

when$\alpha = 1$then equation (15), will be:

$$a_0(1-x)^3 + 3a_1x(1-x)^2 + 3a_2x^2(1-x) + a_3x^3 - 0.05a_0\int_0^x t\,dt - 0.1a_1\int_0^x t\,dt - 0.15a_2\int_0^x t\,dt - 0.2a_3\int_0^x t\,dt = \int_0^x dt - \frac{1}{3}\int_0^x t\,dt.$$

or

$$a_0\left[(1-x)^3 - \frac{0.05}{2}x^2\right] + a_1\left[3x(1-x)^2 - \frac{0.1}{2}x^2\right] + a_2\left[3x^2(1-x) - \frac{0.15}{2}x^2\right] + a_3\left[x^3 - \frac{0.2}{2}x^2\right] = x - \frac{x^2}{6} \qquad (16)$$

Also, substituting $x = 0.1, 0.2, 0.3, and\ 0.4$ into eq.(16), respectively, we get a linear system, that has the following solution,

$a_0 = 0.0001, a_1 = 0.3327, a_2 = 0.6689$, and $a_3 = 0.9921$

Thus, the approximate solution of eq.(13), when $\alpha = 1$ becomes:

$y(x) = 0.0024(1-x)^3 + 0.32 \times 3x(1-x)^2 + 0.712 \times 3x^2(1-x) + 0.841x^3$

Following table (1) represent a comparison between the approximate solution of example (1) when $\alpha = 1$ with the exact solution $y = x$.

x	Approximate Solution	Exact Solution
0.1	0.09996	0.1
0.2	0.1999	0.2
0.3	0.2999	0.3
0.4	0.3999	0.4
0.5	0.4996	0.5
0.6	0.5991	0.6
0.7	0.6982	0.7
0.8	0.7968	0.8
0.9	0.8948	0.9

Table (1) comparison between the approximate solution of example (1) when $\alpha = 1$ with the exact solution.

Case (2)

when$\alpha = 0.5$ into equation (15), we have:

$$a_0(1-x)^3 + 3a_1x(1-x)^2 + 3a_2x^2(1-x) + a_3x^3 - I^{0.5}(0.05a_0x) - I^{0.5}(0.1a_1x) - I^{0.5}(0.15a_2x) - I^{0.5}(0.2a_3x) = I^{0.5}(1) - I^{0.5}(\frac{1}{3}x)$$

which may be written as:

$$a_0\left[(1-x)^3 - 0.05\frac{\Gamma(2)}{\Gamma(2.5)}x^{1.5}\right] + a_1\left[3x(1-x)^2 - 0.1\frac{\Gamma(2)}{\Gamma(2.5)}x^{1.5}\right] + a_2\left[3x^2(1-x) - 0.15\frac{\Gamma(2)}{\Gamma(2.5)}x^{1.5}\right] + a_3\left[x^3 - 0.2\frac{\Gamma(2)}{\Gamma(2.5)}x^{1.5}\right] = \frac{1}{\Gamma(1.5)}x^{0.5} - \frac{1}{3}\frac{\Gamma(2)}{\Gamma(2.5)}x^{1.5} \qquad (17)$$

now, substituting $x = 0.1, 0.2, 0.3$, and 0.4 into eq.(17), respectively, we will a linear system, that has the following solution,

$a_0 = 0.158, a_1 = 0.937, a_2 = 0.691$, and $a_3 = 1.987$
Thus, the approximate solution of eq.(13), when $\alpha = 0.5$ becomes:
$y(x) = 0.158(1-x)^3 + 0.937 \times 3x(1-x)^2 + 0.691 \times 3x^2(1-x) + 1.987x^3$

Following table (2) represent the approximate solution of example (1) when $\alpha = 0.5$.

x	Approximate Solution	Residual
0.1	0.364	0.04
0.2	0.523	0.01
0.3	0.652	0.01
0.4	0.765	0
0.5	0.879	0.04
0.6	1.008	0.14
0.7	1.168	0.31
0.8	1.374	0.59
0.9	1.642	0.96

Table (2) the approximate solution of example (1) when $\alpha = 0.5$.

Case (3)

when $\alpha = 0.25$ into eq.(15), we have:

$$a_0(1-x)^3 + 3a_1x(1-x)^2 + 3a_2x^2(1-x) + a_3x^3 - I^{0.25}(0.05a_0x) - I^{0.25}(0.1a_1x) - I^{0.25}(0.15a_2x) - I^{0.25}(0.2a_3x) = I^{0.25}(1) - I^{0.25}(\tfrac{1}{3}x).$$

which may be written as:

$$a_0\left[(1-x)^3 - 0.05\frac{\Gamma(2)}{\Gamma(2.25)}x^{1.25}\right] + a_1\left[3x(1-x)^2 - 0.1\frac{\Gamma(2)}{\Gamma(2.25)}x^{1.25}\right] + a_2\left[3x^2(1-x) - 0.15\frac{\Gamma(2)}{\Gamma(2.25)}x^{1.25}\right] + a_3\left[x^3 - 0.2\frac{\Gamma(2)}{\Gamma(2.25)}x^{1.25}\right] = \frac{1}{\Gamma(1.25)}x^{0.25} - \frac{1}{3}\frac{\Gamma(2)}{\Gamma(2.25)}x^{1.25} \qquad (18)$$

now, substituting $x = 0.1, 0.2, 0.3$, and 0.4 into eq.(18), respectively, we will get a linear system, that has the following solution,

$a_0 = 0.441, a_1 = 1.218, a_2 = 0.751$, and $a_3 = 2.345$

Thus, the approximate solution of equation (13), when $\alpha = 0.25$ becomes:

$$y(x) = 0.441 \times (1-x)^3 + 1.218 \times 3x(1-x)^2 + 0.751 \times 3x^2(1-x) + 2.345 \times x^3$$

Following table (3) represent the approximate solution of example (1) when $\alpha = 0.25$,

x	Approximate Solution	Residual
0.1	0.6401	0.02
0.2	0.7844	0.01
0.3	0.8937	0
0.4	0.9878	0
0.5	1.0866	0.03
0.6	1.2100	0.1
0.7	1.3776	0.26
0.8	1.6095	0.5
0.9	1.9253	0.86

Table (3) the approximate solution of example (1) when $\alpha = 0.25$.

Following figure (1) represent the approximate solution of example (1) for different values $\alpha = 1, 0.5$, and 0.25.

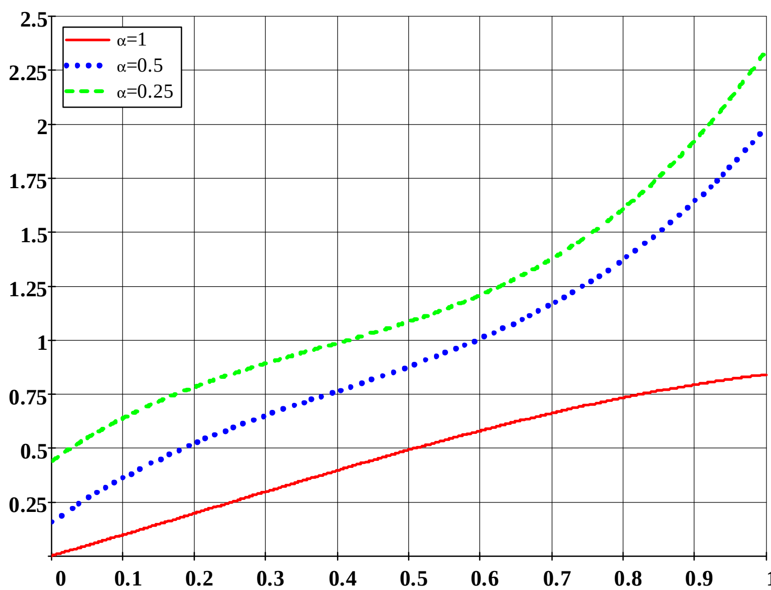

Figure (1) the approximate solution of example (1) for α=1, 0.5, and 0.25.

Example (2)

Consider the fractional Integro-Differential Equation:

$$D_*^{\alpha} y(x) = xe^x + e^x - x + \int_0^1 x\, y(t)dt\,, y(0) = 0, 0 < \alpha \leq 1 \quad (19)$$

By taking the fractional Integration to both sides of eq.(19), we have:

$$y(x) = I^{\alpha}(xe^{x} + e^{x} - x) + I^{\alpha}\left(\int_0^1 x\, y(t)dt\right) \tag{20}$$

To approximate the solution of eq.(19), we set $y(x) = \sum_{i=0}^{3} a_i B_{i,3}(x)$, and substituting into eq.(20), we get:

$$\sum_{i=0}^{3} a_i B_{i,3}(x) - I^{\alpha}\left(\int_0^1 x \sum_{i=0}^{3} a_i B_{i,3}(t)\, dt\right) = I^{\alpha}(xe^{x} + e^{x} - x),$$

or

$$a_0(1-x)^3 + 3a_1x(1-x)^2 + 3a_2x^2(1-x) + a_3x^3 - I^{\alpha}\left[a_0x\int_0^1 (1-t)^3dt + 3a_1x\int_0^1 t(1-t)^2dt + 3a_2x\int_0^1 t^2(1-t)dt + a_3x\int_0^1 t^3dt\right] = I^{\alpha}(xe^{x} + e^{x} - x),$$

hence

$$a_0(1-x)^3 + 3a_1x(1-x)^2 + 3a_2x^2(1-x) + a_3x^3 - I^{\alpha}(0.25a_0x + 0.25a_1x + 0.25a_2x + 0.25a_3x) = I^{\alpha}(xe^{x} + e^{x} - x),$$

and in order to avoid the difficulty of evaluating the fractional integration of e^{x}, we shall substitute it's maclurian series up five terms to get:

$$a_0\left[(1-x)^3 - \frac{0.25}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + a_1\left[3x(1-x)^2 - \frac{0.25}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + a_2\left[3x^2(1-x) - \frac{0.25}{\Gamma(\alpha+2)}x^{\alpha+1}\right] + a_3\left[x^3 - \frac{0.25}{\Gamma(\alpha+2)}x^{\alpha+1}\right] = I^{\alpha}\left(1 + x + \frac{3}{2}x^2 + \frac{4}{6}x^3 + \frac{1}{6}x^4\right) \tag{21}$$

Case (1)

When $\alpha = 1$ into eq.(21) and after substituting $x = 0.1, 0.2, 0.3,$ and 0.4 respectively, we get a linear system that has the following solution,

$a_0 = -2.377 \times 10^{-3}, a_1 = 0.348, a_2 = 0.933,$ and $a_3 = 2.819$

Thus, the approximate solution of eq.(19), when $\alpha = 1$ becomes:

$$y(x) = -2.377 \times 10^{-3} \times (1-x)^3 + 0.348 \times 3x(1-x)^2 + 0.933 \times 3x^2(1-x) + 2.819 \times x^3$$

Following table (4) represent a comparison between the approximate solution of example (2) with the exact solution $y = xe^{x}$:

x	Approximate Solution	Exact Solution
0.1	0.111	0.11052
0.2	0.245	0.24428
0.3	0.405	0.40496
0.4	0.599	0.59673
0.5	0.832	0.82436
0.6	1.112	1.09327
0.7	1.444	1.40963
0.8	1.835	1.78043
0.9	2.291	2.21364

Table (4) comparison between the approximate solution of example (2) when $\alpha = 1$ with the exact solution.

Case (2)

when $\alpha = 0.5$ into eq.(21) and after substituting $x = 0.1, 0.2, 0.3,$ and 0.4 respectively, we shall get a linear system of equations which has the solutions:
$a_0 = 0.145, a_1 = 1.115, a_2 = 1.966,$ and $a_3 = 5.362$
hence the approximate solution of equation (19), when $\alpha = 0.5$ becomes:

$$y(x) = 0.145(1-x)^3 + 1.115 \times 3x(1-x)^2 + 1.966 \times 3x^2(1-x) + 5.362x^3$$

Following table (5) represent the approximate solution of example (2) when $\alpha = 0.5$:

x	Approximate Solution	Residual
0.1	0.435	0.04
0.2	0.734	0.01
0.3	1.058	0
0.4	1.422	0.01
0.5	1.844	0.05
0.6	2.338	0.14
0.7	2.921	0.28
0.8	3.608	0.47
0.9	4.417	0.73

Table (5) approximate solution of example (2) when $\alpha = 0.5$.

Case (3)

when $\alpha = 0.25$ into eq.(21) and after substituting $x = 0.1, 0.2, 0.3,$ and 0.4 respectively, we shall get a linear system of equations which has the solutions:
$a_0 = 0.417, a_1 = 1.791, a_2 = 3.073,$ and $a_3 = 6.992$
hence the approximate solution of eq.(19), when $\alpha = 0.5$ becomes:

$$y(x) = 0.417(1-x)^3 + 1.791 \times 3x(1-x)^2 + 3.073 \times 3x^2(1-x) + 6.992\,x^3$$

Following table (6) represent the approximate solution of example (2) when $\alpha = 0.25$,

x	Approximate Solution	Residual
0.1	0.8292	0.02
0.2	1.2522	0.01
0.3	1.7024	0
0.4	2.1963	0
0.5	2.7501	0.01
0.6	3.3803	0.06
0.7	4.1032	0.14
0.8	4.9352	0.27
0.9	5.8927	0.44

Table (6) the approximate solution of example (2) when $\alpha = 0.25$.
The approximate solution of example (2) will be represented by figure (2) for values of $\alpha = 1, 0.5$, and 0.25.

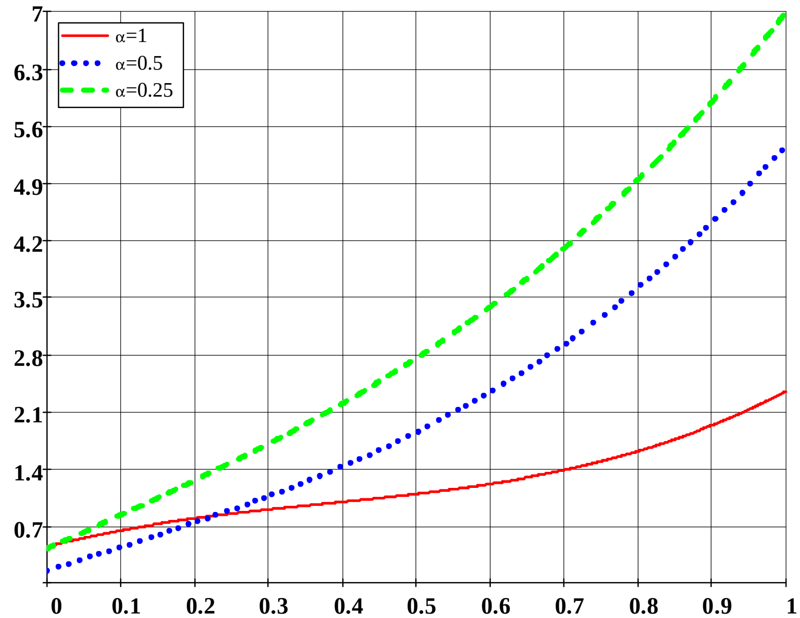

Figure (2) The Approximate Solution of Example (2) for α=1,0.5, and 0.25.

CONCLUSIONS

In this paper a very simple and straight forward method, based on approximation of the unknown function of the fractional integro-differential equation on the Bernstein polynomial basis is presented, its use, produces acceptable results if it is compared with the exact solution when $\alpha = 1$, also it is acceptable for $\alpha = 0.5$, and 0.25 since the Residual is less than one for all values of x.

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