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Case Study

Abundant soliton solution for the time-fractional stochastic Gray-Scot model under the influence of noise and M-truncated derivative

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Abstract

In this study, we investigate the abundant soliton solutions for the time-fractional stochastic Gray-Scot (TFSGS) model analytically. The Gray-Scot model is considered under the influence of M-truncated derivative and multiplicative time noise. This is a reaction–diffusion chemical concentration model that explains the irreversible chemical reaction process. The M-truncated derivative is applied for the fractional version while Brownian motion is taken in the sense of time noise. The novel mathematical technique is used to obtain the abundant families of soliton solutions. These solutions are explored in the form of shock, complicated solitary-shock, shock-singular, and periodic-singular types of single and combination wave structures. During the derivation, the rational solutions also appear. Moreover, we use MATHEMATICA 11.1 tools to plot our solutions and exhibit several three-dimensional, two-dimensional, and their corresponding contour graphs to show the fractional derivative and Brownian motion impact on the soliton solutions of the TFSGS model. We show that the TFDGS model solutions are stabilized at around zero by the multiplicative Brownian motion. These wave solutions represent the chemical concentrations of the reactants.

Article Highlights

- The TFDGS model is considered to find the exact solitsary wave solutions under the random environment.
- The new MEDA method is used to obtain the different form of solutions.
- The different graphical behaviour are drawn to show the effects of noise and fractional derivatives.

Keywords Soliton solutions · M-truncated derivative · Stochastic Gray-Scot (TFSGS) model · New MEDA technique

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1 Introduction

The complex characteristics of dynamical systems are modeled with the help of nonlinear differential equations. The nonlinear evolution equation problems ascend in innumerable fields of the real world such as biology, engineering, medicine, chemistry, physics, astrophysics, electromagnetism, mechanics, kinematics, geo-chemistry, bioengineering, rheology, etc. [1]. In non-linear science, the main challenge is to gain the analytical and exact solution of the system of equations that represents the dynamical system [2, 3]. The theory of derivatives and integrals of non-integers made tremendous progress in the study of nonlinear dynamics because it can describe physical phenomena in a more generalized way. In the last three decades, the exact and solitary wave solutions of nonlinear fractional partial differential equations (NFPDEs) have attained a prominent role due to their importance in explaining the physical system [4–6]. Scientists have used various analytical techniques to gain the solutions of NFPDEs and these types of solutions help the community to understand the physical behavior of each model and are used for the betterment of human life [7].

Reaction–diffusion systems have been increasingly important in recent years in a variety of chemistry and biochemistry domains, including the glycolysis model [8], the lattice Boltzmann model [9], the Brusselator model [10], the Lengyel-Epstein model [11], the Schnakenberg model [12], and others. The Gray-Scott model is a widely used and effective chemical reaction model that explains the irreversible chemical reaction process [13, 14]. In the 1980s, P. Grey and S. K. Scott of the University of Leeds proposed the Gray-Scott model [15]. The way it works is described as

$$\begin{cases} U + 2V \to 3V, rate = k_1 uv^2, \\ V \to Q, rate = k_2 v, \end{cases} \tag{1}$$

where U, V are represents the reactants while Q is the product of the reaction, u, v are represents the chemical concentrations of the reactants U, V respectively. Also, the k_1 and k_2 are the positive constants that represent the reaction rates. Numerous patterns, including self-replicating patterns, the annular pattern emerging from circular spots, self-replicating spots, stationary spots, growing stripes, labyrinthine patterns, spatial–temporal chaos, stripe filaments, travel spots, and many more, have been thoroughly studied in relation to the Gray-Scott model. The dimensionless Gray-Scott [15] model is given below:

$$u_t = \alpha_1 u_{xx} - uv^2 + A(1 - u), \tag{2}$$

$$v_t = \alpha_2 v_{xx} + uv^2 - (A + B)v, \tag{3}$$

where α_1 and α_2 are the diffusion constants, A and B are the reaction rate and feed rate for the system (2–3) respectively. An extension of ordinary partial differential equations (PDEs) to account for randomness or stochasticity is known as a stochastic partial differential equation (SPDE). Systems affected by random processes are described by SPDEs in physics, engineering, economics, and biology, among other domains. The Itô calculus, a stochastic process extension of calculus, is a popular method for describing SPDEs [16–18]. On the other hand, a particular mathematical model that depicts the kinetics of chemical reactions and diffusion in a two-dimensional spatial domain is the Gray-Scott model. This deterministic model was created to investigate how patterns emerge in reaction–diffusion systems. I believe that when you speak to a "stochastic Gray-Scott model," you are referring to a version of the model that adds stochastic influences. One way to achieve this is by adding noise or randomness to the Gray-Scott model's parameters. Molecular fluctuations and external environmental noise are two examples of the many elements that can cause stochasticity in chemical reactions and diffusion. The behavior of such systems under unknown conditions can be understood using stochastic simulations of the Gray-Scott model. The 1-D coupled Gray-Scott model under the influence of the multiplicative time noise as given below [19, 20]:

$$u_t = \alpha_1 u_{xx} - uv^2 + A(1 - u) + \nu_1 ured\dot{B}(t), \tag{4}$$

$$v_t = \alpha_2 v_{xx} + uv^2 - (A + B)v + \nu_2 vred\dot{B}(t), \tag{5}$$

where ν_1, and ν_2 are the Borel functions, stand for the noise strengths. $B(t)$ is the standard Brownian motions, while $\dot{B}(t)$ is the white noise time series. The state variables are independent of the state of the Brownian motion.

In an effort to find both approximate and accurate solutions, many researchers have been working on SPDEs recently. Iqbal et al., [21] conducted a numerical investigation into the nonlinear stochastic Newell-Whitehead-Segel equation. Meanwhile, Yasin et al. focused on the stochastic Fitzhugh-Nagumo model [22] and the stochastic predator–prey model [23]. The reliable numerical analysis was developed by Raza et al. for the stochastic gonorrhea epidemic model [24]. The soliton solutions for the stochastic Konno-Oono system were constructed by Shaikh et al. [25]. Mohammed et al. worked on the stochastic Ginzburg–Landau equation [26] the $(2+1)$-dimensional stochastic chiral nonlinear Schrödinger equation [27] the stochastic exact solutions of the Nizhnik-Novikov-Veselov system [28], the stochastic Burgers' equation [29], and so on.

The authors considered the stochastic Potential-Yu-Toda-Sasa-Fakuyama for the analytical study. They used different techniques to gain a variety of solutions [30]. Hamza et al. considered the fractional stochastic shallow water wave equation. They used He's semi-inverse method and modified extended tanh-function method to gain the rational and trigonometric solutions [31]. Mohammed et al. analyzed the stochastic Korteweg-De Vries equations and gained various families of solutions [32]. The authors worked on the solution of coupled Fokas system [33]. Rehman et al. used the Sardar-subequation method on the strain wave equation to gain the families of solutions [34]. Awan et al. scrutinized the chiral nonlinear Schrödinger's equation with techniques namely: the functional variable method and first integral method and gained solitons. [35]. The authors used the generalized Kudryashov method for the resonant nonlinear Schrödinger equation. They obtained the bright, dark, kink, and singular soliton solutions [36]. The authors worked for the analytical study of physical systems [37–39].

On the other hand, a novel differentiation operator has emerged that combines the ideas of fractal derivative and fractional differentiation. As a result, numerous mathematicians proposed various types of fractional derivatives. The ones put out by Marchaud, Riesz, Caputo, Hadamard, Kober, Erdelyi, He's fractional derivative, Atana-Baleanu's derivative, Grunwald–Letnikov, and Riemann–Liouville are the most well-known. The majority of fractional derivative forms do not adhere to standard derivative formulas such as the product, quotient, or chain rules. The M-truncated derivative (MTD), a novel derivative created by Sousa et al. [40, 41], is a logical progression of the classical derivative. So, using the MTD we convert the system (6–7) into the fractional version as;

$$D_t u = \alpha_1 D_x^{2\epsilon} u - uv^2 + A(1-u) + \nu_1 u\dot{B}(t), \tag{6}$$

$$D_t v = \alpha_2 D_x^{2\epsilon} v + uv^2 - (A+B)v + \nu_2 v\dot{B}(t), \tag{7}$$

where ϵ is the fractional operation which lies between [0,1]. Obtaining the analytical stochastic solutions of the SFS (1) is the aim of this investigation. The stochastic solutions of SFS (1) in the form of rational, elliptic, hyperbolic, and trigonometric functions are obtained by means of a modified mapping approach. Since nonlinear pulse propagation in mono-mode optical fibers is explained by the Fokas system, the derived solutions can be utilized to analyze a wide range of important physical processes. To help with the interpretation of the multiplicative noise effects, the dynamic performances of the different found solutions are represented using both 2D and 3D curves.

This method offers a versatile method for obtaining exact solitary wave solutions. This approach is based on the concept of a auxiliary equation, which is a first-order nonlinear ordinary differential equation. Researchers can generate exact solutions by linking them to particular kinds of nonlinear equations and taking advantage of the intrinsic qualities of the auxiliary equation. A wide variety of nonlinear systems, including those defined by integrable, non-integrable, and partial differential equations, can be investigated thanks to the schemes unified structure. Studying soliton solutions with unique characteristics, such as combine solitons, dark solitons, bright solitons and exact solitary wave solutions, has shown this method to be especially helpful. With the use of this method, scientists can investigate and examine a broad variety of intricate nonlinear events in diverse physical systems. By using this method, researchers can get a wide range of Several soliton solutions displaying a range of behaviors, including dynamics of interactions, amplitude modulation, stability, and propagation properties. The analytical study of factional stochastic Gray-Scot model under the effect of time noise with the help of M-truncated derivative is under-consideration. As it is concentration model and it can be used to examine the concentration of different species. An efficient new MEDA technique is applied to gain abundant solutions such as shock, complicated solitary-shock, shock-singular, and periodic-singular types of single and combination wave structures. The rational solutions are also obtained. The effect of time noise and M-derivative is analyzed. 3D, 2D, and their corresponding contours are plotted for various values of the parameters.

2 Basic definitions

The MTD is consider as [40, 41]

Definition 1 *For the unknown function* $\phi : [0, inf] \rightarrow R$ *for the order* $\alpha \in (0, 1]$, *the M-truncated is taken as.*

$$D_{i,t}^{\alpha,\beta}\phi(t) = \lim_{d\rightarrow 0}\frac{\phi\left(tE_{i,\beta}(dt^{-\alpha})\right) - \phi(t)}{d}, \quad \text{for } t > 0, \tag{8}$$

where $E_{i,\beta}$ is defined such as

$$E_{i,\beta}(z) = \sum_{j=0}^{i}\frac{z^j}{\Gamma(\beta k + 1)}, \quad \text{for } \beta > 0, andz \in C.$$

The following properties for the MTD are as.

Theorem 1 *If there are differentiable function* ϕ *and* ψ *and the real constants are as s, t and* ν, *then the following properties are as follows* [42]

- $D_{i,t}^{\alpha,\beta}(r\phi + s\psi) = rD_{i,t}^{\alpha,\beta}(\phi) + sD_{i,t}^{\alpha,\beta}(\psi)red.$
- $D_{i,t}^{\alpha,\beta}(r\phi + s\psi)t^{\nu} = \frac{\nu}{\Gamma(\beta+1)}t^{\nu-\alpha}.$
- $D_{i,t}^{\alpha,\beta}(\phi\psi) = \phi D_{i,t}^{\alpha,\beta}(\psi) + \psi D_{i,t}^{\alpha,\beta}(\phi)red.$
- $D_{i,t}^{\alpha,\beta}(\phi)(t) = \frac{t^{1-\alpha}}{\Gamma(\beta+1)}\frac{d\phi}{dt}.$
- $D_{i,t}^{\alpha,\beta}(\phi\circ\psi)(t) = \phi'(\psi(t))D_{i,t}^{\alpha,\beta}(\psi(t)).$

Suppose a non-differentiable Wiener process $B(t)$ with the following properties [43]:

$$\lim_{\Delta t\rightarrow 0}\Delta B(t) = 0; \tag{9}$$

$$\lim_{\Delta t\rightarrow 0}\frac{(\Delta B(t))^n}{\Delta t} = \begin{cases} 1, n = 2 \\ 0, n = 3, 4, \cdots \end{cases} \tag{10}$$

Definition 2 *Stochastic process* $B(t)_{t\leq 0}$ *is said a Brownian motion if the following conditions are satisfied* [44];

- $B(t)$ is continues function if $t \leq 0$.
- $B(0)$=0.
- For $\tau_1 < \tau_2$, $B(\tau_2) - B(\tau_1)$ is independent.
- $B(\tau_2) - B(\tau_1)$ has a Gaussian distribution $\kappa(0, \tau_2 - \tau_1)$.

where $B_t = \frac{dB}{dt}$ is the time derivative of Wiener process $B(t)$.

3 Stochastic Wave transformation

Here, in order to obtain the solitary wave solutions, we select the wave transformation [41–43].

$$u(x,t) = U(\rho)e^{\nu_1 B(t) - \frac{1}{2}\nu_1^2 t}, v(x,t) = V(\rho)e^{\nu_2 B(t) - \frac{1}{2}\nu_2^2 t}, where\ \rho = \frac{l}{\varepsilon}x^{\varepsilon} + nt, \tag{11}$$

where $U(\rho)$ and $V(\rho)$ are the deterministic function while l and n are the speed and amplitude of waves. We take the derivatives of Eq. (11) and substituted in the Eqs. (6) & (7) and get

$$-nU^{'} + \alpha_1 l^2 U^{'}{'} - UV^2 e^{2\nu_2 B(t) - \nu_2^2 t} + A(1-U) = 0, \tag{12}$$

$$-nV^{'} + \alpha_2 l^2 V^{'}{'} + UV^2 e^{\nu_1 B(t) - \frac{1}{2}\nu_1^2 t} e^{\nu_2 B(t) - \frac{1}{2}\nu_2^2 t} - (A+B)V = 0. \tag{13}$$

Now, we take expatiations on both sides, we get

$$-nU^{'} + \alpha_1 l^2 U^{'}{'} - UV^2 E(e^{2\nu_2 B(t)}) e^{-\nu_2^2 t} + A(1-U) = 0, \tag{14}$$

$$-nV^{'} + \alpha_2 l^2 V^{'}{'} + UV^2 E(e^{\nu_1 B(t)}) e^{-\frac{1}{2}\nu_1^2 t} E(e^{\nu_2 B(t)}) e^{-\frac{1}{2}\nu_2^2 t} - (A+B)V = 0. \tag{15}$$

$B(t)$ is the time noise, then $E(e^{(\gamma B(t))}) = e^{\frac{1}{2}\gamma^2 t}$ and $E(e^{2(\gamma B(t))}) = e^{\gamma^2 t}$. So, Eqs. (14) and (15) turns into

$$-nU^{'} + \alpha_1 l^2 U^{'}{'} - UV^2 + A(1-U) = 0, \tag{16}$$

$$-nV^{'} + \alpha_2 l^2 V^{'}{'} + UV^2 - (A+B)V = 0. \tag{17}$$

4 New MEDA technique

Now, we consider the solution of Eqs. (16) and (17) in the form of polynomial as follows [45–47],

$$U(\rho) = \sum_{i=0}^{N} b_i \omega^N(\rho), \tag{18}$$

$$V(\rho) = \sum_{i=0}^{N} c_i \omega^N(\rho). \tag{19}$$

where b_i, c_i(i = 0,1,2,3,…N) are the constant that can be found to be later, here $\omega(\rho)$ satisfies the Eqs. (16) & (17) and given as,

$$\omega\prime(\rho) = \ln(C)\left(\delta_0 + \delta_1\omega(\rho) + \delta_2\omega(\rho)^2\right), \tag{20}$$

Now, putting $N = 1$in Eq. (18), then obtain the expression as,

$$U(\rho) = b_0 + b_1\omega(\rho), \tag{21}$$

$$V(\rho) = c_0 + c_1\omega(\rho). \tag{22}$$

A system of equations can be created by taking the derivatives of Eqs. (21) and (22), substituting in Eqs. (16) and (17), by the help of (20), and then gathering the term in the power of $\omega(\rho)$ and putting all of the polynomials' terms equal to zero. Solve this system of equations with the help of Mathematica and get the family of solutions as follows,

Type 1: If $\delta^1 - 4\delta_o\delta_2 < 0$ and $\delta_2 \neq 0$ then we obtained the trigonometric solutions as, where $h = \frac{4c_1^2\delta_0^2}{A+c_0^2} - \frac{\left(A^2+Ac_0^2+4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2\right)^2}{4\alpha_1^2\delta_0^2 l^4\left(A+c_0^2\right)^2\ln(C)^4}$ and $g = \sqrt{-\frac{\frac{\left(-Ab_1^2-b_1^2B+8\alpha_2 b_0^2\delta_2^2 l^2\ln(C)^2\right)^2}{16\alpha_2^2 b_0^2 b_1^2\delta_2^2 l^4\ln(C)^4} - \frac{Ab_1^2+b_1^2B+4\alpha_2 b_0^2\delta_2^2 l^2\ln(C)^2}{\alpha_2 b_1^2 l^2\ln(C)^2}}{\delta_2}}$

$$u_1(x,t) = \left[\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{\left(A+c_0^2\right)^2} + \frac{\left(2\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2\right)}{\left(A+c_0^2\right)^2}\left(-\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{4\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2}\right.\right.$$

$$+\sqrt{\frac{h\left(A+c_0^2\right)}{2c_1^2\delta_0}}\tan\left(\sqrt{\frac{h}{2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\Bigg)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{23}$$

$$v_1(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}\right.\right.$$

$$\left.\left.-\frac{g}{\sqrt{2}}\tan\left(\frac{g\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)}{\sqrt{2}}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{24}$$

$$u_2(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\left(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}\right.\right.$$

$$\left.\left.-\sqrt{\frac{h\left(A+c_0^2\right)}{2c_1^2\delta_0}}\left(\cot\left(\sqrt{\frac{h}{2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{25}$$

$$v_2(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}\right.\right.$$

$$\left.\left.-\frac{g}{\sqrt{2}}\cot\left(\frac{g\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)}{\sqrt{2}}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{26}$$

$$u_3(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\left(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}\right.\right.$$

$$+\sqrt{\frac{h\left(A+c_0^2\right)}{2c_1^2\delta_0}}\left(\tan\left(\sqrt{h}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.\left.\pm\sqrt{pq}\sec\left(\sqrt{h}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{27}$$

$$v_3(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}\right.\right.$$

$$\frac{g}{\sqrt{2}}\left(\tan\left(\sqrt{4\delta_2\delta_o-\frac{\left(-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2\right)^2}{16\alpha_2^2b_0^2b_1^2\delta_2^2l^4\ln(C)^4}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\pm\sqrt{pq}\sec\left(\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\Bigg)\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{28}$$

$$u_4(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\Bigg(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}$$

$$-\sqrt{\frac{h\left(A+c_0^2\right)}{2c_1^2\delta_0}}\Bigg(\Bigg(\cot\Bigg(\sqrt{h}\Bigg(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)$$

$$\pm\sqrt{pq}\csc\Bigg(\sqrt{h}\Bigg(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{29}$$

$$v_4(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\Bigg(-\frac{g}{\sqrt{2}}\Bigg(\cot\Bigg(g_1\Bigg(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)$$

$$\pm\sqrt{pq}\csc\Bigg(g_1\Bigg(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)\Bigg)-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{30}$$

here $g_1=\sqrt{4\delta_2\delta_o-\frac{\left(-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2\right)^2}{16\alpha_2^2b_0^2b_1^2\delta_2^2l^4\ln(C)^4}}$.

$$u_5(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\Bigg(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}$$

$$-\frac{1}{2}\sqrt{\frac{h\left(A+c_0^2\right)}{c_1^2\delta_0}}\Bigg(\tan\Bigg(\frac{1}{2}\sqrt{h}\Bigg(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)$$

$$-\cot\Bigg(\frac{1}{2}\sqrt{h}\Bigg(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{31}$$

$$v_5(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\Bigg(-\frac{1}{2}g_1\Bigg(\tan\Bigg(\frac{1}{2}g_1\Bigg(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)$$

$$-\sqrt{pq}\cot\Bigg(\frac{1}{2}g_1\Bigg(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\Bigg)\Bigg)\Bigg)-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{32}$$

here $g_1=\sqrt{4\delta_2\delta_o-\frac{\left(-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2\right)^2}{16\alpha_2^2b_0^2b_1^2\delta_2^2l^4\ln(C)^4}}$.

Type 2: If $\delta^1 - 4\delta_o\delta_2 > 0$ and $\delta_2 \neq 0$, then we extracted five different types of solutions as,
The Shock solutions are extracted as,

$$u_6(x,t) = \left[\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{\left(A + c_0^2\right)^2} + \frac{\left(2\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2\right)}{\left(A + c_0^2\right)^2}\left(-\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{4\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2}\right.\right.$$
$$\left.\left. - \sqrt{\frac{h\left(A + c_0^2\right)}{2c_1^2 \delta_0}} \tanh\left(\sqrt{\frac{h}{2}}\left(\frac{t\left(A^2 + Ac_0^2 - 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2\right)}{2\delta_0\left(A + c_0^2\right)\ln(C)} + \frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right] e^{\frac{v_1^2}{2}t + v_1 B(t)}, \tag{33}$$

$$v_6(x,t) = \left[\frac{Ab_1^2 + b_1^2 B - 4\alpha_2 b_0^2 \delta_2^2 l^2 \ln(C)^2}{2b_0 b_1^2} - \frac{\left(2\alpha_2 \delta_2^2 l^2 \ln(C)^2\right)}{b_1}\left(-\frac{-Ab_1^2 - b_1^2 B + 8\alpha_2 b_0^2 \delta_2^2 l^2 \ln(C)^2}{8\alpha_2 b_0 b_1 \delta_2^2 l^2 \ln(C)^2}\right.\right.$$
$$\left.\left. - \frac{g_2}{\sqrt{2\delta_2}} \tanh\left(\frac{g_2\left(\frac{b_1(A+B)t}{4b_0\delta_2 \ln(C)} + \frac{l}{\varepsilon}x^{\varepsilon}\right)}{\sqrt{2}}\right)\right)\right] e^{\frac{v_2^2}{2}t + v_2 B(t)}, \tag{34}$$

$$g_2 = \sqrt{\frac{\left(-Ab_1^2 - b_1^2 B + 8\alpha_2 b_0^2 \delta_2^2 l^2 \ln(C)^2\right)^2}{16\alpha_2^2 b_0^2 b_1^2 \delta_2^2 l^4 \ln(C)^4} - 4\delta_2\delta_o}.$$

The singular solutions are extracted as,

$$u_7(x,t) = \left[\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{\left(A + c_0^2\right)^2 + \frac{\left(2\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2\right)}{\left(A + c_0^2\right)^2}}\left(-\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{4\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2} - \sqrt{\frac{h\left(A + c_0^2\right)}{2c_1^2 \delta_0}}\right.\right.$$
$$\left.\left.\left(\coth\left(\sqrt{\frac{h}{2}}\left(\frac{t\left(A^2 + Ac_0^2 - 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2\right)}{2\delta_0\left(A + c_0^2\right)\ln(C)} + \frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right)\right] e^{\frac{v_1^2}{2}t + v_1 B(t)}, \tag{35}$$

$$v_7(x,t) = \left[\frac{Ab_1^2 + b_1^2 B - 4\alpha_2 b_0^2 \delta_2^2 l^2 \ln(C)^2}{2b_0 b_1^2} - \frac{2\alpha_2 \delta_2^2 l^2 \ln(C)^2}{b_1}\left(-\frac{-Ab_1^2 - b_1^2 B + 8\alpha_2 b_0^2 \delta_2^2 l^2 \ln(C)^2}{8\alpha_2 b_0 b_1 \delta_2^2 l^2 \ln(C)^2}\right.\right.$$
$$\left.\left. - \frac{g}{\sqrt{2}} \coth\left(\frac{g_2\left(\frac{b_1(A+B)t}{4b_0\delta_2 \ln(C)} + \frac{l}{\varepsilon}x^{\varepsilon}\right)}{\sqrt{2}}\right)\right)\right] e^{\frac{v_2^2}{2}t + v_2 B(t)}, \tag{36}$$

The complex dark bright solutions are extracted as,

$$u_8(x,t) = \left[\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{\left(A + c_0^2\right)^2} + \frac{\left(2\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2\right)}{\left(A + c_0^2\right)^2}\left(-\frac{A^2 + Ac_0^2 + 4\alpha_1 c_0 c_1 \delta_0^2 l^2 \ln(C)^2}{4\alpha_1 c_1^2 \delta_0^2 l^2 \ln(C)^2}\right.\right.$$

$$-\sqrt{\frac{h(A+c_0^2)}{2c_1^2\delta_0}}\left(\left(\tanh\left(\sqrt{h}G\right)\pm i\sqrt{pq}\operatorname{sech}\left(\sqrt{h}G\right)\right)\right)\Bigg]\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{37}$$

where $G=\left(\frac{\mathrm{t}(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2)}{2\delta_0(A+c_0^2)\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)$.

$$v_8(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\Bigg(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}$$

$$-\frac{g_3\left(\tanh\left(g_1\left(\frac{b_1(A+B)\mathrm{t}}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)\right)\pm i\sqrt{pq}\operatorname{sech}\left(g_1\left(\frac{b_1(A+B)\mathrm{t}}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)\right)\right)}{\sqrt{2}}\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{38}$$

The mixed singular solutions are extracted as,

$$u_9(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\Bigg(\sqrt{\frac{h(A+c_0^2)}{2c_1^2\delta_0}}$$

$$\left(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}-\left(\coth\left(\sqrt{h}G\right)\pm\sqrt{pq}\operatorname{csch}\left(\sqrt{h}G\right)\right)\right)\Bigg)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{39}$$

where $G=\left(\frac{\mathrm{t}(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2)}{2\delta_0(A+c_0^2)\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)$.

$$v_9(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\Bigg(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}$$

$$+-\frac{\sqrt{-\frac{1}{\delta_2}}g_2\left(\coth\left(g_1\left(\frac{b_1(A+B)\mathrm{t}}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)\right)\pm\sqrt{pq}\operatorname{csch}\left(g_1\left(\frac{b_1(A+B)\mathrm{t}}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)\right)\right)}{\sqrt{2}}\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{40}$$

The shock solution is extracted as,

$$u_{10}(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{\left(2\alpha_1c_1^2\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2}\Bigg(-\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{4\alpha_1c_1^2\delta_0^2l^2\ln(C)^2}$$

$$+\sqrt{\frac{h(A+c_0^2)}{2c_1^2\delta_0}}\left(\tanh\left(\frac{1}{2}\sqrt{h}G\right)\pm\coth\left(\frac{1}{2}\sqrt{h}G\right)\right)\Bigg)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{41}$$

where $G=\left(\frac{\mathrm{t}(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2)}{2\delta_0(A+c_0^2)\ln(C)}+\frac{l}{\varepsilon}x^\varepsilon\right)$.

$$v_{10}(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{2\alpha_2\delta_2^2l^2\ln(C)^2}{b_1}\Bigg(-\frac{-Ab_1^2-b_1^2B+8\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{8\alpha_2b_0b_1\delta_2^2l^2\ln(C)^2}$$

$$+\frac{\sqrt{-\frac{1}{\delta_2}}g_2\left(\tanh\left(\frac{1}{2}g_1\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\pm\coth\left(\frac{1}{2}g_1\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)}{\sqrt{2}}\Bigg)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{42}$$

Type 3: If $\delta_2\delta_0 > 0$ and $\delta_1 = 0$, then we obtained the trigonometric solutions as,

$$u_{11}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{(A+c_0^2)^2}+\frac{2\alpha_1c_1^2\delta_0^2l^2\sqrt{\frac{A+c_0^2}{c_1^2}}\ln(C)^2}{(A+c_0^2)^2}\right.$$

$$\left.\tan\left(\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0(A+c_0^2)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{43}$$

$$v_{11}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(\alpha_2\delta_2^2\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\tan\left(\frac{1}{2}\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{44}$$

$$u_{12}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{(A+c_0^2)^2}+\frac{2\alpha_1c_1^2\delta_0^2l^2\sqrt{\frac{A+c_0^2}{c_1^2}}\ln(C)^2}{(A+c_0^2)^2}\right.$$

$$\left.\cot\left(\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0(A+c_0^2)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{45}$$

$$v_{12}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\alpha_2\delta_2^2\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2}{b_1}\right.$$

$$\left.\cot\left(\frac{1}{2}\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{46}$$

$$u_{13}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{2\alpha_1c_1^2\delta_0^2l^2\sqrt{\frac{A+c_0^2}{c_1^2}}\ln(C)^2}{\left(A+c_0^2\right)^2}\left(\tan\left(2\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}\right.\right.\right.$$

$$\left.\left.\left.\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\pm\sqrt{pq}\sec\left(2\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{47}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{13}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2g_4l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\left(\tan\left(J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\pm\sqrt{pq}\sec\left(J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{48}$$

where $J=\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}$.

$$u_{14}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-h_1\left(\cot\left(2\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right.\right.$$

$$\left.\left.\pm\sqrt{pq}\csc\left(2\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{49}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{14}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2g_4l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\left(\cot\left(J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\pm\sqrt{pq}\csc\left(J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}, \tag{50}$$

where $J=\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}$.

$$u_{15}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{1}{2}h_1\left(\tan\left(\frac{1}{2}\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right.\right.$$

$$\left.\left.-\cot\left(\frac{1}{2}\sqrt{\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{51}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{15}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(\alpha_2\delta_2^2\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{2b_1}\right.$$

$$\left.\left(\tan\left(\frac{1}{4}J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)-\cot\left(\frac{1}{4}J\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)},\tag{52}$$

where $J=\sqrt{\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}$.

Type 4: If $\delta_2\delta_0<0$ and $\delta_1=0$, then we extracted different types of solutions as,
The shock solutions are extracted as,

$$u_{16}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\sqrt{-h_1}\tanh\left(\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}.\tag{53}$$

$$v_{16}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\tanh\left(\frac{1}{2}\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)},\tag{54}$$

$$u_{17}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\sqrt{-h_1}\coth\left(\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}.\tag{55}$$

$$v_{17}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\coth\left(\frac{1}{2}g_5\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}.\tag{56}$$

$$u_{18}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\sqrt{-h_1}\left(\tanh\left(2\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right.\right.$$

$$\left.\left.\pm i\sqrt{pq}\operatorname{sech}\left(2\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{57}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{18}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left(\tanh\left(\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.\pm i\sqrt{pq}\operatorname{sech}\left(\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{58}$$

$$u_{19}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\sqrt{-h_1}\left(\coth\left(2\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right.\right.$$

$$\left.\left.\pm\sqrt{pq}\operatorname{csch}\left(2\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{59}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{19}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left(\coth\left(\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.\pm\sqrt{pq}\operatorname{csch}\left(\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{60}$$

$$u_{20}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{1}{2}\sqrt{-h_1}\left(\tanh\left(\frac{1}{2}\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right.\right.$$

$$\left.\left.-\sqrt{pq}\coth\left(\frac{1}{2}\sqrt{-\frac{c_1^2\delta_0^2}{A+c_0^2}}G\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{61}$$

where $G=\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)$.

$$v_{20}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(\alpha_2\delta_2^2\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}l^2\ln(C)^2\right)}{2b_1}\right.$$

$$\left(\tanh\left(\frac{1}{4}\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.-\sqrt{pq}\coth\left(\frac{1}{4}\sqrt{-\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{\alpha_2b_1^2l^2\ln(C)^2}}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{62}$$

Type 5: If $\delta_o=\delta_2$ and $\delta_1=0$, then we extracted the periodic and mixed periodic solutions as

$$u_{21}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{2l^2\ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left.\tan\left(\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{63}$$

$$v_{21}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\tan\left(g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{64}$$

$$u_{22}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{2l^2\ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left.\cot\left(\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{65}$$

$$v_{22}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\cot\left(g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{66}$$

$$u_{23}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left(\tan\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.\pm\sqrt{pq}\sec\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{67}$$

$$v_{23}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{2\alpha_2\delta_2^2l^2\ln(C)^2}{b_1}\left(\tan\left(2\delta_0\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.\right.$$

$$\left.\left.\pm\sqrt{pq}\sec\left(2\delta_0\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{68}$$

$$u_{24}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left(-\cot\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.\pm\sqrt{pq}\csc\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{69}$$

$$v_{24}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\cot\left(2g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.\right.$$

$$\left.\left.\pm\sqrt{pq}\csc\left(2g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{70}$$

$$u_{25}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{1}{2}\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left(\tan\left(\frac{1}{2}\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\left.-\sqrt{pq}\cot\left(\frac{1}{2}\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{71}$$

$$v_{25}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(\tan\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.\right.$$

$$\left.\left.-\sqrt{pq}\cot\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{72}$$

Type 6: If $\delta_0=-\delta_2$ and $\delta_1=0$, then we extracted the different hyperbolic solutions as,

$$u_{26}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left.\tanh\left(\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{73}$$

$$v_{26}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\tanh\left(g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{74}$$

$$u_{27}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left.\coth\left(\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{75}$$

$$v_{27}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\right.$$

$$\left.\coth\left(\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{4\alpha_2b_1^2l^2\ln(C)^2\delta_2}\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{76}$$

$$u_{28}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}\right.$$

$$\left(-\tanh\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\pm i\sqrt{pq}\operatorname{sech}\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{77}$$

$$v_{28}(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\tanh\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\pm i\sqrt{pq}\operatorname{sech}\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{78}$$

$$u_{29}(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}$$

$$\left(-\coth\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\pm\sqrt{pq}\operatorname{csch}\left(2\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{79}$$

$$v_{29}(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(-\coth\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.\pm\sqrt{pq}\operatorname{csch}\left(\frac{1}{2}g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{80}$$

$$u_{30}(x,t)=\Bigg[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}+\frac{1}{2}\frac{2l^2ln(C)^2c_1^2\alpha_1\delta_0^2}{(A+c_0^2)^2}$$

$$\left(\sqrt{pq}\coth\left(\frac{1}{2}\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$\left.+\tanh\left(\frac{1}{2}\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right)\Bigg]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{81}$$

$$v_{30}(x,t)=\Bigg[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(\sqrt{pq}\coth\left(2g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right.$$

$$+\tanh\left(2g_6\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\Bigg]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{82}$$

where $g_6=\frac{Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{4\alpha_2b_1^2l^2\ln(C)^2\delta_2}$.

Type 7: If $\delta_1^2=4\delta_0\delta_2$, there is one solution is extracted as,

$$u_{31}(x,t)=u_{26}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}-\frac{2l^2\ln(C)^2c_1^2\alpha_1\delta_0^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.\tan\left(\delta_0\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{83}$$

$$v_{31}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\frac{\left(16\alpha_1^3c_1^2\delta_0^5l^6\ln(C)^5\right)\left(\frac{\left(A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+lx^{\epsilon}\right)}{2\alpha_1\delta_0l^2\left(A+c_0^2\right)\ln(C)}+2\right)}{\left(A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)^2\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+lx^{\epsilon}\right)}\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{84}$$

Type 8: If $\delta_1=\chi,\delta_0=r\chi(r\neq 0)and\delta_2=0$, there is one solution is extracted as,

$$u_{32}(x,t)=\left[h_2\left(B^{\chi\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+lx^{\epsilon}\right)}-r\right)\right.$$

$$\left.+\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{85}$$

$$v_{32}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}\right.$$

$$\left.-\frac{\left(2\alpha_2\delta_2^2l^2\ln(C)^2\right)}{b_1}\left(B^{\chi\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)}-r\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{86}$$

Type 9: If $\delta_1=0=\delta_2$ there is one solution is extracted as,

$$u_{33}(x,t)=\left[\frac{h_2\left(\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\varepsilon}x^{\varepsilon}\right)\right)}{\ln(C)^2}+\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{87}$$

$$v_{33}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}-\frac{\left(\delta_2\ln(C)^2\left(Ab_1^2+b_1^2B+4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2\right)\right)}{2b_1^3}\right.$$

$$\left.\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{88}$$

Type 10: If $\delta_1=0=\delta_0$ there is one solution is extracted as,

$$u_{34}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\frac{2\alpha_1\delta_0l^2}{\left(A+c_0^2\right)\left(\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right).\right)}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{89}$$

$$v_{34}(x,t)=\left[\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}+\frac{2\alpha_2\delta_2l^2}{b_1\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)}\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{90}$$

Type 11: If $\delta_0=0 and \delta_1\neq 0$ then we extracted the mixed hyperbolic solutions as,

$$u_{35}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\frac{p\left(A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{\left(A+c_0^2\right)^2\left(-\sinh\left(h_3\left(\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right)\right)\right)+\cosh(J)+p\right)}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{91}$$

$$v_{35}(x,t)=\left[\frac{2\alpha_2\delta_2l^2p\epsilon\ln(C)^2}{b_1\left(-\sinh\left(g_7\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right)+\cosh\left(g_7\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right)+p\right)}\right.$$

$$\left.+\frac{Ab_1^2+b_1^2B-4\alpha_2b_0^2\delta_2^2l^2\ln(C)^2}{2b_0b_1^2}\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{92}$$

$$u_{36}(x,t)=\left[\frac{A^2+Ac_0^2+4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right.$$

$$\left.-\frac{2\alpha_1\delta_0l^2\ln(C)^2\left(h_3\sinh(J)+\cosh\left(h_3\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right)\right)\right)}{\left(A+c_0^2\right)\left(2\cosh\left(h_3\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1c_0c_1\delta_0^2l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right)\right)+q\right)}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}, \tag{93}$$

here $g_7 = \frac{-Ab_1^2-b_1^2B+8\alpha_2 b_0^2\delta_2^2 l^2\ln(C)^2}{4\alpha_2 b_0 b_1 \delta_2 l^2 \ln(C)^2}$ and $J = h_3\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1 c_0 c_1 \delta_0^2 l^2\ln(C)^2\right)}{2\delta_0\epsilon\left(A+c_0^2\right)\ln(C)} + lx^{\epsilon}\right)$.

$$v_{36}(x,t) = \left[\frac{2\alpha_2\delta_2 l^2\ln(C)^2\left(g_7\sinh\left(g_7\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right)+\cosh\left(g_7\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right)\right)}{b_1\left(2\cosh\left(g_7\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)\right)+q\right)}\right.$$

$$\left.+\frac{Ab_1^2+b_1^2B-4\alpha_2 b_0^2\delta_2^2 l^2\ln(C)^2}{2b_0b_1^2}\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{94}$$

Type 11: If $\delta_1 = \chi, \delta_2 = r\chi(r \neq 0) and \delta_0 = 0$, then we extract the plane solution as,

$$u_{37}(x,t) = \left[\frac{2\alpha_1 c_1^2\delta_0^2 l^2 p\ln(C)^2 B^{\chi\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1 c_0 c_1\delta_0^2 l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right)}}{\left(A+c_0^2\right)^2\left(p-qrB^{\chi\left(\frac{t\left(A^2+Ac_0^2-4\alpha_1 c_0 c_1\delta_0^2 l^2\ln(C)^2\right)}{2\delta_0\left(A+c_0^2\right)\ln(C)}+\frac{l}{\epsilon}x^{\epsilon}\right)}\right)}\right.$$

$$\left.+\frac{A^2+Ac_0^2+4\alpha_1 c_0 c_1\delta_0^2 l^2\ln(C)^2}{\left(A+c_0^2\right)^2}\right]e^{\frac{v_1^2}{2}t+v_1B(t)}. \tag{95}$$

$$v_{37}(x,t) = \left[\frac{Ab_1^2+b_1^2B-4\alpha_2 b_0^2\delta_2^2 l^2\ln(C)^2}{2b_0b_1^2} - \frac{2\alpha_2\delta_2^2 l^2 p\ln(C)^2 B^{\chi\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)}}{b_1\left(p-qrB^{\chi\left(\frac{b_1(A+B)t}{4b_0\delta_2\ln(C)}+lx^{\epsilon}\right)}\right)}\right]e^{\frac{v_2^2}{2}t+v_2B(t)}. \tag{96}$$

5 Graphical behavior under the effect of noise and M-truncated derivative

This section describes the graphical representations of the obtained results for the TFSGS model. These solutions are successfully gained by using the new MEDA method. The obtained results have randomness and fractional effects in the wave structures. It would be necessary to express the two chemical reactants, u, and v, in a physical description of the

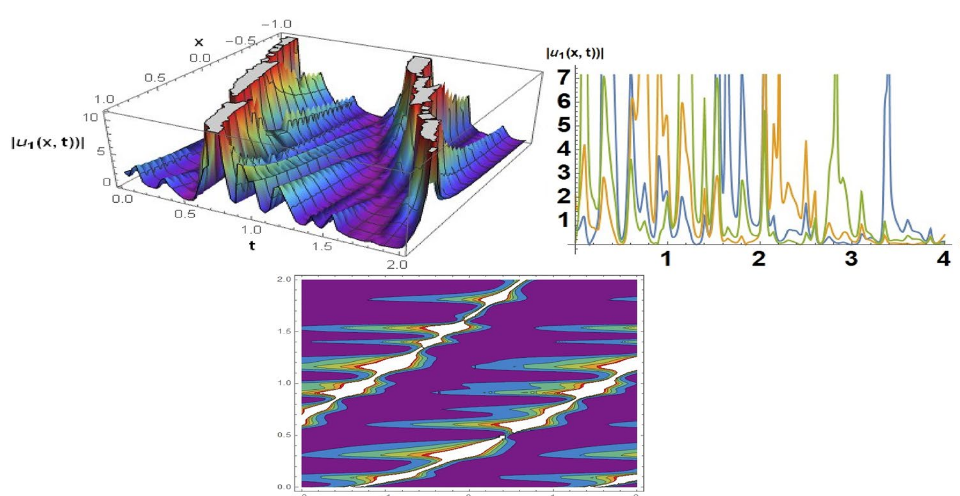


Fig. 1 The 3-dimensional, 2-dimensional, and contour representations for the solution $u_1(x, t)$ under the noise effect using the different values of constants such as $A=0.2$, $\epsilon=0.7$, $\alpha_1=2$, $B=1.9$, $c_0=0.9$, $c_1=1$, $\delta_0=0.5$, $h=0.6$, $l=2$

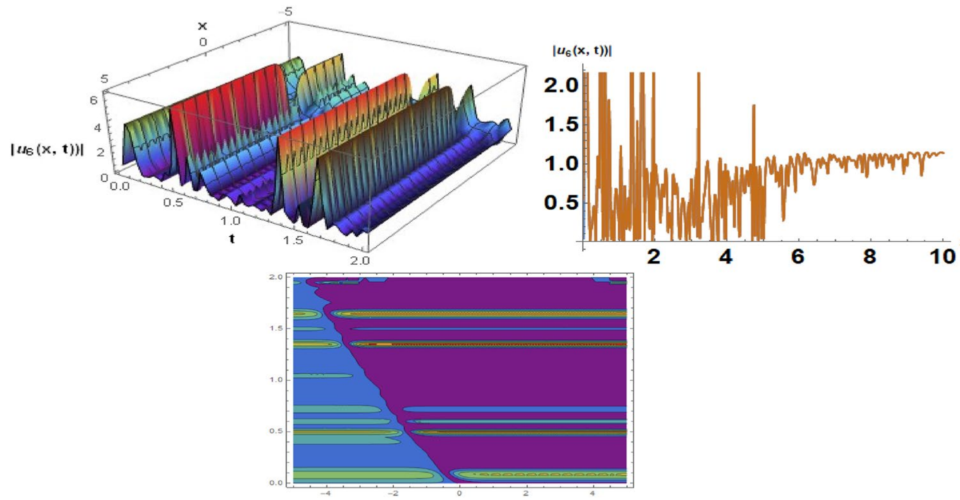


Fig. 2 The 3-dimensional, 2-dimensional, and contour representations for the solution $u_6(x, t)$ under the noise effect using the different values of constants such as $A = 2.2$, $\epsilon = 0.8$, $\alpha_1 = 2.7$, $B = 1.99$, $c_0 = 0.3$, $c_1 = 1.9$, $\delta_0 = 0.5$, $h = 10.001$, $l = 1.2$

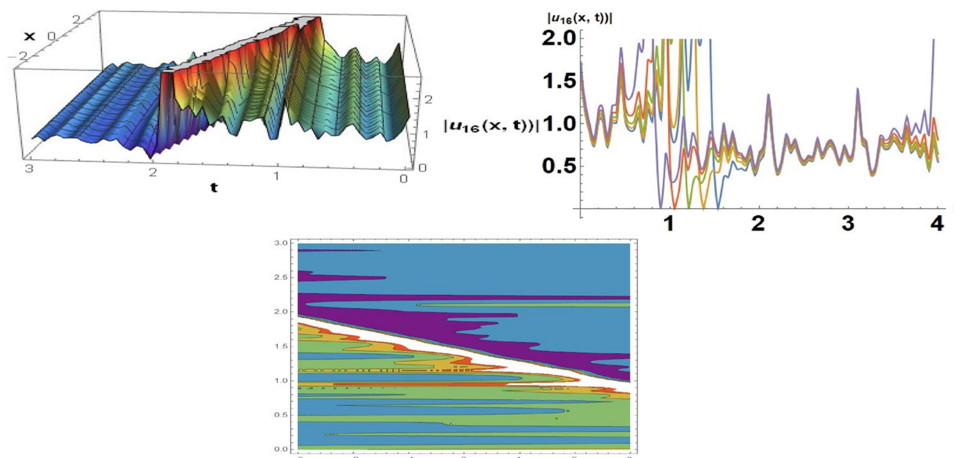


Fig. 3 The 3-dimensional, 2-dimensional, and contour representations for the solution $u_{16}(x, t)$ under the noise effect using the different values of constants such as $A = 1.2$, $\epsilon = 0.9$, $\alpha_1 = 2.7$, $B = 1.3$, $c_0 = 0.03$, $c_1 = 0.5$, $\delta_0 = 0.3$, $h = 1.1$, $l = 1.2$

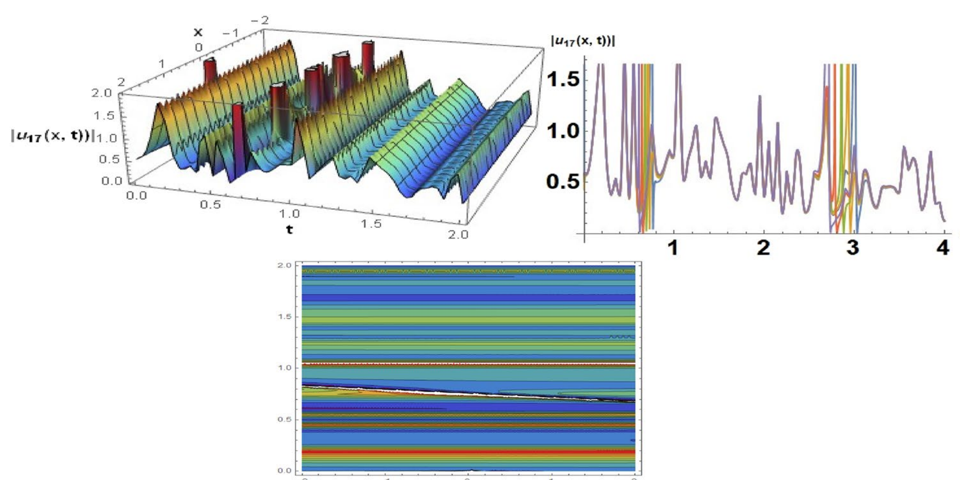


Fig. 4 The 3-dimensional, 2-dimensional, and contour representations for the solution $u_{17}(x, t)$ under the noise effect using the different values of constants such as $A = 1.2$, $\epsilon = 0.5$, $\alpha_1 = 0.7$, $B = 1.3$, $c_0 = 0.3$, $c_1 = 0.9$, $\delta_0 = 0.5$, $h = 1.1$, $l = 0.2$

stochastic Gray-Scott model. These reactants might be conceptualized as many kinds of substances or particles found in an actual system. The two chemical reactants, u and v, would need to be represented physically in order to create a stochastic Gray-Scott model. Reactants are several kinds of substances or particles that are found in a real-world system. There is an additional component of unpredictability in the stochastic Gray-Scott model. The study of pattern generation and complex dynamics in reaction–diffusion systems is a common application of the stochastic Gray-Scott model

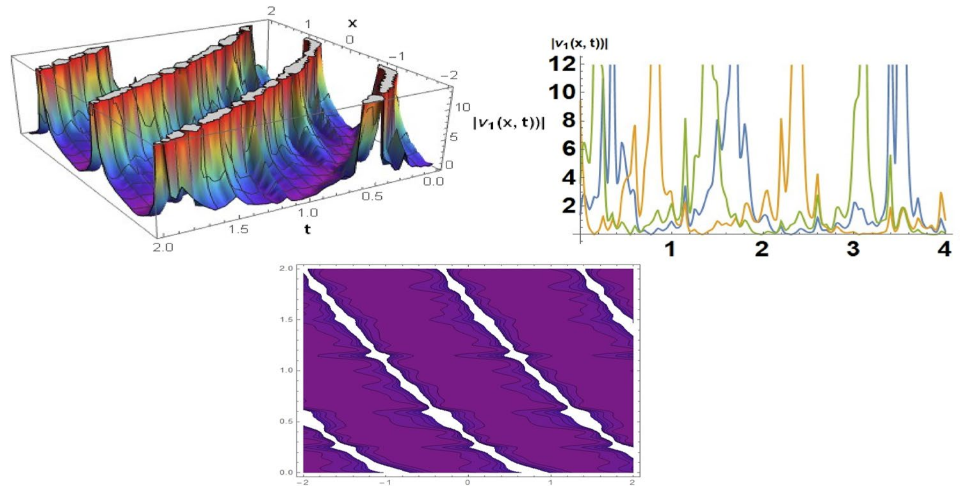


Fig. 5 The 3-dimensional, 2-dimensional, and contour representations for the solution $v_1(x, t)$ under the noise effect using the different values of constants such as $A = 0.2$, $\epsilon = 0.7$, $\alpha_1 = 1$, $\alpha_2 = 2$, $b_1 = 0.9$, $b_0 = 0.5$, $B = 1.9$, $\delta_2 = 0.6$, $l = 2$

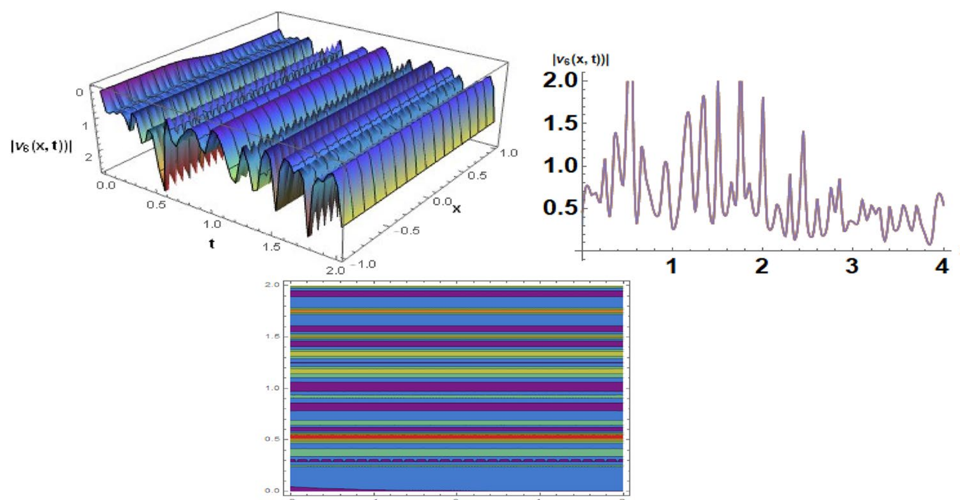


Fig. 6 The 3-dimensional, 2-dimensional, and contour representations for the solution $v_6(x, t)$ under the noise effect using the different values of constants such as $A = 1.2$, $\epsilon = 0.5$, $\alpha_1 = 1$, $\alpha_2 = 2$, $b_1 = 1.4$, $b_0 = 0.9$, $B = 1.4$, $\delta_2 = 0.2$, $l = 3.2$

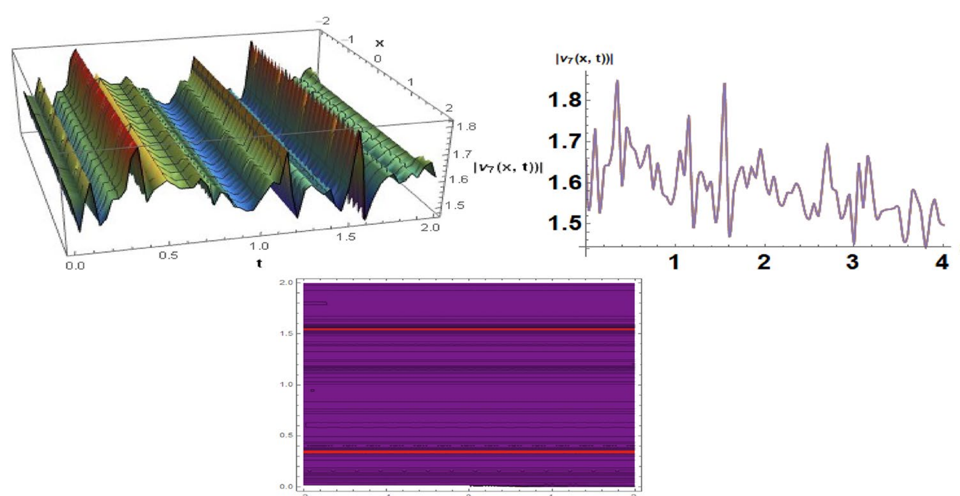


Fig. 7 The 3-dimensional, 2-dimensional, and contour representations for the solution $v_7(x, t)$ under the noise effect using the different values of constants such as $A = 1.9$, $\epsilon = 0.01$, $\alpha_1 = 1$, $\alpha_2 = 2.9$, $b_1 = 0.9$, $b_0 = 0.9$, $B = 3.9$, $\delta_2 = 0.9$, $l = 0.2$

and its physical representation, which finds value in chemistry, biology, and materials science. This could be illustrated by displaying random noise or fluctuations in the system, possibly by varying the reaction or diffusion rates at various times. In the stochastic Gray-Scott model, constructing physical representations of solitary wave solutions is a challenging and computationally demanding operation. Proficiency in data visualization, numerical simulations, and mathematical

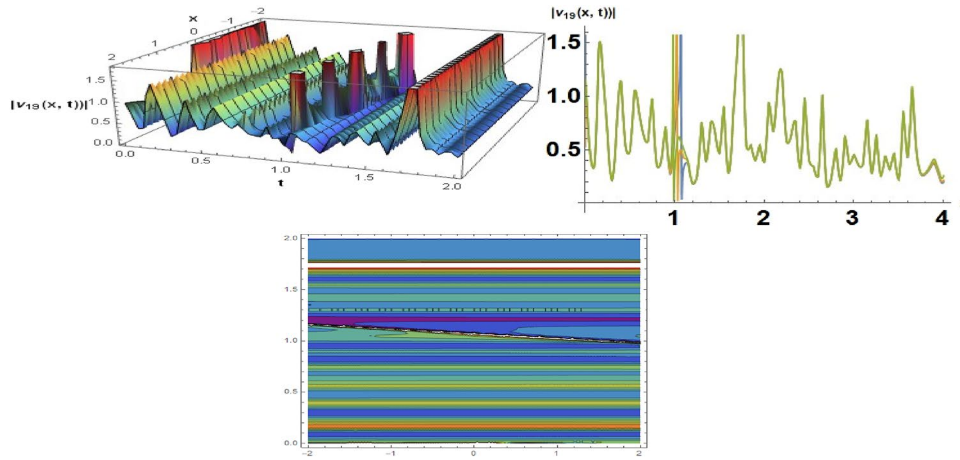
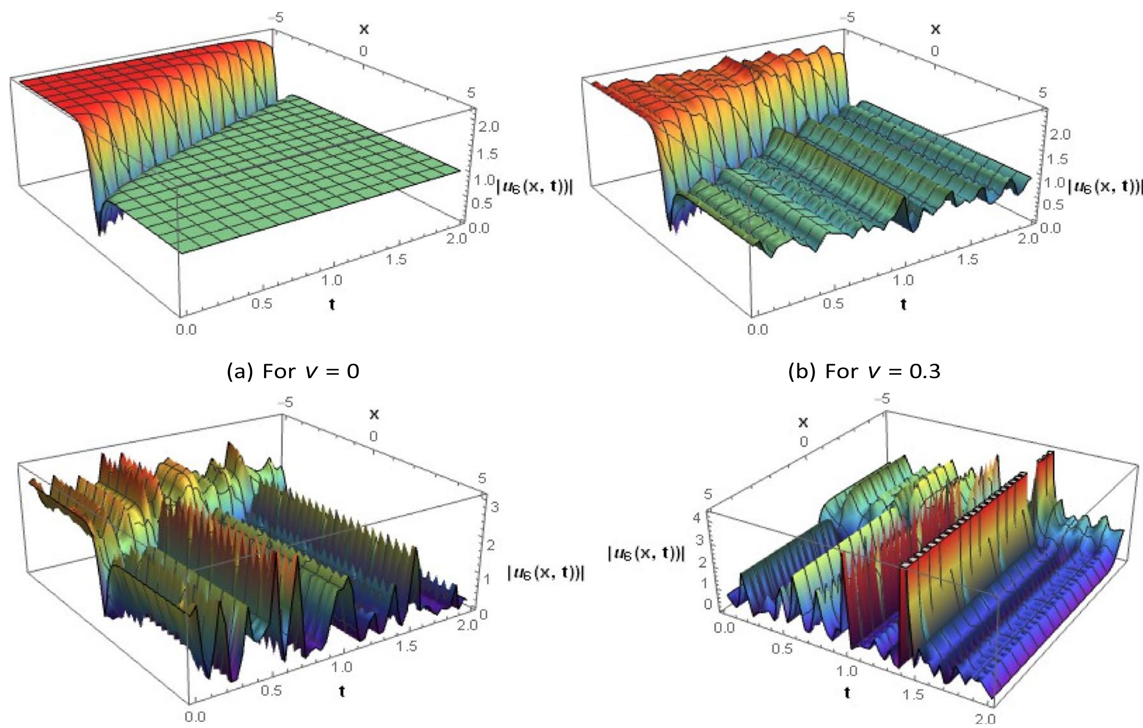


Fig. 8 The 3-dimensional, 2-dimensional, and contour representations for the solution $v_{19}(x, t)$ under the noise effect using the different values of constants such as $A=1.2$, $\epsilon=0.5$, $\alpha_1=1.7$, $B=1.3$, $\delta_0=0.9$, $l=0.2$

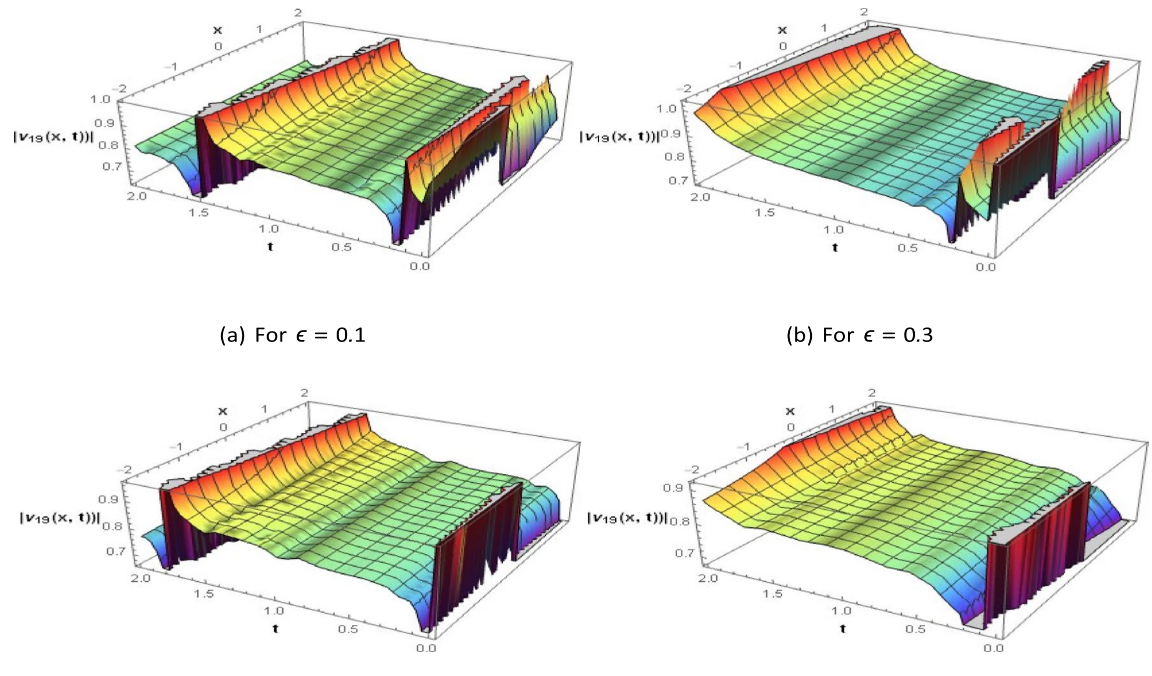


(a) For $v = 0$ (b) For $v = 0.3$

(c) For $v = 0.8$ (d) For $v = 0.9$

Fig. 9 The subfigures (**a**–**d**) show the 3-dimensional behaviors under the different noise strengths

modeling will be required. The different solutions are plotted in the 3-dimensional, 2-dimensional, and their corresponding contour representations respectively. The Fig. 1 is plotted for the solution $u_1(x, t)$ while Fig. 2 for the solution $u_6(x, t)$. Figures 3,4 are plotted for the solutions $u_{16}(x, t)$ and $u_{17}(x, t)$ respectively. Figures 5,6,7,8 are drawn for the solutions $v_1(x, t)$, $v_6(x, t)$, $v_7(x, t)$ and $v_{19}(x, t)$ respectively. Moreover, Fig. 9 shows the different behaviors of the noise which shows that if we increase the values of ν the noise increases in the physical system. Figure 9a is drawn for the $\nu = 0$ which is a classical solution without randomness so the solution provides us the dark soliton solution. Further, we increase the value of $\nu = 0.3$ in Fig. 9b which disturbs the shape a little much and involves the randomness in the behavior. Figures 9c and 9d are drawn for the values $\nu = 0.8$ and $\nu = 0.9$ respectively. To, show the effects of fractional order we dispatched Fig. 10 for the different values of ϵ. Figures 10a,10b,10c,10d are drawn for the $\epsilon = 0.1, 0.3, 0.7, 0.99$ respectively.



(a) For $\epsilon = 0.1$ (b) For $\epsilon = 0.3$

(c) For $\epsilon = 0.7$ (d) For $\epsilon = 0.99$

Fig. 10 The subfigures (**a**–**d**) show the 3-dimensional behaviors under the different fractional effects

6 Conclusions

In this research, we look into the abundant families of soliton solutions for the TFSGS model. The Gray-Scott model is analyzed under the M-truncated derivative and multiplicative time noise. This is a reaction–diffusion chemical concentration model that explains the irreversible chemical reaction process. The different abundant families of solutions are obtained by using the newly modified extended direct algebraic method. These solutions are explored in the form of shock, complicated solitary-shock, shock-singular, and periodic-singular types of single and combination wave structures. These are the exact traveling wave solutions that carry the chemical concentrations for the reactants under the reaction and diffusion process. Additionally, we plot our solutions and display many two-dimensional, three-dimensional, and contour graphs using MATHEMATICA 11.1 that demonstrate the capabilities of the influence of Brownian motion and fractional derivative on the soliton solutions of the TFSGS model. We show that the TFDGS model solutions are stabilized at around zero by the multiplicative Brownian motion. These wave solutions represent the chemical concentrations of the reactants. Further, this study is very helpful for the researchers to analyze this model for the dynamical study. The obtained results are very useful for examining and verifying the analytical solutions using numerical and experimental work in nonlinear dynamics. In future, such work can be applied to take deep insight of the stochastic fractional reaction–diffusion epidemic models, the stochastic fractional reaction–diffusion prey-predator models, stochastic fractional reaction diffusion nutrient algae models.

Author contributions All authors reviewed the manuscript.

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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