Economics Letters 238 (2024) 111681

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Bertrand-Cournot profit reversal in a vertical structure with cross ownership☆

Arijit Mukherjee[a,b,c,f,*], Leonard F.S. Wang[d], Ji Sun[e]

[a] Nottingham University Business School, UK
[b] CESifo, Germany
[c] INFER, Germany
[d] Wenlan School of Business, Zhongnan University of Economics and Law, Wuhan, China
[e] School of Economics and Management, Hubei University of Technology, Wuhan, China
[f] GRU, City University of Hong Kong, Hong Kong

ARTICLE INFO

JEL classifications:
D43
L10
L13

Keywords:
Bertrand competition
Cournot competition
Cross ownership
Profit
Two-tier industry

ABSTRACT

We provide a new reason for Bertrand-Cournot profit reversal. In a two-tier industry with a profit-maximising input supplier and symmetric final good producers, we show that the profit reversal occurs under passive cross ownership among firms.

1. Introduction

The seminal paper by Singh and Vives (1984) suggests that the profits are higher under Cournot competition, while welfare is higher under Bertrand competition. In a two-tier industry with a profit-maximising input supplier and two symmetric final good producers with symmetric passive cross ownership among all firms,[1] we show the following results.

The profits generated in the final good producers' firms are higher under Bertrand competition for high cross ownership and high product differentiation. The profits generated in the input supplier's firm is higher under Bertrand competition for low cross ownership and low product differentiation. The total profits of all firms, consumer surplus, and welfare are higher under Bertrand competition.

Considering labour unions as upstream agents, López and Naylor (2004) show that the profits of the final good producers are higher under Bertrand competition if the upstream agents care more about wages than employment. Arya et al. (2008) show that the profits can be higher under Bertrand competition if a vertically integrated firm supplies inputs to a rival final good producer. Considering a profit-maximising input supplier, Mukherjee et al. (2012) show that the final good producers may earn higher profits under Bertrand competition if they have different production technologies. López (2007) show that a final good producer may earn higher profit under the price strategy when there is a profit-maximising input supplier provided the final goods are vertically differentiated.

Hence, the extant literature concludes that *the profits generated in the final good producers' firms are not higher under Bertrand competition in a two-tier industry if the upstream agents are profit-maximizing firms and the final good producers are symmetric*. We show that this conclusion may not hold in the presence of cross ownership.

Our reason for the profit reversal is different from the reasons for the

☆ We thank an anonymous referee for helpful comments and suggestions. The usual disclaimer applies.Data sharing is not applicable to this article as no new data were created or analyzed in this study.

* Corresponding author at: Nottingham University Business School, Jubilee Campus, Wollaton Rd, Nottingham NG8 1BB, UK.
E-mail address: arijit.mukherjee@nottingham.ac.uk (A. Mukherjee).

[1] Passive cross ownership, which refers to a situation where firms hold non-controlling shares in other firms, can be found in many industries, such as IT (Gilo et al., 2006), and telecommunications (Brito et al. 2014), and banking (Azar et al., 2018).

https://doi.org/10.1016/j.econlet.2024.111681
Received 1 February 2024; Received in revised form 27 March 2024; Accepted 28 March 2024
Available online 29 March 2024

profit reversal in one-tier industries with no strategic input price determination, such as asymmetric costs (Zanchettin, 2006), more than two firms (Häckner, 2000), technology licensing (Mukherjee, 2010), and non-commitment process innovation (Zhang et al., 2024).

2. The model and the results

Consider an industry with a profit-maximising input supplier, firm U, producing a critical input for two symmetric final good producers, firms 1 and 2. Normalise the cost of input production to zero. Assume that firms 1 and 2 require one unit of input to produce one unit of the final good, and they convert the inputs to the final goods without any cost. Like Chen et al. (2023), assume that each firm holds $\alpha \in \left[0, \frac{1}{3}\right)$ fraction of shares in other firms.

The utility function of a representative consumer is $U = q_1 + q_2 - \frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{2}$ for the products of firms 1 and 2, where q_1 and q_2 are the outputs of firms 1 and 2 respectively, and $\gamma \in [0, 1]$ shows the degree of product differentiation. The products are isolated (perfect substitutes) for $\gamma = 0$ ($\gamma = 1$).

Amir et al. (2017) show that if the quadratic utility function is not strictly concave, the direct demand functions need not be well defined. The above-mentioned utility function is strictly concave for $\gamma \in [0, 1)$. Hence, to consider competition between firms 1 and 2, and to avoid the problem mentioned by Amir et al. (2017), we consider $\gamma \in (0, 1)$.

The above-mentioned utility function gives the inverse and direct demand functions as $P_i = 1 - q_i - \gamma q_j$ and $q_i = \frac{1 - P_i - \gamma + \gamma P_j}{1 - \gamma^2}$ respectively, where P_i (P_j) is the price of the ith (jth) final good producer's product, $i,j = 1, 2,\ i \neq j$.

Consider the following game. At stage 1, firm U determines the input price, w. At stage 2, firms 1 and 2 determine outputs (under Cournot competition) and prices (under Bertrand competition) simultaneously, and the profits are realised. We solve the game through backward induction.

2.1. Cournot competition

Given the input price, the ith final good producer maximises $\underset{q_i}{Max}(1 - 2\alpha)(1 - q_i - \gamma q_j - w)q_i + \alpha(1 - q_j - \gamma q_i - w)q_j + \alpha w(q_i + q_j)$, $i,j = 1,2$, $i \neq j$. The equilibrium outputs are $q_1^{C*}(w) = q_2^{C*}(w) = q^{C*}(w) = \frac{1 - w - 2\alpha + 3w\alpha}{2 + \gamma - \alpha(4 + \gamma)}$, where the superscript "C" stands for Cournot competition. The equilibrium profit generated in each final good producer's firm is

$$\pi_1^{C*}(w) = \pi_2^{C*}(w) = \pi^{C*}(w) = \left(1 - q^{C*}(1 + \gamma) - w\right)q^{C*} = \frac{(1 - 2\alpha - w(1 - 3\alpha))(1 - w - \alpha(2 - \gamma) + w\alpha(1 - 2\gamma))}{(2 + \gamma - \alpha(4 + \gamma))^2}.$$

Firm U maximises $\underset{w}{Max}(1 - 2\alpha)2wq^{C*} + 2\alpha\pi^{C*}$. The equilibrium input price is

$$w^{C*} = \frac{2 + \gamma + \alpha(\alpha(8 - \gamma) - 2(4 + \gamma))}{2(1 - 3\alpha)(2 - 3\alpha + \gamma)} > 0 \tag{1}$$

We get $\frac{\partial w^{C*}}{\partial \alpha} = \frac{(1+\gamma)(2+\gamma+3\alpha^2\gamma-2\alpha(2+\gamma))}{2(1-3\alpha)^2(2-3\alpha+\gamma)^2} > 0$, which is similar to Chen et al. (2023). The input supplier's interest in the profits of the final good producers and the collusive behaviour in the final good market that makes the input demand curve flatter tend to reduce the input price. However, the final good producers' interests in the profits of the input supplier's firm tend to increase the input price. The latter effect dominates the former effects to increase the input price for a higher cross ownership.

The equilibrium outputs, prices of the final goods, profits generated in the final good producers' firms, profit generated in the input supplier's firm, consumer surplus and welfare are respectively

$$q_1^{C*} = q_2^{C*} = q^{C*} = \frac{1 - \alpha}{2(2 + \gamma) - 6\alpha} > 0 \tag{2}$$

$$P_1^{C*} = P_2^{C*} = P^{C*} = \frac{3 - \alpha(5 - \gamma) + \gamma}{4 - 6\alpha + 2\gamma} > 0 \tag{3}$$

$$\pi_1^{C*} = \pi_2^{C*} = \pi^{C*} = \frac{(1 - \alpha)(1 - \alpha(6 - \alpha(7 - 2\gamma)))}{4(1 - 3\alpha)(2 - 3\alpha + \gamma)^2} \tag{4}$$

$$\pi_u^{C*} = 2w^{C*}q^{C*} = \frac{(1 - \alpha)(2 + \alpha^2(8 - \gamma) + \gamma - 2\alpha(4 + \gamma))}{2(1 - 3\alpha)(2 - 3\alpha + \gamma)^2} > 0 \tag{5}$$

$$CS^{C*} = \frac{(1 - \alpha)^2(1 + \gamma)}{4(2 - 3\alpha + \gamma)^2} > 0 \tag{6}$$

$$SW^{C*} = \frac{(1 - \alpha)(7 - \alpha(11 - \gamma) + 3\gamma)}{4(2 - 3\alpha + \gamma)^2} > 0 \tag{7}$$

We get $\pi^{C*} > 0$ for $\gamma \in (0, 1)$ provided $\alpha \in [0, 0.2]$. Since $\frac{\partial w^{C*}}{\partial \alpha} > 0$ and a higher α reduces a final good producer's share of profit in its own firm, the final good producers have the incentive to charge prices lower than the input price to increase the input demand and the profit of the input supplier's firm. Hence, the profits in the final good producers' firms are negative for $\alpha \in \left[0.2, \frac{1}{3}\right)$.

Cross ownership is profitable, i.e., $2\pi^{C*}(\alpha > 0) + \pi_u^{C*}(\alpha > 0) > 2\pi^{C*}(\alpha = 0) + \pi_u^{C*}(\alpha = 0)$, for $\gamma \in (0, 1)$ and $\alpha \in \left(0, \frac{1}{3}\right)$.

2.2. Bertrand competition

Given the input price, the ith final good producer maximises

$$\underset{P_i}{Max}\frac{(1 - 2\alpha)(P_i - w)(1 - P_i - \gamma + \gamma P_j)}{1 - \gamma^2} + \frac{\alpha(P_j - w)(1 - P_j - \gamma + \gamma P_i)}{1 - \gamma^2} + \alpha w\left(\frac{1 - P_i - \gamma + \gamma P_j}{1 - \gamma^2} + \frac{1 - P_i - \gamma + \gamma P_j}{1 - \gamma^2}\right),$$

$i, j = 1, 2,\ i \neq j$. The equilibrium prices are $P_1^{B*}(w) = P_2^{B*}(w) = P^{B*}(w) = \frac{w(1-3\alpha)+(1-2\alpha)(1-\gamma)}{2-\alpha(4-\gamma)-\gamma}$, where the superscript "B" stands for Bertrand competition. The equilibrium outputs are $q_1^{B*}(w) = q_2^{B*}(w) = q^{B*}(w) = \frac{1-w(1-3\alpha)-\alpha(2+\gamma)}{(2-\alpha(4-\gamma)-\gamma)(1+\gamma)}$. The equilibrium profit generated in each final good producer's firm is

$$\pi_1^{B*}(w) = \pi_2^{B*}(w) = \pi^{B*}(w) = \left(P^{C*} - w\right)q^{C*} = \frac{(1 - w(1 - \alpha) - 2\alpha)(1 - \gamma)(1 - w(1 - 3\alpha) - \alpha(2 + \gamma))}{(2 - \alpha(4 - \gamma) - \gamma)^2(1 + \gamma)}.$$

Firm U maximises $\underset{w}{Max}(1 - 2\alpha)2wq^{B*} + 2\alpha\pi^{B*}$. The equilibrium input price is

$$w^{B*} = \frac{2 - \gamma - \alpha(2 - \gamma)(4 + \gamma) + \alpha^2(8 - \gamma - \gamma^2)}{2(1 - 3\alpha)(2 - \alpha(3 - \gamma) - \gamma)} > 0 \tag{8}$$

Like Cournot competition, a higher α increases w under Bertrand competition.

The respective equilibrium values are

$$q_1^{B*} = q_2^{B*} = q^{B*} = \frac{1 - \alpha(1 + \gamma)}{2(2 - \alpha(3 - \gamma) - \gamma)(1 + \gamma)} > 0 \tag{9}$$

$$P_1^{B*} = P_2^{B*} = P^{B*} = \frac{3 - 2\gamma - \alpha(5 - 3\gamma)}{4 - 2\alpha(3 - \gamma) - 2\gamma} > 0 \tag{10}$$

$$\pi_1^{B*} = \pi_2^{B*} = \pi^{B*} = \frac{(1 - \alpha(6 - \alpha(7 - \gamma) - \gamma))(1 - \gamma)(1 - \alpha - \alpha\gamma)}{4(1 - 3\alpha)(2 - \alpha(3 - \gamma) - \gamma)^2(1 + \gamma)} \tag{11}$$

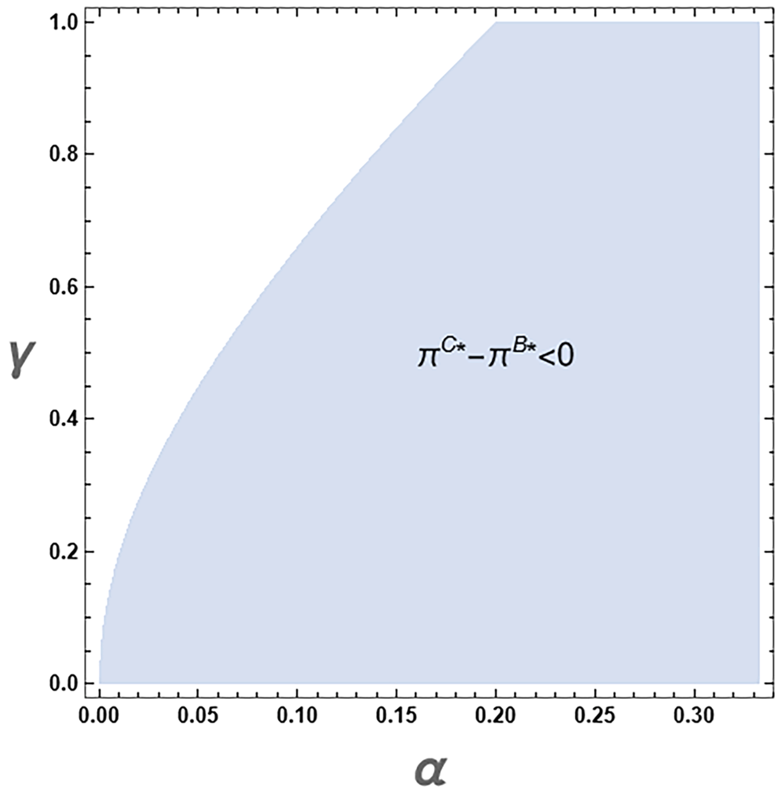

Fig. 1. $\pi^{C*} - \pi^{B*}$.

$$\pi_u^{B*} = 2w^{B*}q^{B*} = \frac{(1-\alpha-\alpha\gamma)(2-\gamma-\alpha(2-\gamma)(4+\gamma)+\alpha^2(8-\gamma-\gamma^2))}{2(1-3\alpha)(2-\alpha(3-\gamma)-\gamma)^2(1+\gamma)} > 0 \tag{12}$$

$$CS^{B*} = \frac{(1-\alpha-\alpha\gamma)^2}{4(2-\alpha(3-\gamma)-\gamma)^2(1+\gamma)} > 0 \tag{13}$$

$$SW^{B*} = \frac{(1-\alpha-\alpha\gamma)(7-4\gamma-\alpha(11-5\gamma))}{4(2-\alpha(3-\gamma)-\gamma)^2(1+\gamma)} > 0 \tag{14}$$

Cross ownership is profitable for $\gamma \in (0,1)$ and $\alpha \in \left(0,\frac{1}{3}\right)$.

We get $\pi^{B*} > 0$ for $\gamma \in (0,1)$ if $\alpha \in \left[0,\frac{1}{7}(3-\sqrt{2})\right]$. If $\alpha \in \left[\frac{1}{7}(3-\sqrt{2}),\frac{1}{3}\right)$, the reason for $\pi^{B*} < 0$ is similar to that of under Cournot competition.

Hence, we have (i) $\pi^{C*} > 0$, $\pi^{B*} > 0$ for $\alpha \in [0,0.2]$, (ii) $\pi^{C*} < 0$, $\pi^{B*} > 0$ for $\alpha \in \left[0.2,\frac{1}{7}(3-\sqrt{2})\right]$, and (iii) $\pi^{C*} < 0$, $\pi^{B*} < 0$ for $\alpha \in \left[\frac{1}{7}(3-\sqrt{2}),\frac{1}{3}\right)$.

2.3. Comparison

Proposition 1. *The equilibrium input prices are lower under Bertrand competition compared to Cournot competition for a positive cross ownership.*

Proof: We get $w^{C*} - w^{B*} = \frac{\alpha\gamma(1-\alpha)(4-4\alpha(2-\gamma)-\gamma(1+\gamma))}{2(1-3\alpha)(2-3\alpha+\gamma)(2-\alpha(3-\gamma)-\gamma)} > 0$ for $\alpha > 0$. ■

Cross ownership makes the input demand curve more elastic under Bertrand competition, which creates a lower input price under Bertrand competition compared to Cournot competition.

Now consider the profits generated in the final good producers' firms. Fig. 1 plots $\pi^{C*} - \pi^{B*}$ for $\alpha \in \left[0,\frac{1}{3}\right]$ and $\gamma \in [0,1]$, and shows $\pi^{C*} - \pi^{B*} < (>) 0$, i.e., the profits generated in the final good producers' firms are higher (lower) under Bertrand competition, in the shaded (white) region. Since we have $\pi^{C*} < 0$, $\pi^{B*} < 0$ for $\alpha \in \left[\frac{1}{7}(3-\sqrt{2}),\frac{1}{3}\right)$, higher profits in this situation will read as lower losses.

To understand the importance of endogenous input price determination and cross ownership for the above result, consider the cases of (i) a competitive input market, creating $w = 0$, and (ii) no cross ownership, implying $\alpha = 0$. If $w = 0$, we get $(\pi^{C*} - \pi^{B*})|_{w=0} = \frac{2(1-3\alpha)^2(1-\alpha)(1-2\alpha)\gamma^3}{(1+\gamma)(4(1-2\alpha)^2-(1-\alpha)^2\gamma^2)^2} > 0$. On the other hand, if $\alpha = 0$, we get $(\pi^{C*} - \pi^{B*})|_{\alpha=0} = \frac{\gamma^3}{2(1+\gamma)(4-\gamma^2)^2} > 0$. Hence, both endogenous input price determination and cross ownership are important for the final good firms' profit-reversal.

Intuitively, for a given input price, fierce competition under Bertrand competition tends to create lower profits in the final good producers' firms under Bertrand competition. However, this effect weakens with higher cross ownership and higher product differentiation. On the other hand, given Proposition 1 and because higher cross ownership increases the difference in input prices between Cournot and Bertrand competition,[2] cross ownership tends to create higher profits in the final good producers' firms under Bertrand competition. Hence, high cross ownership and high product differentiation create higher profits in the final good producers' firms under Bertrand competition.

The following result is from the above discussion.

Proposition 2. *The equilibrium profits generated in the final good producers' firms are higher under Bertrand competition compared to Cournot competition if cross ownership and product differentiation are high.*

Now consider the profits of the input supplier. Plotting $\pi_u^{C*} - \pi_u^{B*}$ in Fig. 2 for $\alpha \in \left[0,\frac{1}{3}\right]$ and $\gamma \in [0,1]$, we get $\pi_u^{C*} - \pi_u^{B*} < 0$ in the shaded region.

If there is endogenous input price determination but no cross ownership, we get $(\pi_u^{C*} - \pi_u^{B*})|_{\alpha=0} = -\frac{\gamma^2}{2(4+4\gamma-\gamma^2-\gamma^3)} < 0$. And, trivially,

[2] We get $\frac{\partial(w^{C*}-w^{B*})}{\partial\alpha} > 0$.

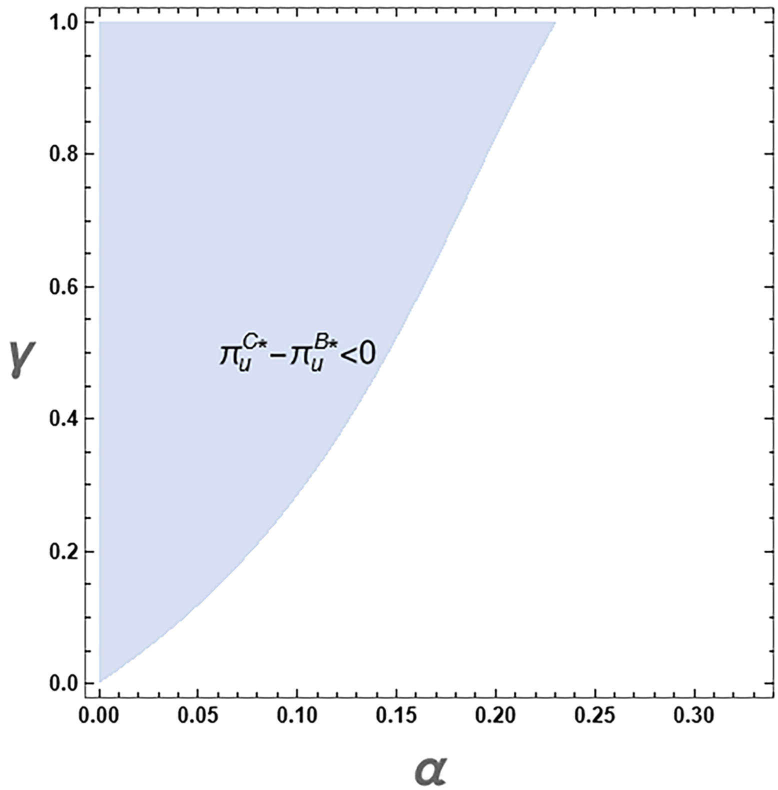

Fig. 2. $\pi_u^{C*} - \pi_u^{B*}$.

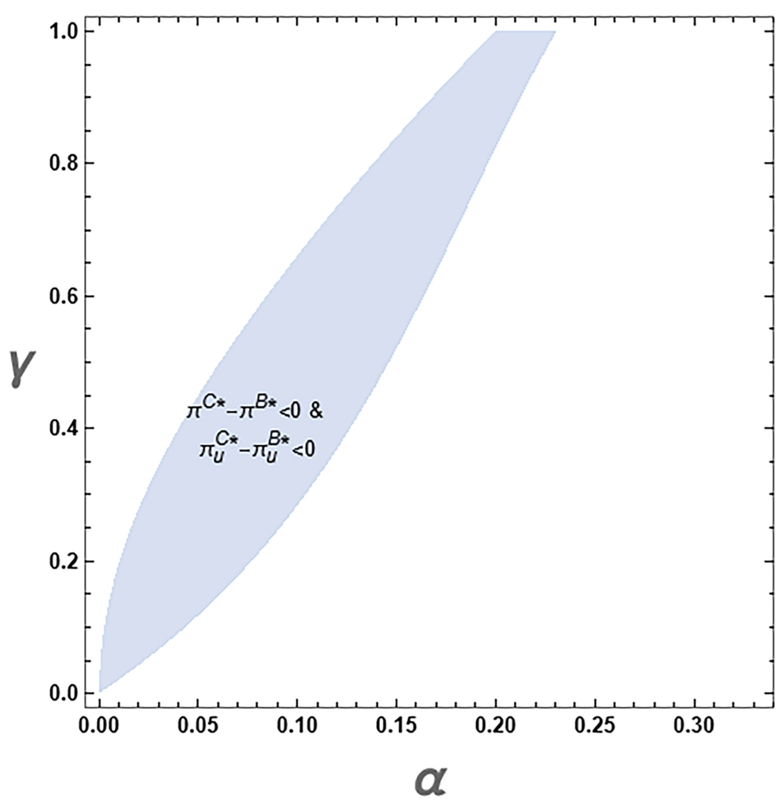

Fig. 3. $\pi^{C*} - \pi^{B*} < 0$ & $\pi_u^{C*} - \pi_u^{B*} < 0$ in the shaded region.

the profit of the input supplier is zero under both Cournot and Bertrand competition if there is a competitive input market, creating $w = 0$. Hence, cross ownership is the important factor for a higher profit in the input supplier's firm under Cournot competition.

Intuitively, since the input price is lower under Bertrand competition, and the input price difference increases with higher α, the profit generated in the input supplier's firm is higher under Bertrand competition if cross ownership and product differentiation are low.

The following result is from the above discussion.

Proposition 3. *The equilibrium profit generated in the input supplier's firm is higher under Bertrand competition compared to Cournot competition if cross ownership and product differentiation are low.*

Fig. 3 considers the overlapping zone in Figs. 1,2 and suggests that the final good producers' firms and the input supplier prefer Bertrand competition in the shaded region.

The total profits of all firms are $2\pi^{C*} + \pi_u^{C*} = \pi_{IND}^{C*} = \frac{(1-\alpha)(3-\alpha(5-\gamma)+\gamma)}{2(2-3\alpha+\gamma)^2}$ and $2\pi^{B*} + \pi_u^{B*} = \pi_{IND}^{B*} = \frac{(1-\alpha-\alpha\gamma)(3-2\gamma-\alpha(5-3\gamma))}{2(2-\alpha(3-\gamma)-\gamma)^2(1+\gamma)}$. It can be found that $\pi_{IND}^{C*} < \pi_{IND}^{B*}$ for $\alpha \in \left[0, \frac{1}{3}\right)$ and $\gamma \in (0,1)$.

If we look at the total profits of all firms, we get $(\pi_{IND}^{C*} - \pi_{IND}^{B*})|_{w=0} = (\pi^{C*} - \pi^{B*})|_{w=0} > 0$ and $(\pi_{IND}^{C*} - \pi_{IND}^{B*})|_{\alpha=0} = -\frac{\gamma^2(4-\gamma(2+\gamma))}{2(1+\gamma)(4-\gamma^2)^2} < 0$. Hence, endogenous input price determination is the important factor for higher total profits under Bertrand competition.

Intuitively, the lower input price under Bertrand competition helps to create higher total profits of all firms under Bertrand competition compared to Cournot competition.

It can be found that $CS^{C*} < CS^{B*}$ for $\alpha \in \left[0, \frac{1}{3}\right)$ and $\gamma \in (0,1)$. Fierce competition and lower input price under Bertrand competition create higher consumer surplus under Bertrand competition compared to Cournot competition.

It is then immediate that the equilibrium welfare, which is the sum of the equilibrium total profits of all firms and the equilibrium consumer surplus, is higher under Bertrand competition compared to Cournot competition.

The following result summarises the above discussion.

Proposition 4. *The equilibrium total profits of all firms, the equilibrium consumer surplus and welfare are higher under Bertrand competition compared to Cournot competition for* $\alpha \in \left[0, \frac{1}{3}\right)$ *and* $\gamma \in (0,1)$.

3. Conclusion

The main contribution of this paper is to show that, in a two-tier industry with a profit-maximising input supplier and symmetric final good producers, the profits in the final good producers' firms can be higher under Bertrand competition compared to Cournot competition if cross ownership and differentiation between the final goods are high.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

Arya, A., Mittendorf, B., Sappington, D.E., 2008. Outsourcing, vertical integration, and price vs. quantity competition. Int. J. Ind. Org. 26, 1–16.

Amir, R., Erickson, P., Jin, J., 2017. On the microeconomic foundations of linear demand for differentiated products. J. Eco. The. 169, 641–665.

Azar, J., Schmalz, M.C., Tecu, I., 2018. Anticompetitive effects of common ownership. J. Fin. 73, 1513–1565.

Brito, D., Cabral, L., Vasconcelos, H., 2014. Divesting ownership in a rival. Int. J. Ind. Org. 34, 9–24.

Chen, L., Matsumura, T., Zeng, C., 2023. Welfare consequence of common ownership in a vertically related market. J. Ind. Eco. (Available online).

Gilo, D., Moshe, Y., Spiegel, Y., 2006. Partial cross ownership and tacit collusion. RAND J. Eco. 37, 81–99.

Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. J. Eco. The. 93, 233–239.

López, M.C., 2007. Price and quantity competition in a differentiated duopoly with upstream suppliers. J. Eco. Man. Str. 16, 469–505.

López, M.C., Naylor, R.A., 2004. The Cournot–Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining. Eur. Eco. Rev. 48, 681–696.

Mukherjee, A., 2010. Competition and welfare: the implications of licensing. Man. Sch. 78, 20–40.

Mukherjee, A., Broll, U., Mukherjee, S., 2012. Bertrand versus Cournot competition in a vertical structure: a note. Man. Sch. 80, 545–559.

Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. RAND J. Eco. 15, 546–554.

Zanchettin, P., 2006. Differentiated duopoly with asymmetric costs. J. Eco. Man. Str. 15, 999–1015.

Zhang, Q., Wang, L.F.S., Mukherjee, A., 2024. Bertrand-Cournot profit reversal under non-commitment process innovation. Man. Sch. (Available online).