Estimating and Testing Long-Run Risk Models: International Evidence

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Andras Fulop[†] Junye Li[‡] Hening Liu[§] Cheng Yan[¶]

Abstract

We estimate and test long-run risk models using international macroeconomic and financial data. The benchmark model features a representative agent who has recursive preferences with a time preference shock, a persistent component in expected consumption growth, and stochastic volatility in fundamentals characterized by an autoregressive Gamma process. We construct a comprehensive dataset with quarterly frequency for ten developed countries and employ an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods to make full econometric inference. Our empirical findings provide international evidence in support of long-run risks, time-varying preference shocks, and countercyclicality of the stochastic discount factor. We show the existence of a global long-run consumption factor driving equity returns across individual countries.

Keywords: Consumption, Equity Premium, Long-Run Risk, Stochastic Discount Factor, Projection Methods, Sequential Monte Carlo Sampler

JEL Classification: *C11, C32, C58, E44, G12*

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Really, the most natural thing to do with the consumption-based model is to estimate it and test it, as one would do for any economic model (p.267).

—— *Cochrane, John H. (2008)*

1. Introduction

The "equity premium puzzle", first documented by Mehra and Prescott (1985), states that the standard consumption-based asset pricing model with constant relative risk aversion (CRRA) would require implausibly high risk aversion to explain the historical equity premium in the US market, given low variation observed in the consumption data. Since then, a rapidly growing literature has emerged to explain the equity premium puzzle, along with other notable behaviors of asset returns such as a low and smooth risk-free rate, high equity volatility, and stock return predictability (e.g., Weil, 1989; Campbell and Cochrane, 1999; Routledge and Zin, 2010; Ju and Miao, 2012; Wachter, 2013). Among those consumption-based models, the long-run risk model proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) has attracted remarkable attention and become a benchmark in the literature.

However, studies on consumption-based models and the long-run risk models in particular so far have been confined to explaining the US market data. Analysis based on macroeconomic and financial data in other developed countries is rather limited, even though it would be interesting for reasons as follows. First, as highlighted by Campbell (2003, 2018), the equity premium puzzle is a global phenomenon that also prominently prevails in other developed countries. Second, given that the heart of the long-run risk models is a slow-moving latent process driving expected consumption growth, we then ask whether the relative positive evidence in support of long-run risk is special for the U.S market? Complementing the US-based finding with evidence from other economies is one promising way to address questions regarding the importance of this process and provides a way to test the model.

Thus, as one contribution of our paper, we construct a comprehensive dataset including quarterly macroeconomic and financial data in the post-war period for a rich set of developed countries and estimate and test long-run risk models using this dataset. Our sample for estimation consists of quarterly data on aggregate consumption, dividends, risk-free rates, and stock market returns for ten developed countries, including the US, the UK, Germany, France, Italy, Japan, Canada, Australia, the Netherlands, and Switzerland.

Moreover, the predominant approach in prior research on consumption-based asset pricing has predominantly employed calibration. This involves selecting values for primitive parameters within a utility function and specifying fundamental processes to match a chosen set of moments related to fundamentals and asset returns. As highlighted earlier, it is natural to formally estimate and test consumption-based models. However, structural estimation studies in asset pricing remain notably scarce. This scarcity is primarily attributed to the formidable challenges in econometric estimation posed by consumption-based models. The complexity arises from the highly nonlinear nature of global solutions to these models concerning state variables. Additionally, the limited availability and low frequency of data on fundamentals, particularly for countries other than the US, further contribute to the difficulty. Consequently, only a handful of studies have undertaken econometric estimation of consumption-based models using US data; see, for example, Bansal, Gallant, and Tauchen (2007), Bansal, Kiku, and Yaron (2016), Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2022). Most of these studies either use moment-based or indirect inference methods (e.g., Bansal, Gallant, and Tauchen, 2007; Bansal, Kiku, and Yaron, 2016; Gallant, Jahan-Parvar, and Liu, 2019), which do not fully exploit information in the likelihood function implied by the original asset pricing models, or crucially rely on the log-linearization method of Campbell and Shiller (1988) to solve for asset prices (e.g., Bansal, Gallant, and Tauchen, 2007; Bansal, Kiku, and Yaron, 2016; Schorfheide, Song, and Yaron, 2018). In recent work, Pohl, Schmedders, and Wilms (2018) demonstrate that the log-linearized solutions to long-run risk models can generate significant numerical errors. They show that using projection methods to solve for global solutions to long-run risk models can account for higher-order effects and effectively reduce numerical errors. In this paper, we conduct full likelihood-based estimation by exploiting the global nonlinear solutions and Bayesian techniques.

We consider a representative agent who has recursive preferences (Epstein and Zin, 1989; Weil, 1989) that allow for the separation between risk aversion and the elasticity of intertemporal substitution (EIS). We follow the long-run risk literature and assume that expected consumption growth contains a slow-moving persistent component that is subject to stochastic changes, and that conditional volatilities of fundamentals are stochastic, capturing time-varying economic uncertainties. Rather than using the autoregressive (AR) process to model conditional variance, as is commonly done in the long-run risk literature, we assume that conditional variances of fundamentals follow autoregressive gamma (ARG) processes to ensure positivity of conditional variances. This assumption leads to reliable solutions to the models with time-varying uncertainty. Furthermore, we follow Albuquerque et al. (2016) to assume that the agent's rate of time preference is subject to stochastic changes. Introducing time-varying preference shocks plays a crucial role for a consumption-based model in reconciling correlation and covariance between stock returns and fundamentals typically observed in the US data as well as in international data. Considering the model with time-varying preference shocks, our estimation naturally takes into account the empirical correlation between stock returns and fundamentals. As a consequence, the parameter estimates and latent states obtained in the estimation are consistent with the estimated law of motion for time preference shocks.

We rely on the collocation projection method to solve for global solutions to our models and make full econometric inference based on an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods. We extend the sequential Monte Carlo square (SMC^2) method used in Fulop et al. (2022) with a more efficient particle filter for likelihood estimation to estimate our models that have more latent states. Different from moment-based methods, our SMC^2 method takes advantage of full information contained in the likelihood function, obtained from running an efficient square-root unscented particle filter, and provides us with the posterior distribution of model parameters and the smoothing distribution of latent states over time that determine fluctuations of asset prices. Different from traditional Bayesian Markov Chain Monte Carlo (MCMC) methods or particle MCMC methods (Andrieu, Doucet, and Holenstein, 2010), a tailor-made version of which is used in Schorfheide, Song, and Yaron (2018) for estimating a linearized model, our SMC^2 method provides us with the marginal likelihood estimates that are necessary statistics for model comparisons and can be easily parallelized, making it computationally convenient to use in estimation.

We want to emphasize that we implement estimation separately for each country, as our main purpose is to examine whether long run risk exists and how the model performs in non-US countries. In other words, we do not examine international risk sharing among developed countries, but instead consider consumption and portfolio autarky for each of the developed countries. A desirable approach would be to estimate the models jointly for a selected set of countries, but at the cost of only a very small number of countries to be considered because of the heavy computational burden in the solution and estimation algorithms. Hence, we leave this interesting direction to future research.

Our empirical findings can be briefly summarized as follows. First, we find that with regard to fitting asset returns, the time-varying preference shock plays a much more important role than a separate stochastic volatility process capturing a time-varying independent risk in the dividend dynamics. For almost all the countries, introducing an independent stochastic volatility process in dividend growth cannot improve the model fit on stock market returns but incur a very high computational cost. Overall, our preferred model is the one that features a time-varying preference shock, a persistent component in expected consumption growth, and a common stochastic volatility process that governs the dynamics of both consumption growth and dividend growth. Such evidence exists almost in all the countries.

Second, our estimation results based on the international analysis clearly indicate values of the EIS greater than 1 (the posterior means are around 2), a presumption that has been emphasized by studies on long-run risks and more broadly, by asset pricing studies based on recursive preferences; see, for example, Bansal and Yaron (2004), Bansal et al. (2014), Ai (2010), Ju and Miao (2012), Wachter (2013), Croce (2014), and Jahan-Parvar and Liu (2014). For all the countries in our analysis, the posterior mean estimates of the relative risk aversion (RRA) parameter range between 5 and 10. Our estimates of relative risk aversion are reasonable and consistent with the prediction of economic theory as well as experimental evidence, but are smaller than values commonly postulated in the calibration studies for the countries other than the US. We find that introducing time-varying preference shocks in the long-run risk model helps deliver economically plausible estimates of risk aversion, not only for the US but also for the other developed economies. Our estimates of RRA and EIS for different countries provide empirical support to investors' preference for early resolution of uncertainty based on international evidence.

Third, we find that for all the countries, expected consumption growth consists of a persistent component, albeit the importance of this long-run risk component varies across countries. For the US, the long run component accounts for a significant amount of time variation in consumption growth, while for the other countries it accounts for less variation in consumption growth. Moreover, there is notable heterogeneity across countries in the level of persistence in stochastic volatility of consumption growth.

Fourth, for most of the countries in our sample, the stochastic discount factor under recursive utility has a countercyclical component. In addition, conditional equity premium and conditional volatility of stock returns also exhibit countercyclical variation to a certain extent. With regard to fitting time series of asset returns, for all the ten countries, our estimation generates fitted risk-free rates that closely track the historical movements of the actual risk-free rates, suggesting high accuracy of our estimates of the stochastic discount factor. Nevertheless, fitted market returns remain less accurate, though they can explain a significant fraction of actual market returns for all the countries.

Finally, we find that the correlations of the long-run consumption components are much stronger than the consumption growth rates across individual countries. Therefore, we construct a global long-run consumption factor based on the filtered long-run consumption components of individual countries. We find that this global long-run consumption factor strongly comoves with a global equity market factor, defined as the equal-weighted average of equity market returns across the ten individual countries or the market factor of the Fama-French developed economies (Fama and French, 2012). The beta estimates of equity market returns of individual countries on the global long-run consumption factor are all larger than 1 and highly statistically significant in the time-series regressions, and the factor carries a significant positive risk premium, at about 0.85% with a p-value of 5%, as estimated in the cross-sectional regression.

Our paper is closely related to two recent papers, Schorfheide, Song, and Yaron (2018) and Fulop et al. (2022), both of which employ likelihood-based Bayesian approaches to estimate their respective versions of the long-run risk model. However, our paper differs from these two in important aspects. First, both studies exclusively focus on the US market, whereas ours implements empirical investigations for ten developed countries and provides international evidence in support of long-run risks. Second, Schorfheide, Song, and Yaron (2018) introduce separate volatility processes, respectively, for consumption growth, the long-run risk component, and dividend growth using conditional log-normal processes. They rely on linearization of the log-volatility processes and the log-linearization method to find linear functions for equilibrium asset prices. It is worth mentioning that there exist nontrivial differences between the approximating model characterizing asset prices and the model used in the estimation. In addition, without using log-linearization, their Bayesian estimation method is too computationally demanding and hard to implement for ten countries. Third, Fulop et al. (2022) consider long-run risk models in which the consumption volatility process is modeled using either an ARG process or an AR process; however, the preference shock is absent in their models, and they don't consider a separate dividend volatility process as well. Fourth, neither of the two studies does model comparisons to evaluate the relative importance of different state variables determining asset prices.

Another related work is Creal and Wu (2020) that estimates consumption-based models with long-run risks, recursive utility, time preference shocks, and a noncentral Gamma volatility process, using the US Treasury yields data. However, different from our structural estimation, they implement estimation using a two-stage procedure, separately for the state dynamics and the pricing kernel, and furthermore, their estimation relies on loglinearization of long-run risk models. In a recent contribution, Nakamura, Sergeyev, and Steinsson (2017) employ Bayesian MCMC methods to estimate a long-run risk model using annual per capita consumer expenditures data on many developed countries for a long sample. However, their identification of long-run risks solely relies on the consumption data, and they do not estimate preference parameters or stochastic rates of time preference as we do in this paper. An earlier work by Rangvid, Schmeling, and Schrimpf (2010) uses the generalized methods of moments (GMM) to estimate a long-run risk model without a dividend process or stochastic consumption volatility for fifteen countries. They heavily rely on log-linearization and only exploit consumption and market returns data at an annual frequency.

The rest of the paper is organized as follows. Section 2 presents long-run risk models considered in our estimation. Section 3 briefly describes the solution method and our econometric inference based on sequential Monte Carlo methods. Section 4 discusses the international macroeconomic and financial data used for model estimation. Section 5 presents estimation results and discusses asset pricing implications. Section 6 concludes the paper. Additional materials are given in the Internet Appendix.

2. Model Framework

2.1. Preferences

We examine an endowment economy, in which a representative agent has recursive preferences as in Epstein and Zin (1989) and Weil (1989). Moreover, following Albuquerque et al. (2016) and Schorfheide, Song, and Yaron (2018), we introduce time preference shocks in the utility function. As shown in Albuquerque et al. (2016), a major role of time preference shocks is to mitigate the strong correlation between stock returns and measurable fundamentals that is typically obtained in consumption-based models without demand shocks. As a result, the agent's recursive utility function is given by

$$V_t = \left[(1-\delta)\lambda_t C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathbb{E}_t(V_{t+1}^{1-\gamma})\right]^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is the time-t consumption, $0 < \delta < 1$ is the agent's time preference parameter, λ_t is the shock to the time rate of preference, γ is the RRA parameter, ψ is the EIS parameter, θ is given by $\theta = \frac{1-\gamma}{1-1/\psi}$, and $\mathbb{E}_t$ denotes conditional expectation with respect to information up to time t.

This class of preferences allows for a separation between RRA and EIS. The agent prefers early (late) resolution of uncertainty when $\gamma > 1/\psi$ ($\gamma < 1/\psi$), and when $\gamma = 1/\psi$, the agent has CRRA preferences and is neutral to the timing of resolution of uncertainty. The agent's utility maximization is subject to the following budget constraint,

$$W_{t+1} = (W_t - C_t)R_{t+1}^W, \tag{2}$$

where W_t is the agent's wealth, and R_t^W is the return on the wealth portfolio.

For any asset i with ex-dividend price $P_{i,t}$ and dividend $D_{i,t}$, the standard Euler equation holds, i.e.,

$$\mathbb{E}_t\left[M_{t+1}R_{i,t+1}\right] = 1, \tag{3}$$

where $R_{i,t+1} = (P_{i,t+1} + D_{i,t+1})/P_{i,t}$, and M_t is the stochastic discount factor (SDF). In particular, for the risk-free asset, we have $R_{f,t} = 1/\mathbb{E}_t[M_{t+1}]$. It can be shown that the SDF for the recursive utility function defined in Equation (1) takes the form[1]

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}, \tag{4}$$

which can be alternatively expressed as

$$M_{t+1} = \delta^\theta \left(\frac{\lambda_{t+1}}{\lambda_t}\right)^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} \left(R_{t+1}^W\right)^{\theta-1}. \tag{5}$$

[1]See Albuquerque et al. (2016) for a derivation.

Thus, the Euler equation (3) implies that for the return on the wealth portfolio, R_t^W, we have

$$\mathbb{E}_t\left[\delta^\theta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(R_{t+1}^W\right)^\theta\right]=1, \tag{6}$$

and for the return on the market portfolio, $R_{m,t}$, we have

$$\mathbb{E}_t\left[\delta^\theta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(R_{t+1}^W\right)^{\theta-1}R_{m,t+1}\right]=1. \tag{7}$$

As in Albuquerque et al. (2016), we assume that the growth rate of preference shocks, defined as $x_{\lambda,t+1}\equiv\ln\left(\lambda_{t+1}/\lambda_t\right)$, follows an AR(1) process,

$$x_{\lambda,t+1}=\rho_\lambda x_{\lambda,t}+\sigma_\lambda\eta_{\lambda,t+1}, \tag{8}$$

where the shock $\eta_{\lambda,t}$ is standard normal, i.e., $\eta_{\lambda,t}\sim N(0,1)$, and is independent of the other shocks in the model.

2.2. Fundamentals

We follow Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) and assume that the log of consumption growth, $\Delta c_{t+1}\equiv\ln\left(C_{t+1}/C_t\right)$, consists of a persistent component, x_t, and a transitory component,

$$\Delta c_{t+1} = \mu+x_t+\sigma_{c,t}\eta_{c,t+1}, \tag{9}$$

$$x_{t+1} = \rho_x x_t+\phi_x\sigma_{c,t}\eta_{x,t+1}, \tag{10}$$

and that dividends are imperfectly correlated with consumption and their log-growth rate, $\Delta d_{t+1}\equiv\ln\left(D_{t+1}/D_t\right)$, has the dynamics of

$$\Delta d_{t+1}=\mu_d+\Phi x_t+\phi_{dc}\sigma_{c,t}\eta_{c,t+1}+\phi_d\sigma_{d,t}\eta_{d,t+1}, \tag{11}$$

where $\eta_{c,t}$, $\eta_{x,t}$, and $\eta_{d,t}$ are i.i.d normal $N(0,1)$, and $\sigma_{c,t}^2$ and $\sigma_{d,t}^2$ are conditional variances of consumption growth and dividend growth, respectively.

In the standard long-run risk model (Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012), consumption variance is assumed to follow a Gaussian AR(1) process. However, this modeling choice suggests that consumption variance can take negative values, which renders the numerical solution to the model problematic. To overcome this issue, Fulop et al. (2022) and Creal and Wu (2015, 2020) use an autoregressive gamma (ARG) process, proposed by Gourieroux and Jasiak (2006), to model consumption variance and show that the ARG-based long-run risk model outperforms the AR-based one in fitting US market data. To this end, we follow Fulop et al. (2022) to model the conditional variances, $\sigma^2_{i,t}$ for $i = \{c, d\}$, using ARG processes with order 1,

$$\sigma^2_{i,t} \sim Gamma(\phi_{is} + \zeta_{i,t}, c_i), \qquad \zeta_{i,t} \sim Poisson\left(\frac{\rho_{is}\sigma^2_{i,t-1}}{c_i}\right), \tag{12}$$

where $Gamma(\cdot)$ and $Poisson(\cdot)$ denote the gamma distribution and the Poisson distribution, respectively, ρ_{is} controls the persistence of each variance process, c_i determines the scale, and to ensure positivity of conditional variances, the Feller condition, $\phi_{is} > 1$, needs to be satisfied. As shown in Gourieroux and Jasiak (2006) and Creal (2017), the transition density of $\sigma^2_{i,t}$ is a noncentral gamma distribution.[2] Its conditional mean and variance are given by $E[\sigma^2_{i,t}|\sigma^2_{i,t-1}] = \bar{\sigma}^2_i(1-\rho_{is}) + \rho_{is}\sigma^2_{i,t-1}$ and $Var[\sigma^2_{i,t}|\sigma^2_{i,t-1}] = \frac{(1-\rho_{is})\bar{\sigma}^2_i}{\phi_{is}}\left((1-\rho_{is})\bar{\sigma}^2_i + 2\rho_{is}\sigma^2_{i,t-1}\right)$, respectively. The stationary distribution of the ARG process is $Gamma(\phi_{is}, c_i/(1-\rho_{is}))$ with the long-run mean given by $\bar{\sigma}^2_i = \phi_{is}c_i/(1-\rho_{is})$. We label this extended long-run risk model as "eLRR2".

When we assume that the same conditional variance enters into dynamics of both consumption growth and dividend growth, i.e., $\sigma_{c,t} = \sigma_{d,t} = \sigma_t$, we have a nested long-run risk model with time preference shocks, which we label as "eLRR1". When we further shut down time preference shocks, we obtain the counterpart of the standard long-run risk model, which is also studied in Fulop et al. (2022), and we simply label this nested

[2] The density function has the form of

$$f(\sigma^2_{i,t}|\sigma^2_{i,t-1}) = \left(\frac{\sigma^2_{i,t}}{\rho_{is}\sigma^2_{i,t-1}}\right)^{(\phi_{is}-1)/2} \frac{1}{c_i}\exp\left(-\frac{(\sigma^2_{it} + \rho_{is}\sigma^2_{i,t-1})}{c_i}\right) I_{\phi_{is}-1}\left(\frac{2\sqrt{\rho_{is}\sigma^2_{i,t-1}\sigma^2_{i,t}}}{c_i}\right),$$

where $I_\zeta(x) = (x/2)^\zeta \sum_{i=0}^{\infty}(x^2/4)^i/\{i!\Gamma(\zeta+i+1)\}$ denotes a modified Bessel function of the first kind.

model as "LRR".

3. Solution and Econometric Inference

3.1. Model Solution

The usual method to solve the long-run risk models is the log-linear approximation method of Campbell and Shiller (1988).[3] In a recent paper, Pohl, Schmedders, and Wilms (2018) show that solving long-run risk models with log-linearization can yield significant numerical errors when state variables are persistent. They advocate using projection methods to account for higher-order effects. The higher-order effects are important for producing reliable asset pricing results. Therefore, in this paper, we solve our models using the collocation projection method (Judd, 1992, 1999). In the Internet Appendix, we also provide the log-linear solutions to the eLRR2 and eLRR1 models.

To illustrate the collocation projection method, we denote the current state of the economy by z and the state in the next period by z'; for example, in the full model of eLRR2, the state vector is $z = \{x_\lambda, x, \sigma_c^2, \sigma_d^2\}$. We solve the models in two steps as follows.

First, we solve the Euler equation for the wealth portfolio and obtain the wealth-consumption ratio. In the projection method, the solution function of the log wealth-consumption ratio, $\varphi_w(z) \equiv \ln\left(\frac{W(z)}{C(z)}\right)$, is approximated by Chebyshev polynomials and a set of associated unknown coefficients. In particular, the approximation is given by $\hat{\varphi}_w(z) = \sum_{k=0}^{n} \alpha_{w,k}\Lambda_k(z)$, where $\Lambda_k(z)$, $k = 0, \ldots, n$, is a set of basis functions, and $\alpha_{w,k}$, $k = 0, \ldots, n$, is a set of unknown coefficients to be determined. The basis functions are constructed as products of Chebyshev polynomials for the relevant state variables. For the Euler equation, the solution function of the log wealth-consumption ratio satisfies

$$\mathbb{E}\left[\exp\left(\theta\left(\ln\delta + x_\lambda' + \left(1-\frac{1}{\psi}\right)\Delta c(z'|z) + \varphi_w(z') - \ln\left(e^{\varphi_w(z)} - 1\right)\right)\right)\middle| z\right] = 1, \quad (13)$$

[3] For example, see Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012), Bansal, Kiku, and Yaron (2016), Beeler and Campbell (2012) and Schorfheide, Song, and Yaron (2018)

where the log-return on the wealth portfolio is given by

$$r_w(z'|z) \equiv \ln\left(\frac{W(z')}{W(z)-C(z)}\right) = \varphi_w(z') - \ln(e^{\varphi_w(z)} - 1) + \Delta c(z'|z). \tag{14}$$

Second, we approximate the solution function of the log price-dividend ratio by $\hat{\varphi}(z) = \sum_{k=0}^{n} \alpha_k \Lambda_k(z)$, where α_k, $k = 0, \ldots, n$, is a set of unknown coefficients to be determined. Equations (3) and (5) imply that the log price-dividend ratio, $\varphi(z) \equiv \ln\left(\frac{P(z)}{D(z)}\right)$, satisfies

$$\mathbb{E}\left[\exp\left(\theta x_\lambda^{'} + \theta \ln\delta - \frac{\theta}{\psi}\Delta c\left(z'|z\right) + (\theta-1)\, r_w(z'|z) + r(z'|z)\right)\Bigg|\, z\right] = 1, \tag{15}$$

where $r\left(z'|z\right)$ is the log-return on an asset with the dividend growth rate of $\Delta d(z'|z)$,

$$r(z'|z) = \ln\left(e^{\varphi(z')} + 1\right) - \varphi(z) + \Delta d(z'|z). \tag{16}$$

We apply the collocation projection method and approximate the solution functions $\varphi_w(z)$ and $\varphi(z)$ using Chebyshev polynomials. For the Gaussian innovation shocks, we use the Gauss-Hermite quadrature to compute conditional expectations. For the ARG specification, we use the importance sampling method to compute conditional expectations. The collocation projection method leads to a square system of nonlinear equations, which can be solved using the standard nonlinear equation solvers to obtain the solutions to the unknown coefficients $\alpha_{w,k}$ and α_k.[4]

To facilitate numerical computation, we first derive the Jacobian of the square system of nonlinear equations with respect to the solution coefficients (via the chain rule) analytically when solving for the wealth-consumption ratio and the price-dividend ratio. We then supply the user-defined Jacobian to the numerical solvers in MATLAB. We find that this practice greatly reduces computational burden and improves numerical efficiency. Even for the model eLRR2 with four state variables that has substantial numerical complexity, the implementation of global solutions is fast. Thus, our numerical

[4] Borovicka and Stachurski (2020) show exact necessary and sufficient conditions for existence and uniqueness of solutions to a class of models with recursive utility. In our estimation, arbitrary parameter values may be generated and do not necessarily satisfy these conditions. We impose these conditions in the estimation as additional restrictions on parameters when solving and simulating our models.

algorithm improves over previous studies in terms of computational cost.[5]

3.2. Estimation

Our models can be cast into the framework of nonlinear and non-Gaussian state-space models. There are four state variables in the full model: the growth rate of preference shocks, $x_{\lambda,t}$, whose dynamics are given in Equation (8), the long-run component, x_t, whose dynamics are given in Equation (10), and the consumption and dividend variance processes, $\sigma^2_{i,t}$ for $i = \{c, d\}$, whose dynamics are given in Equation (12).

Moreover, there are four observables including the consumption growth rates (Δc_t), the dividend growth rates (Δd_t), the stock market returns ($r_{m,t}$), and the risk-free returns ($r_{f,t}$). The dynamics of consumption and dividend growth rates are given in Equations (9) and (11), respectively. For the stock market and risk-free returns, we assume that their dynamics are given by

$$r_{m,t} = f(z_t, z_{t-1}, \Delta d_t, \Theta) + \sigma_m \eta_{m,t}, \tag{17}$$

$$r_{f,t} = g(\tilde{z}_t, \Theta) + \sigma_f \eta_{f,t}, \tag{18}$$

respectively, where $z_t = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}, \sigma^2_{d,t}\}$, $\tilde{z}_t = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}\}$, Θ denotes the set of model parameters, $r_{m,t}$ and $r_{f,t}$ are the observed market and risk-free returns, $f(\cdot)$ and $g(\cdot)$ are two nonlinear functions resulted from the projection method determining the model-implied market and risk-free returns, and the error terms in Equations (17) and (18) capture asset pricing errors, which are assumed to follow independent standard normal distributions with σ_m and σ_f being the standard deviations of the respective pricing errors.

For T periods, we denote all observations as $y_{1:T} = \{\Delta c_t, \Delta d_t, r_{m,t}, r_{f,t}\}_{t=1}^T$ and the latent states as $z_{1:T} = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}, \sigma^2_{d,t}\}_{t=1}^T$. Our aim is to estimate the joint posterior distribution of parameters and latent states, $p(\Theta, z_{1:T}|y_{1:T})$, which can be decomposed

[5]Previous studies such as Pohl, Schmedders, and Wilms (2018) and Fulop et al. (2022) use the cubic spline interpolation method to obtain the solution functions. Because an analytical Jacobian is unavailable with the cubic spline method, both Pohl, Schmedders, and Wilms (2018) and Fulop et al. (2022) cannot achieve numerical efficiency as our algorithm does.

into

$$p(\Theta, z_{1:T}|y_{1:T}) = p(z_{1:T}|\Theta, y_{1:T})p(\Theta|y_{1:T}), \tag{19}$$

where $p(z_{1:T}|\Theta, y_{1:T})$ solves state smoothing, and $p(\Theta|y_{1:T})$ addresses parameter inference.

We extend the SMC^2 method proposed in Fulop et al. (2022) with more efficient numerical methods and particle filter for likelihood estimation to estimate our models. The dimension increase of state variables makes state filtering more challenging. Instead of using the unscented Kalman filter (UKF) of Li (2011) as in Fulop et al. (2022) to generate proposals, we rely on the square-root version that efficiently solves the covariance singularity problem. Both improvements in numerical computation and particle filtering greatly enhance the efficiency of the likelihood estimation and considerably reduce the computational cost.[6]

The SMC^2 method is based on the ideas of particle Markov chain Monte Carlo methods (PMCMC) (Andrieu, Doucet, and Holenstein, 2010) and sequential Monte Carlo samplers (Del Moral, Doucet, and Jasra, 2006). The former shows that MCMC samplers converge to the real posterior distribution of parameters even when the likelihood is approximated by particle filters, and the latter suggests that a bridge can be built between the prior and posterior distributions of parameters by using some MCMC kernels of invariant distribution of parameters. The SMC^2 method delivers exact draws for the joint posterior distribution of parameters and latent states for any given number of the state particles.

Different from moment-based methods, our econometric method exploits full information contained in the likelihood function of the models in estimation. In addition, our method provides us with the posterior distribution of model parameters and the smoothing distribution of latent states over time that determine fluctuations of asset prices. Different from traditional Bayesian MCMC methods or PMCMC methods (Andrieu, Doucet, and Holenstein, 2010),[7] the SMC^2 method can directly deliver the marginal

[6]Nevertheless, the estimation remains computationally challenging for the full model (eLRR2). For example, for the US that has a sample size of 290 quarters with the sample ranging from 1947Q2 to 2019Q3, the estimation takes only about 1.2 hours for LRR (two state variables) and 2.5 hours for eLRR1 (three state variables), whereas it takes more than 25 hours for eLRR2 (four state variables). All estimations are implemented in Matlab on a Dell workstation with Intel Xeon Gold 6238R CPU (46 cores). We make the Matlab codes available for reproducing all our empirical results.

[7]A tailor-made version of PMCMC is used in Schorfheide, Song, and Yaron (2018) for estimating the

likelihood estimates that are necessary statistics for model comparisons and can be easily parallelized, making it computationally convenient to use in estimation. For more details on the SMC^2 method, we refer readers to Chopin, Jacob, and Papaspiliopoulos (2013), Fulop and Li (2013, 2019), and Fulop and Duan (2015).

4. Data

Our dataset can be viewed as an updated and extended version of the international dataset used in Campbell (1999, 2003, 2018). Specifically, we construct quarterly data on real aggregate consumption, dividends, risk-free rates, and stock market returns for each of the following ten countries: Australia (AU), Canada (CA), France (FR), Germany (DE), Italy (IT), Japan (JP), the Netherlands (NL), Switzerland (CH), the UK (UK), and USA (US). For the US and UK, the sample starts from 1947:Q2 and 1965:Q4, respectively; for all the other countries, the sample starts from 1973:Q4; and for all the countries in our analysis, our sample ends in 2019:Q3.

4.1. Macroeconomic Data

Macroeconomic data on real seasonally-adjusted aggregate consumption, population, and consumer price index (CPI) are downloaded from *Datastream*. Following the literature (see, e.g., Campbell, 1999, 2003; Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012), for the US, the UK, and Canada, we use the seasonally adjusted real consumption (per capita) of nondurables and services.[8] However, for the other countries, given data availability, we use private final consumption expenditures to measure aggregate consumption. Specifically, we take real seasonally-adjusted private final consumption expenditures from the Quarterly National Accounts of Organization for Economic Cooperation and Development (OECD) database, which are then divided by the annual population obtained from International Financial Statistics (IFS, line 99) of International Monetary Fund (IMF) to

linearized model.

[8]See the Internet Appendix for the detailed description of the construction of nondurables and services consumption.

yield real seasonally-adjusted consumption per capita.[9] Moreover, as a comparison, we also use the private final consumption data for the US, the UK, and Canada in our estimation. The estimation results are very similar to those obtained using the nondurables and services consumption. For the sake of brevity, these results are presented in the Internet Appendix.

The source of CPI for the US is the Treasury and Inflation database of Wharton Research Data Services (WRDS), and the source of CPI for the other countries is IFS (line 64). We construct quarterly CPI from monthly data by selecting the value of the last month in each quarter for all the countries except for Australia, as the IFS line 64 for Australia is already available at quarterly frequency. We take the first difference of log CPI to construct inflation rates.

4.2. Interest Rate Data

The short-term interest rates are downloaded from *Datastream*. Specifically, we download and construct the following nominal interest rates for each of those countries,

- Australia and Canada: 3-month or 90-day interbank rates from OECD main economic indicators;
- France: average monthly money market rates from Banque de France;
- Germany: 3-month (monthly average) Frankfurt interbank offered rates from European Banking Federation/the Financial Markets Association;
- Italy: 3-month (monthly average) interbank deposit rates from Bank of Italy;
- Japan: overnight uncollaterised call money rate (average) from Bank of Japan;
- Netherlands: average money market rates paid on bankers' call loans from IFS; missing values are replaced by the observations from call money rate from De Nederlandsche Bank (DNB);
- Switzerland: overnight Swiss franc deposit rates in international money markets from IFS; missing values are replaced by the observations from call money/interbank

[9] As consumption data are time-averaged, and the level of consumption is not a point-in-time observation but a flow during a quarter, we face a timing convention problem when computing consumption growth. As such, we follow Campbell (2003) and use the 'beginning-of-quarter' timing convention to calculate the growth rate of consumption per capita.

rate from OECD main economic indicators;

- UK: rates at which 91-day bills are allotted; weighted averages of Friday data from Bank of England;
- US: 90-day US treasury bill rates from the Treasury and Inflation database of WRDS.

Furthermore, to alleviate the concern of the sovereign default risk, we construct default-free interest rates for all the countries by measuring the sovereign default risk using the Moody's sovereign ratings.[10] We run panel regressions of interest rates on sovereign ratings by controlling both country and seasonal fixed effects. The interest rates that remove the component explained by sovereign ratings are regarded as default-free interest rates. The details of construction of default-free interest rates are included in the Internet Appendix.

To construct the real risk-free rates, following the literature (see, e.g., Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012; Schorfheide, Song, and Yaron, 2018), we first construct the *ex post* real risk-free rates by deflating nominal interest rates using inflation rates and then regress the *ex post* real risk-free rates on one-year lagged nominal rates and one-year lagged inflation rates. The predicted values from this regression yield the *ex ante* risk-free rates, which are used in our estimation.

4.3. Stock Market Data

The stock market data for the US are obtained from the Center for Research in Security Prices (CRSP). The market returns are the value-weighted returns on the stock portfolio of NYSE, AMEX and NASDAQ. The dividend growth rates are constructed from the value-weighted returns including and excluding dividends. For the remaining countries, following Rangvid, Schmeling, and Schrimpf (2014), we rely on stock market data from Datastream and obtain nominal dividends by multiplying the market price index by the market dividend yield.

For all the countries, as in Bansal and Yaron (2004) and Schorfheide, Song, and Yaron

[10]We thank a referee for this suggestion.

Table 1: **Summary Statistics**

	$E[r_m]$	$\sigma[r_m]$	$E[r_f]$	$\sigma[r_f]$	$E[\Delta c]$	$\sigma[\Delta c]$	$E[\Delta d]$	$\sigma[\Delta d]$	Sample Period
US	0.0700	0.1621	0.0058	0.0090	0.0185	0.0098	0.0254	0.0466	1947:Q2-2019:Q3
UK	0.0642	0.1873	0.0107	0.0145	0.0077	0.0591	0.0162	0.0380	1965:Q4-2019:Q3
DE	0.0575	0.2007	0.0153	0.0137	0.0153	0.0183	0.0251	0.0563	1973:Q4-2019:Q3
FR	0.0712	0.2223	0.0151	0.0139	0.0140	0.0129	0.0346	0.0499	1973:Q4-2019:Q3
IT	0.0339	0.2530	0.0144	0.0170	0.0135	0.0156	0.0176	0.0998	1973:Q4-2019:Q3
JP	0.0330	0.2074	0.0079	0.0100	0.0153	0.0232	0.0211	0.0472	1973:Q4-2019:Q3
CA	0.0552	0.1587	0.0205	0.0119	0.0131	0.0134	0.0197	0.0537	1973:Q4-2019:Q3
AU	0.0666	0.1940	0.0250	0.0138	0.0172	0.0187	0.0269	0.0559	1973:Q4-2019:Q3
NL	0.0732	0.1985	0.0125	0.0132	0.0119	0.0193	0.0229	0.0626	1973:Q4-2019:Q3
CH	0.0648	0.1888	-0.0004	0.0090	0.0074	0.0115	0.0477	0.0589	1973:Q4-2019:Q3

This table reports summary statistics of the data used for model estimation. The data are sampled at a quarterly frequency for ten developed countries, including the United States (US), the United Kingdom (UK), Germany (DE), France (FR), the Netherlands (NL), Switzerland (CH), Italy (IT), Japan (JP), Canada (CA) and Australia (AU). The sample period for each country is also shown in the table. The summary statistics consists of the mean and standard deviation of equity returns ($E[r_m]$ and $\sigma[r_m]$), the mean and standard deviation of the risk-free rate ($E[r_f]$ and $\sigma[r_f]$), the mean and standard deviation of per capita consumption growth ($E[\Delta c]$ and $\sigma[\Delta c]$), and the mean and standard deviation of dividend growth ($E[\Delta d]$ and $\sigma[\Delta d]$). All variables are in real and log terms.

(2018), we smooth nominal dividends by aggregating their values of the most recent four quarters (including the current quarter). Real stock returns (real dividend growth rates) are calculated by deflating nominal stock returns (nominal dividend growth rates) using quarterly inflation rates.

4.4. Summary Statistics

Table 1 presents the summary statistics of the data used for model estimation. The annualized average real market return ranges from 3.30% (JP) to 7.32% (NL), and its annualized volatility ranges from 15.9% (CA) to 25.3% (IT). In contrast, the mean and volatility of real risk-free rates are much smaller: the annualized average rate ranges from almost 0 (CH) to 2.50% (AU) and the annualized volatility ranges from 0.90% (US and CH) to 1.70% (IT).

In general, the real dividend growth rates are larger and more volatile than the real consumption growth rates. The annualized average real dividend growth rate varies from 1.62% (UK) to 4.77% (CH) and its annualized standard deviation varies from 3.80% (UK) to 9.98% (IT); the annualized average real consumption growth rates are around

2%, ranging from 0.74% (CH) to 1.85% (US), with smaller variations ranging from 0.98% (US) to 5.91% (UK).

The cross-country correlations of consumption growth rates are almost positively correlated, ranging from -0.10 to 0.56. The US consumption growth is positively correlated with consumption growth in the remaining countries, with the correlation being as high as 0.36 with France, whereas the UK consumption growth is very weakly correlated with most of the remaining countries. The average cross-country correlation of consumption growth rates is about 0.19. Relative to consumption growth, the dividend growth rates across the countries show higher correlations. In particular, the dividend growth rates among the European countries have notable comovements, with correlation ranging from 0.25 to 0.55. Both the risk-free rates and stock market returns across the countries are strongly positively correlated.

5. Empirical Results and Implications

5.1. Estimation Results

We estimate three model specifications for each of the ten countries and compare performance across different models for each country. The first model (LRR) is the long-run risk model with one stochastic volatility process and without preference shocks. The second model (eLRR1) differs from the first by taking preference shocks into account. The third model (eLRR2) further allows for a separate volatility process in the dividend growth rates.

Our estimation method needs to be initialized by the prior distributions of model parameters. Our choice of those prior distributions is consistent with the literature; see, e.g., Schorfheide, Song, and Yaron (2018) and Fulop et al. (2022). Importantly, we impose relatively loose priors on key parameters in the utility function and state processes. For instance, the prior on RRA (γ) is a truncated normal with a mean of 6 and a standard deviation of 2, which allows γ to take values larger 10; the prior on EIS is a truncated normal with a mean of 2 and a standard deviation of 0.5, allowing it to take values smaller or larger than 1; both persistence parameters ρ_x and ρ_s have a uniform prior bounded within (-1,1). Furthermore, our priors on the variances of the pricing errors on market returns and risk-free rates are sufficiently loose. The exact functional forms and hyperparameters of the prior distributions are presented in the Internet Appendix.

5.1.1. Model Performance

Table 2 displays several measures used to assess model performance: a statistical measure, i.e., the log marginal likelihood (ML) that measures the overall goodness-of-fit of the model by taking into account both parameter and state uncertainties, and two economic measures, i.e., the standard deviations of the pricing errors in stock market returns and risk-free rates (σ_m and σ_f, respectively) that measure how far the model-implied asset returns are from the observed ones. According to the marginal likelihood estimates and the estimated standard deviations of the pricing errors, we find that the eLRR1 model outperforms the LRR model. The estimated σ_m and σ_f for eLRR1 are smaller than those obtained for LRR for almost all the economies under consideration. The only exception includes Switzerland (CH), for which σ_m is marginally higher under eLRR1. Furthermore, the marginal likelihood estimates are unanimously much higher under eLRR1 than those obtained under LRR for all the economies.

Turning to the comparison of performance between eLRR1 and eLRR2, we could not find affirmative evidence of improvement of eLRR2 over eLRR1. While the log marginal likelihood estimates improve in general when a separate dividend volatility process is introduced, there is little gain in fitting stock returns for almost all the economies. These results suggest that the preference shock is a very important element that leads to better performance in fitting the data, whereas allowing for independent idiosyncratic risks in dividend growth does not seem to improve the overall economic performance of the model. Furthermore, the computational cost of estimating eLRR2 is more than ten times larger than that of estimating eLRR1. Thus, in what follows, we focus on estimation results and discuss asset pricing implications based on the parsimonious model of eLRR1.[11] For

[11] The estimation results for the alternative models LRR and eLRR2, including the posterior estimates of model parameters and the smoothed latent states, are presented in the Internet Appendix.

Table 2: **Model Performance**

	Panel A. LRR				Panel B. eLRR1				Panel C. eLRR2			
	$\sigma_m(\%)$	$\sigma_f(bp)$	ML	h	$\sigma_m(\%)$	$\sigma_f(bp)$	ML	h	$\sigma_m(\%)$	$\sigma_f(bp)$	ML	h
US	8.26	9.85	3.441	1.2	7.10	3.62	3.547	2.5	7.72	3.57	3.563	27.6
	(0.30)	(1.20)			(0.37)	(0.71)			(0.30)	(0.72)		
UK	9.49	3.37	2.152	1.0	8.74	5.62	2.233	2.3	8.66	4.94	2.243	30.6
	(0.39)	(1.01)			(0.45)	(1.28)			(0.44)	(1.13)		
DE	10.6	7.83	1.988	0.6	9.27	3.49	2.130	1.8	8.69	4.90	2.131	22.5
	(0.49)	(0.74)			(0.55)	(0.69)			(0.51)	(0.61)		
FR	10.6	16.4	1.966	0.6	9.13	6.98	2.095	1.5	9.31	8.51	2.139	27.5
	(0.45)	(1.26)			(0.73)	(1.39)			(0.67)	(1.31)		
IT	13.0	15.2	1.791	0.6	11.0	9.81	1.884	1.3	11.8	9.06	1.916	19.9
	(0.58)	(1.38)			(0.79)	(1.89)			(0.75)	(1.77)		
JP	10.5	4.19	1.986	0.7	9.37	5.13	2.069	1.8	9.84	4.45	2.096	59.0
	(0.49)	(1.20)			(0.53)	(1.06)			(0.55)	(1.04)		
CA	6.49	60.9	1.981	0.6	6.14	6.11	2.156	1.6	6.37	6.27	2.168	20.8
	(0.50)	(2.89)			(0.51)	(1.82)			(0.51)	(1.45)		
AU	9.93	8.66	1.940	0.6	8.66	8.65	2.007	1.6	9.07	10.0	2.051	25.0
	(0.45)	(1.30)			(0.69)	(2.47)			(0.53)	(1.91)		
NL	10.8	17.4	1.874	0.6	7.23	16.8	1.940	1.5	8.29	14.3	1.969	23.6
	(0.50)	(1.61)			(0.90)	(2.62)			(0.64)	(3.01)		
CH	4.95	44.3	2.017	0.7	5.47	13.2	2.105	3.0	5.72	11.8	2.121	90.3
	(0.67)	(2.30)			(0.65)	(2.33)			(0.83)	(1.94)		

This table shows estimation results on model performance by comparing three model specifications for the ten countries. LRR refers to the long-run risk model with one stochastic volatility process but without preference shocks. eLRR1 refers to the model with one stochastic volatility process and preference shocks. eLRR2 differs from eLRR1 by further allowing for an independent volatility process in dividend growth. The metrics used for assessing performance of a model include the standard deviations of the measurement errors in stock returns and risk-free rates (σ_m and σ_f respectively) and the log marginal likelihood (ML) in thousands. h presents the computational cost in hours for estimating each model for each country.

the US, UK, and Canada, our discussion is based on estimates using the nondurables and services consumption data. In the Internet Appendix, we present parameter estimates using the private consumption expenditures data and find that the general implications are very similar.

5.1.2. Parameter Estimates

Table 3 presents posterior estimates of the primitive parameters in the recursive utility function and in the dynamics of consumption and dividend growth for all the countries, which result from the eLRR1 model. The posterior mean estimates of the subjective discount factor δ are similar across most of the countries and are well above 0.99 except Germany. The standard deviation and the (5, 95)% percentiles indicate that the estimates are bounded within small intervals and consistent with low real risk-free rates observed in most of those countries.

The posterior estimates of RRA (γ) for the US are largely in line with the long-run risk literature. The posterior mean of γ is around 9, and the (5, 95)% credible interval is about (7, 11). These estimates are similar to those reported in Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2022). For the other countries, the posterior estimates of γ are slightly small: the posterior mean ranges from 5.6 (UK) to 7.8 (FR), and their (5, 95)% percentiles are well within the interval (4,10). Note that the US credible interval overlaps the credible intervals of most of the remaining countries, suggesting that the γ estimate for the US is not highly statistically different from the estimates for the other countries. The values of γ lower than 10 are commonly considered being economically reasonable.

The long-run risk literature advocates values of EIS (ψ) greater than 1. Estimation studies such as Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2022) find empirical support for $\psi > 1$ based on the US data. Our estimation with international data further provides support for typical values of ψ used in the calibration studies based on long-run risks. Table 3 reveals that the posterior mean estimate of ψ is around 2, ranging from 1.6 (CA) to 2.6 (DE) across the ten countries in our study and is slightly larger than the typical value of 1.5 used in the calibration studies. The 5% percentile estimate of ψ is consistently above 1 in all the economies. Together with the estimates of γ, these results suggest that investors in the developed economies have a strong preference for early resolution of uncertainty ($\psi \gg 1/\gamma$). In addition, since our estimation uses both macroeconomic and market data, the estimates of ψ obtained are naturally consistent with the empirical fact that the risk-free rate is not very responsive to expected consumption growth and consumption volatility.

The estimated specification of the growth rate of the preference shock exhibits high persistence for all the countries. The posterior means of the persistence parameter ρ_λ are above 0.9 for all the countries except for the Netherlands (0.89) and Switzerland (0.80). Among those countries, the preference shock of the US economy has the highest level of persistence (0.99). The posterior mean, standard deviation, and (5, 95)% percentiles

Table 3: **Parameter Estimates**

	Mean	Std	5%	95%	Mean	Std	5%	95%
	Panel A: US				Panel B: UK			
δ	0.9988	0.0004	0.9980	0.9993	0.9919	0.0013	0.9896	0.9937
γ	8.9250	1.1462	7.0289	10.912	5.5838	0.5780	4.6212	6.5314
ψ	2.3347	0.2409	1.9412	2.7466	2.4302	0.3177	1.9049	2.9659
ρ_λ	0.9947	0.0019	0.9912	0.9974	0.9199	0.0152	0.8914	0.9420
ϕ_λ	0.0011	0.0001	0.0010	0.0012	0.0017	0.0002	0.0015	0.0020
ρ_x	0.9842	0.0057	0.9744	0.9920	0.8256	0.0277	0.7768	0.8695
ϕ_x	0.2313	0.0265	0.1928	0.2790	0.0421	0.0067	0.0319	0.0533
$\bar{\sigma}$	0.0046	0.0001	0.0043	0.0048	0.0283	0.0006	0.0270	0.0291
ρ_s	0.7493	0.0396	0.6806	0.8053	0.7300	0.0324	0.6767	0.7807
ϕ_s	1.4371	0.1809	1.1570	1.7546	1.7261	0.2569	1.3187	2.1584
Φ	1.7936	0.2558	1.4113	2.2496	6.6574	1.0394	5.0798	8.4375
ϕ_{dc}	0.7352	0.1774	0.4702	1.0233	0.1158	0.0211	0.0856	0.1536
ϕ_d	4.8155	0.3099	4.3329	5.3593	0.3667	0.0284	0.3202	0.4135
	Panel C: DE				Panel D: FR			
δ	0.9758	0.0014	0.9734	0.9781	0.9908	0.0012	0.9887	0.9928
γ	6.1211	1.1471	4.2561	8.0819	7.8204	1.2996	5.7472	10.0032
ψ	2.5502	0.2497	2.1402	2.9506	2.1286	0.3423	1.5691	2.7111
ρ_λ	0.9874	0.0019	0.9839	0.9900	0.9366	0.0113	0.9175	0.9541
ϕ_λ	0.0014	0.0001	0.0012	0.0015	0.0024	0.0002	0.0021	0.0026
ρ_x	0.7831	0.0277	0.7386	0.8291	0.8342	0.0316	0.7767	0.8827
ϕ_x	0.1227	0.0160	0.0976	0.1501	0.1691	0.0244	0.1340	0.2130
$\bar{\sigma}$	0.0083	0.0003	0.0077	0.0087	0.0059	0.0002	0.0055	0.0062
ρ_s	0.5979	0.1029	0.4248	0.7482	0.7251	0.0921	0.5538	0.8504
ϕ_s	2.2423	0.3921	1.6280	2.8823	2.5078	0.5592	1.6274	3.4454
Φ	12.742	1.5315	10.431	15.421	11.031	1.4944	8.8130	13.710
ϕ_{dc}	0.2241	0.0747	0.1236	0.3638	0.2957	0.0865	0.1763	0.4652
ϕ_d	1.7555	0.1694	1.5040	2.0520	2.3772	0.2022	2.0505	2.7142
	Panel E: IT				Panel F: JP			
δ	0.9917	0.0014	0.9893	0.9938	0.9979	0.0009	0.9961	0.9990
γ	6.5814	1.1305	4.7721	8.3589	6.8750	1.1844	4.9134	8.8646
ψ	1.9455	0.2776	1.4828	2.4369	2.0436	0.3374	1.4755	2.5831
ρ_λ	0.9545	0.0114	0.9339	0.9712	0.9476	0.0220	0.9073	0.9776
ϕ_λ	0.0026	0.0002	0.0022	0.0030	0.0017	0.0002	0.0014	0.0020
ρ_x	0.8009	0.0336	0.7421	0.8509	0.8528	0.0286	0.8038	0.8959
ϕ_x	0.2101	0.0278	0.1655	0.2598	0.0778	0.0123	0.0589	0.0991
$\bar{\sigma}$	0.0072	0.0003	0.0067	0.0076	0.0110	0.0003	0.0105	0.0113
ρ_s	0.8276	0.0442	0.7451	0.8914	0.5313	0.0730	0.4130	0.6475
ϕ_s	1.9710	0.4248	1.3132	2.7242	1.6515	0.2237	1.3070	2.0522
Φ	11.939	1.5439	9.4130	14.596	10.886	1.3787	8.8501	13.300
ϕ_{dc}	0.4247	0.1554	0.2197	0.7103	0.2279	0.0509	0.1519	0.3230
ϕ_d	3.7120	0.3563	3.1744	4.3031	1.2463	0.0866	1.1171	1.3968

	Panel G: CA				Panel H: AU			
δ	0.9927	0.0011	0.9907	0.9943	0.9932	0.0019	0.9897	0.9960
γ	7.3618	1.3513	5.2225	9.5506	7.6687	1.3397	5.6417	9.8209
ψ	1.6423	0.3064	1.1846	2.1864	2.2731	0.3133	1.7712	2.7726
ρ_λ	0.9213	0.0143	0.8966	0.9441	0.9452	0.0204	0.9093	0.9724
ϕ_λ	0.0022	0.0002	0.0019	0.0025	0.0025	0.0003	0.0021	0.0030
ρ_x	0.8515	0.0220	0.8131	0.8856	0.7925	0.0315	0.7414	0.8450
ϕ_x	0.1472	0.0221	0.1133	0.1861	0.1313	0.0200	0.0978	0.1662
$\bar{\sigma}$	0.0064	0.0002	0.0060	0.0066	0.0085	0.0003	0.0079	0.0089
ρ_s	0.6888	0.0743	0.5619	0.7957	0.7579	0.0709	0.6268	0.8552
ϕ_s	1.7900	0.3335	1.2922	2.3767	1.5678	0.4747	1.0658	2.5485
Φ	9.6628	1.2461	7.7111	11.841	9.6065	1.4759	7.3703	12.140
ϕ_{dc}	0.1566	0.0589	0.0803	0.2706	0.2507	0.0736	0.1498	0.3764
ϕ_d	2.4358	0.1587	2.1812	2.7082	1.7120	0.1808	1.4455	2.0323
	Panel I: NL				Panel J: CH			
δ	0.9940	0.0019	0.9905	0.9966	0.9992	0.0005	0.9982	0.9997
γ	6.4751	1.3511	4.3457	8.7169	7.5197	1.5592	4.8934	9.9948
ψ	1.9574	0.3115	1.4549	2.4729	1.8697	0.3787	1.2105	2.4811
ρ_λ	0.8911	0.0238	0.8482	0.9293	0.8000	0.0338	0.7504	0.8598
ϕ_λ	0.0030	0.0003	0.0025	0.0036	0.0027	0.0003	0.0022	0.0033
ρ_x	0.8872	0.0319	0.8288	0.9311	0.8915	0.0216	0.8488	0.9211
ϕ_x	0.1466	0.0316	0.1009	0.2025	0.1338	0.0263	0.0926	0.1800
$\bar{\sigma}$	0.0087	0.0003	0.0081	0.0092	0.0054	0.0002	0.0050	0.0056
ρ_s	0.7157	0.0688	0.5930	0.8143	0.6008	0.0968	0.4310	0.7532
ϕ_s	1.8729	0.3357	1.3683	2.4376	1.6553	0.3241	1.1749	2.2318
Φ	6.8215	1.2283	5.0626	9.0170	11.516	1.5961	8.9746	14.214
ϕ_{dc}	0.3239	0.0787	0.2123	0.4711	0.4165	0.1375	0.2367	0.6683
ϕ_d	2.4695	0.2022	2.1433	2.8183	3.6874	0.2710	3.2685	4.1545

This table reports posterior means, standard deviations, 5 and 95 percentiles of model parameters for the long-run risk model with one stochastic volatility process and preference shocks (eLRR1). Parameter estimates are for preference parameters in the recursive utility function and parameters in the processes of consumption growth and dividend growth. The estimation is implemented using the Bayesian SMC^2 method, for the ten countries in our sample.

altogether indicate that the specification of the time preference shock is well identified from international data. The estimates of the volatility parameter ϕ_λ are small, ranging from 0.11% to 0.30%, and are very similar in most of those countries. The magnitude of variation in the growth rate of the preference shock implied from our estimates is in line with that postulated in the calibration of Albuquerque et al. (2016).

However, when we shut down time-varying preference shocks, we obtain very different estimates of RRA and EIS.[12] The posterior mean of γ varies much larger across countries,

[12]For brevity, these results are not reported here. See the Internet Appendix for parameter estimates

ranging from 3.4 (UK) to 8.4 (FR). Furthermore, it seems important to incorporate preference shocks in the model for identifying the EIS parameter when long-run consumption risk is present. Absent from the preference shock, the EIS estimates vary dramatically across the countries and become much smaller in all the economies. Its posterior mean (5% percentile) is below 1 in 5 (6) out of the ten countries. These results suggest that introducing time-varying preference shocks in the long-run risk models helps deliver economically plausible estimates of risk aversion and EIS, not only for the US but also for the other developed economies.

Our estimation based on international data provides empirical support for the presence of a persistent component in consumption growth across different countries. The posterior mean estimates of the persistence parameter (ρ_x) ranges from 0.78 (DE) to 0.98 (US) at the quarterly frequency in these countries. As for the US, we see that the posterior mean of ρ_x is about 0.98 and its the (5, 95)% credible interval is (0.97, 0.99), which are close to the typical values used in the calibration studies. When we shut down time-varying preference shocks, the estimated long-run consumption component becomes even more persistent for all the countries (see the Internet Appendix).

In addition, we find that the importance of the long-run component varies significantly across countries, as is evident from the estimates of ϕ_x. The countries that feature a significant fraction of long-run risk in aggregate consumption include the US and Italy, for which the posterior mean estimates of ϕ_x are about 0.2. The consumption dynamics in the other countries have moderately smaller amounts of long-run risk, with the posterior mean of ϕ_x being about $0.04 - 0.17$. The long-run risk component accounts for the least significant role in consumption dynamics in the UK and Japan. Taking into account our empirical results that the EIS estimates are greater than 1 and that long-run risk drives consumption dynamics, our estimation implies that the long-run risk model is a convincing description of the macroeconomic and market data jointly for global developed economies.

Regarding the consumption volatility specification, the estimates of the long-run

for the LRR model.

mean, $\bar{\sigma}$, are largely in line with the variations of the historical consumption growth in different countries. However, the persistence of the stochastic volatility process varies significantly across countries as the posterior mean of ρ_s ranges from 0.53 (JP) to 0.83 (IT). These estimates are much smaller than the values typically assumed in the calibration studies. Consumption volatility is more persistent in the US, the UK, France, the Netherlands, Italy, and Australia than in Germany, Switzerland, Japan, and Canada. These results therefore cast doubt on the argument usually made in the long-run risk literature (e.g., Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012, 2016) that a very persistent volatility process is required to explain the behavior of market returns. By fully exploiting information in the likelihood function of the asset pricing model, our study does not find global evidence to support this argument.

Turning to the dividend growth process, similar to Abel (1999), our estimates of the leverage parameter, Φ, are all well above 1 (between 1.8 and 12), capturing the "levered" nature of dividends, and are much higher in the other countries than in the US. This result indicates that the long-run risk component plays a more important role in depicting the dividend growth dynamics in countries excluding the US. However, we obtain very different estimates of Φ in the model absent from time-varying preference shocks: its posterior mean is around 1 in most of the countries and its 5% percentile is smaller than 1 in five out of the ten countries (see the Internet Appendix).

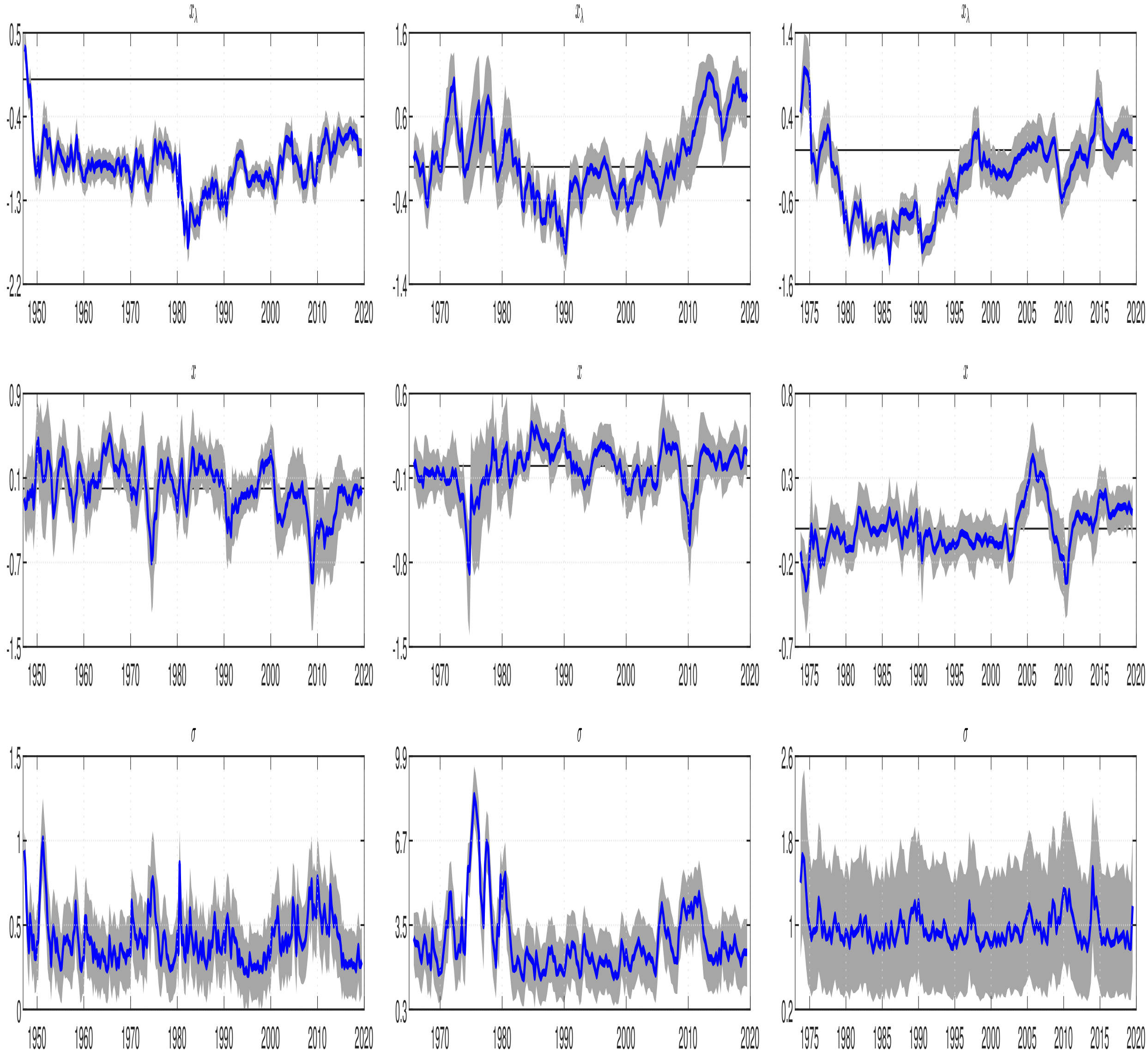
In the estimation, the parameter ϕ_{dc} is primarily identified from the covariation between consumption growth and dividend growth. The posterior mean of ϕ_{dc} ranges from 0.11 (UK) to 0.74 (US). In the US, consumption and dividend have stronger comovement, leading to higher estimates of ϕ_{dc} than those in the other countries. The parameter ϕ_d determines the amount of variation of dividend growth due to the idiosyncratic risk. Due to the empirical result that much of the variation of dividend growth is loaded onto the long-run component in the countries excluding the US, the estimates of ϕ_d are moderately lower in those countries than in the US.

5.1.3. Time Series of Latent States

Our Bayesian method can directly provide us with time series of filtered states, i.e., the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility (σ_t). Those filtered time series naturally take into account both parameter and state uncertainties. Figures 1 displays the posterior means and (5, 95)% credible intervals of the filtered latent states for three selected countries: the US, the UK, and Japan. For the other countries, we present those figures in the Internet Appendix for economizing the space.

As noted by Albuquerque et al. (2016), x_λ determines how the agent trades off current utility versus future utility. All else being equal, an increase in x_λ implies higher valuation of future utility relative to the current utility. The plots of the posterior mean of $x_{\lambda,t}$ reflect time variation in investors's valuation of future versus current utility in different countries. For the US, the posterior mean of $x_{\lambda,t}$ experiences significant declines in several recession episodes such as late 1940s, early 1980s, 1990s, and the 2008 global financial crisis, albeit the average correlation with consumption growth is low.

While our model assumes that $x_{\lambda,t}$ is independent of the other latent states, Bayesian estimation suggests that from the perspective of posteriors the variation in $x_{\lambda,t}$ is partially associated with either the long-run component or stochastic volatility, or both. Interestingly, for the US and the UK, the posterior mean of $x_{\lambda,t}$ is negatively correlated with that of x_t, while the posterior mean of $x_{\lambda,t}$ is positively correlated with that of σ_t for these three countries. This pattern is more noteworthy in the first half of samples for those countries discussed above. A similar pattern also holds for the other countries, whose results are reported in the Internet Appendix. In times either when expected consumption growth is low or when conditional volatility of consumption growth is high, the growth rate of the time preference shock is likely to be high and as such, investors value future utility more relative to the current consumption. Because all of the three driving forces tend to induce investors to save more, asset prices therefore capture these effects altogether. As a consequence, when we use asset returns data in the estimation, our estimation strategy leads to the covariation of the posterior estimates of the three latent states. However,



(a) US (b) UK (c) JP

Figure 1: **Filtered Latent States: US, UK, and Japan**

This figure plots the posterior means of the filtered latent states, the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility state (σ_t), for the US, the UK, and Japan.

the two-stage procedure in Creal and Wu (2020) can hardly capture this covariation.

In addition, Figures 1 shows that the long-run component plays a more significant role in driving the time variation of consumption growth for the US than for the other countries. This observation echoes the parameter estimates for the long-run risk specification reported in Table 3. Not surprisingly, expected consumption growth tends to fall in recessions while rise in booms. The time series of the posterior mean of the stochastic volatility component exhibits the feature of volatility clustering. In the US, the filtered volatility of consumption growth has experienced several upswings in early 1950s, mid 1970s, early 1980s and periods around the 2008 crisis, in which several episodes coincide with the NBER recessions. In the UK, significant increases in the posterior mean of σ_t occur in recession periods around 1975–1980 and 2008–2010. In Japan, the filtered volatility of consumption growth rises in the beginning of 1970s, late 1990s, the 1997 Asian financial crisis, and periods around 2010 and 2014, all of which have witnessed dramatic declines in consumption growth. More discussions for the other countries can be found in the Internet Appendix.

5.2. Asset Pricing Implications

5.2.1. Moment Matching and Asset Return Fitting

In the consumption-based asset pricing literature, the moments matching exercise has been mostly confined to the US data so far. Few studies ever examined performance of matching moments of asset returns for other developed economies. We assess the performance of the estimated long-run risk model in matching moments (means and variances) of asset returns for the countries in our study. In particular, for a specific moment of interest we compute the model-implied analogue for a given parameter set and a latent state path under the joint posterior distribution given the data set. We then report the posterior quantiles of these model-implied moments that account for uncertainties in both the parameters and the latent states.

Table 4 presents moments of asset returns for all the countries, which are generated from the parameter and state particles obtained from our SMC-based Bayesian estimation. The results reveal that the estimated model can well reconcile moments of asset returns for the developed markets in our study. First, the estimated long-run risk model can deliver mean and standard deviation of risk-free rates very close to the moments of the data across all the countries, by means of the 5%, 50% and 95% percentiles. Since the risk-free rate is the reciprocal of conditional expectation of the SDF in the model, our results imply that the behavior of the model-generated SDF is reasonable. Second, the estimated model can closely match the mean and volatility of market returns for six

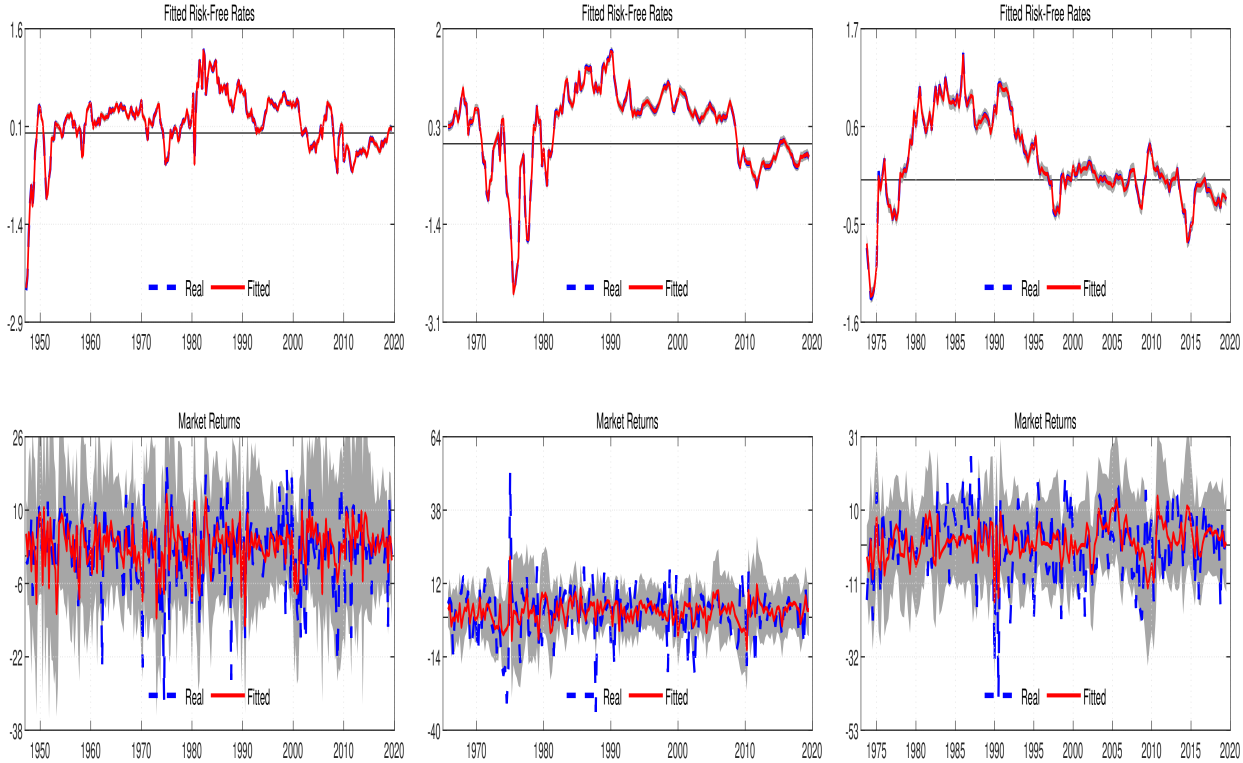
Table 4: **Asset Return Moments**

	Data	5%	50%	95%	Data	5%	50%	95%
	Panel A: US				Panel B: UK			
$E[r_m]$	7.00	8.18	9.16	10.09	6.42	4.95	8.11	11.47
$\sigma[r_m]$	16.21	18.96	23.08	27.30	18.73	13.36	15.80	18.69
$E[r_f]$	0.58	0.56	0.57	0.59	1.07	1.05	1.07	1.10
$\sigma[r_f]$	0.90	0.89	0.90	0.91	1.45	1.42	1.44	1.45
	Panel C: DE				Panel D: FR			
$E[r_m]$	5.75	10.43	11.33	12.28	7.12	5.97	7.33	8.90
$\sigma[r_m]$	20.07	18.19	21.10	24.54	22.23	19.24	23.47	28.45
$E[r_f]$	1.53	1.53	1.54	1.56	1.51	1.48	1.52	1.55
$\sigma[r_f]$	1.37	1.36	1.37	1.37	1.39	1.36	1.38	1.40
	Panel E: IT				Panel F: JP			
$E[r_m]$	3.39	6.63	8.27	10.23	3.30	2.31	3.91	6.34
$\sigma[r_m]$	25.30	26.20	30.81	35.93	20.74	17.62	21.18	24.94
$E[r_f]$	1.44	1.39	1.44	1.48	0.79	0.76	0.78	0.81
$\sigma[r_f]$	1.70	1.67	1.69	1.71	1.00	0.98	1.00	1.01
	Panel G: CA				Panel H: AU			
$E[r_m]$	5.52	4.79	6.20	7.96	6.66	4.78	7.25	9.89
$\sigma[r_m]$	15.87	16.61	19.58	22.82	19.40	16.37	19.44	23.37
$E[r_f]$	2.05	2.02	2.05	2.08	2.50	2.45	2.49	2.53
$\sigma[r_f]$	1.19	1.17	1.19	1.20	1.38	1.35	1.38	1.40
	Panel I: NL				Panel J: CH			
$E[r_m]$	7.32	4.32	6.43	8.57	6.48	4.71	5.02	5.65
$\sigma[r_m]$	19.85	20.51	25.78	31.19	18.88	21.77	25.38	29.30
$E[r_f]$	1.28	1.18	1.26	1.35	-0.04	-0.08	-0.02	0.05
$\sigma[r_f]$	1.32	1.22	1.27	1.32	0.90	0.79	0.85	0.89

This table presents moments of stock returns and risk-free rates implied by the eLRR1 model for the ten countries. The moments of asset returns calculated from the data are also shown for each country. The moments of asset returns implied by the model are calculated from the parameter and state particles in real time obtained from the Bayesian SMC^2 method.

out of the ten developed markets. The (5, 95)% credible intervals of $E[r_m]$ and $\sigma[r_m]$ embrace the corresponding moments of market returns in the data for the UK, France, Japan, Canada, Australia, and the Netherlands. For Germany, while the estimated model overstates the first moment of market returns, the (5, 95)% credible interval of $\sigma[r_m]$ well contains the true market volatility. For Switzerland, the model underestimates the equity premium but overestimates the equity volatility. The model overestimates both equity premium and volatility for the US and Italy.

We also investigate how the model implied asset returns track the observed returns.



(a) US (b) UK (c) JP

Figure 2: **Fitted Risk-Free Rates and Market Returns: US, UK, and Japan**

This figure plots the model-implied risk-free rates and market returns together with the actual returns in the data for the US, the UK, and Japan, respectively. For each country, the model-implied risk-free rates and market returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

Our estimation yields fitted risk-free rates that can closely track the movement of the actual risk-free rates in all the countries. The upper panels of Figure 2 display the related results for the three selected countries: the US, the UK, and Japan, and for the other countries, we present those figures in the Internet Appendix. In the model, either an increase in expected consumption growth or a reduction in conditional volatility leads to a lower risk-free rate. As a result, our Bayesian estimation identifies the association of the variations in risk-free rates with those in x_t and σ_t for the countries in our analysis. For instance, for the UK in Figure 2, the dramatically low risk-free rates observed in 1970s are consistent with the contemporaneous high volatility of consumption growth. Nevertheless, we observe that the pricing errors are significant in fitting market returns for all the countries in the analysis, a fact that can be observed from the pricing error standard deviations in Table 2 as well.

5.2.2. *Cyclical Variations of SDF*

We next examine the cyclical variation of the estimated SDF. For this purpose, we decompose the SDF under recursive utility into two components as follows,

$$M_{t+1} = \underbrace{\delta e^{x_{\lambda,t+1}} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}}_{M_{1,t+1}} \cdot \underbrace{\delta^{\theta-1} \left(\frac{C_{t+1}}{C_t}\right)^{\gamma-\frac{\theta}{\psi}} \left(R^W_{t+1}\right)^{\theta-1}}_{M_{2,t+1}}, \tag{20}$$

where $M_{1,t+1}$ is the SDF under the CRRA utility, and $M_{2,t+1}$ arises due to the separation between RRA and EIS. We compute the time series estimates of M_{t+1}, $M_{1,t+1}$ and $M_{2,t+1}$ using the posterior means of the model parameters and the filtered latent states for each country. Table 5 reports the overall correlations of the SDF and its components with consumption growth for each country. We find that the estimated SDF displays countercyclicality: the correlations between the SDF and consumption growth are all negative, ranging from -0.14 (IT) to -0.75 (UK). We also find that the countercyclicality of M_1 is much stronger than that of M_2 for all the countries. For example, the correlation between $M_{1,t}$ ($M_{2,t}$) and consumption growth is about -0.52 (-0.14) for Germany, and the correlation between $M_{1,t}$ ($M_{2,t}$) and consumption growth is about -0.53 (-0.24) for Switzerland.

Figures 3 plots the time series of the estimated M_{t+1}, $M_{1,t+1}$ and $M_{2,t+1}$, along with the consumption growth data, for the three selected countries: the US, the UK, and Japan. For the US, we observe that the estimated SDF has a notable countercyclical component, which rises in recessions and falls in booms. The correlation between the SDF and consumption growth is about -0.50 over the sample. A similar finding has been found for the US by Chen, Favilukis, and Ludvigson (2013), though they obtain dramatically different estimates of preference parameters than ours. This result implies that for an asset whose payoff is procyclical, its risk premium tends to be positive in a setting where consumption growth contains a very persistent component and stochastic volatility is also persistent. Both components of the SDF account for its countercyclical variations. The SDF under the CRRA utility, $M_{1,t+1}$, is strongly countercyclical because the variation

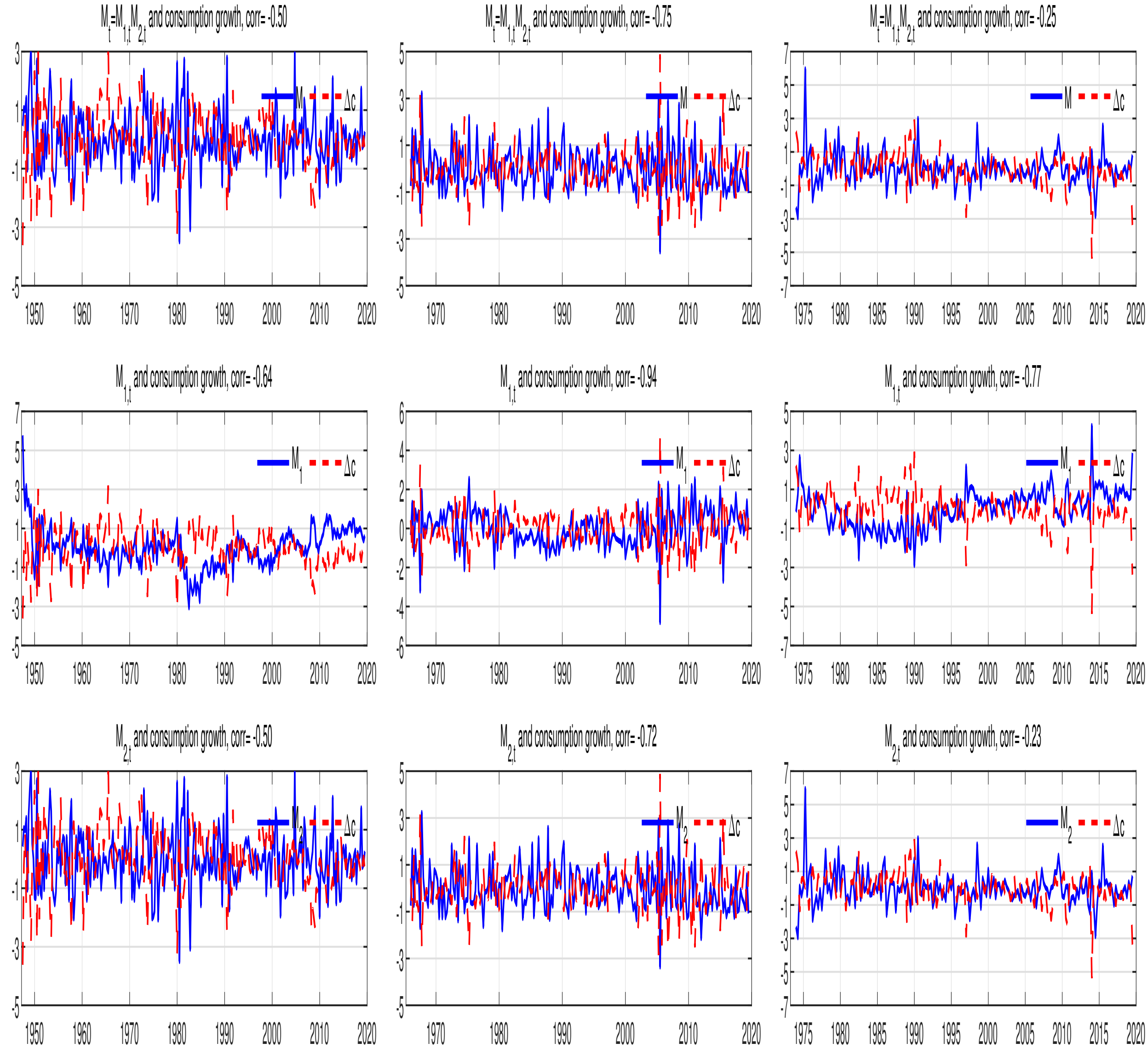
Table 5: **Cyclical Variations of SDF**

	US	UK	DE	FR	IT	JP	CA	AU	NL	CH
M	-0.50	-0.75	-0.15	-0.24	-0.14	-0.25	-0.22	-0.17	-0.24	-0.26
M_1	-0.64	-0.94	-0.52	-0.35	-0.51	-0.77	-0.48	-0.55	-0.55	-0.53
M_2	-0.50	-0.72	-0.14	-0.24	-0.12	-0.23	-0.21	-0.16	-0.22	-0.24
$E_t[r_{m,t+1} - r_{f,t}]$	-0.23	-0.11	0.09	0.08	-0.05	-0.14	-0.16	-0.06	-0.17	-0.23
$\sigma_t[r_{m,t+1}]$	-0.26	-0.11	-0.09	0.08	0.04	-0.14	-0.16	-0.08	-0.17	-0.21

This table presents, for each of the ten countries, 1) correlations of the SDF (M_t) and its components ($M_{1,t}$ and $M_{2,t}$) with per capita consumption growth respectively, and 2) correlations of conditional equity premium ($E_t[r_{m,t+1}-r_{f,t}]$) and conditional volatility ($\sigma_t[r_{m,t+1}]$) of equity returns with per capita consumption growth respectively. The SDF and conditional moments of equity returns are computed based on the posterior means of the model parameters and the filtered latent states, both of which are estimated using the Bayesian SMC^2 method.

of the preference shock is low according to our estimation. Compared to $M_{1,t+1}$ that is independent of long-run risk, volatility risk and investors' attitudes toward intertemporal substitution, $M_{2,t+1}$ has a dominant effect in determining the SDF in the long-run risk model. It is noteworthy that the estimated $M_{2,t+1}$ is also significantly countercyclical, taking into account long-run risk, volatility risk and investors' preferences toward the timing of the resolution of uncertainty. For the UK, the countercyclicality of the SDF is very strong: the correlation between the SDF and the consumption growth is as large as -0.75. The figure shows that the countercyclicality of the SDF is also significant in Japan, again due to correlation of $M_{2,t+1}$ with consumption growth. Results for the other countries can be found in the Internet Appendix.

Table 5 reports the overall correlations of consumption growth with conditional equity premium ($E_t[r_{m,t+1}-r_{f,t}]$) and conditional volatility ($\sigma_t[r_{m,t+1}]$) of equity returns implied by our long-run risk model for all the countries. The conditional equity premium and conditional volatility of equity returns are computed based on the posterior means of the model parameters and the filtered latent states. Except for France, Italy (for equity volatility), and Germany (for equity premium), all correlations are negative: the strongest negative correlation between equity premium and consumption growth is for the US and Switzerland (-0.23), and the strongest negative correlation between conditional volatility and consumption growth is for the US (-0.26). Figure 4 plots conditional equity premium and conditional volatility of equity returns implied by our long-run risk model, along with

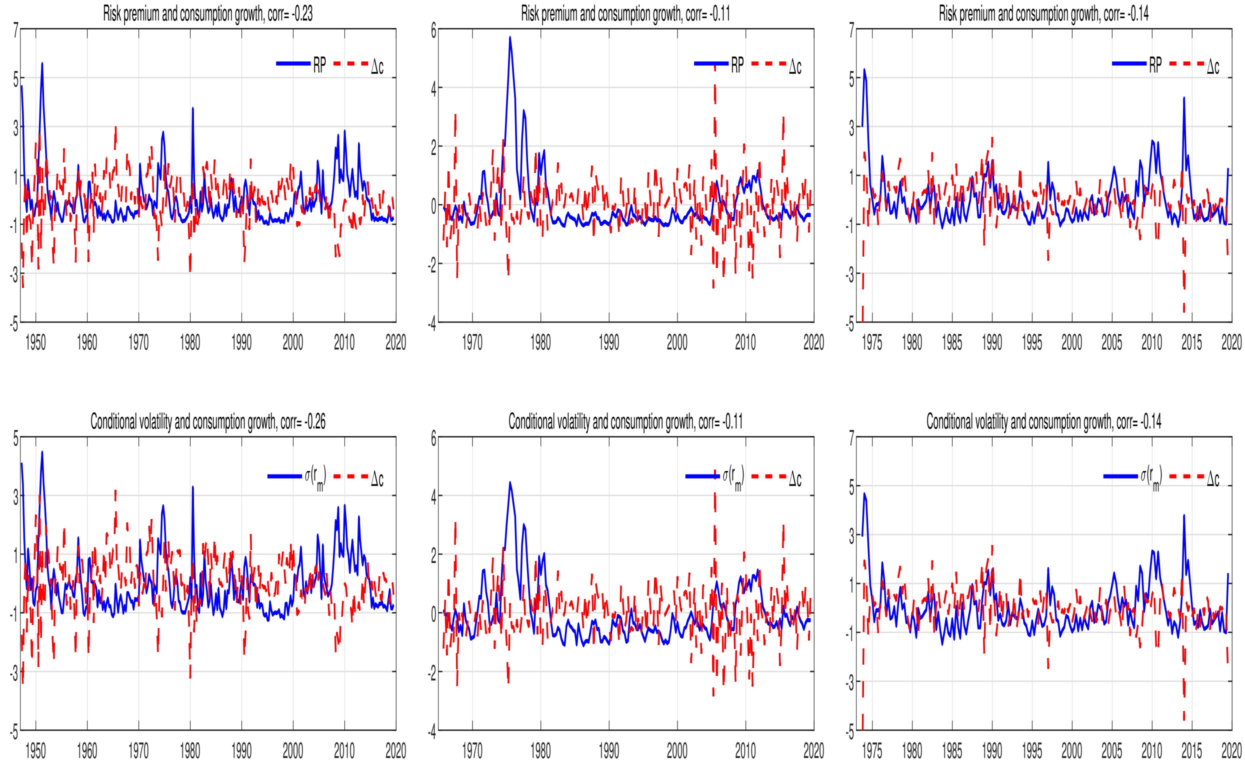


(a) US (b) UK (c) JP

Figure 3: **Stochastic discount factor: US, UK, and Japan**

This figure plots the model-implied SDF and its components, together with per capita consumption growth for the US, the UK, and Japan, respectively. For each country, the model-implied SDF is computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

the consumption growth, for the above-mentioned three countries. The plots suggest that both conditional equity premium and conditional volatility of equity returns have a countercyclical component in those three countries.



(a) US (b) UK (c) JP

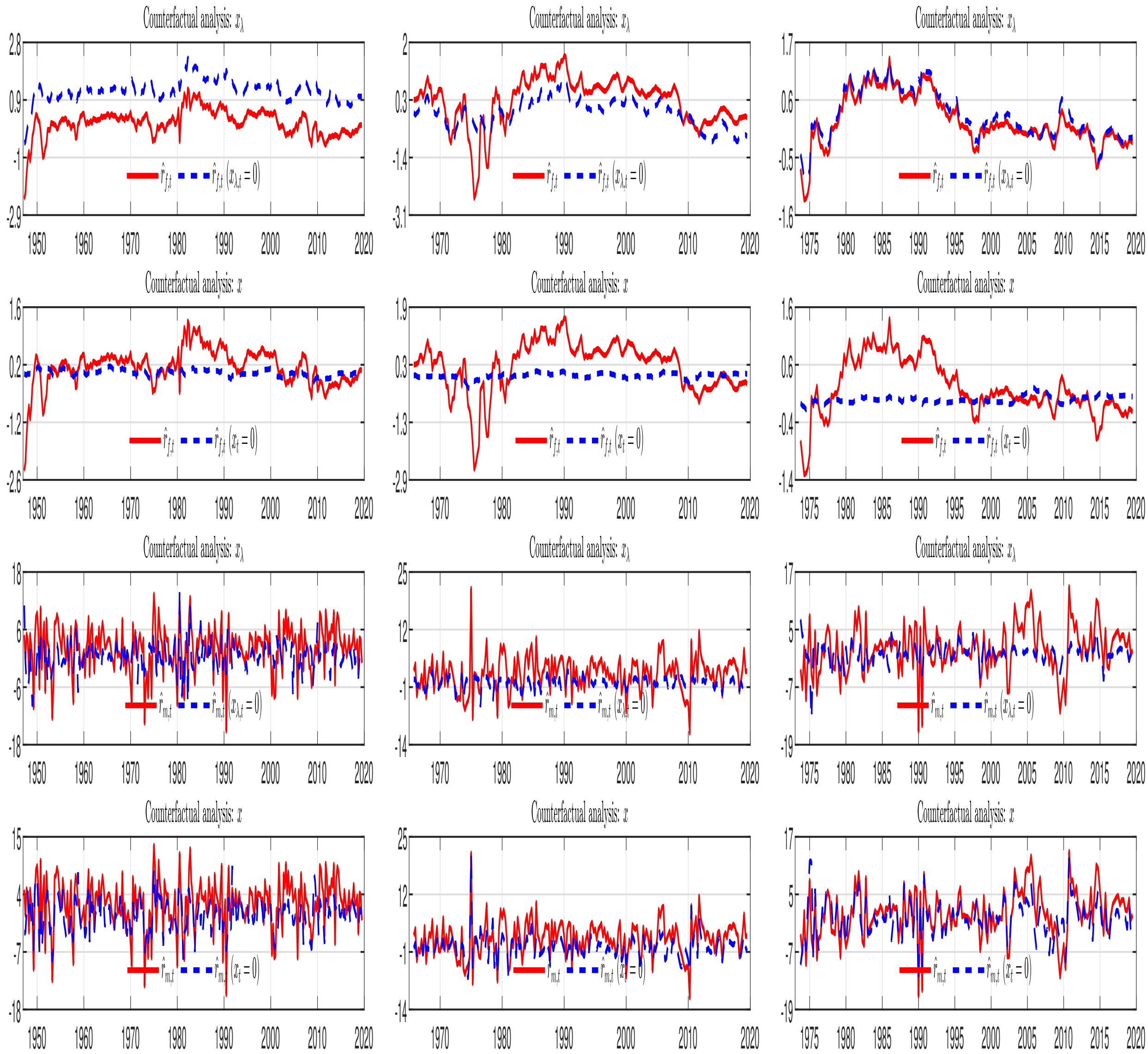
Figure 4: **Conditional risk premium and volatility: US, UK, and Japan**

This figure plots the model-implied conditional equity premium and conditional volatility of equity returns, together with per capita consumption growth for the US, the UK, and Japan, respectively. For each country, the conditional equity premium and conditional volatility of equity returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

5.2.3. Counterfactual Analysis

To emphasize the importance of the preference shock and the long-run component, we perform counterfactual analyses on fitted risk-free rate and market returns generated from our estimation. In particular, we compute counterfactual risk-free rates and market returns that would be obtained either in the absence of the preference shock or the long-run component. For all the countries, the impacts of the preference shock and the persistent component in expected consumption growth on the risk-free rate and market returns are remarkable. Figure 5 presents the corresponding results for the three selected countries: the US, the UK, and Japan. Additional plots for the other countries can be found in the Internet Appendix.

It turns out that for the US the preference shock matters notably not only for the



(a) US (b) UK (c) JP

Figure 5: **Counterfactual analysis: US, UK, and Japan**

This figure plots counterfactual risk-free rates and market returns for the US, the UK, and Japan that would be obtained either in the absence of the preference shock ($x_{\lambda,t}$) or the long-run risk component (x_t). For each country, the results are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

level of the risk-free rate but also for its time variation. The risk-free rate without the preference shock is too high and too smooth relative to the risk-free rate implied by the benchmark model eLRR1. This finding complements the analysis of Schorfheide, Song, and Yaron (2018) who find that the preference shock mainly accounts for the time variation in the observed risk-free rate. In contrast, the impact of the preference shock on the risk-free rate is less important for the UK and Japan. This is a distinct feature for most of countries other than the US, which, however, has not been documented in previous studies. The preference shock also has crucial effects on equity returns. For the US, the UK, and Japan, the equity returns implied by the model abstracted from the preference shock are too low and too smooth compared to the returns implied by the eLRR1 model; such a result can also be found for the other countries. This finding is consistent with the mechanism illustrated by Albuquerque et al. (2016) that the preference shock generates additional risk premium.

Turning to the role of the long-run component, we find that the risk-free rate that would prevail without the the long-run component is too smooth compared to the fitted risk-free rate in the eLRR1 model. In addition, equity returns become lower and less volatile in the absence of the long-run component. Figure 5 illustrates such impacts for the three selected countries.

5.3. A Global Long-Run Consumption Factor

A key insight in the long-run risk model is that a long-run consumption component plays a crucial role in explaining equity risk premium. We have found international evidence in support of existence of long-run risk in the ten developed countries. In addition, we also find that the correlations of the long-run consumption components become much stronger across individual countries than the correlations of consumption growth, and the former range from 0.14 to 0.70 with the average cross-country correlation being 0.47. These empirical findings motivate us to construct a global long-run consumption factor as follows. First, we take the equal-weighted average of the filtered long-run consumption components across individual countries at each point in time, i.e.,

$$f_{c,t} = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{t,i}, \quad t = 1, \dots, T. \tag{21}$$

Second, we take innovations of $f_{c,t}$ (first difference, $\Delta f_{c,t}$) as a global long-run consumption risk factor. Figure 6 presents the time series of this global consumption factor, which spans from 1973:Q4 to 2019:Q3. Since the global consumption factor is constructed from

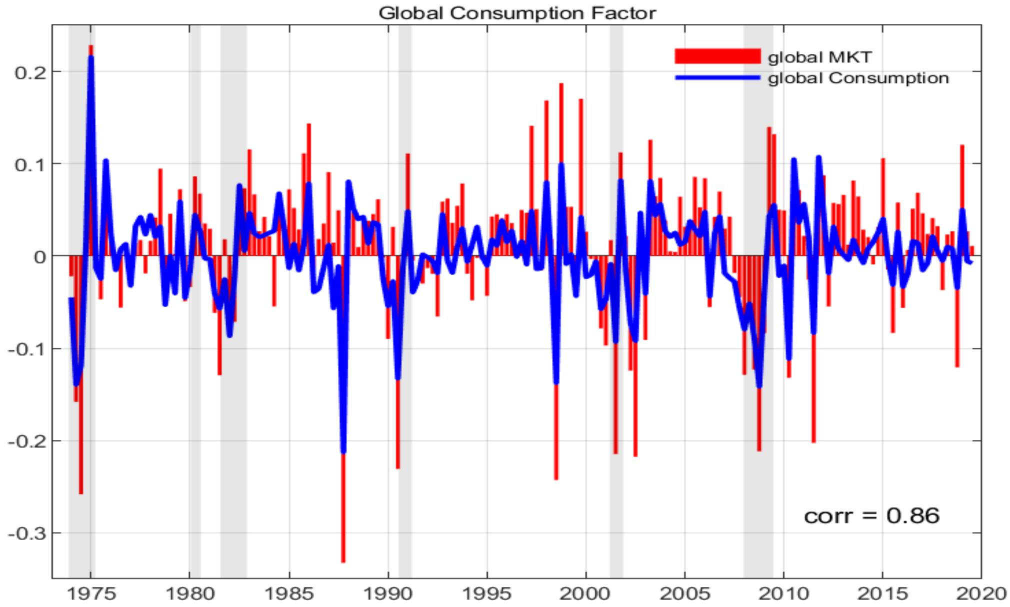


Figure 6: **A Global Consumption Factor**

This figure plots the time series of the global consumption factor (solid line) that is constructed first by taking the equal-weighted average of the filtered long-run consumption components across individual countries at each point in time and then taking the first difference. The global consumption factor is multiplied by 100. The bars represent a global equity market factor that is defined as the equal-weighted average of equity market returns across the ten individual countries. These two factors are in quarterly frequency, spanning from 1973:Q4 to 2019:Q3. The shaded areas are NBER recession periods.

structural estimation of the long-run risk model, it not only captures macroeconomic conditions reflected in the fundamentals data but also encapsulates relevant information contained in asset prices. For comparison, we also superimpose a global equity market factor, defined as the equal-weighted average of equity market returns across the ten individual countries. We see that our global long-run consumption factor strongly comoves with this global equity market factor, with a correlation of as high as 0.86.

We examine how this global consumption factor performs in explaining variations of equity premium across countries. Following Cochrane (Chapter 12, 2005), we implement the two-pass regressions procedure. Panel A of Table 6 presents results from the time-series regressions of equity returns in individual countries on the global consumption factor ($\Delta f_{c,t}$) with a constant. We find that for all the countries the beta (slope) estimates are larger than 1 and are highly statistically significant. The adjusted R^2 ranges from 29% (CA) to 62% (NL) (above 40% for 9 countries and above 50% for 6 countries). We then run the cross-sectional regression without a constant to estimate the risk premium associated with the global consumption factor. Panel B of Table 6 presents the estimate ($\hat{\lambda}_c$) and χ^2 tests for assessing the model's overall performance in the cross-sectional regression. For comparison, we use three approaches, namely, ordinary least square (OLS), Shanken (1992)'s method, and Hansen (1982)'s GMM, to compute standard errors and χ^2 statistics. We find that the estimate of the factor risk premium, $\hat{\lambda}_c$, is about 0.85% and that the p-value of $\hat{\lambda}_c$ is about 5% regardless of the approach used to calculate the standard error. These results indicate that the global consumption factor carries a significant positive risk premium. We also find that the p-values of the χ^2 tests are all large, being around 0.7 under the three approaches, and that the mean absolute pricing error (MAE) of the model is about 0.26%.[13]

We further examine how our global consumption factor is related to the Fama-French six developed markets factors (Fama and French, 2012), which are constructed based on stock market data of 23 developed countries that contain the 10 countries considered in this paper. Those factors include the market factor (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW), the investment factor (CMA), and the momentum factor (MOM). However, the Fama-French six developed markets factors are only available from July 1990.[14] Panel C of Table 6 presents results from regressions of quarterly individual Fama-French factors on our global consumption factor. We find that the developed market factor (MKT) is closely related to the global consumption factor, as the coefficient on the global consumption factor is about 1.38, which is highly statistically significant ($t = 14.7$), and the adjusted R^2 is about 67%. The slope coefficient is larger than one, suggesting that the global equity market represents a levered claim on the the global consumption factor. The global consumption factor also explains to some extent the profitability (RMW) and investment (CMA) factors, as the coefficients

[13]Following the same fashion, we also construct a global consumption volatility factor based on the filtered consumption volatility of individual countries. However, we find that both the beta estimates of this factor in the time-series regressions and the factor risk premium estimate in the cross-sectional regression are not statistically significant.

[14]Those data are available at Ken French's online data library, https://mba.tuck.dartmouth.edu /pages/faculty/ken.french/data_library.html. We construct quarterly factors by summing up monthly factor returns in each quarter.

Table 6: **The Global Consumption Factor and Equity Risk Premia**

A. Time-Series Regressions

	US	UK	DE	FR	IT	JP	CA	AU	NL	CH
100×Const.	1.32	1.29	0.98	1.33	0.39	0.80	0.62	0.95	1.45	1.61
	(2.96)	(2.61)	(1.80)	(2.25)	(0.44)	(1.58)	(0.94)	(1.57)	(2.69)	(3.37)
β_c	1.32	1.42	1.47	1.70	1.65	1.09	1.10	1.26	1.54	1.39
	(16.1)	(11.5)	(10.0)	(15.2)	(11.0)	(9.58)	(7.81)	(8.05)	(11.2)	(10.8)
Adj R^2	0.60	0.57	0.55	0.61	0.44	0.48	0.29	0.43	0.62	0.57

B. Cross-Sectional Regression

	Estimate	OLS	Shanken	GMM
$\hat{\lambda}_c$	0.85%	0.05	0.05	0.05
χ^2		0.69	0.71	0.70
MAE	0.26%			

C. Global Consumption Factor and Developed FF Six Factors

	MKT	SMB	HML	RMW	CMA	MOM
100×Const.	1.42	0.19	0.77	1.05	0.63	1.74
	(3.02)	(0.57)	(1.33)	(4.31)	(1.54)	(2.67)
Δf_c	1.45	0.03	-0.16	-0.25	-0.39	-0.34
	(14.7)	(0.49)	(-1.12)	(-3.77)	(-3.32)	(-1.74)
Adj R^2	0.67	-0.01	0.01	0.18	0.18	0.05

Panel A presents results from the time-series regressions of equity returns in individual countries on the global consumption factor ($f_{c,t}$) with a constant. Panel B presents the estimate of the factor risk premium ($\hat{\lambda}_c$) and χ^2 tests. p-values are computed based on the standard errors of the estimate and the χ^2 statistics using the OLS method, Shanken (1992)'s method, and the generalized methods of moments (GMM). In both panels, the sample spans from 1973:Q4 to 2019:Q3. Panel C presents results from regressions of the Fama-French six developed markets factors on the global consumption factor. The Fama-French six developed markets factors are only available from July 1990, and we construct quarterly factors by summing up monthly factors returns in each quarter. In all panels, the $t-$statistics are reported in parentheses.

on the global consumption factor in both regressions are highly statistically significant, and the adjusted R^2s are about 18%. The global consumption factor seems marginally related to the momentum factor (MOM), as the coefficient is only marginally statistically significant ($t = -1.74$) and the corresponding R^2 is only about 5%.

6. Conclusions

The long-run risk model of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) has attracted remarkable attention and has become a benchmark in the consumption-based asset pricing literature. Despite the success of the long-run risk model in characterizing dynamics of fundamentals and asset returns in the US market, its performance with regard to other developed countries is yet to be examined. Furthermore, the vast majority of consumption-based asset pricing studies have relied on the calibration approach, and studies on structural estimation of asset pricing models remain very limited. The main cause for the sparsity in this research is that efficient econometric estimation of consumption-based models is challenging primarily due to that global solutions to these models are highly nonlinear functions of state variables and that data on fundamentals are often observed in very low frequencies and are hard to obtain for countries other than the US.

We estimate and test long-run risk models by employing an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods for international economies. Our benchmark model features a representative agent who has recursive preferences with a time preference shock, a persistent component in expected consumption growth, and stochastic volatility in fundamentals characterized by an autoregressive Gamma process. We construct a comprehensive dataset including macroeconomic and financial data in the post-war period for ten developed countries, including the US, the UK, Germany, France, Italy, Japan, Canada, Australia, the Netherlands, and Switzerland. We use the quarterly data on consumption, dividends, and asset returns to implement estimations.

Our estimation provides international evidence in support of long-run risks in expected consumption growth and a countercyclical component in the stochastic discount factor. We find that the introduction of time-varying preference shocks in the long-run risk model helps deliver economically plausible estimates of relative risk aversion and the elasticity of intertemporal substitution, not only for the US but also for the other developed economies. We also find that the importance of the persistent component varies significantly across the countries. In addition, our estimated stochastic volatility process, which reflects time-varying economic uncertainty, is less persistent than those postulated in the calibration studies. Our estimation yields model-fitted risk-free rates that closely track the historical movements of the actual risk-free rates for the countries. We show the existence of a global long-run consumption factor, which strongly comoves with the global equity market factor and carries a significant positive risk premium.

An interesting future research direction would be to undertake the joint modeling and estimation of LRR models on the country panel, allowing us to have a principled discussion of cross-country correlations in consumption and asset prices in different classses.

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