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Managing Physical Inventory and Return Policies for Omnichannel Retailing

Abstract: As the retail industry increasingly adopts an omnichannel strategy amidst the growth of e-commerce, this study explores the interplay between inventory and return policies in such a setting. Specifically, we focus on "return losses," defined as the losses incurred by retailers due to customer product returns. We target how these return losses impact retailers' profits and physical inventories. While previous research has mainly concentrated on cross-channel returns, little attention has been paid to how omnichannel return policies affect store inventory and profit. We posit that profit-maximizing retailers allow consumers to select their purchase channels based on utility but require returns to follow specific policies. In this paper, we model four return policies based on the channel of return: original purchasing channel return, offline return, online return, and cross-channel return. We use a newsvendor model to construct an optimal profit function that accounts for the additional profit from offline return, uncertainty demand, and inventory cost. Our analysis identifies the conditions under which certain return policies are beneficial or detrimental to omnichannel retailers. We discover that the retailer's product pricing, return losses, and consumer return hassle costs are the main factors influencing the best return policy and inventory policy decisions. Moreover, whether to increase or decrease store inventory and the relationship between physical inventory and return loss depend on the return policy, price, and return hassle cost. Numerical simulations support our findings, which offer practical guidance for omnichannel retailers aiming to optimize their inventory and return policies.

Keywords: omnichannel retailer; profit maximizing; return policies; return loss; physical inventory; optimization

1. Introduction

The advent of e-commerce presents a significant challenge to traditional offline retail channels in catering to diverse consumer demands (Bell et al., 2014; Neves-Moreira & Amorim, 2024; Rigby, 2011). In response, the retail industry has seen a growing trend towards omnichannel retailing, an approach integrating both online and offline channels for enhanced consumer convenience and competitiveness. Retail giants such as J.C. Penney, Walmart, Macy's, and JD.com have implemented such a mode. The main goal of omnichannel retail management is to give consumers a seamless, consumer-focused shopping experience, no matter where they are making purchases (e.g., online, physical stores, mobile apps, or websites). The basic concept behind this method is that it lets consumers freely move between channels. Previous empirical research conducted by Ansari et al. (2008) investigated how consumers transition between channels in a multi-channel environment. Further, Gallino et al. (2017) explored an omnichannel fulfillment option—shipping to stores—and found it led to an increase in the sales share of bottom-selling items across inventory units.

Certain challenges in retailing, such as the costly consumer returns that detrimentally affect profitability, are common across both traditional and omnichannel approaches. This issue is particularly pronounced in the evolving domain of omnichannel retailing, where the high return probability and associated costs of product returns further exacerbate their adverse impact on retailers' profit (Ofek et al., 2011). Recent surveys have highlighted a significant increase in retail return losses in the United States, with figures rising from approximately $428 billion in 2020 to $761 billion in 2021 (Appriss Retail, 2020; National Retail Federation, 2021). This trend underscores the critical need for retailers to develop and implement reasonable and efficient return policies. Current return policy research finds that cross-channel return is a typical policy, allowing consumers to return products purchased from any channel either to the offline store or by mail to the retailer online. Retailers offer two main types of return policies: in-store and online (i.e., mail-back, usually to a processing location). In practice, most retailers also adopt cross-channel return policies. For example, Walmart consumers who are not satisfied with the product they purchased can return the product to Walmart online or offline, based on their policy (Walmart, 2023). Macy's consumers can return online-purchased products through online and offline channels, but they can only return offline-purchased products to the store (Macy, 2023).

Most recent studies of returns focus on the factors that influence returns and return policies, including improving product quality to increase consumer satisfaction and reduce returns (Lawton, 2008; Ma et al., 2020) and offering free returns on consumer return behavior (Walsh & Möhring, 2020). However, these studies do not take into account how return policies affect inventory in physical stores. Research has been limited to examining the effects of different services and order quantities on optimal inventory and did not consider return policies. Our Suning case study reveals a significant challenge in omnichannel retailing: the inability to apply a universal return policy across all product types due to varying return probabilities and costs. This demonstrates the importance of tailoring return policies based on product categories and balancing operational efficiency with customer preferences. Our research aims to analyze the effects of these diverse return policies on inventory and profitability. We focus on identifying effective inventory strategies and the impact of consumer return losses on store inventories and omnichannel retailers' profits, thus extending the analysis of return policies within an omnichannel context.

This study primarily focuses on the impact of consumer return losses on store inventories and omnichannel retailers' profits. Since online consumers can only virtually assess a product before purchasing, they are unable to actually investigate products. Instead, they rely on pictures and product reviews, which may result in discrepancies between expectations and the actual product in terms of color, material, and size, leading to returns due to judgment errors. In contrast, offline consumers can physically evaluate products but still return them during the return period if quality issues are detected after purchase. Notably, prior research on omnichannel returns often overlooks the potential for products purchased during offline shopping to be returned (Gao & Su, 2017a, 2017b; Nageswaran et al., 2020). Therefore, the first aim of this study is to ascertain the optimal inventory strategies and profit for omnichannel retailers, considering different return policies.

While the literature suggests that consumers typically do not experience product shortages when shopping online (Gao & Su, 2017a, 2017b), there is a high rate of product returns in online purchases, with studies indicating that about one-third of online sales result in returns (Banjo, 2013). This phenomenon might reflect the unique aspects of online shopping experiences rather than a shortage of products. This high return rate is partly attributed to the inability of consumers to verify a product's qualities online. As a response, some consumers opt to purchase from physical stores to minimize the likelihood of returns (Dzyabura & Jgabathula, 2018; Kim et al., 2019). However, this shift to offline purchasing presents another challenge: physical stores often have limited inventory, which might not always meet every consumer's demand. Consequently, some consumers who visit physical stores may encounter out-of-stock situations, leading them either to resort to online shopping or to forgo their purchases. The second research question this study addresses is the influence of consumer returns on determining the optimal inventory levels under various return policies.

To cover a range of application scenarios, this study explores four return policies: original purchasing channel returns (i.e., returning a product through the original purchasing channel), offline returns (i.e., returning products in stores, irrespective of the purchase channel), online returns (i.e., returning products online, irrespective of the purchase channel), and cross-returns (i.e., consumers freely choose return channels). We aim to answer two key questions: (1) How do return hassle costs and return policies impact consumer channel choice and profits? (2) How do return losses and return policies affect retailers' physical inventories and profits? To answer these questions, we consider an omnichannel retailer that provides both online and offline channels. We assume that consumers are rational, basing their purchasing decisions on a utility comparison between the two channels.

Furthermore, research suggests that physical store visits lead to additional demand, presenting retailers with the opportunity to generate extra revenue (Nageswaran et al., 2020; Samorani et al., 2019). A study by UPS (2015) indicated that 45% of consumers visiting stores have additional consumption demands. Most sales in physical stores are generated through cross-selling (Gao, 2020). In addition, when returning products, the inconvenience, or "hassle cost", also affects consumers' choice of return channel (Nageswaran et al., 2020). When consumers purchase products online, they can return them by mailing them back to designated physical stores but may be required to pay a shipping fee (e.g., H&M charges a $5.99 return fee; H&M, 2023). Although consumers can return items to stores to avoid shipping costs, this may prove inconvenient. Hence, the aim is to investigate the influence of different return hassle costs on retailers' return policy, optimal profit, and consumers' channel choice.

In this study, we set up an omnichannel consumer utility model, utilizing the newsvendor model, to analyze how retailers' adoption of different return policies impacts consumer behavior and retailer profitability. Unlike previous research, our model uniquely integrates three critical factors: consumer return hassle cost, retailer return loss, and product pricing. This approach allows us to examine the influence of these factors on retailers' inventory management strategies and profitability, as well as on the effectiveness of various return policies. Our innovation lies in providing a better understanding of how return policies affect both consumers and retailers in an omnichannel setting. Consumers are given the freedom to choose their purchasing channels based on the total utility received, while simultaneously being constrained by the return policies set by retailers. By comparing profits under different return policies, we enable retailers to strategically select the most advantageous policy, thereby maximizing their profits and optimizing inventory management in response to both retailers' return losses and consumers' return hassle costs.

Our study provides two main contributions. First, we delve into the relationship between optimal inventory policies and return losses, which we define as the losses retailers incur due to customer product returns. In this exploration, we identify the most effective inventory levels under various return policies. Our research reveals that profit-maximizing retailers often benefit from reducing their inventory when confronted with return losses. However, this logic does not always apply to the offline return policy. Interestingly, we find it may be counterintuitive for retailers to increase offline inventory, particularly in scenarios where return losses escalate despite a low probability of returns. Additionally, our findings challenge traditional perceptions, suggesting that retailers might need to reduce their offline inventory even in circumstances where offline return losses are lower but accompanied by a high probability of returns. These novel insights provide a fresh perspective on how return losses affect inventory management in an omnichannel retail setting, challenging conventional wisdom.

Our study's second contribution is identifying the analytical, closed-form boundary conditions for optimal profit under different return policies. While offline returns generate additional demand incentives and might yield higher profits, retailers must consider channel prices, consumers' hassle costs, and return losses to make optimal policy choices The results indicate that consumers prefer purchasing products through channels with lower prices. Although several factors impact consumers' purchasing decisions and retailers' profits, retailers only need to compare return losses on the two channels to determine optimal return policies when consumers choose their purchase channel. The findings suggest that retailers should always adopt a return policy with fewer return losses. The findings suggest that retailers should always adopt a return policy with fewer return losses.

The rest of the paper is organized as follows. Section 2 presents the literature review on omnichannel retailing and inventory and product returns. In Section 3, we describe our model. Section 4 analyzes the return policy and its effects on retailers' physical inventory and profits under four return policies. The results and proposed optimal return policy based on the synthesized data are provided in Section 5. Section 6 extends our research by exploring cases where preferences for offline channels come into play. Section 7 concludes the paper, presenting a summary of the study and recommendations for additional research.

2. Literature Review

Our research closely aligns with two aspects of the omnichannel literature: pricing and inventory, and return policies. The literature review is thematically organized into these two categories.

2.1. Pricing and inventory

In omnichannel retailing, pricing encounters two significant challenges, as outlined by an eMarketer survey (2013): consumer sensitivity to price and price competitiveness from other retailers. Studies have shown that the direct channel can play a strategic role in driving down the price of the retail channel, making the manufacturer profitable (Chiang et al., 2003). Liu (2006) investigated the decision making of a physical retailer considering opening an online department, assuming consistent or different prices across channels. This study focuses on omnichannel retailers' optimal inventory and profit under different return policies and channel prices. Cattani (2006) found that different pricing methods should be adopted for different market structures, both online and offline, under the influence of online direct selling channels. Ofek et al. (2011) discussed how multi-channel operations affect pricing strategies in retailers that operate online channels and provide offline sales compensation. Kireyev et al. (2017) examined whether online and offline products from multi-channel retailers should have the same prices and the impact of self-matching pricing strategies on online and offline prices. Harsha et al. (2019) analyzed cross-channel fulfillment and optimal pricing policies for stores fulfilling online orders from various locations. Li and Jiang (2019) identified the optimal pricing for the suppliers and retailers and the effect of consumers' sensitivities to return policies on pricing strategies, product demand, and enterprise performance. Joshi et al. (2023) developed the best pricing techniques for retailers with three modes for selling their products: physical, online, and both.

In their study, Gallino et al. (2017) discovered that sales increased by offering free ship-to-store services, and thus, retailers increased inventories in their stores. Gao and Su (2017a, 2017b), presuming that online channels were not out of stock, focused on optimal inventories in physical stores. Kumar et al. (2019) examined store-participation effects and store-return effects and determined that allowing consumers to visit the store to evaluate products boosted offline purchases. However, none of these studies analyzed the retailers’ inventory with returns. Hendalianpour et al. (2020) examined perishables' demand, incorporating factors such as sales, reference pricing, product freshness, and safety inventory, using double-interval gray values for product sales price and safety inventory. Hu et al. (2022) explored the effects of Buy Online and Pick Up In-Store (BOPS) programs on store operations, particularly from an inventory perspective. Gökbayrak et al. (2023) investigated optimal inventory control policies for retailers in the context of product returns, addressing the challenges posed by the growth of e-commerce. Das et al. (2023) considered a retailer with multiple physical stores and an online presence and how an omnichannel fulfillment program to fulfill online orders can lead to greater cost reductions. Our paper extends the existing research by examining three key factors in the analysis of consumer return policies: consumer return hassle cost, retailer return loss, and product price. We aim to identify the impact of these factors on retailers' inventory management, profitability, and return policy.

2.2. Return policies

Previous research has investigated the impact of product quality and consumer satisfaction on return rates, and the relationship between return policies and sales. For example, Mukhopadhyay et al. (2007) noted that higher product quality led to increased consumer satisfaction, which in turn reduced the probability of return. Lawton (2008) found that product returns were triggered by defective quality or consumer dissatisfaction. Yoo (2014) investigated a joint decision on the return policy and product quality in a buyer-supplier supply chain. Zhou and Hinz (2016) concluded that offering a return policy can further increase sales. Letizia et al. (2017) suggested that return policies and sales channel structures are tightly interlinked and should be studied within a unified framework. Ertekin (2018) noted that a satisfactory in-store return experience could influence consumers’ exchange behavior during a return event and increase future purchases.

Additional research has explored various factors that contribute to product returns, including defective quality, dissatisfaction with products, and the impact of return policies on sales. Walsh and Möhring (2017) found that product reviews reduced return rates, and providing free return services had no impact on consumers’ return behavior. Akturk et al. (2018) studied the impact of ship-to-store on both retailer sales and consumer returns and showed that this approach increased cross-channel consumer returns. According to Khouja et al. (2019), consumers sometimes returned items or repurchased them to secure a discount when they were dissatisfied with the product or noted a price reduction. Ma et al. (2020) suggested that retailers offer a short-return window with a lower price to increase demand and profit when product quality decreases. Zhang and Choi (2021) examined the impact of sales tax on multi-channel strategies and social welfare and showed that consumer showrooms reduce product returns and might therefore increase social welfare. Patel et al. (2021) explored whether free returns can create economic value for online retailers and found that the gross profit generated by free returns was lower than the associated cost. Lei et al. (2022) investigated the impact of the presence of consumer resale on retailers’ return policies and profits and noted that retailers do not offer any returns when consumer resale transaction costs are lower than those of retailers, but retailers can profit from consumer resale. Balaram et al. (2022) observed that size issues are a common reason for clothing returns. Kalantary et al. (2023) designed four retail policies based on retailer and consumer preferences to examine their impact on retailers' optimal profits. Xiong and Liu (2023) explored how return insurance influences retailers' strategic decisions. Finally, Yang et al. (2023) examined consumer purchase and return decisions under the Buy Online and Return to Store (BORS) model, focusing on its impact on retailers' profit and inventory.

Recent studies have delved into the impact of sales tax, consumer resale, and offline return strategies on retailers' profits and return rates, as well as the effectiveness of offering free returns for online retailers. Ertekin and Agrawal (2021) determined that changes to a return-period policy do not influence sales and return rates for online stores. Cao et al. (2020) explored the optimal offline return policy and channel selection of retailers in contexts of both information symmetry and asymmetry. Hwang et al. (2022) found that partnerships between online retailers and physical stores offering consumers store return options increased the unique number of consumers, the quantity of goods sold, and net revenues across both channels. Liu et al. (2022) probed whether dual-channel merchants should allow returns through both channels and if they need to engage in contractual agreements with manufacturers to cover additional return costs. Gao et al. (2022) proposed that high return rates induced by online purchases motivate retailers to downsize their larger physical stores. Nageswaran et al. (2023) elucidated that stores and online retailers offering differentiated products have an incentive to collaborate on returns when it proves mutually beneficial.

Returns often result in losses for retailers, but strategic return policies can help mitigate these losses and even bring additional benefits. Gao and Su (2017b) pointed out the potential for additional purchases when consumers buy in stores. Samorani et al. (2019) suggested that consumers might purchase substitute products each time they return an item until they finally decide to keep (or abandon) the product. Zhao et al. (2020) compared the applicability and effectiveness of four return policies, contrasting those that required consumers to pay a return fee with policies that put the cost on retailers. Nageswaran et al. (2020) researched partial versus full refunds and their impact on retailers' pricing strategies and profits, finding that full refunds can generate extra income for retailers. Building on this line of research, we aim to explore the impact of such additional income on retailers' return policies.

This study, in contrast to the studies noted above, investigates how inventory changes when an omnichannel retailer adopts various return policies and considers the impact of retailer return losses as well as consumer return hassle costs. Our model gives consumers the freedom to select their purchasing channels according to the total utility they derive from these channels, but they are forced to adhere to the return policies set by retailers. By using the newsvendor model, we are able to quantify the profit function of retailers under different return policies and calculate their optimal inventory. The difference between this study and the previous research is summarized in Table 1.

Table1: A comparative summary of this study and previous publication

Literature	Return	Omnichannel	Inventory	Pricing	Research questions
Joshi et al. (2023)	×	×	×	✓	The channel choice and pricing decision for multichannel retailers
Chiang et al. (2003)	×	×	×	✓	Effect of direct sales channel on
Hendalianpour et al. (2020)	×	×	✓	✓	Impact of inventory levels and pricing on retailers' optimal decisions
Das et al. (2023)	×	✓	✓	×	Omnichannel Fulfillment Program
Nageswaran et al. (2020)	✓	✓	×	✓	Pricing and Return Strategies for Omnichannel Retailers
Hu et al. (2020)	×	✓	✓	×	Effect of the BOPS model on retailer profits
Gökbayrak et al. (2023)	✓	×	✓	×	Inventory control policy
Ofek et al. (2011)	✓	×	×	✓	Product Return
Lei et al. (2022)	✓	×	×	×	Consumer resale
Hwang et al. (2022)	✓	✓	×	×	Online-off-line return partnerships between third-party retailers and brick-and-mortar stores
Kireyev et al. (2017)	×	×	×	✓	Multichannel pricing strategy
Nageswaran et al. (2023)	✓	✓	×	×	Returns strategy
Gao et al. (2022)	✓	✓	×	×	The effect of multichannel and omnichannel retailing on physical
Gao and Su (2017a)	✓	✓	×	×	Buy-online-and-pick-up-in-store
Gao and Su (2017b)	×	✓	✓	×	How omnichannel retailers can effectively deliver online and offline
Kalantary et al. (2023)	✓	×	×	✓	Return policy
Zhang and Choi (2021)	✓	×	×	×	Multichannel strategies
Xiong and Liu (2023)	✓	×	×	✓	Return insurance
Yang et al. (2023)	✓	×	✓	✓	Returns and inventory strategy
Ertekin (2018)	✓	×	×	×	In-store return experience.
This study	✓	✓	✓	✓	The optimal returns and inventory strategy in an omnichannel context

3. Model Setup

In this study, we examine an omnichannel retailer that offers two channels for product sales: online and offline. To manage inventory costs, the retailer restricts the inventory available in the physical store. Drawing on the assumption by Gao and Su (2017a, 2017b), we anticipate that the offline channel may experience stockouts. Retailers sell products to consumers in a single period of time (Liu et al., 2021), when consumers encounter shortages offline, they may switch to online purchases where stockouts are not expected. Resale of returned products (Chiang et al., 2003; Ertekin & Agrawal, 2021; Liu et al., 2021) is not considered in this study. We assume that consumers can freely choose the channel they prefer to purchase products based on their individual preferences but must adhere to the retailer's return policies when returning unsatisfactory products. To maintain the model' s tractability and investigate the impact of return policies, we develop four distinct policies to optimize the omnichannel retailer's estimated profit.

1. The original purchasing channel return policy (i.e., consumer returns the product to the channel through which it was purchased, U_{BORO} and U_{BSRS}^{BORO});
2. Offline return policy (i.e., consumers return products offline no matter which channel they buy through, U_{BORS} and U_{BSRS}^{BORS});
3. Online return policy (i.e., consumers return products online no matter which channel they buy through, U_{BORO} and U_{BSRO}^{BORO});
4. Cross-channel return policy (i.e., consumers choose a return channel according to the principle of maximizing their own utility, U_{BORO}, U_{BORS}, U_{BSRS}^{BORO}, U_{BSRS}^{BORS}, U_{BSRO}^{BORS}, and U_{BSRO}^{BORO}).

To effectively differentiate consumer behaviors across various purchasing channels and return policies, our study employs a unique notation system in representing the total utility function, denoted as 'U_X^Y'. This system simplifies the understanding of diverse purchasing and return scenarios. Specifically, online purchases are represented by a utility function with a subscript 'X' only, comprising four capital letters. The structure of 'X' is designed to convey specific details of the transaction: the first two letters indicate the purchasing channel, and the last two letters represent the return policy. In contrast, offline purchases are denoted by a utility function with both a subscript 'X' and a superscript 'Y', with 'Y' mirroring the structure of 'X'. For instance, the utility function U_{BORO} represents a scenario where a consumer makes an online purchase and follows the retailer's online return policy, commonly known as Buy Online Return Online (BORO). Another example is the utility function U_{BSRS}^{BORS}, which denotes that the consumer initially purchases offline (indicated by the subscript BSRS, signifying an offline purchase and return). If the offline store is out of stock, the consumer then opts for an online purchase, as denoted by BORS (Buy Online, Return to Store). This notation system is integral in analyzing and interpreting the varied consumer behaviors in our omnichannel retail study.

Consumers will acquire a valuation for the product when they make a purchase. The rapid development of omnichannel has revolutionized consumers' online purchasing experiences. Because of this, some consumers (Frambach et al., 2007; Liang & Huang, 1998) prefer to make in-person purchases over online options, while others believe that online consumption is more convenient (Shankar et al., 1999; Cook & Coupey, 1998; eMarketer, 2019). Our study differs from those of Gao and Su (2017a, 2017b) in that we consider heterogeneity in consumer perceptions of the product valuation, while they assumed homogeneity. To ensure that the model will remain analytically tractable, this paper employs the conventional assumption (Chiang, 2003; Nageswaran et al., 2020), and assumes both online and offline V has the same distribution to build the consumers' expected utility, where V is uniformly distributed on the interval [0,1]. Suppose the utility of consumers' purchasing from the online and offline channels is U .When purchasing online, consumers browse the product website to place an order, and we assume that the online channel will meet their needs. When purchasing offline, consumers arrive at the store to choose products, provided that the store is not out of stock; however, if the store is out of stock, consumers turn to online purchases, and the online channel will satisfy their needs (Gan & Su, 2017a, 2017b). Consumers' beliefs that the offline store has inventory is represented as ξ , and the probability that consumers purchase in the store is ϑ (here, $\vartheta \in \{0,1\}$). When rational consumers buy online, $\vartheta = 0$; when they buy offline, $\vartheta = 1$.

In the omnichannel retail context, we assume that the offline inventory quantity of retailers is q (Liu et al., 2021). If consumers return a product through the online channel, they mail the product back to a designated location; if consumers return through the offline channel, they need to take it to a physical store. Both return channels incur return hassle costs for consumers. When returning goods offline, consumers need to visits the physical store or communicate with a sales associate, collectively referred to as the hassle cost of offline return and represented by h_s . Similarly, when returning goods online, consumers need to log in to the website and wait for consumer service to process the return so that the consumer can bring the goods to, for example, UPS, which is collectively referred to as the hassle cost of online return, as represented by h_o (Hsiao & Chen, 2014).

In this paper, p_0 and p_s represent the online and offline unit price of the product, respectively, and c_0 and c_s represent the online and offline inventory cost, respectively (without loss of generality, there is $c_o < c_s < \min\{p_o, p_s\}$). Here, r_o and r_s represent online and offline unit return losses for retailers, respectively (Granot, 2005). Assuming that the total demand for consumers in the market is *D*, the corresponding density function of demand is $f(D)$, and the demand distribution function is $F(D)$ (Hendalianpour et al., 2021). Note consumers may find products out of stock when they visit the store; thus, ED_{in} represents the expected demand for products in stock when consumers arrive at the store, and ED_{out} denotes the expected demand for products when consumers arrive at the store and they are out of stock. Following Nageswaran et al. (2020), the study assumes that consumers make an additional purchase when they purchase in a physical store. Thus, the additional demand stimulated factor from the offline purchase is α (here, $\alpha \geq 1$). Table 3 provides a summary of our notation for parameters and variables in modeling (Please refer to Appendix A for details).

4. Optimal Return Policy Analysis

This section analyzes four return policies, including the retailers' adoption of online and offline operation modes. Section 4.1 discusses the optimal inventory and profit under the original purchasing channel return policy. The unified offline return policy is analyzed in Section 4.2, and the online return policy is examined in Section 4.3. Further, Section 4.4 investigates the scenario where consumers are free to return goods (i.e., cross-channel) based on their own utility. The omnichannel retailers' corresponding optimal store inventory and expected profit are proposed. The optimal profit for the four return modes is compared to provide an optimal return policy for an omnichannel retailer, and insights are summarized in Section 4.5.

4.1. Original purchasing channel return policy

In the realm of retail, the implementation of an original channel return policy can significantly shape consumer behavior and logistics. A case in point is the policy adopted by Jingdong (detailed at https://help.jd.com/user/issue/327-988.html), which requires consumers to comply with specific return procedures. Under this policy, if a consumer decides to return a product, they are required to do so through the same channel through which they made the original purchase. Consequently, consumers have two options:

1. Buy online and (possibly) return online. In this situation, consumers buy online directly if the product fails to satisfy consumers' expectations and will be returned; hence, the consumers' expected utility is $U_{BORO} = E_{\max}\{V, p_o - h_o\} - p_o$ (Nageswaran et al., 2020).
2. Buy offline and (possibly) return offline. There are two situations: (1) when consumers arrive at the store and the store has inventory, and (2) when consumers arrive at the store but the store is out of stock, in which case some consumers will switch to online purchase and (possibly) return online. If the product fails to meet consumers' expectations, they might return it, yielding the expected utility, as shown by $U_{BSRS}^{BORO} = \xi\{E_{\max}\{V, p_s - h_s\} - p_s\} + (1-\xi)[U_{BORO}]$.

Figure 1 shows that due to the uncertainty of the consumer's valuation V of the product, the consumer may perceive a high product valuation or a low product valuation. Therefore, the consumer will choose to return or retain the product based on his or her valuation of the product. The utility achieved when returning the product is the price p minus the hassle cost h of returning the product. When the consumer values the product less than the utility of returning it ($p-h$; i.e., $V \le p-h$), he or she chooses to return the product. Conversely, if the consumer's valuation of the product is more than the utility of returning the product ($p-h$; i.e., $V > p-h$), the consumer opts to keep it. Where $p_o - h_o$ and $p_s - h_s$, the terms also represent the probability that a consumer returns the product online and offline, respectively. To account for the heterogeneity in consumers' valuations, we assume that the variable V is uniformly distributed over the interval [0, 1].

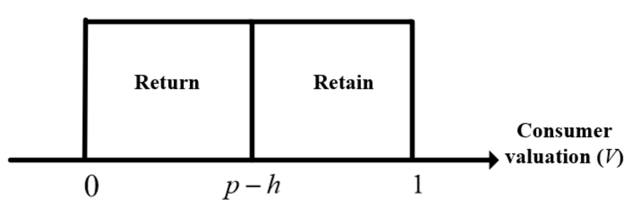

Figure 1. Consumer heterogeneity valuation of products

Hence, considering the uncertainty of the consumer's perception of the product's value, price, and return hassle costs, the utility obtained from purchasing through the two channels is shown in equation (1):

$$\begin{cases} U_{BORO} = \underbrace{E_{\max}\{V, p_o - h_o\} - p_o}_{purchase\ \ online} \\ \quad = \int_0^{p_o - h_o} (p_o - h_o) dv + \int_{p_o - h_o}^{1} V dv - p_o = \left((p_o - h_o)^2 + 1\right)/2 - p_o \\ U_{BSRS}^{BORO} = \underbrace{\xi\{E_{\max}\{v, p_s - h_s\} - p_s\}}_{purchase\ \ in-store} + \underbrace{(1-\xi)[U_{BORO}]}_{out\ \ of\ \ stock\ \ online\ \ purchase} \\ \quad = \xi\left\{\int_0^{p_s - h_s} (p_s - h_s) dv + \int_{p_s - h_s}^{1} V dv - p_s\right\} + (1-\xi)[U_{BORO}] \\ \quad = \xi\left\{\left((p_s - h_s)^2 + 1\right)/2 - p_s\right\} + (1-\xi)\left((p_o - h_o)^2 + 1\right)/2 - p_o \end{cases} \tag{1}$$

When consumers choose not to buy the product, the corresponding utility is 0. Due to the difference in price and return cost of different channels and whether purchasing online when the store is out of stock, the utility of consumers varies across different channels. A rational consumer's choice of shopping channel is governed by the total utility derived from each option. If the utility gained from online shopping exceeds that obtained from offline channels, the consumer will opt for an online purchase. Conversely, if offline shopping offers greater utility, they will favor visiting brick-and-mortar stores. In equation (1), U_{BORO} and U_{BSRS}^{BORO} denote the utility for the consumer who purchases online and offline, respectively. In this case, in equation (1), $\xi = \min E\left(q, \vartheta D\right)/E\left(\vartheta D\right)$ (Gao & Su, 2017a). Rational consumers, when $U_{BORO} > U_{BSRS}^{BORO}$ and $U_{BORO} \geq 0$ hold, use online channels for purchases and return products; and if $U_{BORO} \leq U_{BSRS}^{BORO}$ and $U_{BSRS}^{BORO} \geq 0$ hold, they opt to buy a product in a store, but they may return it to the store due to the uncertainty in the product valuation. If the product is out of stock in the store, the consumer switches to an online purchase and returns the product to the online channel. When $U_{BORO} = U_{BSRS}^{BORO}$, consumers receive the same utility on two channels. For simplicity, we assume that consumers bought offline under this condition.

Thus, the expected profit functions for the retailer are shown as follows:

$$\begin{aligned} \pi_1(q) = {} & \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ & + \vartheta[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ & + (1-\vartheta)[(p_o - c_o - r_o(p_o - h_o))ED] \end{aligned} \tag{2}$$

Following Nageswaran et al. (2020), we assume that α indicates the additional demand stimulated factor generated by consumers' offline purchases. Given a store's inventory level, the expected profit earned by the retailer through the store channel is shown in the first two terms of equation (2). The first term represents the retailers' profit when consumers arrive at the store and the inventory is available. The second term shows the retailers' profit when consumers arrive at the store, but the product is out of stock. The third term represents the profit for the retailer when consumers buy online directly.

Here, π_1 denotes the profit for the omnichannel retailer operating under an original channel return policy. Under this policy, the retailer anticipates that "$D_{in} = \min\{q, \vartheta D\}$" number of consumers will find their desired product in stock at the store. However, another group of consumers, totaling "$D_{\text{out}} = (\vartheta D - q)^+$", will encounter an out-of-stock situation in the store. The total number of consumers, is thus the sum of these two groups, expressed as $D_{in} + D_{out} = D$. This yields Lemmas 1 and 2.

Lemma 1. The consumer's purchasing channel choice is conditional:

(i) When $h_o < p_o - \sqrt{2p_o - 1 + 2\{((p_s - h_s)^2 + 1)/2 - p_s\}}$ holds, $\max\{U_{BORO}, U_{BSRS}^{BORO}, 0\} = U_{BORO}$; in this case, all consumers choose online purchasing;

(ii) When $p_o - \sqrt{2p_o - 1 + 2\{((p_s - h_s)^2 + 1)/2 - p_s\}} < h_o < p_o - \sqrt{2p_o - 1 - 2\frac{\xi}{1-\xi}\{((p_s - h_s)^2 + 1)/2 - p_s\}}$ holds, $\max\{U_{BORO}, U_{BSRS}^{BORO}, 0\} = U_{BSRS}^{BORO}$, and all consumers prioritize purchasing from the store.

Lemma 1 indicates that the consumer's return hassle cost plays a crucial role in determining purchasing channels. Therefore, consumers will choose online purchasing if the return hassle cost meets the condition in part (i) of Lemma 1, to minimize potential losses. However, if the hassle cost of consumers meets the condition in part (ii) of Lemma 1, consumers are likely to purchase from offline stores first. By optimizing equation (2), the following results were generated (see the appendix for the derivations):

Lemma 2. The optimal inventory and profit of the retailers under the original purchasing channel return policy are:

1) If consumers purchase online (i.e., satisfy the condition of part (i) of Lemma 1), the optimal inventory and optimal profit are:

$$\begin{cases} q_1^{O*} = 0 \\ \pi_1^{O*}(q_1^{O*}) = (p_o - c_o - r_o(p_o - h_o))ED \end{cases} \tag{3}$$

2) If consumers purchase in-store (i.e., satisfy the condition of part (ii) of Lemma 1), the optimal inventory and optimal profit are:

$$\begin{cases} q_1^{S*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha[p_s - r_s(p_s - h_s)] - (p_o - c_o - r_o(p_o - h_o))}\right) \\ \pi_1^{S*}(q_1^{S*}) = \alpha\{[p_s - r_s(p_s - h_s)]ED_{in} - c_s q_1^{S*}\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \end{cases} \tag{4}$$

To distinguish profit in different situations, we express profit as π_λ^μ and inventory as q_λ^μ. The different values of μ in the superscript indicate consumers' purchasing from different channels. An instance of $\mu = O$, $\mu = O_1$, or $\mu = O_2$ indicates that rational consumers buy online, and $\mu = S$, $\mu = S_1$, or $\mu = S_2$ indicates that rational consumers buy offline first. Furthermore, the value in the subscript indicates that $\lambda = 1$, $\lambda = 2$, $\lambda = 3$, and $\lambda = 4$, which means that the retailer provides an original purchasing channel return policy, offline return policy, online return policy, and cross-channel return policy, respectively. Here, $x = \overline{F}^{-1}(a)$ denotes the solution of $1 - F(x) = a$ with respect to x (Gao & Su, 2017a).

Lemma 2 reveals the optimal store inventory characteristic with additional income and online and offline return loss. Further, the following properties can be obtained in Proposition 1.

Proposition 1. *The properties of optimal inventory of the retailers under the original purchasing channel return policy are:*

1) *When consumers purchase online, the optimal store inventory is 0.*
2) *When consumers buy offline, the optimal inventory satisfies the following results under the original channel return policy.*
   i. *The optimal inventory (q_1^{S*}) increases in online return loss r_o (i.e., $\partial q_1^{S*} / \partial r_o > 0$).*
   ii. *The optimal inventory (q_1^{S*}) decreases with offline return loss r_s (i.e., $\partial q_1^{S*} / \partial r_s < 0$).*
   iii. *The optimal inventory (q_1^{S*}) increases in additional demand stimulated factor α (i.e., $\partial q_1^{S*} / \partial \alpha > 0$).*
   iv. *The optimal inventory (q_1^{S*}) increases in offline channel price p_s (i.e., $\partial q_1^{S*} / \partial p_s > 0$).*

The first part of Proposition 1 shows that, when all consumers choose to buy products online, there is no need for physical inventory in stores. The second part of Proposition 1 states that with price increases, retailers should increase inventory in their stores to ensure that consumers who choose offline channels can find the products in stores. In addition, with offline return losses increasing, retailers should reduce store inventory. When consumers opt for in-store purchasing, it may stimulate additional demand for retailers. Therefore, with an increase in additional demand, retailers should increase store inventory. The more that consumers purchase offline, the higher extra revenue and the more physical inventory; hence, retailers should increase the inventory of offline stores to achieve more profit, based on the assumption that consumers will make additional purchases when they purchase in-store and the inventory is available.

Proposition 1 illustrates how offline prices affect the ideal inventory. Increasing store inventory is essential because, when consumers decide to buy in person, the rise in offline pricing can increase retailers' profit per unit of product. Retailers might have a limit on how high they can raise their prices, however, to avoid consumer losses. Consumers do not know about the online return loss of retailers, so their choice to buy offline or online would not be related to the return loss of the retailer. Instead, they make their choice based on the principle of utility maximization.

4.2. Offline return policy

When an omnichannel retailer adopts an offline return policy, as exemplified by Amazon (https://www.amazon.com/gp/help/customer/display.html?ref_=hp_left_v4_sib&nodeId=GL7EJJE5PXVGT6XS), consumers must return any unsatisfactory merchandise to a Hub Center physical store, regardless of the original purchase channel. The consumer utility function under this scenario is detailed in Eq. (5).

$$\begin{cases} U_{BORS} = E_{\max}\{V, p_o - h_s\} - p_o = \left((p_o - h_s)^2 + 1\right)/2 - p_o \\ U_{BSRS}^{BORS} = \xi\{E_{\max}\{V, p_s - h_s\} - p_s\} + (1-\xi)[U_{BORS}] \\ \qquad\quad = \xi\left\{\left((p_s - h_s)^2 + 1\right)/2 - p_s\right\} + (1-\xi)\left\{\left((p_o - h_s)^2 + 1\right)/2 - p_o\right\} \end{cases} \tag{5}$$

Unlike what is shown in equation (1), here, consumers can return goods only through the offline channel, so the consumer's return hassle cost is h_s, no matter whether consumers purchased through an online or offline channel. Therefore, with this return policy, the retailer only needs to consider the offline return losses. Similarly, the profit function for the retailer is illustrated in equation (6).

$$\begin{aligned} \pi_2(q) = {} & \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ & + \vartheta[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\ & + (1-\vartheta)[(p_o - c_o - r_s(p_o - h_o))ED] \end{aligned} \tag{6}$$

Then, given the consumer utility function and the retailer's profit function, the consumers' channel selections, optimal inventory, and profit of the retailer can be analyzed. Thus, Lemmas 3 and 4 can be obtained as follows (see the appendix for the derivations):

Lemma 3. The conditional consumer's purchasing channel choice under the offline return policy is:

(i) When $p_o < p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$ hold, $\max\{U_{BORS}, U_{BSRS}^{BORS}, 0\} = U_{BORS}$, and all consumers choose online purchasing;

(ii) When conditions $p_o \ge p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$ are met, $\max\left\{U_{BORS}, U_{BSRS}^{BORS}, 0\right\} = U_{BSRS}^{BORS}$, and all consumers choose physical store purchasing first.

Similar to Lemma 1, Lemma 3 suggets that consumers will prefer an online purchase if their hassle costs match the conditions in part (i) of Lemma 3. If their hassle costs on two channels satisfy part (ii) of Lemma 3, however, consumers are more likely to choose offline stores first.

Lemma 4. The optimal inventory and profit for the retailers under the offline channel return policy are:

1) If consumers purchase online (i.e., the constraint fits part (i) of Lemma 3), the optimal physical inventory and the optimal profit of retailers are:

$$\begin{cases} q_2^{O*} = 0 \\ \pi_2^{O*}(q_2^{O*}) = (p_o - c_o - r_s(p_o - h_o))ED \end{cases} \tag{7}$$

2) If consumers purchase in-store (i.e., the constraint fits part (ii) of Lemma 3), the optimal inventory and the optimal profit for retailers are represented in equation (8).

$$\begin{cases} q_2^{S*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))}\right) \\ \pi_2^{S*}(q_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out} \end{cases} \tag{8}$$

For an offline return policy, the retailer's optimal store inventory has no relationship with online return losses. Similarly, based on the optimal inventory of Lemma 4, the following further relationships between the optimal inventory and offline return losses are shown in Proposition 2.

Proposition 2. *Under the offline return policy, the following features of the retailers' optimal inventory are:*

1) *If the store price is higher than the online price (i.e., $p_o < p_s$), all consumers will choose to purchase online. In this case, the physical store inventory is zero, and the retailer's profits are affected by a combination of offline return losses, return hassle costs, and price.*
2) *If the offline channel price is lower than the online price (i.e., $p_o \geq p_s$), all consumers will initially opt to purchases from a physical store, which results in the outcomes listed below:*
   i. *The optimal inventory (q_2^{S*}) increases in the offline channel price (i.e., $\partial q_2^{S*} / \partial p_s > 0$).*
   ii. *The optimal inventory (q_2^{S*}) increases in the additional demand stimulated factor (i.e., $\partial q_2^{S*} / \partial \alpha > 0$).*

   The relationship between the optimal inventory and offline return losses is demonstrated as follows:
   iii. *if $(p_o - h_o) > \alpha(p_s - h_s)$, the optimal inventory q_2^{S*} increases with r_s (i.e., $\partial q_2^{S*} / \partial r_s > 0$).*
   iv. *if $(p_o - h_o) \leq \alpha(p_s - h_s)$, the optimal inventory q_2^{S*} decreases with r_s (i.e., $\partial q_2^{S*} / \partial r_s < 0$).*

Proposition 2 illustrates that when all consumers choose offline purchase, the relationship between optimal physical inventory and offline return loss is affected by online and offline prices and offline return hassle costs. In this case, consumers who favor buying offline would switch to the online channel when the product is out of stock in the store. When all consumers buy online, however, retailers suffer high losses due to the high probability of online returns. As a result, retailers' physical inventory increases with offline return losses to reduce the loss of returns from consumers who move online due to stock shortages. In contrast, if there is a low probability of online returns (i.e. $(p_o - h_o) \leq \alpha(p_s - h_s)$), as offline return losses rise, the retailer should reduce physical store inventory to encourage more consumers to use the online channel. This lowers the return rate and prevents the retailer from suffering greater losses as a result of higher offline return losses.

Similar to Proposition 1, if consumers opt to make their purchases in physical stores and the additional demand stimulated factor is high, the additional demand created by consumers who make their purchases in physical stores can generate considerable profit for retailers; thus, the inventory should be increased.

4.3. Online return policy

Similarly, in a scenario where only online return is permitted, such as on Alibaba's 1688 platform (a prominent e-commerce platform under Alibaba), consumers are required to return products through mailing services. An example can be seen in their return policy (https://rule.1688.com/?spm=a26304.12183230.0.0.6691665bJRaM1F&type=detail&ruleId=11005545&c

Id=3032#/rule/detail?ruleId=11005545&cId=3032), where unsatisfactory products must be returned via mail. The corresponding expected profit function for the retailer in this case is outlined as follows:

$$\begin{aligned}\pi_3(q) = {} & \vartheta\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ & + \vartheta[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ & + (1-\vartheta)[(p_o - c_o - r_o(p_o - h_o))ED]\end{aligned} \tag{9}$$

Given the consumer utility function and the omnichannel retailer's profit function, the consumers' channel selections, optimal inventory, and profit of the retailer can be analyzed. This provides us with Lemmas 5 and 6 (see Appendix A for the derivations):

Lemma 5. The conditional consumer's purchasing channel choices under the online return policy are:

(i) When conditions $p_o < p_s$ and $h_o < p_o - \sqrt{2p_o - 1}$ are met, $\max\left\{U_{BORO}, U_{BSRO}^{BORO}, 0\right\} = U_{BORO}$, and all consumers choose online purchasing;

(ii) When conditions $p_o \geq p_s$ and $h_o < p_o - \sqrt{2p_o - 1}$ are met, $\max\left\{U_{BORO}, U_{BSRO}^{BORO}, 0\right\} = U_{BSRO}^{BORO}$, and all consumers choose to enter physical stores for purchasing first.

Similar to Lemma 3, if conditions of part (i) of Lemma 5 are satisfied, all consumers will opt for online purchasing. If conditions of part (ii) of Lemma 5 are fulfilled, however, all consumers are more likely to purchase from offline stores first.

Lemma 6. The optimal physical inventory and expected profit of the omnichannel retailers under the online channel return policy are:

1) If the consumers purchase online (i.e., satisfy conditions of part (i) of Lemma 5), the optimal physical inventory and the optimal expected profit are shown as follows:

$$\begin{cases} q_3^{O*} = 0 \\ \pi_3^{O*}(q_3^{O*}) = (p_o - c_o - r_o(p_o - h_o))ED \end{cases} \tag{10}$$

2) If all consumers prefer purchasing in-store (i.e., meet conditions of part (ii) of Lemma 5), the optimal physical inventory and optimal expected profit are:

$$\begin{cases} q_3^{S*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))}\right) \\ \pi_3^{S*}(q_3^{S*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \end{cases} \tag{11}$$

If retailers offer an online return policy, the optimal store inventory has no relationship with offline return losses. Proposition 3, based on the optimal inventory of Lemma 6, explains the relationships between the ideal inventory and online return losses.

Proposition 3. *Under the online return policy, the following features of the retailers' optimal inventory are:*

1) *If the offline channel price is higher than the online price, consumers will naturally choose online purchasing. In this case, the physical store inventory is zero. Then, the retailer's profits are affected by a combination of online return losses, return hassle costs, and price.*
2) *If the store price is lower than the online price, consumers will prefer physical stores to make a purchase. Then, the following results will be obtained:*
   i. *The optimal inventory (q_3^{S*}) increases with the offline channel price (i.e., $\partial q_3^{S*} / \partial p_s > 0$).*
   ii. *The optimal inventory (q_3^{S*}) increases with the additional demand stimulated factor (i.e., $\partial q_3^{S*} / \partial \alpha > 0$).*

   The relationship between the optimal inventory and online return losses is demonstrated as follows:
   iii. *if $(p_o - h_o) > \alpha(p_s - h_s)$, the optimal inventory q_3^{S*} increases with r_o (i.e., $\partial q_3^{S*} / \partial r_o > 0$).*
   iv. *if $(p_o - h_o) \le \alpha(p_s - h_s)$, the optimal inventory q_3^{S*} decreases with r_o (i.e., $\partial q_3^{S*} / \partial r_o < 0$)*

Similarly, Proposition 3 indicates that, when retailers adopt the online return policy, if there is a high probability of online returns (i.e., $(p_o - h_o) > \alpha(p_s - h_s)$), to mitigate the loss caused by returns, retailers will increase inventory in physical stores to satisfy consumers as much as possible. In contrast, retailers should decrease their physical inventory to encourage consumers to make online purchases if the probability of an online return is small (i.e., $(p_o - h_o) \le \alpha(p_s - h_s)$).

As indicated in Propositions 1, 2, and 3, if consumers choose to buy offline and a retailer adopts an original purchasing channel return policy, the retailer's physical store inventory should decrease (resp., increase) with the offline (resp., online) return losses increase (i.e., $\partial q_1^{S*} / \partial r_s < 0$ and $\partial q_1^{S*} / \partial r_o > 0$), thereby reducing the potential loss from returns. When a retailer adopts an offline return policy, if the probability of offline returns is low, the retailer should increase inventory with the increase of offline return losses (i.e., $\partial q_2^{S*} / \partial r_s > 0$) to avoid consumers' switching to online purchasing, resulting in a large number of returns. Similarly, when retailers adopt an online return policy and when the probability of online returns is low, to lower the loss caused by returns, physical store inventory should be reduced with the increase of the online return loss (i.e., $\partial q_3^{S*} / \partial r_o < 0$), thus inducing consumers to switch to the online channel by taking advantage of the lack of physical store inventory, avoiding a large number of offline returns.

4.4. Cross-channel return policy

In reality, as retailing evolves to be more consumer-centric, the trend of allowing consumers the freedom to choose their own buying and returning channels is increasing. For instance, Walmart (https://www.walmart.com/help/article/start-return-online/a760300a0bfb4cd4891e43f092c8bd18) employs a cross-channel return approach, enabling consumers to select their preferred channels for both purchasing and returning. Under this approach, it is assumed that all rational consumers will make their purchase and return channel selections based on their utility. Therefore, recalling equations (1), (5), and (9), the consumer's expected utility functions now include a new option U_{BSRO}^{BORS} in addition to including U_{BORO},

U_{BORS}, U_{BSRS}^{BORO}, U_{BSRS}^{BORS}, and U_{BSRO}^{BORO}. For instance, consumers might typically choose to purchase from physical retailers first and return the unsatisfactory product online. Alternatively, when there is a shortage, consumers will switch to the online channel for purchasing and return the unsatisfactory product offline.

Therefore, recalling equations (1), (5), and (9), the new utility function of consumers is constructed as shown in equation (12).

$$\begin{aligned} U_{BSRO}^{BORS} &= \xi\{E_{\max}\{V, p_s - h_o\} - p_s\} + (1-\xi)[U_{BORS}] \\ &= \xi\left\{\left((p_s - h_o)^2 + 1\right)/2 - p_s\right\} + (1-\xi)\left\{\left((p_o - h_s)^2 + 1\right)/2 - p_o\right\} \end{aligned} \tag{12}$$

The optimal inventory and store profit are examined by utilizing the consumer utility function and the channel prices. Thus, Lemmas 7 and 8 can be obtained as follows (see the appendix for the derivations):

Lemma 7. The conditional consumer's purchasing channel choices under the cross-channel return are:

(i) When $p_o < p_s$ and $h_o < h_s < p_o - \sqrt{2p_o - 1}$ hold, $U_{\max} = U_{BORO}$, and all consumers choose to buy online and return online.

(ii) When $p_o \geq p_s$ and $h_o < h_s < p_o - \sqrt{2p_o - 1}$ exist, $U_{\max} = U_{BSRO}^{BORO}$, and all consumers prioritize purchasing offline. When the product is out of stock in the store, consumers switch to online channels. Finally, consumers will return online, regardless of which channel they buy products from.

(iii) When $p_o < p_s$ and $h_s \leq h_o < p_o - \sqrt{2p_o - 1}$ hold, $U_{\max} = U_{BORS}$, and all consumers choose to buy online and return offline.

(iv) When $p_o \geq p_s$ and $h_s \leq h_o < p_o - \sqrt{2p_o - 1}$ hold, then $U_{\max} = U_{BSRS}^{BORS}$, and all consumers prioritize offline purchases. When the product is out of stock in the store, consumers switch to online channels. Ultimately, consumers will return offline, regardless of which channel they buy products from.

Lemma 7 demonstrates that when consumers are free to choose purchase and return channels, the differences in price and hassle cost of return between two channels significantly impact consumer choice outcomes. Lemma 7 states that, if omnichannel retailers allow consumers to make their own decisions regarding the purchase and return of items, then, when the online price is lower than the store prices (i.e., $p_o < p_s$), rational consumers will make the purchase online. If the online return hassle cost is lower than the store return hassle cost (i.e., $h_o < h_s$), consumers will be more eager to return products online; alternatively, if the online return hassle cost is higher, consumers will be more likely to return items to stores. Consumers will, nevertheless, prefer shopping in-store if the online price is higher than the store price (i.e., $p_o \geq p_s$). Consumers will choose an online return if the offline return hassle cost is greater than the online return hassle cost (i.e., $h_o < h_s$), regardless of whether the product is out of stock in the store; otherwise, if the offline return hassle cost is less than the online return hassle cost (i.e., $h_o \geq h_s$), then consumers will return offline.

Lemma 8. The optimal physical inventory and expected profit of the omnichannel retailers under the online channel return policy are:

1) If consumers purchase and return online, and the prices and return hassle cost satisfy the conditions of part (i) of Lemma 7, the optimal physical inventory and optimal profit are given by:

$$\begin{cases} q_4^{O_1*} = 0 \\ \pi_4^{O_1*}(q_4^{O_1*}) = (p_o - c_o - r_o(p_o - h_o))ED \end{cases} \tag{13}$$

2) If consumers prioritize purchasing in-store (i.e., regardless of stock status), when the product is out of stock, consumers switch to online channels; otherwise, consumers will purchase in-store and return online. When the return loss and hassle cost satisfy the conditions of part (ii) of Lemma 7, the optimal physical inventory and the optimal profit are expressed by:

$$\begin{cases} q_4^{S_1*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))}\right) \\ \pi_4^{S_1*}(q_4^{S_1*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \end{cases} \tag{14}$$

3) If consumers purchase online and return offline and the channel prices and return hassle cost satisfy part (iii) of Lemma 7, the optimal physical inventory and optimal profit are:

$$\begin{cases} q_4^{O_2*} = 0 \\ \pi_4^{O_2*}(q_4^{O_2*}) = (p_o - c_o - r_s(p_o - h_o))ED \end{cases} \tag{15}$$

4) If consumers prioritize purchasing in-store and returning online (i.e., when the product is out of stock, consumers switch to online channels), the channel prices and return hassle costs align with the conditions of Lemma 7. Then, the optimal inventory and the optimal profit are:

$$\begin{cases} q_4^{S_2*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))}\right) \\ \pi_4^{S_2*}(q_4^{S_2*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_2*}\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out} \end{cases} \tag{16}$$

Similar to Lemmas 4 and 6, based on the optimal inventory in Lemma 8, the relationship between optimal inventory and online and offline return losses are shown in Propositions 2 and 3.

4.5. Optimal return policy analysis

Compared to the optimal inventory policy in the preceding four return policies, retailers pay more attention to their maximum profits. Based on the findings in Sections 4.1, 4.2, 4.3, and 4.4, this section provides a comparison of the optimal profits under the four return policies (see the appendix for the derivations). The following propositions provide the conditions under which retailers will achieve the maximum profit by employing the related return policy.

Proposition 4. *The relationship for the maximum profit of the retailers under the four return policies are:*

(i) If $p_o < p_s$, $h_o < p_o - \sqrt{2p_o - 1}$, *and* $h_s < p_o - \sqrt{2p_o - 1}$ *hold, consumers choose to buy products online under the four policies:*

When $h_o < h_s$ *holds,*

a) If $r_o < r_s$, *we find* $\pi^* = \pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) = \pi_4^{O_1*}(q_4^{O_1*})$;

b) If $r_o \geq r_s$, *we find* $\pi^* = \pi_2^{O*}(q_2^{O*})$.

Considering $h_s \leq h_o < p_o - \sqrt{2p_o - 1 + 2\{((p_s - h_s)^2 + 1)/2 - p_s\}}$,

c) If $r_o < r_s$, $\pi^* = \pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*})$;

d) If $r_o \geq r_s$, $\pi^* = \pi_2^{O*}(q_2^{O*}) = \pi_4^{O_2*}(q_4^{O_2*})$.

(ii) Assuming $p_o \geq p_s$, $h_o < p_o - \sqrt{2p_o - 1}$, *and* $h_s < p_o - \sqrt{2p_o - 1}$, *consumers choose to buy products offline under the four policies:*

When $p_o - \sqrt{2p_o - 1 + 2\{((p_s - h_s)^2 + 1)/2 - p_s\}} < h_o < h_s$,

e) If $r_o < r_s$, *we have* $\pi^* = \pi_3^{S*}(q_3^{S*}) = \pi_4^{S_1*}(q_4^{S_1*})$;

f) If $r_o \geq r_s$, *we have* $\pi^* = \pi_2^{S*}(q_2^{S*})$.

Assuming $h_s \leq h_o$,

g) If $r_o < r_s$, *we have* $\pi^* = \pi_3^{S*}(q_3^{S*})$;

h) If $r_o \geq r_s$, *we have* $\pi^* = \pi_2^{S*}(q_2^{S*}) = \pi_4^{S_2*}(q_4^{S_2*})$.

According to the findings above, all consumers opt to buy online when the conditions in part (i) are satisfied. In contrast, when part (ii) prerequisites are met, all consumers purchase offline.

Part (i) of Proposition 4 shows that when a consumer chooses to purchase a product online based on utility maximization, if the offline return hassle cost for the consumer is higher than the online return hassle cost and the retailer's online return loss is less than the offline return loss, the retailer should adopt either an original purchase channel return policy, an online return policy, or a cross-channel return policy in order to maximize its profit. Retailers will allow consumers to return goods in-store in order to obtain higher benefits and reduce costs. On the contrary, when online return loss is higher than offline return loss, it is important for retailers to adopt an offline return policy so as to prevent high return loss from damaging profits. Similarly, if the consumer's hassle cost of the online return is higher than that of the offline return and when the loss of the online return is lower than that of the offline return, the retailer should choose to let consumers return goods online or return to the original purchasing channel. When retailers' online return loss is higher than their offline return loss, however, retailers will choose to let consumers return goods in stores to lower losses and achieve higher profit. In this case, the offline return policy or cross-return policy should be adopted.

When consumers choose to purchase in-store, part (ii) in Proposition 4 states that if the offline return hassle cost is higher than the online return hassle cost and the online return loss is less than the offline return loss, the retailer should adopt the online return policy or cross-return policy. In contrast, when the online return loss is higher than the offline return loss, a profit-maximizing retailer will allow consumers to return products offline to obtain higher profits. In this case, the retailer should adopt the offline return policy. Similarly, suppose the consumers' online return hassle cost is higher than the offline return hassle cost, and the retailer's online return loss is less than the offline return loss. In that case, the retailer should adopt the online return policy. When the retailer's online return loss is higher than their offline return loss, however, the profit-maximizing retailer will allow consumers to return offline to achieve the maximum profit. Therefore, the retailer should adopt the offline or cross-return channel policy.

The above results indicate that the stimulated additional demand does not affect the optimal profit relationships under the four return policies. When all consumers prefer to buy online, additional offline income is not generated, so the optimal profit has no relationship with the additional income under the different return policies. Different return policies, however, cause different return losses to retailers; thus, both online and offline return losses affect optimal profit. If all consumers prioritize purchasing in-store, the omnichannel retailer's profit comes from two types of consumers: those who buy offline when the store has inventory and those who switch to online purchases due to offline out-of-stock inventory. Based on the assumption that consumers who place orders online without bringing additional income to the retailer, the retailer's profit is affected by the relationship between return losses on the two channels. The results lead to managerial insight in this study.

Managerial insight: *Consumers base their choice between online and in-store purchases on the overall utilities of products available through these channels. Retailers, aiming to maximize profits, should encourage online returns when loss from these is lower than from offline returns, and vice versa. This strategy is in line with the core insight that minimizing return losses is key to maximizing profits. Furthermore, the probability of returns in each channel, alongside the return policy, significantly influences retailers' decisions on whether to increase or decrease store inventory in response to rising return losses. Specifically, under an original channel return policy with increasing offline return losses, a revenue-maximizing retailer should reduce inventory to encourage online purchases. Conversely, if the probability of online returns is higher than offline returns, the retailer should increase inventory even with rising offline return losses when employing an offline return policy.*

The managerial insights and optimal scheduling proposed in this section depend primarily on the traditional operation mode. The results for the omnichannel retailers under the four return policies are presented in Table 2.

Table 2 Summarizes the optimal return policy under different circumstances.

Condition	Hassle cost	Return loss	Optimal profit	Optimal policy

$p_o < p_s$ $h_o < \theta_2$	$h_o < h_s$	$r_o < r_s$	$\pi_1^{O*}(q_1^{O*}), \pi_3^{O*}(q_3^{O*}), \pi_4^{O_1*}(q_4^{O_1*})$	Original purchase channel return, online return, or cross-channel return
$h_s < \theta_2$		$r_o \ge r_s$	$\pi_2^{O*}(q_2^{O*})$	Offline return
	$h_s \le h_o < \theta_1$	$r_o < r_s$	$\pi_1^{O*}(q_1^{O*}), \pi_3^{O*}(q_3^{O*})$	Original purchase channel return or online return
		$r_o \ge r_s$	$\pi_2^{O*}(q_2^{O*}), \pi_4^{O_2*}(q_4^{O_2*})$	Offline return or cross-channel return
$p_o \ge p_s$	$\theta_1 < h_o < h_s$	$r_o < r_s$	$\pi_3^{S*}(q_3^{S*}), \pi_4^{S_1*}(q_4^{S_1*})$	Online return or cross-channel return
$h_o < \theta_2$		$r_o \ge r_s$	$\pi_2^{S*}(q_2^{S*})$	Offline return
$h_s < \theta_2$	$h_s \le h_o$	$r_o < r_s$	$\pi_3^{S*}(q_3^{S*})$	Online return
		$r_o \ge r_s$	$\pi_2^{S*}(q_2^{S*}), \pi_4^{S_2*}(q_4^{S_2*})$	Offline return or cross-channel return

Where, $\theta_1 = p_o - \sqrt{2p_o - 1 + 2\left\{\left((p_s - h_s)^2 + 1\right)/2 - p_s\right\}}$, $\theta_2 = p_o - \sqrt{2p_o - 1}$.

Table 2 shows that the optimal return policies design differs significantly among the four policies under a traditional omnichannel retailer. The relationship between the return policies and inventory results is novel and insightful and provides omnichannel retailers an additional set of considerations.

5. Numerical Simulation and Summary

In Section 4, we develop analytical, closed-form expressions for omnichannel retailers' optimal profit under several assumptions, the results of which provide corresponding managerial insights. Following this, Section 5 features a detailed numerical simulation to both validate our theoretical model and verify the proposed optimal return policy, thereby demonstrating its practical implications.

This paper substantiates its research perspective through an in-depth case study of Suning (https://www.suning.com/), a leading omnichannel retailer in China. Our methodology included field interviews and an extensive understanding of Suning's operational strategies. These investigations revealed that Suning has not settled on a universally optimal return policy for all product categories. Instead, their approach to return policies varies significantly, influenced by factors such as product type, sales cost, and service models. Suning implements different return policies based on product categories. Perishable items like fresh fruits typically have an offline return policy, focusing on immediate resolution. High-value items such as jewelry and watches also require offline returns, often necessitating an on-site inspection. Conversely, large items like furniture, kitchen, and bathroom supplies are generally returned online, likely due to logistical convenience. Products like digital 3C (Computers, Communications, and Consumer Electronics) items and daily necessities are more versatile, allowing returns either offline at any store or online, depending on the consumer's preference, under a cross-channel return policy. Based on the results of our interviews, we established four types of return policies for companies to use as a reference. These variations underscore the complexity and diversity in return strategies across different product types.

To realistically represent the omnichannel retail environment, our study incorporates synthesized data closely aligned with real-world scenarios, as observed in our in-depth case study of Suning's operations. This approach is crucial to accurately capturing the diverse and complex return policies across different product categories that Suning implements. By generating synthetic data in line with the conditions of Proposition 4, we effectively represent Suning's return strategies in our analysis.

For our simulation, specific assumptions are made to mirror real-world conditions: the unit product costs for online and offline channels are set at $c_o = 0.4$ dollars and $c_s = 0.2$ dollars, respectively. The selling prices for online and offline units are $p_o = 0.5$ dollars and $p_s = 0.6$ dollars. We also account for the return hassle costs for online and offline consumers, which are $h_o = 0.2$ dollars and $h_s = 0.4$ dollars, respectively. The probability of a product being available in-store is assumed to be $\xi = 0.5$. When a consumer visits the store, they generate additional demand, represented by a stimulation coefficient denoted as $\alpha = 1.7$. In line with Gupta et al. (2020), demand is assumed to follow a uniform distribution $D \sim U[0,80]$.

Case 1: When $p_o < p_s$ and $h_o < h_s < \theta_2$ hold, the optimal profit of the retailers under the four return policies (i.e., the relationship between optimal profit and return loss) can be seen below, in Figure 2.

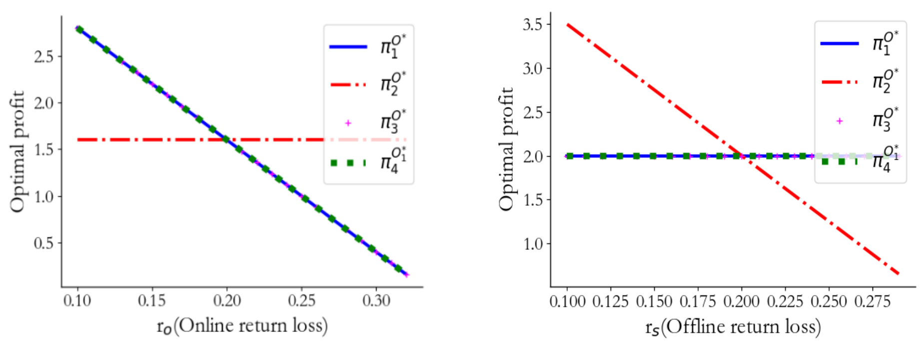

Figure 2. Relationship between return loss and optimal profit under Case 1

The graph on the left side of Figure 2 indicates that, when the condition of Case1 is satisfied, and the offline unit return loss is fixed (i.e., $r_s = 0.1$ dollars), regardless of whether the retailer adopts the original purchasing channel return, online return, or cross-return policy, consumers choose to purchase online and return online. The retailer obtains the same optimal profit under these three return policies (i.e., $\pi_1^{O^*}(q_1^{O^*}) = \pi_3^{O^*}(q_3^{O^*}) = \pi_4^{O_1^*}(q_4^{O_1^*})$). Suppose the offline return policy is adopted. In this case, the retailer's profit $\pi_2^{O^*}(q_2^{O^*})$ is not affected by the online return loss because the offline return loss is a fixed value. The left side of the intersection in Figure 2 illustrates the situation in which, as the online return loss increases, when the online return loss is lower than the offline return loss, the retailer can obtain the maximum profit by employing the original purchasing channel return policy, online return policy, or cross-return policy, and thus achieve profit maximization. When the online return loss is equal to the offline return loss, the four curves intersect at a point that indicates that the retailers will achieve the same maximum profit, regardless of which return policy they choose. The right side of the intersection in Figure 2 shows that a retailer should implement the offline return policy when the online return loss is higher than the offline return loss.

Similarly, the graph on the right side of Figure 2 indicates that, when the value of the online unit return loss is fixed (i.e., $r_o = 0.2$ dollars). Suppose the offline original channel return, online return, or cross-return policy is adopted. In this case, the retailer's profit $\pi_1^{O^*}(q_1^{O^*}), \pi_3^{O^*}(q_3^{O^*}), \pi_4^{O_1{}^*}(q_4^{O_1{}^*})$ is not affected by the offline return loss because the online return loss is a fixed value. As the offline return loss increases, the left part of the intersection in Figure 2 shows that, when the offline return loss is lower than the online return loss, the retailer can maximize profit by adopting the offline return policy. The four curves converge at one point, indicating that retailers will achieve the same profit, regardless of their return policy, when the return losses are equal on both offline and online channels. The right part of the intersection in Figure 2 shows that, when the offline return loss is higher than the online return loss, the retailer can maximize profit by adopting the original channel return policy, online return policy, or cross-return policy.

Case 2: When $p_o < p_s$ and $h_s \le h_o < \theta_1$, the optimal profit curves of the retailers under the four return policies can be seen below, in Figure 3.

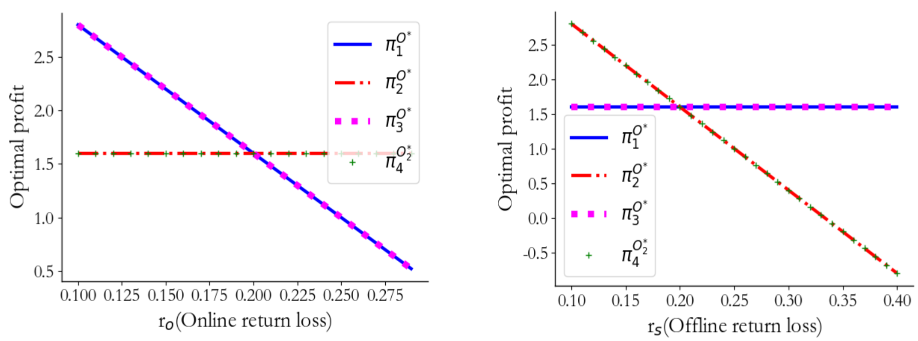

Figure 3. Relationship between return loss and optimal profit under Case 2

In Figure 3, we assume that the online and offline unit return hassle costs are $h_o = 0.2$ dollars and $h_s = 0.1$ dollars. Unlike Case 1, the relationship between optimal profit and return losses under the four different return policies, when the online price is lower than the offline price, and the online return hassle cost is between the offline return hassle cost and the threshold θ_1, is shown in Figure 3. From the curves on the left part, we can see that, if the value of the offline unit return loss is fixed, $r_s = 0.2$ dollars, and when the conditions of Case 2 are satisfied, if the retailers adopt either the original channel return policy or the online return policy, they will achieve the maximum profit (i.e., $\pi_1^{O^*}(q_1^{O^*}) = \pi_3^{O^*}(q_3^{O^*})$); in contrast, when the retailers adopt either the offline return policy or the cross-return policy, the retailer achieves the maximum profit (i.e., $\pi_2^{O^*}(q_2^{O^*}) = \pi_4^{O_2{}^*}(q_4^{O_2{}^*})$).

If the offline return policy or cross-return policy is applied, the retailer's profits (i.e., $\pi_1^{O^*}(q_1^{O^*}), \pi_3^{O^*}(q_3^{O^*})$) are not affected by the loss of online returns because the loss of offline returns is a fixed value. The left side of the intersection in Figure 3 shows that, as the online return loss increases, when the online return loss is lower than the offline return loss, the optimal return policy for the retailer is either the original purchasing channel return policy or the online return policy, which creates the maximum profit. The retailer achieves the same optmal profit under all four return policies when the loss from online returns is equivalent to the loss from offline returns. The right side of the intersection in Figure 3 demonstrates that, when the online return loss is higher than the offline return loss, then the offline return policy or the cross-return policy provides the retailer with the highest profit.

Similarly, the graph on the right side of Figure 3 illustrates that, when the online unit return loss is fixed (i.e., $r_o = 0.2$ dollars), and if the original purchasing channel return policy or online return policy is adopted, the retailer's profits ($\pi_2^{O^*}(q_2^{O^*}), \pi_4^{O_2^*}(q_4^{O_2^*})$) are not affected by the online return loss.

Case 3: When $p_o \geq p_s$ and $\theta_1 < h_o < h_s < \theta_2$, the optimal profit of the retailers under the four return policies can be seen below, in Figure 4.

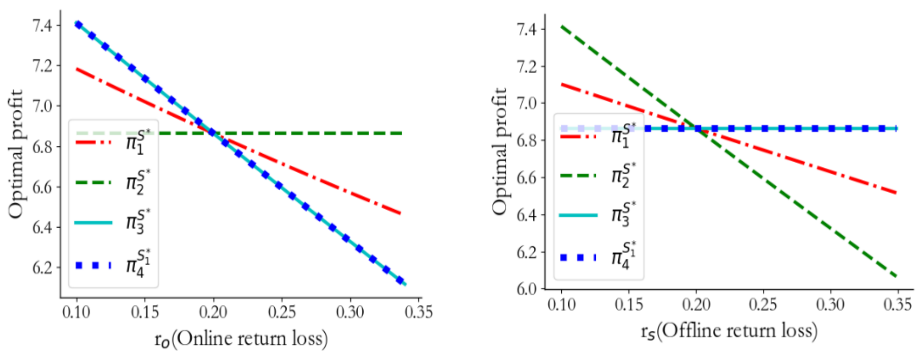

Figure 4. Relationship between return loss and optimal profit under Case 3

In Figure 4, the online and offline unit prices are denoted as $p_o = 0.5$ dollars and $p_s = 0.4$ dollars, and both the online and offline unit return hassle costs are $h_o = 0.25$ dollars and $h_s = 0.35$ dollars. Unlike Cases 1 and 2, the relationship between optimal profit and return loss under the four different return policies when Case 3 conditions are satisfied is plotted in Figure 4.

From the graph on the left side, we can see that, if the value of the offline unit return loss is fixed (i.e., $r_s = 0.1$ dollars), and when the retailer adopts an online return policy or a cross-return policy, the consumer will choose to return the goods online whether he or she purchases in-store or switches to online; thus, the retailer will obtain the highest profit (i.e., $\pi_3^{S^*}(q_3^{S^*}) = \pi_4^{S_1^*}(q_4^{S_1^*})$). Suppose the offline return policy is adopted, the retailer's profit $\pi_2^{S^*}(q_2^{S^*})$ is not affected by the online return loss. Therefore, as the online return loss increases, the left side of the intersection in Figure 4 indicates that, when the online return loss is lower than the offline return loss, the retailer should employ the online return policy or the cross-return policy to achieve the maximum profit. When the online return loss is equal to the offline return loss, the profit is the same, regardless of the return policy adopted by the retailer. The right side of the intersection in Figure 4 shows that the offline return policy gives the retailer the most profit when the online return loss is higher than the offline return loss.

The graph on the right side of Figure 4 shows that, if the online unit return loss value is fixed, $r_o = 0.2$ dollars, and if the online return policy or cross-return policy is implemented, the online return loss will not have an impact on the retailer's optimal profits ($\pi_3^{S*}(q_3^{S*}), \pi_4^{S_1*}(q_4^{S_1*})$). The results are consistent with the graph on the left side of Figure 4.

Case 4: When $p_o \geq p_s$ and $h_s < h_o < \theta_2$, the optimal profit of the retailers under the four return policies can be seen below, in Figure 5.

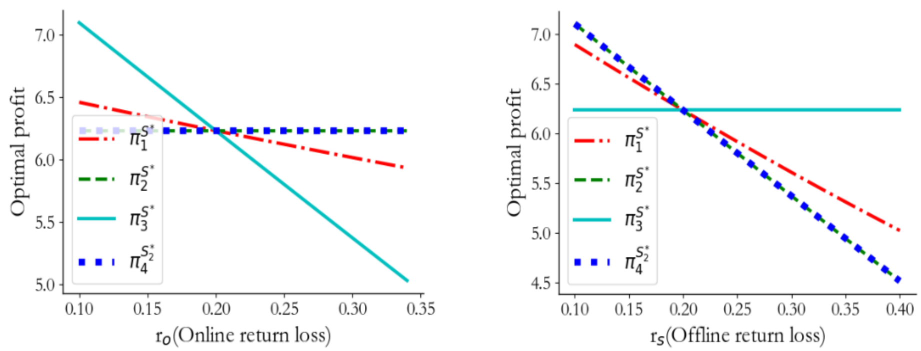

Figure 5. Relationship between return loss and optimal profit under Case 4

Online and offline return unit hassle costs are assumed to be $h_o = 0.35$ dollars and $h_s = 0.25$ dollars, respectively, in Figure 5. The relationship between the optimal profit of retailers and return losses under the four different return policies and when the online price is higher than the offline price and the online return hassle cost is between the offline return hassle cost and the threshold θ_2 are shown in Figure 5.

The graph on the left side shows that, if the value of the offline return loss is fixed and when the retailer adopts either an offline return policy or a cross-return policy, the consumer will choose to return the goods offline whether he or she buys in-store or switches from the store to an online channel, and the retailer will earn the same maximum profit (i.e., $\pi_2^{S*}(q_2^{S*}) = \pi_4^{S_2*}(q_4^{S_2*})$). If the offline return policy or cross-return policy is adopted, the retailer's profit $\pi_2^{S*}(q_2^{S*}), \pi_4^{S_2*}(q_4^{S_2*})$ is not affected by the online return loss. As a result, the left side of the intersection in Figure 5 shows that the online return policy generates the highest profit when the online return loss is less than the offline return loss. When the online return loss is equal to the offline return loss, the retailer will achieve the same profit, regardless of the return policy. The right side of the intersection in Figure 5 shows that, when the online return loss is higher than the offline return loss, then the offline return or cross-return policy benefits the omnichannel retailer who operates both offline and online channels. Similar to the graph on the left, the graph on the right illustrates how, for a specific online unit return loss (i.e., $r_o = 0.2$ dollars), if the online return policy is implemented, the online return loss has no impact on the retailer's profit $\pi_3^{S*}(q_3^{S*})$.

Our empirical findings highlight that there is no universally optimal return policy for all product types at Suning, reinforcing the significance of the research conducted in this paper. This observation underscores a common challenge faced by omnichannel retailers globally. In response, our numerical simulations, closely aligned with the practical scenarios at Suning, offer robust, real-world foundations for our theoretical analyses. These simulations reflect Suning's varied return policies and their operational implications, providing insights that are both theoretically innovative and practically applicable.

Our research suggests that due to varying costs, sales prices, return probabilities, and return losses associated with different products, retailers like Suning do not benefit from a 'one-size-fits-all' return policy. Instead, we propose that omnichannel retailers tailor their return strategies to different product types. We explored four distinct return strategies, analyzing their impact on optimal profits and store inventory. Additionally, we identified the boundary conditions for profit maximization under each return policy and their optimal solutions. This tailored approach, consistent with Suning's diverse return policies, highlights the necessity for retailers to customize their return strategies based on the unique characteristics of each product category.

6. Extension: Incorporating the Offline Channel Preference

Section 4 outlines results based on the premise that consumers lack a channel preference. Previous studies revealed that returned goods purchased online account for a third of online sales (Banjo, 2013). As consumers are unable to assess the product quality online, some opt for physical stores to avoid returns (Dzyabura Jgabathula, 2018; Kim et al., 2019). However, only some studies have considered purchasing channel preferences concerning omnichannel retailers' optimal inventories and return policies. In this section, we specifically analyze the impact of customer-based return losses on retailers' optimal inventory, profits, and return policies, with a concentrated focus on customers' preferences offline channel purchases (Kamble et al., 2021). This examination aims to solve how these offline channel preferences influence retailers' operational decisions and financial performance, providing insights into managing returns and inventory in an omnichannel retail environment.

Unlike Section 4, the model presented in this section categorizes consumers into two types: (i) those without channel preferences and (ii) those with offline channel preferences. Consequently, retailer profits derive from two groups: those with channel preference (a fraction denoted as θ), who purchase products solely from offline channels, and those without channel preference (a fraction denoted as $1-\theta$), who select the purchase channel based on a comparison of the utilities of online and offline channels. Thus, the profit function of the omnichannel retailer under the four return policies can be defined as follows:

(i) When the retailer adopts the original channel return policy, then form $\tilde{\pi}$ represents the retailer's profit when there are channel preference consumers. Similar to Lemma 1, the expected profit gained by consumers purchasing from retailers according to various online and offline channels is denoted as follows:

$$\begin{cases} \tilde{\pi}_1^{O*}(\tilde{q}_1^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{O*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\ \tilde{\pi}_1^{S*}(\tilde{q}_1^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED_{out} \end{cases} \tag{18}$$

(ii) When the retailer adopts the offline return policy, similar to Lemma 3, the profit function of the retailer's sales in different channels is as follows:

$$\begin{cases} \tilde{\pi}_2^{O*}(\tilde{q}_2^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{O*}\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED \\ \tilde{\pi}_2^{S*}(\tilde{q}_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \end{cases} \tag{19}$$

(iii) When the retailer chooses the online return policy, akin to the conditions in Lemma 5, the retailer's profit functions in various situations are represented as:

$$\begin{cases} \tilde{\pi}_3^{O*}(\tilde{q}_3^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_3^{O*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\ \tilde{\pi}_3^{S*}(\tilde{q}_3^{S*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} \\ \qquad + (1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \end{cases} \tag{20}$$

(iv) Lastly, when the retailer opts for the cross-return policy, the profit functions gained by the retailer in four different scenarios are:

$$\begin{cases} \tilde{\pi}_4^{O_1*}(\tilde{q}_4^{O_1*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{O_1*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\ \tilde{\pi}_4^{S_1*}(\tilde{q}_4^{S_1*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} \\ \qquad + (1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ \tilde{\pi}_4^{O_2*}(\tilde{q}_4^{O_2*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{O_2*}\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED \\ \tilde{\pi}_4^{S_2*}(\tilde{q}_4^{S_2*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_2*}\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \end{cases} \tag{21}$$

We can infer from the above equations that the optimal profit function of the retailer changes, and there will always be a θ proportion of some consumers with an offline channel preference who opt to buy the product offline, even when the retailer chooses to sell the product online. The remaining consumers select the channel based on the utility function (All proofs are given in Appendix B).

Our first observation is that when consumers decide to purchase online, if the offline return hassle cost is higher and the online return loss is less than the offline return loss, the retailer should implement the original purchase channel return policy, the online return policy, or the cross-channel return policy to maximize profit. However, the retailer should utilize the offline return policy if the online return loss exceeds the offline return loss. Second, if the hassle cost for consumers to return online surpasses the hassle cost for offline returns, and when the online return loss is lower, retailers should allow consumers to return online or to the original purchase channel. Nevertheless, when the retailer's online return loss is substantial, the retailer should opt for an offline return policy or a cross-return policy. When consumers purchase products online, retailers still need to consider inventory management. Third, when consumers opt to purchase in-store, the conclusion aligns with Proposition 4. In conclusion, the significant findings of this paper still hold when retailers have channel preferences.

7. Conclusion and Future Work

The main contribution of this paper is the examination of the interplay between inventory policies and return policies in omnichannel retailing, with a focus on how return losses impact retailers' profits and physical inventories. This study pioneers the analysis of how retailers manipulate optimal return policy structures and provides insight into when traditional online and offline purchase channels are used. We assumed that offline purchases would yield additional profit for retailers if the store had inventory. By determining optimal actions for both retailers and consumers, we evaluated the benefits of different return policies, physical inventories, and profit levels for the retailer.

The study establishes that the relationship between return losses and physical inventory depends on the type of return policy adopted by the retailer. For instance, if the retailer adopts an offline return policy, increasing the physical store inventory can be beneficial to reduce the probability of online returns. In such cases, retailers can increase store prices to earn higher profits. However, if the retailer uses the original purchasing channel return policy, lowering online prices to increase inventory may not be the best policy.

The results concerning the relationship between physical inventory and return loss show that if the retailer opts for the original purchasing channel return policy, from the perspective of maximizing the retailer's profit, the retailer should decrease inventory as offline return losses increase. In contrast, if online return losses increase, the optimal physical inventory should be increased. Consider a scenario in which a retailer decides to implement an offline return policy due to the relationship between price and the return hassle, particularly when the probability of an offline return is minimal. To avoid incurring higher return losses, the retailer should increase inventory in physical stores as offline return losses rise, thereby decreasing the online return probability. When an online return policy is applied, given the lower probability of online returns, the physical store inventory should be decreased as online return losses rise. This will encourage online purchases and help retailers avoid losses, as there will be a decrease in offline returns. This also holds even when retailers adopt a cross-return policy. These findings challenge conventional wisdom but are reasonable in the context of different return losses of retailers and consumers' probabilities regarding both offline and online channels. This study identified the conditions under which the retailers should either increase or decrease their inventory in response to a rise in return loss.

This research reveals that maximizing retailer profit depends on the relationship between online and offline return losses and online and offline return hassle costs. In other words, when the loss due to offline returns is low, even if online channels are better for consumers, the retailer should let all consumers return to the store. Otherwise, a profit-maximizing retailer should drive all consumers to return online. In short, retailers should take advantage of the relationship between the return loss and consumer return hassle cost to formulate the optimal return method and increase revenue. Considering consumers with offline purchase channel preferences, we found that consumers' channel preferences did not impact the retailers' return policy. When the online return losses were higher than the offline return losses, the offline return policy benefitted retailers' profits. Otherwise, the original purchasing channel return policy was better.

To create a valuable and tractable framework and derive valuable insights, we assumed that online and offline consumers perceive the same product value. However, with the expansion of e-commerce, online goods may better serve the needs of consumers who prefer online channels. Future research could relax these assumptions and broaden our understanding of the problem. Instead of analyzing the traditional online + offline operation mode, another avenue is to evaluate the physical showroom mode adopted by the retailers. Additionally, this study examined the retailers' physical inventories and profits. A more comprehensive understanding of omnichannel retailers' operations requires an analysis of how retailers' return policies and consumers' utilities affect channel pricing. Finally, conducting a sensitivity analysis to examine how variations in key parameters impact our model's outcomes would provide valuable insights, particularly in understanding the robustness of our findings under different market conditions and assumptions. This analysis could form an essential aspect of future work in this domain, offering a more profitable, multifaceted view of omnichannel retail operations.

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Appendix

Table 3. Notation for parameters and variables

Symbol	Definition
c_o	Online marginal inventory cost of production for retailers
c_s	Offline marginal inventory cost of production for retailers
h_o	Hassle cost of consumer returns online
h_s	Hassle cost of consumer returns to the physical store
p_o	Online price charged for the product
p_s	Offline price charged for the product
q	Offline inventory quantity
V	Consumer's valuation for the product, following uniform distribution on the interval [0,1]
ξ	Probability that the store has inventory
r_o	Online unit return losses for retailers
r_s	Offline unit return losses for retailers
α	Additional demand stimulated factor from offline purchase

Appendix A: Analytical Analysis

Utility Comparison

When we compare U_{BORO} and U_{BORS}, we can get:

$$U_{BORO} = \frac{(p_o - h_o)^2 + 1}{2} - p_o > \frac{(p_o - h_s)^2 + 1}{2} - p_o = U_{BORS} \Leftrightarrow h_o < h_s. \quad \text{(X1)}$$

Therefore, if there is $h_o < h_s$, $U_{BORO} > U_{BORS}$ holds; if there is $h_o > h_s$, we will get $U_{BORO} < U_{BORS}$.

In the following proof, we compare the utility relationship in four different situations.

Inventory and Profit Comparison

First of all, we know that there are $ED_{in} = E\min\{q, D\}$ and $ED_{out} = E(D-q)^+$.

Hence, we can obtian the following results:

$$\min\{q, D\} = \begin{cases} q, \ if D > q \\ D, \ if\, 0 < D < q \end{cases} \text{ and } (D-q)^+ = \begin{cases} D-q, if D > q \\ 0, if\, 0 < D < q \end{cases}. \quad \text{(X2)}$$

Therefore, we can obtain the expected expression of availability and shortage when the retailer adopts the originally purchased channel to return products, as shown below:

$$ED_{\text{in}} = \int_0^q Df(D)dD + \int_q^\infty qf(D)dD; \ ED_{out} = \int_q^\infty (D-q)f(D)dD. \quad \text{(X3)}$$

Here, $f(D)$ is the probability density function of demand D, and $0 < f(D) < 1$.

Proof of Lemma 1 (Original purchasing channel return policy)

In the first situation, when the retailer adopts original purchasing channel return policy, comparing the unitlies of U_{BORO} and U_{BSRS}^{BORO}, we can get:

$$\begin{aligned} &U_{BORO} = \frac{(p_o - h_o)^2 + 1}{2} - p_o > \xi\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\} + (1-\xi)[U_{BORO}] = U_{BSRS}^{BORO} \\ &\Leftrightarrow \ \frac{(p_o - h_o)^2 + 1}{2} - p_o > \frac{(p_s - h_s)^2 + 1}{2} - p_s \Rightarrow h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} \end{aligned}. \quad \text{(X4)}$$

Given that $U_{BORO} > 0$ and $U_{BSRS}^{BORO} > 0$, we can say that $h_o < p_o - \sqrt{2p_o - 1}$ and $h_o < p_o - \sqrt{2p_o - 1 - 2\frac{\xi}{1-\xi}\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ are holding, respectively.

If $U_{BORS} > 0$ holds, it can yield the following solution:

$$p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < p_o - \sqrt{2p_o - 1} < p_o - \sqrt{2p_o - 1 - 2\frac{\xi}{1-\xi}\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} \quad \text{(X5)}$$

Therefore, we will get the following results:

1) If $h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ holds, threre is $\max\{U_{BORO}, U_{BSRS}^{BORO}, 0\} = U_{BORO}$; in this case, all consumers choose online purchasing;

2) If $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} \le h_o < p_o - \sqrt{2p_o - 1 - 2\frac{\xi}{1-\xi}\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ holds, we can get $\max\{U_{BORO}, U_{BSRS}^{BORO}, 0\} = U_{BSRS}^{BORO}$, and all consumers give priority to purchasing from the store.

Proof of Lemma 2 (Original purchasing channel return policy)

1) Recall the proof of Lemma 1, if there is $h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$,we can get $U_{BORO} > U_{BSRS}^{BORO}$, and consumers are choosing to buy online. Therefore, we have $\vartheta = 0$, the profit is shown:

$$\begin{aligned}
\pi_1^O(q) &= \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\
&\quad + \vartheta[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\
&\quad + (1-\vartheta)[(p_o - c_o - r_o(p_o - h_o))ED] \\
&= (p_o - c_o - r_o(p_o - h_o))ED
\end{aligned} \tag{X6}$$

We can obtain the optimal inventory $q_1^{O*} = 0$.

2)When $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < h_o < p_o - \sqrt{2p_o - 1 - 2\frac{\xi}{1-\xi}\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ exists, we can get $U_{BORO} < U_{BSRS}^{BORO}$, which means all consumers are choosing to buy offline. Therefore $\vartheta = 1$, In this case, the profit function is shown as follows:

$$\begin{aligned}
\pi_1^S(q) &= \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\
&\quad + \vartheta[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\
&\quad + (1-\vartheta)[(p_o - c_o - r_o(p_o - h_o))ED] \\
&= \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out}
\end{aligned} \tag{X7}$$

The best respone functon (first-order derivative) of $\pi_1^S(q)$ on inventory q is:

$$\partial\pi_1^S(q)/\partial q = \left\{\alpha\left[p_s - r_s(p_s - h_s)\right] - (p_o - c_o - r_o(p_o - h_o))\right\}(1 - F(q)) - \alpha c_s . \tag{X8}$$

Where, $\begin{cases} \partial ED_{in}/\partial q = qf(q) + \int_q^\infty f(D)dD - qf(q) = \int_q^\infty f(D)dD \\ \partial ED_{out}/\partial q = -qf(q) - \int_q^\infty f(D)dD + qf(q) = -\int_q^\infty f(D)dD \end{cases}$.

We also have the following function

$$\partial \pi_1^S(q)/\partial q^2 = -\left\{\alpha\left[p_s - r_s(p_s - h_s)\right] - (p_o - c_o - r_o(p_o - h_o))\right\} f(q) < 0 \tag{X9}$$

Since the second-order derivative is negative (i.e., $\partial \pi_1^S(q)/\partial q^2 < 0$), the optimal result can be obtained by using $\partial \pi_1^S(q)/\partial q = 0$. The optimal inventory is:

$$q_1^{S*} = \overline{F}^{-1}\left(\frac{\alpha c_s}{\alpha\left[p_s - r_s(p_s - h_s)\right] - (p_o - c_o - r_o(p_o - h_o))}\right) \tag{X10}$$

Where, $p_s - r_s(p_s - h_s) - c_s$ is the profit that can be obtained from offline sales of a product, and $p_o - c_o - r_o(p_o - h_o)$ is the profit that can be received from online sales of a product.

If merchants open an offline channel, there are $p_s - r_s(p_s - h_s) - c_s > p_o - c_o - r_o(p_o - h_o)$ and $\alpha \geq 1$ based on the assumption. Thus, it is intuition that $\alpha\left[p_s - r_s(p_s - h_s)\right] - (p_o - c_o - r_o(p_o - h_o)) > 0$ holds.

Proof of Propostion 1

Since we have $\partial\left\{\alpha[p_s - r_s(p_s - h_s)] - (p_o - c_o - r_o(p_o - h_o))\right\}/\partial p_o = (r_o - 1) < 0$, the function of $\dfrac{\alpha c_s}{\alpha\left[p_s - r_s(p_s - h_s)\right] - (p_o - c_o - r_o(p_o - h_o))}$ increases with the online price p_o. Therefore, the optimal physical inventory of q_1^{S*} decreases with the price p_o.

Similarly, there is $\partial\left\{\alpha[p_s - r_s(p_s - h_s)] - (p_o - c_o - r_o(p_o - h_o))\right\}/\partial p_s = \alpha(1 - r_s) > 0$. In this case, the optimal inventory of q_1^{S*} increases with offline price p_s.

Obviously, the optimal inventory of q_1^{S*} increases with online channel return loss r_o but decreases with offline channel return loss r_s.

Proof of Lemma 3 (Offline return policy)

In the second case, if the omnichannel retailer implements the offline return policy, comparing the utilities between U_{BORS} and U_{BSRS}^{BORS}, we can get:

$$\begin{gathered} U_{BORS} = \frac{(p_o - h_s)^2 + 1}{2} - p_o > \xi\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\} + (1 - \xi)[U_{BORS}] = U_{BSRS}^{BORS} \\ \Leftrightarrow \frac{(p_o - h_s)^2 + 1}{2} - p_o > \frac{(p_s - h_s)^2 + 1}{2} - p_s \end{gathered}. \tag{X11}$$

We start by defining an auxiliary function $f_1(x) = \dfrac{(x - h_s)^2 + 1}{2} - x$ and taking its derivative on *x*.

If $x > h_s + 1$ is true, the auxiliary function increases as x increases. If $x \leq h_s + 1$ is true, on the other hand, the auxiliary function decreases as x increases. Because we have $p_s < 1 < h_s + 1$ and $p_o < 1 < h_s + 1$, therefore, if there is $p_o < p_s$,then $f_1(p_o) > f_1(p_s)$ (i.e., $U_{BORS} > U_{BSRS}^{BORS}$) holds; if $p_o \geq p_s$ holds,then $f_1(p_o) \leq f_1(p_s)$ is obtained (i.e., $U_{BORS} \leq U_{BSRS}^{BORS}$).

For the condition $U_{BSRS}^{BORS} > 0$, it can yield the following solution:

$$\begin{aligned} & U_{BSRS}^{BORS} > 0 \\ & \Leftrightarrow \xi\left\{\frac{(p_s-h_s)^2+1}{2}-p_s\right\}+(1-\xi)[U_{BORS}]>0 \\ & \Leftrightarrow \frac{(p_o-h_s)^2+1}{2}-p_o > -\frac{\xi}{1-\xi}\left\{\frac{(p_s-h_s)^2+1}{2}-p_s\right\} \Leftrightarrow f_1(p_o) > -\frac{\xi}{1-\xi} f_1(p_s) \end{aligned} \quad . \tag{X12}$$

Since there are $U_{BORS} = f_1(p_o) = \frac{(p_o-h_s)^2+1}{2} - p_o > 0$ and $-\frac{\xi}{1-\xi} < 0$, hence, when $p_o \geq p_s$, we can get

$0 < f_1(p_o) \leq f_1(p_s)$ and $\frac{(p_o-h_s)^2+1}{2} - p_o > 0 > -\frac{\xi}{1-\xi}\left\{\frac{(p_s-h_s)^2+1}{2}-p_s\right\}$ (i.e., $U_{BSRS}^{BORS} > 0$).

Hence, we can get $\max\{U_{BORS}, U_{BSRS}^{BORS}, 0\} = U_{BORS}$ when $p_o < p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$ hold, and all consumers choose online purchasing. When $p_o \geq p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$ hold, we can get $\max\{U_{BORS}, U_{BSRS}^{BORS}, 0\} = U_{BSRS}^{BORS}$, and all consumers choose physical store purchasing first.

Proof of Lemma 4 (Offline return policy)

1) If there is $p_o < p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then we get $U_{BORS} > U_{BSRS}^{BORS}$, which means consumers choose to buy online. Therefore, we have $\vartheta = 0$. In this case, the profit function is given by:

$$\begin{aligned} \pi_2^O(q) &= \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\ &+ (1-\vartheta)[(p_o - c_o - r_s(p_o - h_o))ED] \\ &= (p_o - c_o - r_s(p_o - h_o))ED \end{aligned} \tag{X13}$$

We can obtain the optimal inventory of $q_2^{O*} = 0$.

2) If there is $p_o > p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then $U_{BORS} < U_{BSRS}^{BORS}$ holds, which indicates that all consumers are choosing to buy in- store. Therefore, we get $\vartheta = 1$, and the profit is given by:

$$\begin{aligned} \pi_2^S(q) &= \vartheta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\ &+ (1-\vartheta)[(p_o - c_o - r_s(p_o - h_o))ED] \\ &= \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out} \end{aligned} \quad . \tag{X14}$$

The first-order derivative of profit function $\pi_2^S(q)$ with the respect to inventory q is:

$$\partial\pi_2^S(q)/\partial q = \left\{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))\right\}(1 - F(q)) - \alpha c_s$$

Where, $\begin{cases} \partial ED_{\text{in}}/\partial q = qf(q) + \int_q^\infty f(D)dD - qf(q) = \int_q^\infty f(D)dD \\ \partial ED_{\text{out}}/\partial q = -qf(q) - \int_q^\infty f(D)dD + qf(q) = -\int_q^\infty f(D)dD \end{cases}$.

We also have the following second-order derivative function

$$\partial \pi_2^S(q)/\partial q^2 = -\{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))\} f(q) < 0 .$$

Since $\partial\pi_2^S(q)/\partial q^2 < 0$, the optimal inventory can be obtained by using $\partial\pi_2^S(q)/\partial q = 0$. The optimal solution that maximizes the profit function is:

$$q_2^{S*} = \overline{F}^{-1}\left(\frac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))}\right) \tag{X15}$$

Proof of Propostion 2

For simplify, we first get $\partial\{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))\}/\partial p_o = (r_s - 1) < 0$, so the function of $\frac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))}$ increases with the online price p_o. On this bais, the optimal inventory of q_2^{S*} decreases with p_o.

Similarly, since $\partial\{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))\}/\partial p_s = \alpha(1 - r_s) > 0$ holds, the optimal inventory of q_2^{S*} increases with offline price p_s.

In addition, we can also get the following relationship:

$$\partial\{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))\}/\partial r_s = (p_o - h_o) - \alpha(p_s - h_s) .$$

Thus, if $(p_o - h_o) > \alpha(p_s - h_s)$ holds, the function $\frac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))}$ decreases with offline channel return loss r_s, the optimal inventory of q_2^{S*} will increase with r_s; if there is $(p_o - h_o) \le \alpha(p_s - h_s)$, we can get the optimal inventory q_2^{S*} decreases with offline return loss r_s.

Obviously, the optimal inventory q_2^{S*} increases with α.

Proof of Lemma 5 (Online return policy)

In the third scenario, when retailer adopts the online return policy, by comparing U_{BORO} and U_{BSRO}^{BORO}, we get:

$$\begin{aligned}&\mathrm{U}_{BORO}=\frac{(p_o-h_o)^2+1}{2}-p_o>\xi\left\{\frac{(p_s-h_o)^2+1}{2}-p_s\right\}+(1-\xi)[\mathrm{U}_{BORO}]=U_{BSRO}^{BORO}\\&\Leftrightarrow\frac{(p_o-h_o)^2+1}{2}-p_o>\frac{(p_s-h_o)^2+1}{2}-p_s\end{aligned}\tag{X16}$$

We define an auxiliary function $f_2(x)=\frac{(x-h_o)^2+1}{2}-x$ and take its first derivative. If $x>h_o+1$, this function increases with x. On the contrary, if $x\le h_o+1$, this function of $f_2(x)$ decreases with x. Since $p_s<1<h_o+1$ and $p_o<1<h_o+1$ are holding, thus, if there is $p_o<p_s$, we have $f_2(p_o)>f_2(p_s)$ (i.e., $U_{BORO}>U_{BSRO}^{BORO}$); if there is $p_o\ge p_s$, $f_2(p_o)\le f_2(p_s)$ is obtianed (i.e., $U_{BORO}\le U_{BSRO}^{BORO}$).

For the condition of $U_{BSRO}^{BORO}>0$, it can yield the following solution:

$$\begin{aligned}&U_{BSRO}^{BORO}>0\\&\Leftrightarrow\xi\left\{\frac{(p_s-h_o)^2+1}{2}-p_s\right\}+(1-\xi)[U_{BORO}]>0\\&\Leftrightarrow\frac{(p_o-h_o)^2+1}{2}-p_o>-\frac{\xi}{1-\xi}\left\{\frac{(p_s-h_o)^2+1}{2}-p_s\right\}\Leftrightarrow f_2(p_o)>-\frac{\xi}{1-\xi}f_2(p_s)\end{aligned}\tag{X17}$$

According to $U_{BORO}=f_2(p_o)=\frac{(p_o-h_o)^2+1}{2}-p_o>0$ and $-\frac{\xi}{1-\xi}<0\xi$, hence, when $p_o\ge p_s$ holds, we can get $0<f_2(p_o)\le f_2(p_s)$ and $\frac{(p_o-h_o)^2+1}{2}-p_o>0>-\frac{\xi}{1-\xi}\left\{\frac{(p_s-h_o)^2+1}{2}-p_s\right\}$ (i.e., $U_{BSRO}^{BORO}>0$).

Hence, if the retailer employs the online return policy, when $p_o<p_s$ and $h_o<p_o-\sqrt{2p_o-1}$ are holding, we can get $\max\{U_{BORO},U_{BSRO}^{BORO},0\}=U_{BORO}$, and all consumers choose online purchasing; however, when $p_o\ge p_s$ and $h_o<p_o-\sqrt{2p_o-1}$ exist, there is $\max\{U_{BORO},U_{BSRO}^{BORO},0\}=U_{BSRO}^{BORO}$; in this case, all consumers choose to shop in the physical store.

Proof of Lemma 6 (Online return)

1) If there are $p_o<p_s$ and $h_o<p_o-\sqrt{2p_o-1}$, then we have $U_{BORO}>U_{BSRO}^{BORO}$. In this case, all consumers are choosing to buy online (i.e., $\vartheta=0$). The profit in this case is given by:

$$\begin{aligned}\pi_3^O(q)&=\vartheta\alpha\{[p_s-r_o(p_s-h_s)]ED_{in}-c_sq\}\\&+\vartheta[(p_o-c_o-r_o(p_o-h_o))ED_{out}]\\&+(1-\vartheta)[(p_o-c_o-r_o(p_o-h_o))ED]\\&=(p_o-c_o-r_o(p_o-h_o))ED\end{aligned}\tag{X18}$$

We can obtain the optimal inventory $q_3^{O*}=0$.

2) If there are $p_o > p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then we have $U_{BORO} < U_{BSRO}^{BORO}$, which indicates all consumers are choosing to buy in the physical store(i.e., $\vartheta=1$). In this case, the profit is given by:

$$\begin{aligned}\pi_3^S(q) &= \vartheta\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ &+ (1-\vartheta)[(p_o - c_o - r_o(p_o - h_o))ED] \\ &= \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out}\end{aligned} \tag{X19}$$

The first-order derivative of the profit function $\pi_3^S(q)$ with respect to inventory q is:

$$\partial\pi_3^S(q)/\partial q = \left\{\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))\right\}(1 - F(q)) - \alpha c_s.$$

Where, $\begin{cases}\partial ED_{\text{in}}/\partial q = qf(q) + \int_q^\infty f(D)dD - qf(q) = \int_q^\infty f(D)dD \\ \partial ED_{out}/\partial q = -qf(q) - \int_q^\infty f(D)dD + qf(q) = -\int_q^\infty f(D)dD\end{cases}.$

The second-order derivative function is shown:

$$\partial\pi_3^S(q)/\partial q^2 = -\left[\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))\right]f(q) < 0.$$

Because $\partial\pi_3^S(q)/\partial q^2 < 0$ exists, the equation (X19) has a unique optimal solution, which can be found by using $\partial\pi_3^S(q)/\partial q = 0$ as follows:

$$q_3^{S*} = \overline{F}^{-1}\left(\frac{\alpha c_s}{\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))}\right) \tag{X20}$$

Proof of Propostion 3

Similar to Lemmas 2 and 4, we find that the optimal inventory of q_3^{S*} decreases with p_o but increases with p_s. In addition, if $(p_o - h_o) > \alpha(p_s - h_s)$ holds, the optimal inventory q_3^{S*} increases with online return loss r_o; however, if $(p_o - h_o) \le \alpha(p_s - h_s)$ holds, q_3^{S*} decreases with r_o.

Obviously, the optimal inventory q_3^{S*} increases with α.

Proof of Lemma 7 (Cross-channel return)

In the fouth scenario, consumers choose the purchase and return channels based on their own utility.

(1) First, we compare the utilities of U_{BORO} and U_{BORS}, U_{BSRS}^{BORO} and U_{BSRS}^{BORS}, U_{BSRO}^{BORO} and U_{BSRO}^{BORS}, respectively. Referring the proofs of Lemmas 1, 3 and 5, if $h_o < h_s$ holds, we can get $U_{BORO} > U_{BORS}$, $U_{BSRS}^{BORO} > U_{BSRS}^{BORS}$, and $U_{BSRO}^{BORO} > U_{BSRO}^{BORS}$.

Next, we compare the utilities of U_{BSRS}^{BORO} and U_{BSRO}^{BORO} with the following expressions:

$$\begin{cases} U_{BSRS}^{BORO} = \xi\left(\dfrac{(p_s - h_s)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_o)^2 + 1}{2} - p_o\right) \\ U_{BSRO}^{BORO} = \xi\left(\dfrac{(p_s - h_o)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_o)^2 + 1}{2} - p_o\right) \end{cases} \tag{X21}$$

According to the functons in equation (X21), when $h_o < h_s$, we have $U_{BSRO}^{BORO} > U_{BSRS}^{BORO}$.

Third, we compare the utilities of U_{BORO} and U_{BSRO}^{BORO}, where,

$$\begin{cases} U_{BORO} = \dfrac{(p_o - h_o)^2 + 1}{2} - p_o \\ U_{BSRO}^{BORO} = \xi\left(\dfrac{(p_s - h_o)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_o)^2 + 1}{2} - p_o\right) \end{cases} \tag{X22}$$

Similar to the proof of Lemma 5, we can get: i) when $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$ hold, there is $\max\{U_{BORO}, U_{BSRS}^{BORO}, U_{BSRO}^{BORO}\} = U_{BORO}$, which indicates all consumers will choose online purchase and return online; ii) when $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$ exist, we have $\max\{U_{BORO}, U_{BSRS}^{BORO}, U_{BSRO}^{BORO}\} = U_{BSRO}^{BORO}$, which represents all consumers choose to buy offline and return online, and choose to return online and buy online when the store is out of stock.

(2) When we compare the utilities of U_{BORO} and U_{BORS}, U_{BSRS}^{BORO} and U_{BSRS}^{BORS}, U_{BSRO}^{BORO} and U_{BSRO}^{BORS}, respectively. If there is $h_o \geq h_s$ holding, then, we can get the following $U_{BORO} \leq U_{BORS}$, $U_{BSRS}^{BORO} \leq U_{BSRS}^{BORS}$ and $U_{BSRO}^{BORO} \leq U_{BSRO}^{BORS}$.

Next, we compare the utilities of U_{BSRS}^{BORS} and U_{BSRO}^{BORS}, the expressions are shown as follows:

$$\begin{cases} U_{BSRS}^{BORS} = \xi\left(\dfrac{(p_s - h_s)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_s)^2 + 1}{2} - p_o\right) \\ U_{BSRO}^{BORS} = \xi\left(\dfrac{(p_s - h_o)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_s)^2 + 1}{2} - p_o\right) \end{cases} \tag{X23}$$

According to the functions in equation (X23), when $h_o \geq h_s$, we will have $U_{BSRS}^{BORS} \geq U_{BSRO}^{BORS}$.

In the third step, we compare the utilities of U_{BORS} and U_{BSRS}^{BORS}:

$$\begin{cases} U_{BORS} = \dfrac{(p_o - h_s)^2 + 1}{2} - p_o \\ U_{BSRS}^{BORS} = \xi\left(\dfrac{(p_s - h_s)^2 + 1}{2} - p_s\right) + (1-\xi)\left(\dfrac{(p_o - h_s)^2 + 1}{2} - p_o\right) \end{cases} \tag{X24}$$

Similar to the proof of Lemma 3, we can get: i) when $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$ exist, we can find $\max\{U_{BORS}, U_{BSRS}^{BORS}, U_{BSRO}^{BORS}\} = U_{BORS}$; in this case, all consumers will choose purchase online and return in the store; ii) when there are $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$, $\max\{U_{BORS}, U_{BSRS}^{BORS}, U_{BSRO}^{BORS}\} = U_{BSRS}^{BORS}$, consumers will choose to buy and return offline, however, they will choose to return offline after buying online when the store is out of stock.

Proof of Lemma 8 (Cross-channel return)

1) Recall the proof of Lemmas 2, 4 and 6; if there are $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$, we will get $U_{\max} = U_{BORO}$,which means consumers will buy online (i.e., ϑ=0). In this case, the profit is given by：

$$\pi_4^{O_1}(q) = (p_o - c_o - r_o(p_o - h_o))ED \tag{X25}$$

We can obtain the optimal inventory $q_4^{O_1*} = 0$ in equation (X25).

2) If there are $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$, we will get $U_{\max} = U_{BSRO}^{BORO}$, then all consumers are choosing to purchase in-store (i.e., ϑ=1). In this case, the profit is the same as Lemma 6:

$$\pi_4^{S_1}(q) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \tag{X26}$$

The optimal inventory $q_4^{S_1*} = \overline{F}^{-1}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_o(p_s - h_s)) - (p_o - c_o - r_o(p_o - h_o))}\right)$ can be obtained.

Similar to Lemma 6, the optimal inventory $q_4^{S_1*}$ decreases with p_o but increases with p_s. Meanwhile, if $(p_o - h_o) > \alpha(p_s - h_s)$ holds, $q_4^{S_1*}$ increases with r_o and α ; while if there is $(p_o - h_o) < \alpha(p_s - h_s)$, then $q_4^{S_1*}$ decreases with r_o.

3) Recall the proof of Lemmas 2, 4 and 6, we get: if there are $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$, we can obtain $U_{\max} = U_{BORS}$, which indicates that all consumers choose to buy online (i.e., ϑ=0). In this case, the profit function is as follows:

$$\pi_4^{O_2}(q) = (p_o - c_o - r_s(p_o - h_o))ED \tag{X27}$$

We can obtain the optimal inventory $q_4^{O_2*} = 0$

4) If there are $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$, then $U_{\max} = U_{BSRS}^{BORS}$. Under this assumption, consumers purchase in-store and return offline regardless of whether the store is out of stock or not. In this case, same as lemma 4, the profit is given by:

$$\pi_4^{S_2}(q) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out} \tag{X28}$$

We can obtain the optimal inventory $q_4^{S_2*} = \overline{F}^{-1}(\dfrac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (p_o - c_o - r_s(p_o - h_o))})$.

Similar to the proof of Lemma 4, the optimal inventory $q_4^{S_2*}$ decreases with p_o but increases with p_s. Meanwhile, if there is $(p_o - h_o) > \alpha(p_s - h_s)$, then $q_4^{S_2*}$ increases with r_s and α ; if $(p_o - h_o) < \alpha(p_s - h_s)$ holds, then $q_4^{S_2*}$ decreases with r_s.

Proof of Propostion 4 (Optimal profit comparison)

Based on the proof of Lemmas 2, 4, 6, and 8, the optimal profits under the four return policies are as follows:

$$\begin{cases}\pi_1^{O*}(q_1^{O*}) = [p_o - c_o - r_o(p_o - h_o)]ED \\ \pi_1^{S*}(q_1^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\ \pi_2^{O*}(q_2^{O*}) = [p_o - c_o - r_s(p_o - h_o)]ED \\ \pi_2^{S*}(q_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out} \\ \pi_3^{O*}(q_3^{O*}) = [p_o - c_o - r_o(p_o - h_o)]ED \\ \pi_3^{S*}(q_3^{S*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\ \pi_4^{O_1*}(q_4^{O_1*}) = [p_o - c_o - r_o(p_o - h_o)]ED \\ \pi_4^{S_1*}(q_4^{S_1*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\ \pi_4^{O_2*}(q_4^{O_2*}) = [p_o - c_o - r_s(p_o - h_o)]ED \\ \pi_4^{S_2*}(q_4^{S_2*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_2*}\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out}\end{cases} \tag{X29}$$

Where $\pi_4^{O_1*}(q_4^{O_1*})$ and $\pi_4^{S_1*}(q_4^{S_1*})$ represent that when the retailer adopts the cross-return policy, consumers choose to return all products online. Therefore, the retailer's profit in this case is equal to the profit of adopting the online return policy (i.e., $\pi_4^{O_1*}(q_4^{O_1*}) = \pi_3^{O*}(q_3^{O*})$ and $\pi_4^{S_1*}(q_4^{S_1*}) = \pi_3^{S*}(q_3^{S*})$). Similarly, Where $\pi_4^{O_2*}(q_4^{O_2*})$ and $\pi_4^{S_2*}(q_4^{S_2*})$ represent that when the retailer adopts the cross return policy, consumers choose to return all products offline. Therefore, the retailer's profit in this case is equal to the profit of adopting the offline return policy (i.e., $\pi_4^{O_2*}(q_4^{O_2*}) = \pi_2^{O*}(q_2^{O*})$ and $\pi_4^{S_2*}(q_4^{S_2*}) = \pi_2^{S*}(q_2^{S*})$).

Recall the proof of Lemma 1, when the retailer adopts the originally purchased channel return policy, comparing the conditions $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ and h_s, we get the following results:

$$\begin{aligned} & p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} - h_s > 0 \\ & \Leftrightarrow p_o - h_s > \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} \Leftrightarrow \frac{(p_o - h_s)^2 + 1}{2} - p_o > \frac{(p_s - h_s)^2 + 1}{2} - p_s \end{aligned} \tag{X30}$$

Recalling the proof of Lemma 3, we have the following results:

If $p_o < p_s$, there is $\frac{(p_o - h_s)^2 + 1}{2} - p_o > \frac{(p_s - h_s)^2 + 1}{2} - p_s$ (i.e., $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} > h_s$);

If $p_o \geq p_s$, there has $\frac{(p_o - h_s)^2 + 1}{2} - p_o < \frac{(p_s - h_s)^2 + 1}{2} - p_s$ (i.e., $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < h_s$).

i) For the condition that $p_o < p_s$, we obtain the following result：

(1) If there is $h_o < h_s$, all consumers will choose to purchase online. Under this scenario, the optimal profits under four return policies are $\pi_1^{O*}(q_1^{O*})$, $\pi_2^{O*}(q_2^{O*})$, $\pi_3^{O*}(q_3^{O*})$, and $\pi_4^{O_1*}(q_4^{O_1*})$.

If $r_o < r_s$, we have $\pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) = \pi_4^{O_1*}(q_4^{O_1*}) > \pi_2^{O*}(q_2^{O*})$; however, if $r_o \geq r_s$, we obtain $\pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) = \pi_4^{O_1*}(q_4^{O_1*}) \leq \pi_2^{O*}(q_2^{O*})$.

(2) If there is $h_s \le h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$, comparing the optimal proifts of $\pi_1^{O*}(q_1^{O*})$, $\pi_2^{O*}(q_2^{O*})$, $\pi_3^{O*}(q_3^{O*})$, and $\pi_4^{O_2*}(q_4^{O_2*})$, we can get:

If $r_o < r_s$, we will get $\pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) > \pi_2^{O*}(q_2^{O*}) = \pi_4^{O_2*}(q_4^{O_2*})$; however, if $r_o \ge r_s$, we have $\pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) \le \pi_2^{O*}(q_2^{O*}) = \pi_4^{O_2*}(q_4^{O_2*})$.

ii) For the condition that $p_o \ge p_s$, we get the following result：

(3) If $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < h_o < h_s$ holds, all consumers will choose to purchase in-store. In this scenario, the optimal profits under four return policies are $\pi_1^{S*}(q_1^{S*})$, $\pi_2^{S*}(q_2^{S*})$, $\pi_3^{S*}(q_3^{S*})$, and $\pi_4^{S_1*}(q_4^{S_1*})$.

If $r_o < r_s$, recalling the proof of Lemma 2, we get:

$$\begin{aligned}
&\pi_1^{S*}(q_1^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\
&\ge \pi_1^{S*}(q_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\
&> \pi_2^{S*}(q_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (p_o - c_o - r_s(p_o - h_o))ED_{out}
\end{aligned}$$

Therefore, we get $\pi_1^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_2^{S*}) > \pi_2^{S*}(q_2^{S*})$.

Similarly, comparing the values of $\pi_1^{S*}(q_1^{S*})$ and $\pi_3^{S*}(q_3^{S*})$, we can get:

$$\begin{aligned}
&\pi_3^{S*}(q_3^{S*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\
&\ge \pi_3^{S*}(q_1^{S*}) = \alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out} \\
&> \pi_1^{S*}(q_1^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (p_o - c_o - r_o(p_o - h_o))ED_{out}
\end{aligned}$$

Therefore, we obtian $\pi_3^{S*}(q_3^{S*}) \ge \pi_3^{S*}(q_1^{S*}) > \pi_1^{S*}(q_1^{S*})$

In conclusion, we have $\pi_3^{S*}(q_3^{S*}) \ge \pi_3^{S*}(q_1^{S*}) > \pi_1^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_2^{S*}) > \pi_2^{S*}(q_2^{S*})$.

Therefore, the optimal profit is $\pi^* = \pi_3^{S*}(q_2^{S*}) = \pi_4^{S_1*}(q_4^{S_1*})$.

If $r_o \ge r_s$, recalling proof of Lemmas 2, 4 and 6, we obtain the following results:

$$\pi_2^{S*}(q_2^{S*}) \ge \pi_2^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_3^{S*}) \ge \pi_3^{S*}(q_3^{S*}).$$

To sum up, the optimal profit is $\pi^* = \pi_2^{S*}(q_2^{S*})$ in this case.

(4) If $h_s \le h_o < p_o - \sqrt{2p_o - 1}$, by comparing $\pi_1^{S*}(q_1^{S*})$, $\pi_2^{S*}(q_2^{S*})$, $\pi_3^{S*}(q_3^{S*})$, and $\pi_4^{S_2*}(q_4^{S_2*})$, we obtain the following results:

If there exists $r_o < r_s$, we get $\pi_3^{S*}(q_3^{S*}) \ge \pi_3^{S*}(q_1^{S*}) > \pi_1^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_2^{S*}) > \pi_2^{S*}(q_2^{S*}) = \pi_4^{S_2*}(q_4^{S_2*})$

Therefore, the optimal profit is $\pi^* = \pi_3^{S*}(q_3^{S*})$;

If $r_o \ge r_s$ holds, we have $\pi_4^{S_2*}(q_4^{S_2*}) = \pi_2^{S*}(q_2^{S*}) \ge \pi_2^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_1^{S*}) \ge \pi_1^{S*}(q_3^{S*}) \ge \pi_3^{S*}(q_3^{S*})$

Therefore, the optimal profit is $\pi^* = \pi_2^{S*}(q_2^{S*}) = \pi_4^{S_2*}(q_4^{S_2*})$.

In conclusion

If $p_o < p_s$,

when there is $h_o < h_s$, and

if $r_o < r_s$ also holds, then the retailer should allow consumers to return using the originally purchased channel return policy, online return policy, or cross-channel return policy to achieve maximum profit, and the related profits are $\pi^* = \pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*}) = \pi_4^{O_1*}(q_4^{O_1*})$;

if $r_o \geq r_s$ also holds, the profit-maximizing omnichannel retailer should utilize offline return policy, the corresponding optimal profit is $\pi^* = \pi_2^{O*}(q_2^{O*})$.

when there is $h_s \leq h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$, and

if $r_o < r_s$, then $\pi^* = \pi_1^{O*}(q_1^{O*}) = \pi_3^{O*}(q_3^{O*})$, and the omnichannel retailer should emply the originally purchased channel return policy or online return policy.

if $r_o \geq r_s$, then $\pi^* = \pi_2^{O*}(q_2^{O*}) = \pi_4^{O_2*}(q_4^{O_2*})$, and the omnichannel retailer should adopt the offline return policy or cross-channel return policy.

If $p_o \geq p_s$,

when $p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < h_o < h_s$, and

if $r_o < r_s$, then $\pi^* = \pi_3^{S*}(q_3^{S*}) = \pi_4^{S_1*}(q_4^{S_1*})$, and the omnichannel retailer should emply online return policy or cross-channel return policy.

if $r_o \geq r_s$, then $\pi^* = \pi_2^{S*}(q_2^{S*})$, and the omnichannel retailer should adopt offline return policy.

when $h_s \leq h_o < p_o - \sqrt{2p_o - 1}$, and

if $r_o < r_s$, then $\pi^* = \pi_3^{S*}(q_3^{S*})$, and the omnichannel retailer should emply online return policy.

if $r_o \geq r_s$, then $\pi^* = \pi_2^{S*}(q_2^{S*}) = \pi_4^{S_2*}(q_4^{S_2*})$, and the omnichannel retailer should implement offline return policy or cross-channel return policy.

Appendix B: Considering consumer's offline channel preference (Extension)

Proof of Lemma 9 (Original purchasing channel return policy incorporating channel preference)

When considering the issue of consumer heterogeneity, the utility of the product purchased by consumers in different channels does not change. Thus:

1) Recall the proof of Lemma 1, if there is $h_o < p_o - \sqrt{2p_o - 1 + 2\left\{\frac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$,we can get $U_{BORO} > U_{BSRS}^{BORO}$, and consumers are choosing to buy online. Therefore, we have ϑ=0, and the profit is shown as:

$$\begin{aligned}\tilde{\pi}_1^O(q) &= \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ &+ (1-\vartheta)(1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED] \\ &= \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED\end{aligned} \tag{E1}$$

We can obtain the optimal inventory $\tilde{q}_1^{O*} = \overline{F}^{inv}\left(\dfrac{c_s}{p_s - r_s(p_s - h_s)}\right)$.

3) When $p_o - \sqrt{2p_o - 1 + 2\left\{\dfrac{(p_s - h_s)^2 + 1}{2} - p_s\right\}} < h_o < p_o - \sqrt{2p_o - 1 - 2\dfrac{\xi}{1-\xi}\left\{\dfrac{(p_s - h_s)^2 + 1}{2} - p_s\right\}}$ exists, we can get $U_{BORO} < U_{BSRS}^{BORO}$, which means all consumers are choosing to buy offline. Therefore $\vartheta = 1$, and in this case, the profit function is shown as follows:

$$\begin{aligned}\tilde{\pi}_1^S(q) &= \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta\alpha(1-\theta)\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\ &+ (1-\vartheta)(1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED] \\ &= \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED_{out}\end{aligned} \tag{E2}$$

The best respone functon (first-order derivative) of $\tilde{\pi}_1^S(q)$ on inventory q is:

$$\partial\tilde{\pi}_1^S(q)/\partial q = \left\{\alpha\left[p_s - r_s(p_s - h_s)\right] - (1-\theta)(p_o - c_o - r_o(p_o - h_o))\right\}(1 - F(q)) - \alpha c_s \tag{E3}$$

We also have the following function

$$\partial\tilde{\pi}_1^S(q)/\partial q^2 = -\left\{\alpha\left[p_s - r_s(p_s - h_s)\right] - (1-\theta)(p_o - c_o - r_o(p_o - h_o))\right\}f(q) < 0$$

Since the second-order derivative is negative (i.e., $\partial\tilde{\pi}_1^S(q)/\partial q^2 < 0$), the optimal result can be obtained by using $\partial\tilde{\pi}_1^S(q)/\partial q = 0$. The optimal inventory is:

$$\tilde{q}_1^{S*} = \overline{F}^{inv}\left(\frac{\alpha c_s}{\alpha\left[p_s - r_s(p_s - h_s)\right] - (1-\theta)(p_o - c_o - r_o(p_o - h_o))}\right) \tag{E4}$$

Where, $p_s - r_s(p_s - h_s) - c_s$ is the profit that can be obtained from offline sales of a product, and $p_o - c_o - r_o(p_o - h_o)$ is the profit that can be received from online sales of a product.

If merchants open an offline channel, there are $p_s - r_s(p_s - h_s) - c_s > (1-\theta)(p_o - c_o - r_o(p_o - h_o))$ and $\alpha \geq 1$ based on the assumption. Thus, it is intuitive that $\alpha\left[p_s - r_s(p_s - h_s)\right] - (1-\theta)(p_o - c_o - r_o(p_o - h_o)) > 0$ holds.

Proof of Lemma 10 (Offline return policy incorporating offline channel preference)

1) Recall the proof of Lemma 3, if there is $p_o < p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then we get $U_{BORS} > U_{BSRS}^{BORS}$, which means consumers choose to buy online. Therefore, there is $\vartheta=0$. In this case, the profit function is shown as:

$$\begin{aligned}\tilde{\pi}_2^O(q) &= \vartheta\theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+\vartheta(1-\theta)\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+\vartheta(1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\ &+(1-\vartheta)(1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED] \\ &= \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED\end{aligned} \tag{E5}$$

We can obtain the optimal inventory of $\tilde{q}_2^{O*} = \overline{F}^{inv}\left(\dfrac{c_s}{p_s - r_s(p_s - h_s)}\right)$.

2) If there is $p_o > p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then $U_{BORS} < U_{BSRS}^{BORS}$ holds, which indicates that all consumers are choosing to buy in- store. Therefore, we will get $\vartheta=1$ and the profit is given by:

$$\begin{aligned}\tilde{\pi}_2^S(q) &= \vartheta\theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+\vartheta(1-\theta)\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ &+\vartheta(1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\ &+(1-\vartheta)(1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED] \\ &= \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}]\end{aligned} \tag{E6}$$

The first-order derivative of profit function $\tilde{\pi}_2^S(q)$ on inventory q is:

$$\partial\tilde{\pi}_2^S(q)/\partial q = \left\{\alpha(p_s - r_s(p_s - h_s)) - (1-\theta)(p_o - c_o - r_s(p_o - h_o))\right\}(1 - F(q)) - \alpha c_s$$

We also have the following second-order derivative function

$$\partial\tilde{\pi}_2^S(q)/\partial q^2 = -\left\{\alpha(p_s - r_s(p_s - h_s)) - (1-\theta)(p_o - c_o - r_s(p_o - h_o))\right\} f(q) < 0$$

Since $\partial\tilde{\pi}_2^S(q)/\partial q^2 < 0$, the optimal inventory can be obtained from $\partial\tilde{\pi}_2^S(q)/\partial q = 0$. The optimal solution when the profit function is maximized is:

$$\tilde{q}_2^{S*} = \overline{F}^{inv}\left(\frac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (1-\theta)(p_o - c_o - r_s(p_o - h_o))}\right) \tag{E7}$$

Proof of Lemma 11 (Online return incorporating offline channel preference)

1) Recall the proof of Lemma 5, if there are $p_o < p_s$ and $h_o < p_o - \sqrt{2p_o - 1}$, then we have $U_{BORO} > U_{BSRO}^{BORO}$. In this case, all consumers are choosing to buy online (i.e., $\vartheta=0$).The profit in this case is given by：

$$\begin{aligned}\tilde{\pi}_3^O(q) &= \vartheta\theta\alpha\left\{\left[p_s - r_s(p_s-h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)\alpha\left\{\left[p_s - r_o(p_s-h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)[(p_o - c_o - r_o(p_o-h_o))ED_{out}] \\ &+ (1-\vartheta)(1-\theta)[(p_o - c_o - r_o(p_o-h_o))ED] \\ &= \theta\alpha\left\{\left[p_s - r_s(p_s-h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_o(p_o-h_o))ED\end{aligned} \tag{E8}$$

We can obtain the optimal inventory $\tilde{q}_3^{O*} = \overline{F}^{inv}\left(\dfrac{c_s}{p_s - r_s(p_s-h_s)}\right)$.

2) If there are $p_o > p_s$ and $h_s < p_o - \sqrt{2p_o - 1}$, then we have $U_{BORO} < U_{BSRO}^{BORO}$, which indicates all consumers are choosing to buy in the physical store(i.e., ϑ=1). In this case, the profit is given by:

$$\begin{aligned}\tilde{\pi}_3^S(q) &= \vartheta\theta\alpha\left\{\left[p_s - r_s(p_s-h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)\alpha\left\{\left[p_s - r_o(p_s-h_s)\right]ED_{in} - c_s q\right\} \\ &+ \vartheta(1-\theta)[(p_o - c_o - r_o(p_o-h_o))ED_{out}] \\ &+ (1-\vartheta)(1-\theta)[(p_o - c_o - r_o(p_o-h_o))ED] \\ &= \theta\alpha\left\{\left[p_s - r_s(p_s-h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s-h_s)\right]ED_{in} - c_s q\right\} \\ &+ (1-\theta)[(p_o - c_o - r_o(p_o-h_o))ED_{out}]\end{aligned} \tag{E9}$$

The first-order derivative of the profit function $\pi_3^S(q)$ on inventory q is:

$$\begin{aligned}\partial\tilde{\pi}_3^S(q)/\partial q &= \left\{\theta\alpha(p_s - r_s(p_s-h_s)) + (1-\theta)\alpha\left(p_s - r_o(p_s-h_s)\right) - (1-\theta)(p_o - c_o - r_o(p_o-h_o))\right\} \\ &\quad * (1-F(q)) - \alpha c_s\end{aligned} \tag{E10}$$

The second-order derivative function is shown as:

$$\partial\tilde{\pi}_3^S(q)/\partial q^2 = -\left\{\theta\alpha(p_s - r_s(p_s-h_s)) + (1-\theta)\alpha\left(p_s - r_o(p_s-h_s)\right) - (1-\theta)(p_o - c_o - r_o(p_o-h_o))\right\} f(q) < 0$$

Because $\partial\tilde{\pi}_3^S(q)/\partial q^2 < 0$ exists, the equation (E10) has a unique optimal solution, which can be found using $\partial\tilde{\pi}_3^S(q)/\partial q = 0$ as follows:

$$\tilde{q}_3^{S*} = \overline{F}^{inv}\left(\frac{\alpha c_s}{\left\{\theta\alpha(p_s - r_s(p_s-h_s)) + (1-\theta)\alpha\left(p_s - r_o(p_s-h_s)\right) - (1-\theta)(p_o - c_o - r_o(p_o-h_o))\right\}}\right) \tag{E11}$$

Proof of Lemma 12 (Cross-channel return incorporating offline channel preference)

1) Recall the proof of Lemmas 9, 10, and 11; if there are $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$, we will get $U_{\max} = U_{BORO}$,which means consumers will buy online (i.e., ϑ=0). In this case, the profit is given by：

$$\tilde{\pi}_4^{O_1}(q) = \theta\alpha\left\{\left[p_s - r_s(p_s-h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_o(p_o-h_o))ED \tag{E12}$$

We can obtain the optimal inventory $\tilde{q}_4^{O_1*} = \overline{F}^{inv}\left(\dfrac{c_s}{p_s - r_s(p_s-h_s)}\right)$ from equation (E11).

2) If there are $h_o < h_s < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$, we will get $U_{\max} = U_{BSRO}^{BORO}$, then all consumers are choosing to purchase in-store(i.e., $\vartheta = 1$). In this case, the profit is the same as Lemma 11:

$$\begin{aligned}\tilde{\pi}_4^{S_1}(q) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q\right\} \\ + (1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}]\end{aligned} \tag{E13}$$

The optimal inventory can be obtained as:

$$\tilde{q}_4^{S_1*} = \overline{F}^{inv}\left(\frac{\alpha c_s}{\left\{\theta\alpha(p_s - r_s(p_s - h_s)) + (1-\theta)\alpha\left(p_s - r_o(p_s - h_s)\right) - (1-\theta)(p_o - c_o - r_o(p_o - h_o))\right\}}\right).$$

Similar to Lemma 11, the optimal inventory $\tilde{q}_4^{S_1*}$ decreases with p_o but increases with p_s. Meanwhile, if there is $(p_o - h_o) > \alpha(p_s - h_s)$, $\tilde{q}_4^{S_1*}$ increases with both r_o and α; while if there is $(p_o - h_o) < \alpha(p_s - h_s)$, then $\tilde{q}_4^{S_1*}$ decreases with r_o but decreases with r_s.

3) Recall the proof of Lemmas 9, 10 and 11, we get: if there are $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o < p_s$, we can obtain $U_{\max} = U_{BORS}$, which indicates that all consumers choose to buy online (i.e., $\vartheta = 0$). In this case, the profit function is shown as follows:

$$\tilde{\pi}_4^{O_2}(q) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED \tag{E14}$$

We can obtain the optimal inventory $\tilde{q}_4^{O_2*} = \overline{F}^{inv}\left(\dfrac{c_s}{p_s - r_s(p_s - h_s)}\right)$.

4) If there are $h_s < h_o < p_o - \sqrt{2p_o - 1}$ and $p_o \geq p_s$, then $U_{\max} = U_{BSRS}^{BORS}$. Under this assumption, consumers purchase in-store and return offline regardless of whether the store is out of stock or not. In this case, similar to Lemma 10, the profit is given by:

$$\tilde{\pi}_4^{S_2}(q) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \tag{E15}$$

We can obtain the optimal inventory $\tilde{q}_4^{S_2*} = \overline{F}^{inv}\left(\dfrac{\alpha c_s}{\alpha(p_s - r_s(p_s - h_s)) - (1-\theta)(p_o - c_o - r_s(p_o - h_o))}\right)$.

Similar to the proof of Lemma 10, the optimal inventory $\tilde{q}_4^{S_2*}$ decreases with p_o but increases with p_s. Meanwhile, if there is $(1-\theta)(p_o - h_o) > \alpha(p_s - h_s)$, then $\tilde{q}_4^{S_2*}$ increases with r_s and α; if $(1-\theta)(p_o - h_o) < \alpha(p_s - h_s)$ holds, then $\tilde{q}_4^{S_2*}$ decreases with r_s.

Proof of Proposition 5 (Optimal profit comparison incorporating offline channel preference)

Based on Lemmas 9,10,11, and 12, the optimal profits under the four return policies are as follows:

$$\begin{cases}
\tilde{\pi}_1^{O*}(\tilde{q}_1^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{O*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\
\tilde{\pi}_1^{S*}(\tilde{q}_1^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_1^{S*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED_{out} \\
\tilde{\pi}_2^{O*}(\tilde{q}_2^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{O*}\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED \\
\tilde{\pi}_2^{S*}(\tilde{q}_2^{S*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_2^{S*}\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}] \\
\tilde{\pi}_3^{O*}(\tilde{q}_3^{O*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_3^{O*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\
\tilde{\pi}_3^{S*}(\tilde{q}_3^{S*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_3^{S*}\right\} \\
\qquad + (1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\
\tilde{\pi}_4^{O_1*}(\tilde{q}_4^{O_1*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{O_1*}\right\} + (1-\theta)(p_o - c_o - r_o(p_o - h_o))ED \\
\tilde{\pi}_4^{S_1*}(\tilde{q}_4^{S_1*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} + (1-\theta)\alpha\left\{\left[p_s - r_o(p_s - h_s)\right]ED_{in} - c_s q_4^{S_1*}\right\} \\
\qquad + (1-\theta)[(p_o - c_o - r_o(p_o - h_o))ED_{out}] \\
\tilde{\pi}_4^{O_2*}(\tilde{q}_4^{O_2*}) = \theta\alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{O_2*}\right\} + (1-\theta)(p_o - c_o - r_s(p_o - h_o))ED \\
\tilde{\pi}_4^{S_2*}(\tilde{q}_4^{S_2*}) = \alpha\left\{\left[p_s - r_s(p_s - h_s)\right]ED_{in} - c_s q_4^{S_2*}\right\} + (1-\theta)[(p_o - c_o - r_s(p_o - h_o))ED_{out}]
\end{cases} \tag{E16}$$

Where $\tilde{\pi}_4^{O_1*}(\tilde{q}_4^{O_1*})$ and $\tilde{\pi}_4^{S_1*}(\tilde{q}_4^{S_1*})$ represent that when the retailer adopts the cross-return policy, consumers choose to return all products online. Therefore, in this case, the retailer's profit is equal to the profit of adopting the online return policy (i.e., $\tilde{\pi}_4^{O_1*}(\tilde{q}_4^{O_1*}) = \tilde{\pi}_3^{O*}(\tilde{q}_3^{O*})$ and $\tilde{\pi}_4^{S_1*}(\tilde{q}_4^{S_1*}) = \tilde{\pi}_3^{S*}(\tilde{q}_3^{S*})$). Similarly, where $\tilde{\pi}_4^{O_2*}(\tilde{q}_4^{O_2*})$ and $\tilde{\pi}_4^{S_2*}(\tilde{q}_4^{S_2*})$ represent that when the retailer adopts the cross-return policy, consumers choose to return all products offline. Therefore, the retailer's profit, in this case, is equal to the profit of adopting the offline return policy (i.e., $\tilde{\pi}_4^{O_2*}(\tilde{q}_4^{O_2*}) = \tilde{\pi}_2^{O*}(\tilde{q}_2^{O*})$ and $\tilde{\pi}_4^{S_2*}(\tilde{q}_4^{S_2*}) = \tilde{\pi}_2^{S*}(\tilde{q}_2^{S*})$).

The analysis shows that the result is the same as Proposition 8 when consumers have channel preferences.