

Aperture Efficiency of Non-Uniform Antenna Arrays with Controlled Sidelobe Level

AHMED ALIELDIN^{1,2}, YI HUANG¹, FELLOW, IEEE

¹The Department of Electrical Engineering and Electronics, The University of Liverpool, Liverpool, L69 3GJ, U.K.

²The Egyptian Technical Research and Development Centre, Cairo, 11618, Egypt

CORRESPONDING AUTHOR: Ahmed Alieldin (e-mail: ahmed.alieldin@alexu.edu.eg).

ABSTRACT The non-uniform distribution of the weight vector of the elements in an antenna array has attracted much attention in order to obtain such as the desired beam pattern, null locations, and performance objectives. This topic has been of interest for many decades. Several mathematical models and optimization algorithms have been developed to achieve a controllable sidelobe level. However, few researchers have considered the effect of the non-uniform distribution on the aperture efficiency. This paper presents a comprehensive study and proposes accurate mathematical modelling of the aperture efficiency of an antenna array which is divided into two components: power-loss efficiency (which dominates total aperture efficiency) and power-distribution efficiency to gain a better understanding. The paper also shows how the sidelobe level of an antenna array is linked to these efficiencies and how they can be improved. It is proved that the aperture efficiency exponentially decays when lowering the sidelobe level. However, it converges to a certain value when the sidelobes vanish. Moreover, a validation example through simulations is presented to verify the mathematical modelling of the aperture efficiency.

INDEX TERMS Antenna array, aperture efficiency, non-uniform distribution, sidelobe level, weight vector

I. INTRODUCTION

One of the main goals of deploying antenna arrays is to improve the directivity of the overall antenna at a certain direction(s). However, due to extending the aperture of the antenna to form an array, sidelobes are generated in other directions. These sidelobes are often undesirable. They may represent a waste of transmitted energy in unwanted directions, cause severe confusion in direction-finding systems or permit reception from unwelcome sources or deliberate jammers [1].

In a uniformly distributed antenna array, all elements have equal weights (amplitude and/or phase). This results in a symmetric and evenly distributed radiation pattern. However, in many cases, it is desirable to have a non-uniform distribution of weights to achieve specific objectives. Some reasons for using non-uniform weight distributions include:

- (1) Beamforming: to shape the main beam of the antenna array into a specific pattern. By assigning complex weights to certain elements, the beam can follow any desired shape, enhancing the antenna's sensitivity to signals coming from a specific angle(s).
- (2) Side lobe control: to reduce the side lobe levels in the antenna radiation pattern. By assigning tapered weights to certain elements, the radiation in undesired directions can be minimized, leading to improved antenna performance.

- (3) Nulling: to create nulls in the antenna's radiation pattern. Nulling is the intentional reduction or suppression of signal reception from certain directions. By assigning negative weights to specific elements, the array can attenuate or cancel signals arriving from those directions.
- (4) Adaptive arrays: to be employed in adaptive antenna arrays, where the weights are adjusted dynamically based on changing environmental conditions or interference sources. Adaptive algorithms optimize the weights to maximize the desired signal and minimize interference or noise.

To control the weight vector of the elements in an antenna array, several techniques have been utilized to reduce the sidelobes level (SLL) of an antenna array by applying either one of the well-known weight vectors such as Taylor or Dolph-Chebyshev [2] [3] or untraditional weight vectors using optimization algorithms such as genetic or particle swarm [4] [5]. Moreover, many other synthesis techniques have been used to control the SLL such as the k-means algorithm and convex programming which could achieve a SLL as low as -50 dB [6] - [9]. In all cases, the weights of the antenna elements are non-uniformly distributed. They are often tapered from large, at the central elements, to small, at the edge elements, to shrink the level of peak sidelobes which are mostly (but not necessarily) adjacent to the main lobe.

As a direct consequence of using this tapering distribution to reduce the SLL, a decay in the aperture efficiency takes place.

This decay is noticed as an undesirable reduction in the directivity of the antenna array [10], but how the aperture efficiency is linked to the SLL is not well studied although it is known there is a trade-off between them. This paper is to address this issue. In the next sections, the aperture efficiency is discussed along with its calculations and its link to the SLL.

II. UNIFORM ANTENNA ARRAY

The antenna elements of a uniform antenna array have an equal feeding as shown in FIGURE 1. The directivity (D) of the antenna array (assuming isotropic sources) is linked to its number of elements (M), the normalized-to-wavelength distance between any two adjacent elements (d), the observation angle measured from the array axis (θ) and progressive phase shift (β) by [11]:

$$D = \sum_{m=1}^M w_m e^{-j(m-1)(2\pi d \cos\theta + \beta)} \quad (1)$$

where w_m is the amplitude of the weight of the m^{th} antenna element measured in the electric field unit (V/m). For a uniform feeding, w_m is assumed to be w_0 and equals 1 V/m as shown in FIGURE 2 as an example of a 16-element antenna array. The peak SLL in this case is $(2/3\pi) \approx -13.5$ dB as shown in FIGURE 3 (taking $d = 0.7$ and $\beta = 0^\circ$) [2]. This value is the worst SLL obtained from a uniform antenna array. However, the aperture efficiency is 100% (0 dB) as all the elements contribute uniformly to maximize the antenna directivity. The maximum directivity (D_{max}) in this case is calculated as [12]:

$$D_{max} = \left(\sum_{m=1}^M w_0 \right)^2 / \sum_{m=1}^M (w_0)^2 = M^2 / M = M \quad (2)$$

For a 16-element antenna array, $D_{max} \approx 24.1$ dBi. Note that the total power is represented by the denominator of equation (2) which equals to $\sum_{m=1}^M (w_0)^2 = M$.

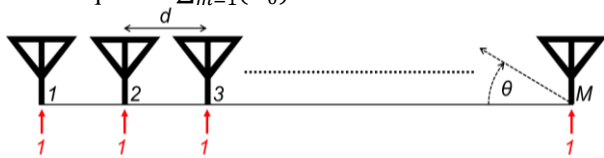


FIGURE 1. Uniform M-element antenna array.

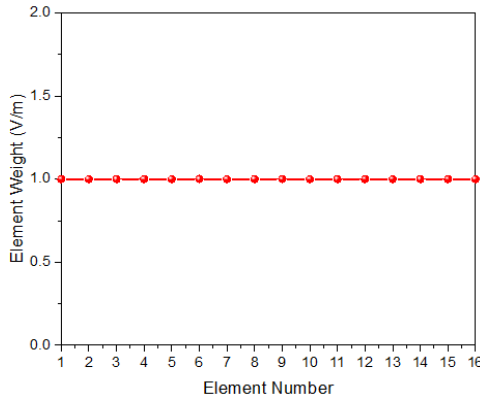


FIGURE 2. Weight vector of a uniform 16-element antenna array.

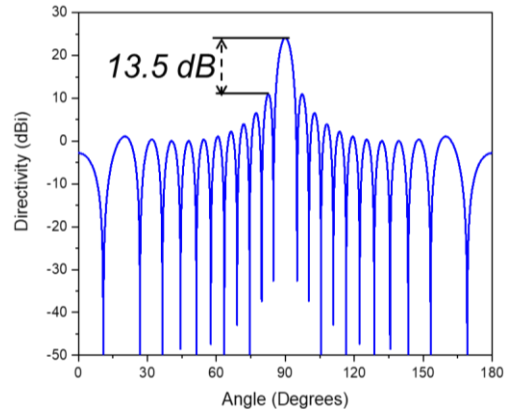


FIGURE 3. Directivity of a uniform 16-element antenna array.

III. NON-UNIFORM ANTENNA ARRAY

The term non-uniform comes from having unequal amplitudes at the antenna elements in order to reduce the SLL [13]. Therefore, a variable attenuator at each port of the antenna elements is utilized. These attenuators are meant to tune the feeding amplitude of each element to taper their weights so that they follow the desired weight vector. The power loss in those attenuators causes a shrink in the maximum directivity of the antenna array and hence the aperture efficiency is not 100% anymore. It is worth noting that the weights of the middle elements are kept at 1 V/m to minimize the attenuation in the overall feeding structure as shown in FIGURE 4. In that case, the maximum achievable aperture efficiency occurs when all the sidelobes have the same power level. In other words, if one (or more) of these sidelobes should have less power level than the others, more attenuation is required for one or more antenna elements and hence less aperture efficiency.

Taking the same example of the 16-element antenna array with isotropic sources, Pseudo Inverse Synthesis (PIS) is applied to generate a non-uniform weight vector for the elements as shown in FIGURE 5 to achieve an SLL of -40 dB with equal sidelobes as shown in FIGURE 6 [14]. It is noticed that the maximum directivity in this case (D_{max_PL}) is 19.3 dBi. A total aperture efficiency (η_{AP}) of -4.8 dB ($\approx 33\%$) is achieved. This drop in the aperture efficiency is due to two types of efficiencies: power-loss efficiency and power-distribution efficiency. These two types are discussed in detail in the following sections.

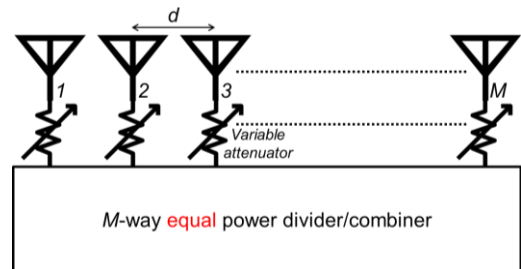


FIGURE 4. Non-uniform M-element antenna array using attenuators.

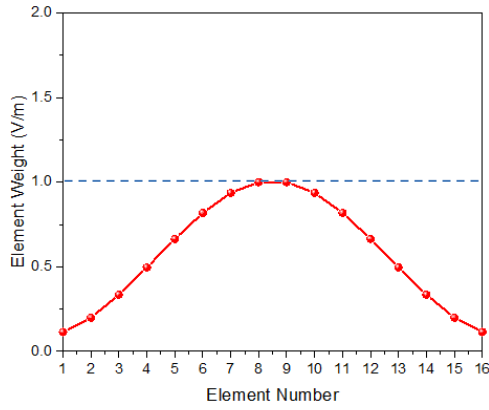


FIGURE 5. Weight vector of a non-uniform 16-element antenna array using attenuators.

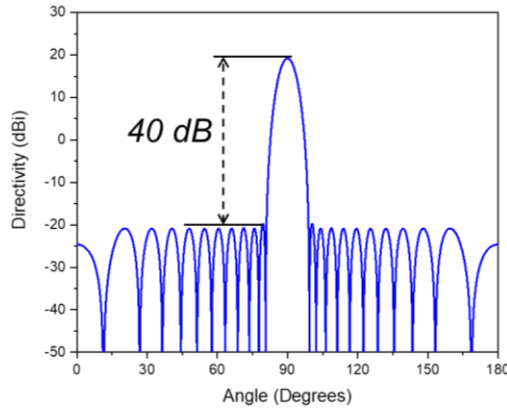


FIGURE 6. Directivity of a non-uniform 16-element antenna array using attenuators.

A. POWER-LOSS EFFICIENCY (η_{PL}):

This type of efficiency is caused by the ohmic loss introduced by the attenuators. It possesses most of the total aperture efficiency. Its value in linear scale (not in dB) can be simply calculated as the ratio between the supplied power after the attenuation to the power provided before the attenuation (which equals the power of the uniform feeding).

$$\eta_{PL} = \frac{\sum_{m=1}^M (w_m)^2}{M} \quad (3)$$

Therefore, in the previous example, $\eta_{PL} = -3.7$ dB ($\approx 42.5\%$).

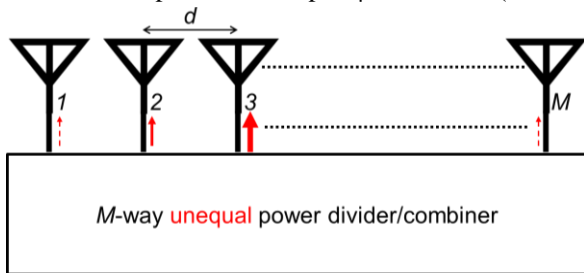


FIGURE 7. Non-uniform M-element antenna array using redistribution.

Power-loss efficiency can be improved by eliminating the attenuators from the feeding structure. In this case, the full power (without attenuation) is redistributed among the antenna elements to keep the ratio of the weight vector at the same level. Therefore, some values of the weight vector may exceed 1 V/m. This can be, practically, accomplished by constructing a feeding network based on a combination of equal and unequal power dividers/combiners with precalculated power-division ratios to keep the SLL at its same value as shown in FIGURE 7 [15]. The main drawback of this configuration is that the antenna array loses its beam reconfigurability as once the feeding network is designed, the weight vector (w_{dis}) is kept permanent and so is the SLL. In this case, since 100% of the feeding power is distributed and delivered to the antenna elements similarly to the uniform antenna array (apart from being unequally distributed), w_{dis} is defined such that the total power after redistribution equals the total power of the uniform feeding (M) as:

$$\sum_{m=1}^M (w_{dis})^2 = M \quad (4)$$

For such a case, the power-loss efficiency (η_{PL}) is 100% (0 dB). w_{dis} is calculated based on w_m as:

$$w_1^2 + w_2^2 + \dots + w_M^2 = \sum_{m=1}^M (w_m)^2 \quad (5)$$

By multiplying both sides in equation (5) by M

$$M(w_1^2 + w_2^2 + \dots + w_M^2) = M \sum_{m=1}^M (w_m)^2 \quad (6)$$

$$\left(\frac{M}{\sum_{m=1}^M (w_m)^2} \right) (w_1^2 + w_2^2 + \dots + w_M^2) = M \quad (7)$$

By equating the RHS of equations (4) and (7) and decomposing $\sum_{m=1}^M (w_{dis})^2$ to its power factors, then

$$w_{dis} = \sqrt{M} w_m / \sqrt{\sum_{m=1}^M (w_m)^2} = w_m / \sqrt{\eta_{PL}} \quad (8)$$

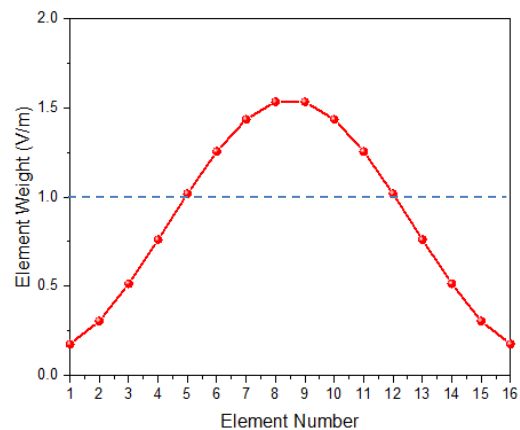


FIGURE 8. Weight vector of a non-uniform 16-element antenna array using redistribution.

B. POWER-DISTRIBUTION EFFICIENCY (η_{dis}):

The weight vector in the previous example of the 16-element antenna array is redistributed using equation (8) and presented in FIGURE 8. It is noticeable that the feeding at the few central elements, now, exceeds 1 V/m. The directivity of the antenna array is shown in FIGURE 9. The SLL is kept at the level of -40 dB. Moreover, the maximum directivity has improved to be ($D_{max_dis} \approx 23$ dBi) (an increase in the aperture efficiency by 3.7 dB). This is obviously due to achieving 100% power-loss efficiency. However, D_{max_dis} is still less than D_{max} . This can be explained as a drop in the aperture efficiency due to the power distribution efficiency (η_{dis}). This is mathematically known as *Cauchy – Schwarz inequality* [16] which states that the absolute value of the dot product of any two (or more) vectors is less than or equal to the product of their lengths. So, if U and V are two sets of non-negative real numbers where $U = [u_1, u_2, \dots, u_N]$ and $V = [v_1, v_2, \dots, v_N]$, then

$$\sum_{i=1}^N (u_i v_i) \leq \sqrt{\sum_{i=1}^N u_i^2 \sum_{i=1}^N v_i^2} \quad (9)$$

By substituting U and V by w_{dis} and 1 respectively in equation (9), we get

$$\sum_{m=1}^M (w_{dis} \cdot 1) \leq \sqrt{\sum_{m=1}^M (w_{dis})^2 \sum_{m=1}^M 1^2} = \sqrt{M \cdot M} = M \quad (10)$$

So, based on equation (2),

$$D_{max_dis} \leq D_{max} \quad (11)$$

Thus, D_{max_dis} is always less than D_{max} except for the case when both U and V are composed of ones (i.e., uniform distribution) or zeros (i.e., no radiation exists). The power distribution efficiency (η_{dis}) can be mathematically calculated as the squared ratio between D_{max_dis} and D_{max} . So,

$$\eta_{dis} = \left(\frac{D_{max_dis}}{D_{max}} \right)^2 = \left(\frac{\sum_{m=1}^M w_{dis}}{M} \right)^2 / M^2 \quad (12)$$

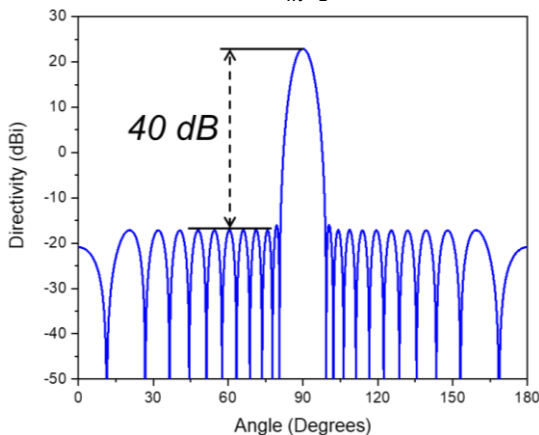


FIGURE 9. Directivity of a non-uniform 16-element antenna array using redistribution.

By substituting equation (8) into (12), we get

$$\eta_{dis} = \left(\frac{\sum_{m=1}^M w_m}{M} \right)^2 / \sum_{m=1}^M (w_m)^2 \quad (13)$$

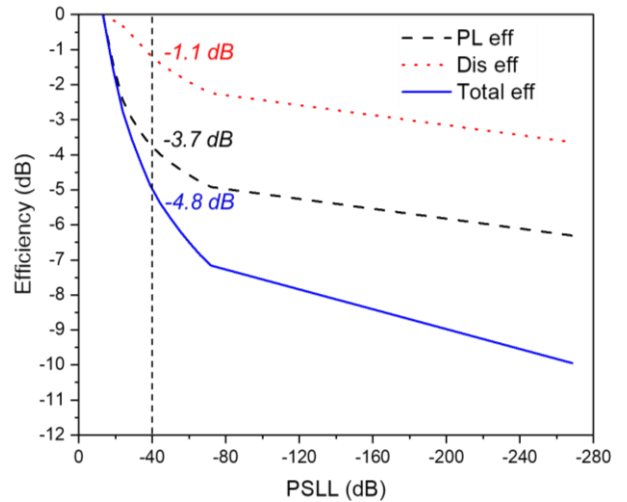
the total aperture efficiency can be calculated as:

$$\eta_{AP} = \eta_{PL} \times \eta_{dis} \quad (14)$$

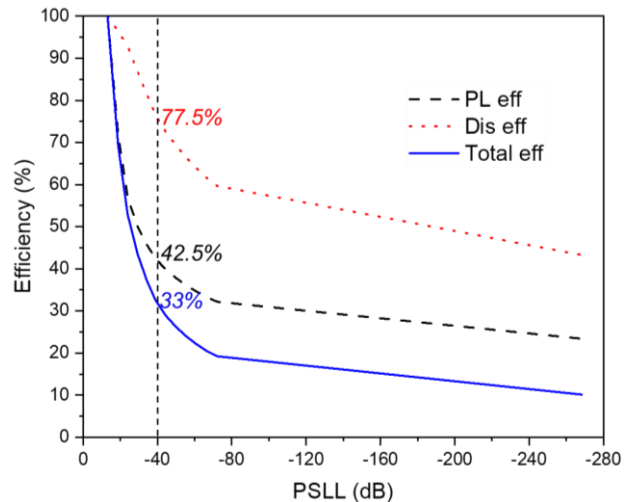
By substituting equations (3) and (13) to (14), we get

$$\eta_{AP} = \left(\frac{\sum_{m=1}^M w_m}{M} \right)^2 / M^2 \quad (15)$$

Which can also be defined as the squared ratio between D_{max_PL} and D_{max} .



(a)



(b)

FIGURE 10. Aperture efficiency against SLL in (a) dB scale (b) linear scale

In the previous example, and using equations (13), (14), and (15), $\eta_{dis} = -1.1$ dB ($\approx 77.5\%$). Therefore, the total aperture efficiency (η_{AP}) = $-3.7 - 1.1 = -4.8$ dB ($42.5\% \times 77.5\% \approx 33\%$). The relations between the power-loss efficiency, the power-distribution efficiency, and the total aperture efficiency against the SLL are shown in FIGURE 10 in both the dB-scale and linear scale. It is noticeable that the efficiencies exponentially decay when the SLL becomes smaller. The reason is that to achieve lower SLL, more tapering should be applied to the weight vector and hence less efficiency is obtained. The values of the SLL presented on the X-axis are marked when all sidelobes have the same energy level. So, it can be claimed that the corresponding efficiencies are the maximum values that could be achieved. Lower efficiencies are obtained if one (or more) of the sidelobes is less than the others. In other words, the curves in FIGURE 10 represent the upper boundaries of the aperture efficiency when $M = 16$. The three curves represent the maximum values for each type of efficiency. That is due to having equal sidelobes in each case. Meaning, that if one or more of the sidelobes is less than the others, more tapering is needed and hence smaller efficiencies (than the ones in the graph) are obtained.

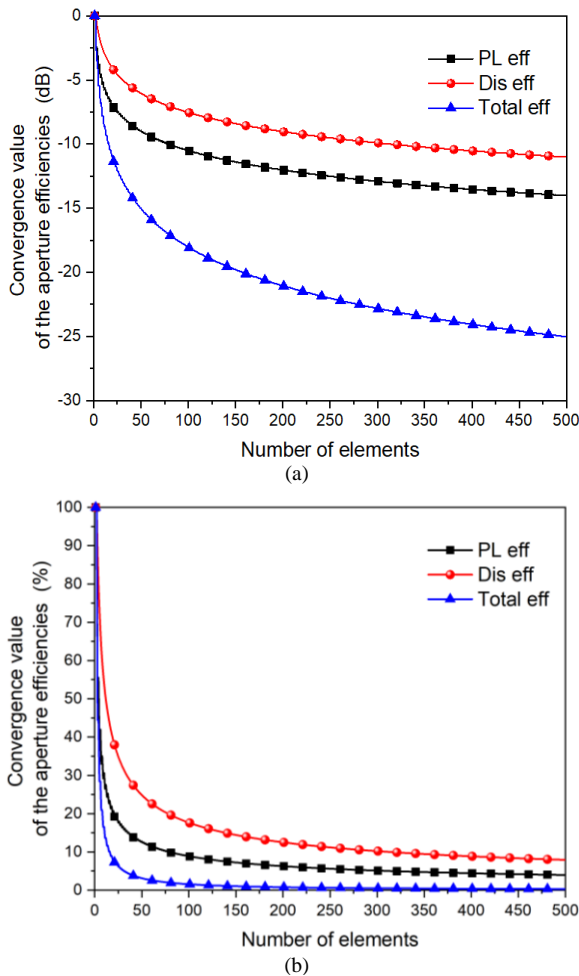


FIGURE 11. Convergence values of the aperture efficiencies at different numbers of elements when SLL approaches $-\infty$ in (a) dB scale (b) linear scale

It is also noticed that the power-loss efficiency dominates most of the total efficiency compared to the power-distribution efficiency due to deliberate attenuation. However, the power-loss efficiency can be improved by redistributing the power among the antenna elements meanwhile the decay in the power-distribution efficiency to obtain lower SLL is unavoidable.

It is also noted that, due to its exponential behaviour, the total aperture efficiency converges toward a constant value when the SLL approaches $-\infty$. In our case, where $M = 16$, the total aperture efficiency converges toward -10 dB (10%). However, arrays with a different number of elements have different convergence values of the aperture efficiency when the SLL approaches $-\infty$. The relations between the number of elements in an antenna array and the corresponding convergence value of the aperture efficiencies are shown in FIGURE 11 in both the dB-scale and linear scale. These relations are also exponential where the total aperture efficiency converges toward -25 dB ($\approx 0.3\%$) when the antenna array is extremely large.

IV. VALIDATION

To validate this mathematical modelling, a 16-element antenna array has been designed and constructed through simulations based on 16 identical half-wavelength dipoles as antenna elements. Each dipole resonates at 3 GHz and has a uniform omnidirectional radiation pattern as shown in FIGURE 12. The dipoles are spaced by 70 mm ($0.7\lambda_0$, where λ_0 is the free-space wavelength at the resonant frequency). The antenna elements have been fed in three different scenarios using the three different weight vectors shown in FIGURE 2, FIGURE 5 and FIGURE 8 respectively. The simulations have been done using the CST Microwave Studio CAD tool. The directivity in each of the three scenarios is shown in FIGURE 13 and the three scenarios can be discussed as follows:

- In the 1st scenario, where uniform feeding is applied, D_{max} is 16 dBi with an SLL of -13.5 dB and a total aperture efficiency of 0 dB (100%).
- In the 2nd scenario, D_{max_PL} is 11.2 dBi with an SLL of -40 dB. The aperture efficiency is -4.8 dB ($\approx 33\%$) where both η_{PL} and η_{dis} contribute to the total aperture efficiency by -3.7 dB and -1.1 dB respectively.
- In the 3rd scenario, D_{max_dis} is 14.9 dBi with an SLL of -40 dB. The aperture efficiency is -1.1 dB ($\approx 77.5\%$) where only η_{dis} contributes to it while η_{PL} is 100%. TABLE I summarizes the three scenarios.

TABLE 1. The Performance of the 16-Element Antenna Array at Different Feeding Scenarios

Scenario	SLL (dB)	Directivity (dBi)	η_{PL}	η_{dis}	η_{AP}
1 st	-13.4	16.0	100%	100%	100%
2 nd	-40.0	11.2	42.5%	77.5%	33%
3 rd	-40.0	14.9	100%	77.5%	77.5%

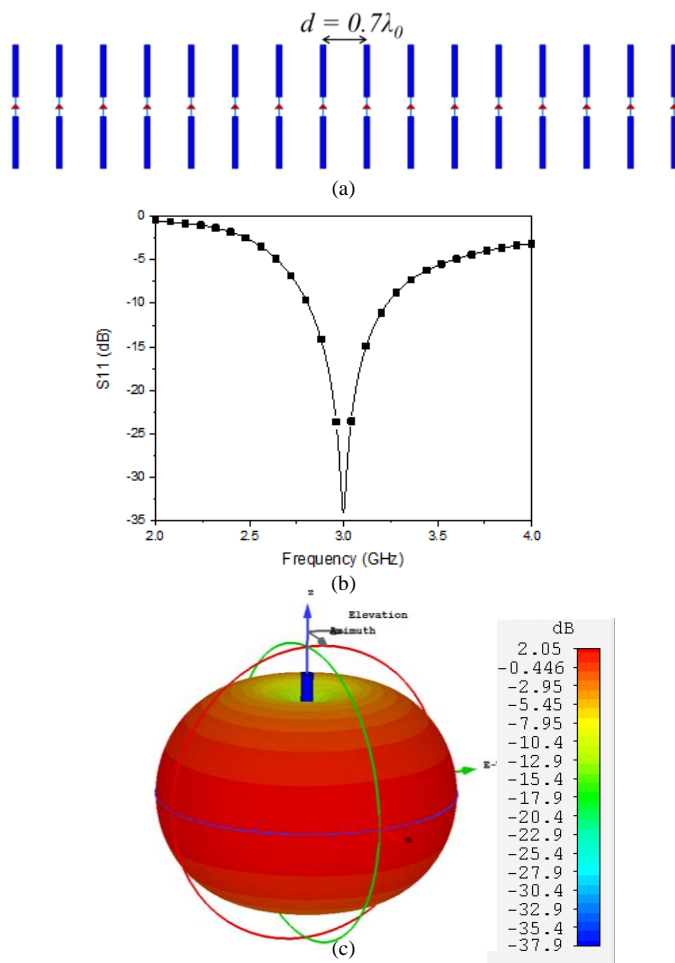


FIGURE 12. Simulated antenna array (a) structure (b) dipole return-loss (c) dipole 3D radiation pattern

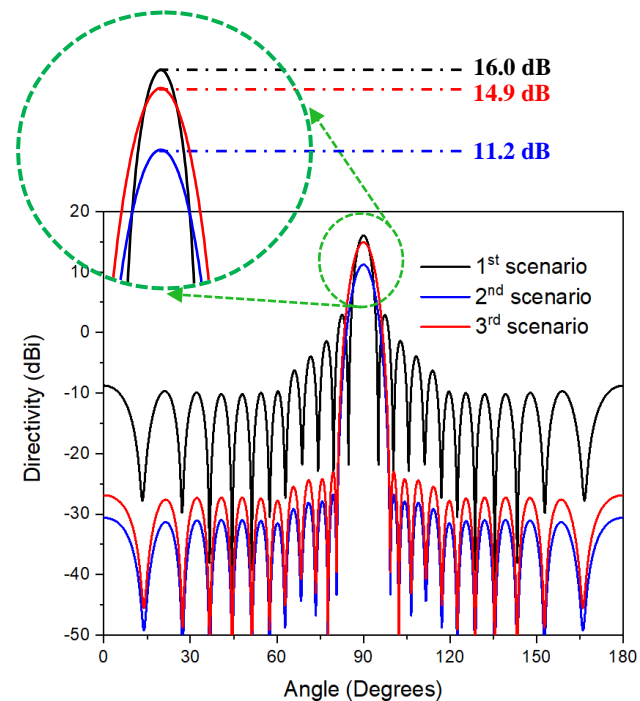


FIGURE 13. Directivity of a simulated 16-element antenna array at different feeding scenarios.

V. CONCLUSIONS

In this paper, mathematical modelling to study the relationship between the SLL of an antenna array and its aperture efficiency has been conducted. The total aperture efficiency of a non-uniform antenna array is composed of two types of efficiencies: power-loss efficiency due to the ohmic loss in the attenuators and power-distribution efficiency due to the non-uniformity of the weight distribution, their relationships are shown in FIGURE 10: for a given SLL, we can obtain the maximum total aperture efficiency which is dominated by the power-loss efficiency. However, the latter can be improved by eliminating the effect of the attenuators. The aperture efficiency exponentially decays when lowering the SLL. It converges to a certain value which depends on the number of elements in the antenna array when the SLL approaches $-\infty$. A validating model through simulations has been proposed to verify the mathematical model. This study has revealed a fundamental relationship between the antenna array efficiency and the SLL and is useful in supporting antenna designs.

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AHMED ALIELDIN (*Member, IEEE*) received a B.Sc. degree in radar engineering from the Military Technical College, Egypt, in 2005, an M.Sc. (Eng.) degree in antenna and microwave propagation from the University of Alexandria, Egypt, in 2013 and the PhD degree in antennas and electromagnetics from the University of Liverpool, U.K., in 2019. His academic research activities and PhD were centred on antenna designing and measurements with an emphasis on mobile communication applications. His academic research works include novel textile antennas, multiple-input multiple-output antennas, base station antennas, satellite antennas, transparent antennas, and phased-MIMO radar antennas. In addition to working in academia, he also held various positions throughout more than fifteen years of working in the industry. He has authored/coauthored many articles in leading international journals and conference proceedings. He has also filed a patent. He serves as a technical reviewer for leading academic journals and conferences.



Yi Huang (Fellow, IEEE) received a B.Sc. degree in physics and an M.Sc. (Eng) degree in microwave engineering from Wuhan University, Wuhan, China, and D.Phil. degree in communications from the University of Oxford, Oxford, U.K., in 1994. He was with NRIET, Nanjing, China, as a Radar Engineer and for various periods with the Universities of Birmingham, Birmingham, U.K., Oxford, and Essex as a Research Staff Member. In 1994, he joined as a Research Fellow at British Telecom Labs, London, U.K. and then joined the Department of Electrical Engineering and Electronics, the University of Liverpool, as a Lecturer, in 1995, where he is currently a Chair in Wireless Engineering, Deputy Head of Department, Head of High-Frequency Engineering Research Group and M.Sc. Program Director. He has authored over 300 refereed papers in leading international journals and conference proceedings, and authored books on Antennas: from Theory to Practice (John Wiley, 2008) and Reverberation Chambers (Wiley 2016). His current research interests include radio communications, applied electromagnetics, radar and antennas since 1987. Prof. Huang has received many research grants from research councils, government agencies, charities, the EU and industry, acted as a consultant to various companies, and served on a number of the National and international technical committees (such as the IET, EPSRC, European ACE, COST-IC0603, and COST-IC1102 and EurAAP). He has been an Editor, an Associate Editor, or a Guest Editor of four international journals. He has been a Keynote/Invited Speaker and Organizer of many conferences and workshops (e.g., IEEE iWAT, WiCom, and LAPC). He is currently an Editor-in-Chief of Wireless Engineering and Technology (ISSN 2152-2294/2152-2308), an Associate Editor of the IEEE Trans on AP, a College Member of EPSRC, U.K./Ireland Delegate to EurAAP, and a Distinguished Lecture of AR-S.