DESIGNING ONLINE ACTIVITIES FOR TEACHING QUANTUM MECHANICS

A Research-Based Approach

by

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Designing Online Activities for Teaching Quantum Mechanics: A Research-Based Approach

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We present a research basis for the development of ACE Physics—a new website that hosts online tutorials (i.e., interactive worksheets) for teaching quantum mechanics. Unlike most physics tutorials, we have designed ours for completion outside the classroom and without instructor facilitation. This makes them easier for instructors to adopt, but introduces various tradeoffs and challenges, which we explore. These challenges underscore the importance of research as a tool for informing the design and goals of our activities as well as for evaluating their effectiveness.

Our tutorials aim to help students develop intuition, so we begin with a study of student perspectives on intuition in quantum mechanics, which can inform instructional choices as well as future physics education research into the elusive but essential role of intuition in physics. We find that students have varied views on intuition and identify several ways that it can be integrated into instruction. We then report an investigation into student understanding of basis and change of basis—key ideas in any quantum course—which informed the creation of a tutorial about these topics. Next, we examine student behaviors during in-class tutorials. We present a framework for analyzing video data from this context, report findings from applying the framework, and explore how these findings can guide modifications to tutorials for use outside class. Finally, we evaluate the success of ACE Physics tutorials assigned as homework using two methods. First, we report content learning gains associated with two different tutorials as measured by pre- and post-testing. Second, we investigate student engagement with and feedback about ACE Physics using various survey data as well as interaction data collected by the website. We find mixed but largely positive results, suggesting that we should continue developing these activities and conduct further studies of their effectiveness. We conclude with an overview of the strategies we have developed for designing online physics tutorials, which synthesizes our findings as they apply to curriculum development.

Dedication

Physics is for everyone.

Acknowledgements

A physics PhD is a privilege and a struggle, but for me it was people who made it worthwhile. I wouldn't have attempted it without my parents—Rachel, who insists I can do anything, and Ralph, who reminds me I don't have to—and wouldn't have completed it without my brother Tejo's advice to take time off (he made up for that by getting me to run a marathon days before my defense).

I met some of my best friends in Boulder: Alex, Ameya, Claire, Dalton, Sarang, and many others. They've given me adventure, dance, good food, overly complex games, and support, and they help me push myself (Dalton required that I use the word "sesquipedalian" in my thesis). I relied on Hannes and Minah's unwavering friendship as well, as I have for years.

Several mentors and collaborators were critical to my work and experience, including Homeyra Sadaghiani, Ben Schermerhorn, Bianca Cervantes, Tyler Garcia, and Alan Cortez. Noah Finkelstein's PER class continues to shape my work. Bethany Wilcox supported me throughout my PhD, including advising my first ever research project; I value her candor and precision. Gina Passante generously lent insight and clarity to anything I shared with her. ACE Physics would likely not exist if not for Bryan Knauber and Zach Snow, mentors from my life as a software engineer.

Most significantly, I'm grateful for Steve Pollock. I could thank Steve for many things, but in short it seems the best possible advisor is one who is also a friend. I will have accomplished a lot if this work encodes a small fraction of his intuition for teaching.

Finally, I must thank the entire CU PER group, my committee, and all the students who played a role in my research. The National Science Foundation supported the work in this dissertation via Grants No. PHY-2012147 and 2011958, as well as DUE 1626280, 1626594, and 1626482.

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Knowledge flows from and emerges in activity, so the design of activity should be the first priority of those who would aid learners. [29]

Chapter 1

Introduction

What makes an instructional activity successful? In physics classes, activities such as worksheets typically target a set of content learning goals; for example, a quantum mechanics worksheet might help students learn to calculate expectation values. Activities can also emphasize more general skills or practices important for physicists, such as collaboration, metacognition, or sense-making and intuition. Strong activities will address a variety of such goals while also attending to student confidence or affect. Research can help activity designers select, refine, and approach their goals as well as assess whether activities successfully achieve these goals.

We can evaluate an activity's success in terms of its goals, but this does not tell us *why* the activity succeeded. We might address this question by investigating student behaviors: what does the process of learning during the activity look like? We can also examine the design of successful activities: what qualities (e.g., format, presentation of content, tone, etc.) do they share? Physics education research (PER) about instructional activities often, though not always,[1] emphasizes research demonstrating the need for an instructional intervention as well as research measuring the intervention's success, but gives less attention to the design of the activity itself, and less attention still to the process of designing it. Research attending to these latter questions can support the creation of new activities while illuminating features of productive learning environments.

This dissertation presents research surrounding one variety of instructional activity that has proven especially effective in physics classrooms: *tutorials*. Chapter 2 provides background on the

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[1] Examples include [83] and [124] while exceptions include [13] as well as our own work [115]. See Chapter 2 for a more comprehensive review.

tutorial format and its goals and efficacy, and introduces the specific set of quantum mechanics-focused tutorials considered in the rest of the dissertation. The next two chapters relate to the goals of tutorials (but also have implications for research and instruction beyond tutorials). Chapter 3 presents an examination of student perspectives on intuition in quantum mechanics, which applies to tutorials because they often aim to help students develop intuition. The study identifies six facets of intuition—distinct ideas that arise when students discuss intuition—that can be accounted for in instructional materials intended to support intuitive thinking. Chapter 4 is a more traditional investigation into student content understanding that informed the development of a single tutorial about basis and change of basis in quantum mechanics.

The first half of Chapter 5 studies student behaviors during tutorials as they are typically deployed: on paper and in a classroom where students work together in groups and instructors float between groups supporting students as needed. The second half of Chapter 5 considers how to apply the findings from the first half to the design of a new format for tutorials: interactive online versions that students can complete outside of class.

Successful activities share one obvious feature that does not require research to understand: for an activity to do anything for students, students have to do the activity. This statement is self-evident, but adoption poses the most significant barrier when it comes to improving physics education with interactive engagement techniques such as tutorials.[2] A variety of systemic and cultural factors constrain the adoption of these techniques in physics classes, but the primary barrier is the simple fact that interactive pedagogy is typically more resource-intensive than traditional lecture (or is at least perceived to be). Activities that are easier for instructors to implement have the potential to reach more students.

Recognizing this, and aiming to make the benefits of tutorials available to more students, this dissertation introduces a new website—ACE Physics (`acephysics.net`)—that hosts interactive tutorials for upper-division quantum mechanics. The online tutorials can be assigned as homework

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[2] Interactive engagement refers to any pedagogical techniques that actively engage students in thought or discussion, in contrast with traditional lectures, which at best only engage students passively. It is a well-established result in PER that interactive engagement in any form is nearly always more effective than traditional lecture alone [48, 53].

because they include dynamic elements that provide students with guidance and feedback, as compared with static, on-paper tutorials that rely on instructors and group discussion for this purpose. Removing the requirements of class time and instructor facilitation makes the online tutorials significantly easier to adopt than their on-paper counterparts, but also introduces a variety of tradeoffs discussed throughout the dissertation. The lack of instructor facilitation means the success of these tutorials depends more heavily on their design, which underscores the importance of using research as a basis for that design.

In addition to discussing tutorials generally, Chapter 2 describes the ACE Physics project in more detail, including motivation, background, and framing. We return to the topic of ACE Physics in Chapter 6, which reports research measuring the content learning gains after two ACE Physics tutorials that were assigned as homework, one of which is the tutorial about change of basis motivated by Chapter 4. Chapter 7 presents a complementary evaluation of the success of ACE Physics tutorials assigned as homework that focuses on student engagement instead of content learning gains.

Chapter 8 documents the various design patterns and guidance strategies implemented in the ACE Physics tutorials. This documentation can support the creation of more online physics activities (including other ACE Physics tutorials) as well as research into the effectiveness of different patterns. Finally, Chapter 9 presents conclusions and discusses future directions for the project. Footnotes on chapter and section headings indicate which content has been published elsewhere.[3]

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[3] During my dissertation I authored two publications in Physical Review [17, 24] and co-authored two others [115, 149]. In addition, I authored two peer-reviewed conference proceedings publications [16, 18] and co-authored another [10]. I additionally presented multiple posters and talks related to ACE Physics at several American Association of Physics Teachers Conferences as well as several Physics Education Research Conferences, and I presented on both ACE Physics and intuition at the Oslo PER Summer Institute.

Chapter 2

The ACE Physics project: Background, motivation, and tradeoffs

2.1 Introduction[1]

Quantum mechanics is an essential course for undergraduate physics majors, but the material is known to be conceptually challenging for students [44, 99, 100, 134, 144]. Research-based *tutorials*—guided-inquiry worksheets that assist students in constructing physics knowledge for themselves—have repeatedly proven to be a highly effective complement to lectures in physics courses, and are especially targeted at supporting students' conceptual understanding [33, 37, 60, 62, 67, 70, 73–76, 83, 124, 125, 130]. Our collaboration has developed a collection of tutorials for upper-division quantum mechanics that has been adopted by numerous instructors nationwide [107]. We call these the "ACEQM tutorials".

Designed to be completed in groups with facilitators present to guide students, tutorials require institutional and instructional resources and buy-in: an effective implementation relies on trained facilitators [37, 85]. These requirements compete with trends of increased student-instructor ratios and use of online materials [25]. In upper-division physics courses such as quantum mechanics, tutorials often face additional barriers: limited class time, lack of a recitation section, and greater instructor autonomy, which makes it difficult to establish a tradition of running tutorials every semester. The ACE Physics project aims to side-step these barriers by adapting tutorials to an online format that may be easily administered outside of class and does not require real-time instructor intervention or a dedicated physical space.

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[1] Portions of this section were previously published in [18].

The freely available ACE Physics ("Adaptable Curricular Exercises for Physics") website, `acephysics.net`, hosts interactive online versions of all the existing ACEQM tutorials, as well as some new tutorials. Whereas the original ACEQM tutorials were designed to be done by students working on paper and—more significantly—in groups, in the classroom, and with instructors present, the ACE Physics versions are designed to be done by students working on their computers, outside of class, without instructors present, and (possibly but not necessarily) alone. These differences require the ACE Physics versions to vary meaningfully from the original on-paper ACEQM versions. For example, the paper tutorials often include intentionally open-ended, possibly ambiguous, and challenging questions appropriate for the instructor-supported, group work environment, which must be streamlined for the ACE Physics version. More significantly, the ACE Physics versions contain a variety of interactive feedback elements that guide students according to the answers they submit.

We intend for our tutorials—both on-paper and on ACE Physics—to provide formative (as opposed to summative) experiences for students. We do not grade tutorials (except perhaps for participation credit), and aim to provide space for students to comfortably engage with and refine their ideas regardless of initial correctness. A typical on-paper tutorial presents a sequence of short, mostly conceptual questions about a given topic. A typical ACE Physics tutorial does the same, but intersperses dynamic guidance elements that appear conditionally based on student responses. Although we frequently ask open-ended questions, we do *not* attempt to parse students' written responses; dynamic guidance logic is based on multiple-choice and numeric responses only. Example guidance elements include: hints, follow-up questions, messages prompting students to compare two of their answers that logically disagree, or messages explicitly telling students if they are correct or incorrect. To some extent, we design ACE Physics guidance elements to model the conversations we have with students when facilitating in-class tutorials.

Although most of the activities it houses are derivative of previously existing paper tutorials, the ACE Physics website itself was created from scratch as part of this dissertation. Chapter 8 discusses design considerations involved in translating paper tutorials to ACE Physics, and technical documentation as well as the source code for the website is available upon request.

2.2 Background and prior work

This dissertation builds on an existing basis of physics education research (PER) in the topical area of quantum mechanics. These projects have studied student understanding of multiple quantum mechanics subtopics, including notation [43, 44, 72, 73, 129, 133, 146], expectation values [44, 75, 112, 113, 116, 155], time dependence [35, 71, 97, 99, 131], and measurement and probability [35, 144, 145, 155–157]. See [134] for an overview of PER related to student understanding in quantum mechanics. Later chapters in this dissertation will review additional literature when relevant.

This dissertation also builds on a tradition of research-based instructional activity development in PER. A well-known example is the *Tutorials for Introductory Physics* [84] produced by the University of Washington's PER group, who have numerous publications detailing the research base for these and other tutorials, including some focused on quantum [56, 57, 83, 124, 125, 152, 153]. Other researchers have also produced tutorials using a variety of research-based approaches (e.g., [110, 139]), as well as other types of instructional activities, such as "invention activities" [7, 120, 121] and simulations [87, 140]. In addition to interactive simulations, which are typically published online, there is also precedent in PER for the development of online instructional worksheets that include dynamic guidance elements [88, 111]. There have also been a variety of instructional activities and simulations designed specifically for teaching quantum mechanics, some of which are used in or motivated ACEQM tutorials [62, 67, 105, 107, 132, 140]. However, unlike the ACE Physics project, none of these quantum mechanics activities are designed for standalone use outside the classroom.

2.3 What makes a tutorial?

A tutorial, like other instructional activities, aims to intellectually engage students in the process of learning some set of skills, concepts, and practices. We can roughly divide these goals into two categories: *content learning goals*—ideas related to the physics topic covered in class that week; and *meta learning goals*—broader skills important for physicists, such as practice collaborating with peers, development of expert-like attitudes about the nature of scientific models, or formation of a habit to reflect on the physical viability of solutions. Research helps us select learning goals and operationalize them as statements like, "After completing the tutorial, students should be able to. . . " [11, 103, 127].

What distinguishes tutorials as a category of instructional activity is *how* they achieve their goals (or attempt to). Tutorial worksheets share common characteristics, as do the classroom environments in which in-class tutorials are run. However, tutorials also tend to share a common set of meta learning goals (though not all tutorials emphasize all such goals equally), and the format and structure of tutorial worksheets and classroom environments work in service of these goals. Therefore we cannot discuss the design of tutorials independent of their goals.

McDermott and others at the University of Washington (UW) developed the first instructional physics tutorials to address conceptual difficulties they found were common among introductory physics students (even those who could calculate solutions to typical exam-style problems) [81–84, 124]. The UW tutorials implement a specific paradigm, known as *elicit/confront/resolve* [57, 124], whereby sequences of tutorial questions draw out and then help students resolve a common reasoning error. Our tutorials [23, 107] (like others [6, 32]) do not use this exact paradigm (see Section 2.4) and also differ because they are designed for upper-division courses, but they do share some of the same motivating principles as the UW tutorials [82].[2] First, "questions that require qualitative reasoning and verbal explanation are essential" for students to develop a functional understanding[3] of physics topics. Second, "students need to participate in the construction of qualitative models" to develop a coherent conceptual framework. Third, instruction must explicitly address difficult concepts by providing students with "multiple challenges in different contexts" [82].

In accordance with these principles, tutorials consistent of a sequence of exercises or questions meant to actively engage students with the intricacies of a specific physics concept or the relationships between multiple concepts. The questions, which are often (but not always) qualitative in nature,

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[2] These motivating principles, found in Chapter 5 of [82], are presented in the context of a different curriculum development project called *Physics by Inquiry*. However, McDermott later (in Chapter 7) states that the *Tutorials* were built on the same principles.

[3] McDermott defines "functional understanding" as "the ability to do the reasoning needed to apply appropriate concepts and physical principles in situations not previously encountered" [81].

are selected to build on one another and are generally of a smaller scale or grain size than a typical homework or exam problem. Homework or exams may ask students to synthesize what they have already learned, whereas tutorials seek to help students construct a refined, functional understanding one step at a time. Ideally, each subsequent question falls within the student's *zone of proximal development* [143], which entails what a student can learn next by solving a new problem (with some external guidance) given what they already know. Most questions will ask students to articulate their reasoning, and a given tutorial will often ask similar questions, which focus on related concepts, in a variety of contexts (e.g., different physical scenarios).

As discussed in greater detail below (Section 2.5), in-class tutorials are defined not just by the content of the worksheet but also by the classroom context in which they are run. Students work in groups, which naturally encourages them to articulate their reasoning to their peers as the group comes to consensus about a particular question (which instructors encourage groups to do). When students get stuck or make errors, instructors engage the group in Socratic dialogue. Notably, tutorial instructors attempt to *guide* students to a resolution and rarely *tell* them the answers directly (at least not immediately). This guidance style is employed because it keeps students actively engaged in the process of constructing knowledge for themselves instead of simply receiving it from the instructor, which is typically less effective [48, 53, 109].

Our quantum mechanics tutorials share several common learning goals that we believe the tutorial format is well-suited to address. First, we generally expect students to exhibit some confusion about topics recently covered in lecture (even when lecture includes interactive engagement elements, such as clicker questions). This confusion might surface when solving homework problems, but the small grain size of tutorial questions can help students pinpoint exactly where their confusion lies, and the regular feedback they receive during the tutorial can help them resolve it. Through this process, and especially because tutorials require students to articulate their reasoning often, we also intend for students to practice the metacognitive ability of evaluating their own understanding [68].

A second goal common to many of our tutorials is for students to develop fluency with multiple representations. We often discuss quantum mechanics using a variety of representations, including Dirac notation, matrix notation, and function notation, as well as graphs and other visual representations like the Bloch Sphere, and students need practice making sense of and moving between these representations [26, 59, 62, 69, 72, 115, 146, 147].[4] Tutorials can provide students with opportunities to explore features of different representations and how they relate to one another. Our tutorials often also use varied language (or notation) so that students will be comfortable communicating about quantum mechanics in various contexts.

Finally, we also intend for our tutorials to encourage students to practice various habits of mind. When students solve a problem, they should "reflect" on their solution and decide if it is sensible such as by checking a numerical answer's order of magnitude or by testing whether an algebraic result exhibits the expected relationship between two quantities [148]. Our tutorials often ask students to do this reflection explicitly. Explicit sense-making is useful at every stage in the problem-solving process, not just at the end [40, 41, 94], and we intend for our tutorials to encourage it generally. Intuition is useful for sense-making and reflection because students can check the answers they calculate against their intuitive expectations (see Chapter 3). Our tutorials aim to help students develop intuition through various means, such as through visualization or by connecting new concepts to ones the students already understand (see Table 3.3). In so doing, we also hope to impart that one *can* develop intuition for quantum mechanical concepts, because we know students may expect quantum mechanics to be nonintuitive (see Table 3.4).

2.4 Strengths-based activity design

It has been documented that the cognitive theories adopted (either explicitly or tacitly) by curriculum developers influence curriculum development generally and tutorial design specifically [6]. When designing ACE Physics activities (as well as the on-paper ACEQM tutorials), we intentionally lean towards a *strengths-based* approach to investigating student thinking and crafting tutorial questions. A strengths-based approach involves engaging with student ideas holistically and helping students refine their understanding by building on the ideas they already have.

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[4] See also Chapter 4.

Strengths-based activity design aligns well with a *resources* perspective on cognition [49]. Resources describe the set of knowledge elements and cognitive tools one can apply when solving a physics problem [49]. The resources perspective suggests that instructors can help students learn by drawing out and building upon the resources students have that can be productively applied to the physics at hand.

Strengths-based activities are in contrast with activities that focus primarily on drawing out and correcting student difficulties and misconceptions. For a comparison of the two approaches in the context of quantum mechanics curricula, see [33]. As discussed in the reference, both approaches have merits and they have both proven effective. We believe the strengths-based approach is best-suited for the ACE Physics activities, which are designed to be completed outside the classroom and thus lack social interactions that can—hopefully—buoy student affect in the face of a tutorial intended to elicit and correct one's misconceptions.

2.5 Challenges of moving tutorials outside class: A sociocultural view

ACE Physics aims to export some of the benefits of tutorials from the in-class environment to homework, which is completed outside class. This introduces numerous challenges. The most obvious challenge, addressed throughout the dissertation, is that instructors (and to a lesser extent, other students) are not available to give students feedback and guide their understanding. Although this guidance occurs via interpersonal interactions, this framing of the problem is fundamentally cognitive because it treats instructors primarily as a tool for guiding (or correcting) student thinking. In this view, ACE Physics simply introduces alternative tools for this purpose in the form of dynamic guidance elements built into the online tutorials. Clearly, though, viewing students' instructors and peers exclusively as stewards for student cognition is reductionist; this perspective overlooks the rich social and cultural features of the in-class tutorial environment as well as significant contextual differences between it and the outside class environment. Most of these features cannot be replicated by ACE Physics.

We can capture some of these contextual differences by identifying the "embedded levels of

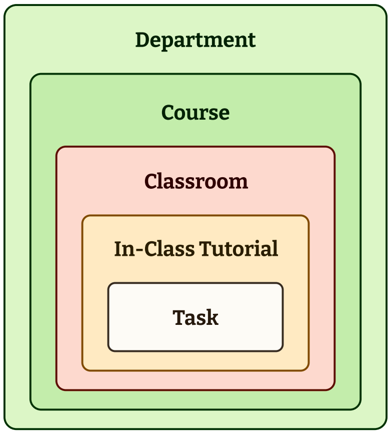

Figure 2.1: Frames of context for in-class tutorials. Similar to the frames presented in [37].

context" [38] relevant in each of the tutorial environments. In this framework, levels—or "frames"—of context fall into three categories [38]. The first, most tightly scoped frame, is the *task*. In the case of tutorials, the tasks are individual questions, which might involve doing algebra, sketching a graph, or performing some conceptual reasoning. Tasks live inside the next frame in the framework: the *situation*, which, in this case, is the tutorial activity itself. Finally, the situation occurs within a cultural context shaped by a hierarchy of frames known as *idiocultures*—a collection of customs, behaviors, and norms localized to a specific context. One idioculture relevant to tutorials is that of the physics course in which the tutorials are assigned. There is a bidirectional relationship between the levels in this framework. The outer levels affect the inner levels: the idiocultures influence the ways students engage with the specific situation, which in turn shapes how they interact with specific tasks. Conversely, the inner levels constitute the outer levels: situations are made up of multiple related tasks, and idiocultures of multiple related situations or smaller idiocultures. In this way the inner levels can influence the outer levels as well, but to a lesser degree than the reverse [38].

2.5.1 Frames of context for in-class and outside class tutorials

Finkelstein and Pollock apply the embedded levels of context framework to in-class tutorials [37]. In addition to the three frames named above (*task*, *tutorial*, and *course*), they introduce an additional

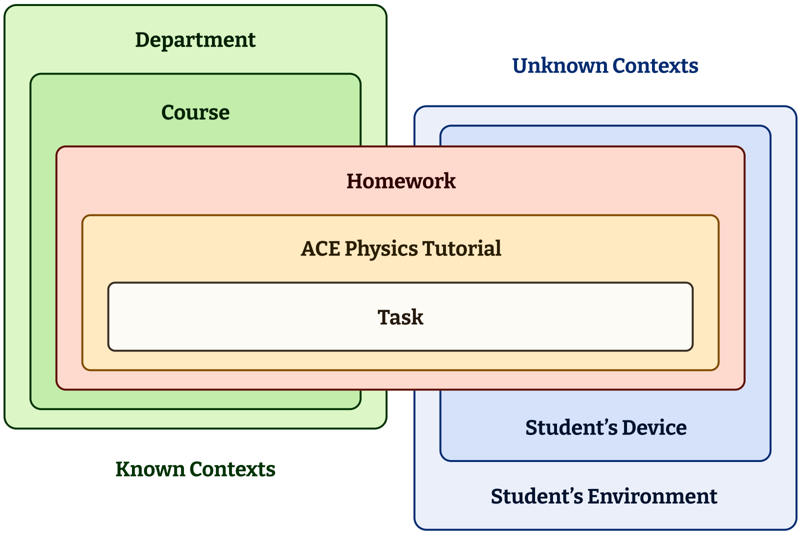

Figure 2.2: Frames of context for outside class tutorials.

idioculture at the outermost level: the *department*. A physics department, which is made up of a collection of physics courses, has some global norms and expectations for how students behave that shape the course-level idiocultures to some degree. (Idiocultural slices can be drawn at any level, but as one continues to zoom out they become less distinctive.) We adopt the same frames of context to characterize the in-class tutorial environment, but we insert a new frame—the *classroom*—as shown in Figure 2.1. In the original set of frames, the features of the classroom are considered part of the *situation* of the tutorial [37]. In our version, the *classroom* and *in-class tutorial* frames combine to form this situation, but we have separated these frames to facilitate comparison with the outside class tutorial environment. This separation also allows for the possibility that the classroom is not exclusively used for tutorials (as was the case in [37]) and therefore may carry idiocultural baggage such as norms established during lecture, which may conflict with norms established during tutorials. Figure 2.2 presents frames that characterize the outside class environment.

There are two differences between Figures 2.1 and 2.2. The first difference is that the *classroom* frame is replaced by a *homework* frame for the outside class environment. The course may define norms that apply to both the classroom and homework contexts, but there may also be customs or expectations that are unique to either of these two contexts. For example, group work might be expected in class but uncommon on homework. In addition, the division of the *homework* and *ACE Physics tutorial* frames captures the fact that a typical homework assignment will likely include a variety of physics problems in addition to a tutorial.

The more significant difference between the two figures is the introduction of "unknown contexts" for the outside class environment (Figure 2.2). The *homework* frame is certainly influenced by the course it is part of, but it is not wholly contained within the *course* frame because instructors do not control the environment in which students do their homework. Idiocultural features of the unknown student environment influence whether students work on their homework alone or with peers and impact the time and focus they apply to the assignment. Students may do homework during a help session, or perhaps in a common space with fellow physics majors, or maybe in a quiet study space at home. Alternatively, they may do homework during a long public bus ride, in the break room of their part-time retail job, or in a loud room crowded with peers or family members. Instructors, and especially departments and universities, can provide resources and accommodations for students to help them find comfortable, productive homework environments, but such efforts are limited. Practically speaking, we do not know the environment in which students will work on ACE Physics tutorials. The outermost frame labeled "student's environment" encompasses all of these possibilities.

The second unknown context presented in Figure 2.2 is labeled "student's device". We argue that this device represents an idioculture—one that is incidentally embedded in the frame of whichever physical environment students use the device in, and one that influences whatever activities students use the device for. The actual computing device students use (perhaps a fancy new laptop, perhaps an outdated netbook or tablet) is a relevant technical consideration when it comes to developing the ACE Physics website, but the physical device alone does not constitute an idioculture—if anything the physical device itself is part of the *situation* of the activity, being an "associated tool" [38]. The virtual world students access through the software on their devices, though, does demonstrate features of an idioculture (if not a tapestry of many idiocultures).

As referenced in [38], Fine defines idiocultures as consisting of "particular examples of behavior or communication that have symbolic meaning and significance for members of a group" [36]. Users and software designers together form a group with a common set of significant behaviors and communication patterns. Software designers impart meaning to users through established patterns, such as using the color red to indicate an error. Similarly, there are certain user behaviors that mean the same thing across different programs; for example, the interaction of swiping right to go "back" (i.e., to the previous screen). Users have expectations for how software should communicate and behave. If a piece of software breaks these expectations (a programmer could just as well use green to indicate errors and make swiping left go back), it can feel alien—external to the idioculture—and users may have a difficult time using it.

The example of one atypical piece of software feeling unfamiliar to users demonstrates how the set of software a user uses affects their expectations for all software. When using a new piece of software, users might apply behaviors previously learned using other software. Such behaviors could include simple interactions like swiping right, but could also meaningfully affect the way users engage with the new software. For example, if a student commonly multitasks when browsing the internet on their device, they may apply this same behavior when working on an ACE Physics tutorial, to the detriment of their focus on the tutorial.

We have considered *student's environment*, *student's device*, and *homework* each as individual contexts, but these frames may in fact encompass multiple, possibly conflicting idiocultures. For example, many students likely use their computing devices both for schoolwork and also for entertainment—two tasks associated with different sets of norms and behaviors. Similarly, traditional homework problems, which are graded for correctness and often involve long calculations, may prompt somewhat different behaviors as compared with tutorials, which are not graded for correctness and focus on conceptual reasoning. Each student who does an ACE Physics tutorial will do so within an idiosyncratic cocktail of contexts that may both mesh and conflict with the goals of the tutorial in different ways.

2.5.2 Implications for ACE Physics

What does the shift from the in-class environment (Figure 2.1) to the outside class environment (Figure 2.2) imply for the ACE Physics project? Most significantly, this shift means the loss of the classroom frame. The following quotation, which describes the classroom environment for tutorials in an introductory physics course, captures the elements lost with this frame [37]:

> *How students engage in tutorial tasks. . . is significantly shaped by students' interaction with each other and with the TAs [teaching assistants]. Students support each other's understanding by making reasoning explicit; TAs support student engagement in tasks by not disclosing answers or grading student work. . . . Other elements of the situation enable these productive interactions: locating students in the same room, providing appropriate desks for students to work at, eliciting student conceptions in advance of tutorials. . . . Each component of these situations is shaped by TA, student, and instructor understanding of the norms of this environment. . .*

When working on ACE Physics as part of homework students may work alone,[5] but even if they do collaborate they will nonetheless do so in a context (i.e., *student's environment*) that lacks established norms specifically optimized for tutorials. Instructors (including TAs) do help guide student cognition, but, perhaps more significantly, they cultivate norms that encourage students to engage fully with the tutorials and to support one another's learning. It is the classroom frame that provides these features.

Without the classroom frame, then, what hope does ACE Physics have to create effective tutorials? To some extent this remains an open question—the fundamental question of the project—although Chapters 6 and 7 report some mixed but largely encouraging results. The theoretical framework we have discussed does, however, allow for mechanisms whereby the norms from the classroom tutorial environment can, to a lesser extent, be established on homework as well.

Firstly, norms cascade from outer levels in the framework into the inner levels. Although the *homework* frame is not wholly contained within the *course* frame, norms established in the course can still influence the way students approach homework and specifically the ACE Physics tutorials assigned on homework. Therefore, if instructors establish norms such as encouraging sense-making

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[5] In fact, they often do. See Chapter 7.

and valuing student ideas and debate in the classroom, these norms can seep into the *homework* and *ACE Physics tutorial* frames as well—especially if instructors explicitly encourage this.

Secondly, any level in the framework has the capacity to define its own norms and also, to some extent, to influence its containing frames. The design of an ACE Physics tutorial and the specific questions (tasks) it asks can promote certain norms; for example, conceptual questions that require students to explain their reasoning before they are allowed to continue to the next question could promote sense-making. There is even limited potential to overcome difficulties introduced by the unknown contexts (*student's device* and *student's environment*). For example, an engaging visual design could diminish the distracting effect of other software available on the student's device.

Without the classroom frame, the design of the tutorials becomes significantly more important, because the tutorials themselves become responsible for shaping norms and encouraging specific student behaviors in addition to trying to provide cognitive feedback to students. Moreover, because ACE Physics is a piece of software, it must engage with the cultural expectations students have for how software should function and how they should interact with it. Chapter 8 explores some design strategies we have developed to serve these goals.

2.6 Benefits and tradeoffs of ACE Physics

We believe well-implemented in-class tutorials will always exceed even the best online imitation in terms of supporting both content and meta learning goals. Hardcoded guidance pathways cannot compete with an instructor reacting in real time to the needs of a student, nor can they mimic the rich, productive interactions of in-class group work. In addition, online tutorials lack the carefully cultivated norms present in the classroom tutorial environment. We encourage quantum instructors to run in-class tutorials using the variety of available resources (e.g., [33, 62, 67, 105, 107, 132, 140]). However, this is not always possible; even instructors interested in running in-class activities must be selective about their use of class time. In practice, most quantum courses will not offer in-class tutorials for many topics. ACE Physics is meant to provide a new option here. If an instructor cannot or will not administer a tutorial in class, they can consider assigning the ACE Physics version as a homework problem instead. An ACE Physics tutorial should not be compared against its in-class counterpart; it should be compared against the alternative of not running the tutorial at all.

Some cases where the ACE Physics option may be especially compelling are large courses with low instructor-student ratios or courses taught in classrooms where the physical space is not conducive to student group work. Another example is topics that students could use extra practice with but are not so high-priority as to merit dedicating class time for this practice. Unlike in-class tutorials, students may take a variable amount of time completing ACE Physics activities, meaning students less comfortable with a topic can spend more time on an ACE Physics tutorial if they want additional practice. Another use for ACE Physics is that instructors uninterested in tutorials still have the option to share the website with their students as an optional resource; doing so only requires distributing a link in a class announcement. All of these scenarios exemplify how ACE Physics can make more tutorials available to more students more easily.

Although we consider it unproductive to compare ACE Physics tutorials with in-class tutorials, we can compare the dynamic, interactive medium of the screen with the static medium of paper. Paper tutorials have some upsides as compared with their on-screen counterparts. The clearest advantage of paper is the input device: a pen is vastly superior to a mouse and keyboard because it allows freeform input, thus enabling students to sketch and write mathematical notation easily. (ACE Physics works well on touchscreen devices but not all students have such devices, so it does not include any touchscreen-specific features such as an integrated sketchpad.) The loss of freeform input profoundly impacts the design of ACE Physics tutorials, as discussed in Section 8.4. A second advantage of paper tutorials is that printouts are typically provided by the instructor, meaning each student does not need an internet-enabled device of their own. (ACE Physics will work on small devices like smartphones, but the experience is optimized for larger screens such as tablets or laptops.) Finally, as discussed in the previous section, computer screens typically come with distractions introduced by other programs, such as notifications from messaging software, whereas paper does not.

Conversely, on-screen tutorials have some benefits over their on-paper counterparts. The fundamental benefit is that screens are dynamic, meaning the content of the tutorial can respond to student input. When referring to the ACE Physics version of a tutorial we mean a version designed to be completed by students outside of class, but instructors can just as well run ACE Physics versions in-class, as long as every student (or every group) has an internet-enabled device with them. Instructors might do this if they have a large class, because the feedback built into ACE Physics can provide students with guidance while the instructor is busy helping other groups. If the classroom is virtual, the ACE Physics versions will likely work better than paper tutorials.[6] Finally, the ACE Physics versions are likely more accessible to students with vision difficulties thanks to the suite of accessibility features available on modern computing devices. For example, the majority of ACE Physics, including equations, should be navigable via screen readers, which is typically not the case for PDFs.

2.7 Why quantum mechanics?

The subject of quantum mechanics has no particular significance when it comes to the general idea of the ACE Physics project—creating online physics activities that students can use productively on their own. Indeed, we optimistically named the project ACE *Physics*, not ACE *Quantum Mechanics*, to accommodate the possibility of one day including tutorials about other physics subjects. The simple reality is that there is demand in the community for quantum-focused instructional materials and PER, and the collaboration with which I work already had a set of on-paper quantum mechanics tutorials ready to be moved online. There are, however, some features of quantum mechanics that make it an especially good candidate for ACE Physics tutorials.

The first feature is that quantum mechanics is (typically) an upper-division course. As described above, ACE Physics tutorials are not graded for correctness. Although the design of the activities can support this to some extent, the website relies on students willing to engage with the questions in earnest and able to regulate and reflect on their learning. It is reasonable to expect

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[6] This was our experience during emergency remote teaching necessitated by the COVID-19 pandemic, as described in Section 5.2.

that 100 junior-level physics majors will demonstrate these qualities to a greater extent than will 1,000 introductory physics students from a variety of majors, especially in a core physics course such as quantum mechanics.

The second feature is that, compared with other upper-division physics courses, quantum mechanics may have a stronger conceptual focus, because an upper-division quantum course is often the first time students are expected to develop a strong conceptual understanding of the postulates of quantum mechanics. As a counterexample, students are typically expected to enter upper-division electricity and magnetism with a base of conceptual understanding from their introductory course, and as a result the upper-level class usually focuses more on mathematical techniques. This is not to say that concepts are not important throughout the physics curriculum (they are), that tutorials cannot be used in other upper-division courses (they have been), or that such tutorials could not be adapted for ACE Physics (perhaps one day they will be). However, the canonical idea of a "tutorial" in PER is a *concept-focused* worksheet, and upper-division quantum mechanics is especially ripe to benefit from such activities.

A final reason is that there is growing interest in the community for instructional materials around quantum computing and quantum information science topics. Courses on these topics are being developed at many institutions, and, although they are relevant to physicists, they are often either situated or cross-listed in other departments (e.g., computer science). These courses often include conceptual elements, but, especially because efforts to develop them are scattered across institutions and disciplines, it is unlikely that many of these courses will included dedicated tutorial sessions. Hence there is an opportunity for developing and deploying new, easy to adopt online activities about these topics. Creation of such materials is a natural extension of the existing ACE Physics topic set, and is in fact underway, as described in Chapter 9.

We could broaden the question of "why quantum mechanics?" to "why physics?"—why ACE *Physics* and not ACE *Biology*, ACE *Comparative Literature*, or just ACE *College*? Some of our findings and strategies for designing online instructional activities could reasonably apply in other disciplines, and the skills that our activities foster may help students with non-physics topics.[7] However, ACE Physics emphasizes a certain combination of skills, practices, and habits of mind distinct to physics formed over centuries of the field's existence and explored and refined by decades of physics-specific education research. ACE Physics is deeply situated within the physics context, both topically and culturally.

2.8 Conclusion

We have created a new website, ACE Physics, that houses freely available interactive tutorials for teaching quantum mechanics. Unlike most physics tutorials, we have designed these activities to be completed by students outside class, making them significantly easier for instructors to implement. Removing instructor facilitation from tutorials introduces significant challenges because instructors are not present to guide student thinking and foster productive norms. This places the burden on the design of the ACE Physics tutorials to overcome these challenges, which highlights the importance of using research as a basis for this design and as a tool for evaluating its success. We will return to design considerations in Chapter 8, but the remainder of this dissertation presents novel studies that form part of the research basis for ACE Physics. One goal common to our tutorials is to help students develop *intuition* for quantum mechanical concepts. The next chapter explores what intuition means.

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[7] As any proud physicist will point out.

Chapter 3

Is physics intuitive?
Student perspectives in upper-division quantum mechanics[1]

3.1 Introduction

Among other goals, our tutorials (on ACE Physis and otherwise) aim to help students develop intuition, yet as a concept, intuition is elusive and defies definition. Nonetheless, if it plays any role in learning and doing physics, we are compelled both to communicate our intuitions when teaching and to communicate about intuition when conducting education research. This work aims to lend clarity—and the perspective of students—to this communication, with a focus on quantum physics.

Quantum theory deviates from classical physics in profound ways. For example, observation affects subsequent outcomes, incompatible observables are unknowable simultaneously, and entanglement leads to non-local effects. Does this make quantum mechanics nonintuitive? These ideas are far removed from our macroscopic lived experiences. Does this make them counterintuitive?

A common claim in education research about quantum mechanics is that the subject matter is not intuitive. References [33, 34, 63, 64, 87, 90, 102, 104, 108, 114, 136] refer to quantum mechanics or certain subtopics as *counter*intuitive (meaning contrary to one's intuition). Others refer to the subject as *un*intuitive (meaning one lacks intuition) or simply *non*intuitive (meaning either *counter* or *un*) [70, 74, 135]. Often, these statements are made in passing, and authors may not intentionally use one term over another. Still, as a whole, the literature seems to agree: quantum mechanics

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[1] This chapter was previously published in Physical Review [17] and received the editors' suggestion. We are grateful to Jessica Hoehn for her feedback on the paper, which we have modified slightly here. The motivation for this project stemmed from our investigation of student discomfort in quantum mechanics [16], which is not reported in this dissertation.

topics are often counterintuitive (or at least unintuitive), and this is a relevant consideration when teaching.

This raises multiple questions, and this chapter focuses on the following three:

(1) Do students consider quantum mechanics as counterintuitive as the literature suggests they might?
(2) What does "intuitive" mean to students?
(3) How do students' views on intuition affect their approach to learning quantum mechanics?

Investigating these questions will illuminate students' perspectives on intuition in quantum mechanics with possible implications for future research and for instruction.

To address these research questions, we interviewed and surveyed students drawn from two semesters of undergraduate quantum mechanics. We will present quantitative survey results, but focus more on qualitative analysis. We adopt a phenomenographic approach for our qualitative analysis [78, 79]: we set out to characterize the different ways that students perceive and experience intuition when learning quantum mechanics. We do so without an *a priori* categorization scheme in mind (i.e., we employ emergent coding), meaning we can address our research questions without establishing a definition of intuition. We employ this strategy because our objective is to forefront student perspectives.

In line with this goal, we avoid value judgements of student statements about intuition. For example, we consider the view that quantum mechanics is intuitive to be neither inherently expert-like nor inherently novice-like. Instruction often aims to help students develop intuition, and we share this goal. We also recognize that some physicists perceive the differences between classical and quantum physics as nonintuitive even when—or perhaps because—they understand those differences well. In this work, we investigate distinctions beyond an intuitive/nonintuitive binary.

We begin by grounding this study in the existing literature (Section 3.2) and describing our methodology (Section 3.3). We then present our findings (Section 3.4) in order of our research questions. We conclude with discussion of possible instructional implications as well as potential future research directions (Sections 3.5 to 3.7).

3.2 Intuition in PER literature

There is a large body of work discussing intuition both within physics education research (PER) and more broadly: “Philosophers, writers, mystics, psychologists, biologists, and educators have all contributed to the great conversation” [14]. A comprehensive review is beyond the scope of this chapter, but a brief survey of PER literature will contextualize the present study. Discussions of intuition in PER literature vary in content, context, and purpose; we organize our summary of these discussions using four categories. The categories are not mutually exclusive, and this chapter fits best in the fourth category. The discussions in the first two categories do not probe the nature of intuition directly but do investigate its presence in student thinking and use in the classroom.

Intuitions as barriers to understanding or problem solving. In some cases, students’ initial, intuitive reactions to a given physics problem may cause errors in their solution strategy. This can possibly be explained in terms of dual-process theories, which posit that reasoning consists of two modes: a “fast”, intuitive process, and a “slow”, deliberate, or rational process [39, 65, 66, 151]. If students fail to activate a slow process when solving problems, they can be misled by their initial intuitions, which may be incorrect or incomplete.

The *elicit, confront, resolve* paradigm for instructional activities is one strategy for addressing this problem. Used most notably in the *Tutorials* produced by the University of Washington [84], the strategy aims to helps students “overcome deeply-rooted ideas” that are incorrect, which may arise from students’ intuitions [124]. The broad body of research into student “misconceptions” overlaps with this perspective on intuition.

Intuition as an instructional goal. Some instructional strategies or materials are designed with the explicit goal of helping students develop intuition. For example, Elby discusses guiding students through the process of “refining raw intuitions” when teaching introductory mechanics [32], and Whittmann and Morgan describe the design of an “Intuitive Quantum Physics” course for nonscience majors that foregrounds “everyday” thinking [150]. Bao and Redish discuss the “difficulty in building intuition” in quantum physics, but name helping students in doing so as an explicit instructional goal [3].

Ontology: Intuition as a specific category of knowledge or type of thought process. Research in this category attempts to characterize or define *intuitive thinking* or *intuitive knowledge*. The dual-process theories mentioned above undertake this effort [39, 65, 66, 151], as does diSessa's work on "phenomenological primitives", which are fundamental, intuitive ideas or thought patterns from which more systematic knowledge may be constructed [28]. Other examples include work from Chi and Slotta, who argue that intuitive knowledge has more structure than suggested by diSessa [12], from Brock, who presents intuition as a form of "tacit knowledge" [8], and from Sherin, who studies the role of intuition in physics problem solving [126]. In contrast, Singh examines physicists' thought processes explicitly when they have minimal intuition about a given problem [128].

Epistemology: Practitioners' perspectives on intuition. Regardless of the exact definition of the term, if intuition is a part of knowing physics, then physicists' views on intuition constitute part of their epistemologies (beliefs about the nature of knowledge). Research in this category treats intuition as a part of knowledge or type of thought process, but focuses on people's perspectives on knowledge instead of attempting to classify or define it directly.

Intuition is a consideration in work studying students' epistemologies in quantum physics [27, 31, 51], but little work in PER has focused primarily on students' views on intuition. Outside of PER, Marton *et al.* have investigated the views on scientific intuition expressed—not by students—but by Nobel prize-winners in yearly panel discussions [80]. Their findings include that the discussion participants: (a) did, for the most part, consider scientific intuition to exist as a process distinct from analytic thinking; (b) used the word "intuition" to describe a person's capability (e.g., someone with a knack for intuitive understanding), something that happens (e.g., being struck by intuition), or an outcome (e.g., having an intuition); (c) believed that intuition can be developed through experience, but that it comes more naturally for some; and (d) considered intuition to be something that one can feel or experience.

Similar in both spirit and methodology to the work of Marton *et al.*, this chapter investigates the views on intuition expressed by students in a quantum mechanics course. Students' epistemologies

Instrument	Administration & Timing
Question about incoming expectations for intuition in quantum	Once per semester, on the first pre-lecture survey, due on the first day of class
Intuition characterization question (Figure 3.1)	Every other week on pre-lecture surveys
Questions about struggle	Every other week on pre-lecture surveys
Interviews (Figure 3.2)	Week 9 of the fall semester
Question about the role of intuition in quantum	Once, on the pre-lecture survey for week 6 of the spring semester
Question about developing intuition in quantum	Once, on the pre-lecture survey for week 10 of the spring semester

Table 3.1: Summary of data collection. Other than interviews, all data were collected from weekly pre-lecture surveys ("preflights") administered during two semesters of an upper-division quantum mechanics course. The surveys and interviews are described in the text.

affect their learning in physics generally [1, 50] and in quantum mechanics specifically [27, 51, 58]. Moreover, as discussed in the introduction, intuition is perhaps especially pertinent in quantum mechanics. Hence, learning what students have to say about intuition can illuminate this element of students' epistemologies to complement theoretical work, inform further research, and guide instruction that seeks to help students develop or refine their intuition.

3.3 Methodology

We employed a mixed-methods approach to investigate students' perspectives on intuition in quantum mechanics, including surveys and interviews. In this section, we describe the courses from which interviewees and survey respondents were drawn, detail the survey questions and interview protocol, and summarize our analysis strategy. Table 3.1 provides a summary of data collection.

3.3.1 Quantum mechanics course

Surveys were administered in two semesters of the junior-level quantum mechanics course at a large, R1 university. Interviewees were drawn from the first semester only. The first semester (Fall

2020) had 63 enrolled students, and the second semester (Spring 2021) had 99 enrolled students. During both semesters, the class was taught by the same PER faculty (author SP). Both times, the class was taught fully remotely due to the COVID-19 pandemic, meaning the lectures were held synchronously over Zoom and were also recorded and made available to students asynchronously. Lectures included interactive elements such as frequent clicker questions.

The vast majority of students in the class were physics majors, with some astrophysics majors as well. The class reflects the demographic breakdown of the upper-division physics and astrophysics majors, which the university reports as follows. Approximately 20% of majors are reported to be female.[2] The major is split evenly between in-state and out-of-state students, and 14% are first-generation college students. 58% of majors are identified as White, an additional 22% as international, 9% as Hispanic/Latino, 7% as Asian, and below 5% as any other race/ethnicity.

The class, which is the first half of a two-semester quantum mechanics sequence for physics majors, was taught using a "spins-first" instructional paradigm following McIntyre's *Quantum Mechanics* text [86]. The postulates of quantum mechanics were first presented in the context of a two-state, spin-$1/2$ system, and only later applied to position-space wave functions. Prerequisites included a linear algebra course, and the department's modern physics course, which includes a unit on quantum mechanics. This means students had some prior classroom exposure to quantum topics.

Three lectures were held each week. Additionally, one optional weekly recitation section was held (also over Zoom) in which students worked on conceptual worksheets (tutorials) in small groups under the guidance of the professor and assistant instructors. Approximately 30% of students attended the optional tutorials. The tutorials, as well as homework assignments and clicker questions, are available at [107]. In both semesters, the course included two midterm exams and a final exam.

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[2] The university's statistics only report gender as a binary.

How would you characterize your intuition about the ideas/concepts you learned this week?
Check ALL that apply.

☐ The ideas seem **intuitive**

☐ Some of the ideas seem **COUNTER**intuitive (they disagreed with my intuition about the world)

☐ Some of the ideas seem **UN**intuitive (I don't have any intuition about them one way or another)

☐ I'm not worried about/interested in how intuitive the ideas were

Figure 3.1: This question was administered on online pre-lecture surveys (preflights) approximately every other week during each of two semesters of undergraduate quantum mechanics. The preflights were mandatory and students received participation credit for completing them. Preflights typically also included physics content questions.

3.3.2 Surveys

In both semesters, the course included online pre-lecture surveys assigned once per week, which we call "preflights".[3] The preflights, which were graded for participation only, typically consisted of brief conceptual questions about recently covered material. They additionally included survey questions for this research study, which were always asked first, before any physics content questions.

On the first preflight of each semester, which was due on the first day of class, we asked the following question to gauge students' expectations for intuition in quantum mechanics:

> Think back on your previous physics courses: did you consider what you learned in those courses to be "intuitive" or "unintuitive" (or even "counterintuitive")? Why? (Maybe you felt differently about different courses.) How do you expect Quantum Mechanics to compare to your previous courses, in terms of how intuitive (or not) you will find the material?

Students were able to respond in a single open-ended text box.

To probe whether students considered quantum topics intuitive, the question in Figure 3.1 was included on preflights biweekly during both semesters. Students were asked to characterize their intuition about the topics they had learned that week, and could check boxes corresponding with *intuitive, unintuitive, counterintuitive,* or *unconcerned.* Students were allowed to check multiple boxes, because in any given week some new ideas may feel intuitive while others do not.

During the first semester, every time the intuition characterization question was asked it was followed by:

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[3] The use of "preflights" was motivated by [93].

> Optional: Explain your response to the previous question about intuition.

After a few weeks, few students (roughly 20%) provided explanations. During the second semester, we only included the explanation option the first time we asked the intuition characterization question. On two later preflights that semester, we asked each of the following once:

> FOR YOU PERSONALLY, how important is intuition to your learning in physics? What role does it play?

and

> Do you feel like you're developing intuition about quantum mechanics in this class? If so, what has helped you develop it? If not, why not?

Both were asked directly following the intuition characterization question for the given week.

In addition to questions about intuition, we also asked students about their "struggle" with different aspects of the material on preflights. Approximately biweekly, students were asked on preflights to rank their struggle with: the physics; the math; connecting the math and the physics; notation; language; and quantum "weirdness". For each category, students chose from an ordinal scale of: no struggle (0); mild struggle (1); moderate struggle (2); severe struggle (3). The struggle questions were asked as a follow-up to our work characterizing student discomfort with the material in quantum mechanics [16], but also allowed us to investigate the potential relationship between students' sense of intuition and their self-reported sense of struggle with the content. In the first semester, the struggle questions were asked on the same preflights as the intuition questions; they were asked on alternating weeks in the second semester so the preflight length would be more consistent every week.

3.3.3 Interviews

To study student thinking on intuition more deeply, we conducted 11 interviews with students from the Fall 2020 semester's quantum mechanics class. Student volunteers each received a $20 gift card, and all students in the class were invited to participate in interviews. Interviews, which lasted 30 minutes and were held over Zoom, were conducted about halfway through the semester (around week 9 of a 15-week semester). The protocol consisted of 12 guiding questions as shown in Figure 3.2. Interviews were recorded and later transcribed.

The interviewees were representative of the class in terms of their course performance: of the 11 interviewees, 5 averaged between 90–100% on exams, 2 between 80–90%, and 3 between 60–70%. The 11th interviewee withdrew from the class some time after the interview. The overall class exam average was 75%. We report exam averages and not overall course grades because the course grade distribution was unusual compared with previous semesters due to considerations for difficulties caused by emergency remote teaching and the COVID-19 pandemic.

3.3.4 Analysis

The multiple-choice preflight question asking students to characterize their intuition lent itself to a quantitative analysis; we report various counts and correlation coefficients in Section 3.4.1. Additionally, students' struggle rankings allowed us to investigate possible correlations between students' self-reported struggle and sense of intuition, as discussed in Section 3.4.3. Our remaining data required qualitative analysis.

We employed a phenomenographic approach for our qualitative analysis. Phenomenography is a methodology for drawing out and characterizing different perceptions of the same phenomena [78, 79]. In this study, the phenomenon in question is intuition (or specifically intuition in quantum mechanics), and the themes or categories that arise describe different ways students perceive or experience it. Notably, the categories are of student statements, not of students themselves.

Phenomenography sometimes produces a set of themes or categories that are arranged hierarchically (see [101] for an example from PER); however, this is not required. We do not consider any perspective on intuition to be inherently expert- or novice-like, meaning there is no obvious hierarchy to apply to these perspectives. Our application of phenomenography is informed by Marton *et al.*, who implemented this methodology to (non-hierarchically) categorize the ways Nobel laureates perceive "scientific intuition" [80].

We applied emergent coding to the interview transcripts and identified themes relevant to

Interview Protocol

(1) Do you think physics is “intuitive?” Why? What are the things that make it “intuitive” or not?

(2) What does “intuitive” mean?

(3) How important do you think “intuition” is in your understanding in physics classes generally? What about in your problem solving (e.g., doing homework or taking exams)?

(4) How about in Quantum Mechanics this semester specifically?

(5) Do you think Quantum Mechanics is “intuitive?” Why? What are the things that make it “intuitive” or not?

(6) What does “intuitive” mean for you in your QM course specifically?

(7) Do you feel like you are developing/have developed a sense of intuition in QM this semester? Can you describe what that process is like for you? What helps you develop a sense of intuition?

(8) In previous interviews, some students distinguished “counterintuitive” (something that is different from what you would intuitively expect) from “unintuitive” (something that you don’t have an intuition for one way or another). What do you think about this distinction?

(9) Do you feel like you’ve had to “unlearn” things when developing intuition in QM? (In other words, did you find things “counterintuitive?”)

(10) Can something simultaneously “make sense” but be “unintuitive?”

(11) Can something simultaneously “make sense” but be “counterintuitive?”

Figure 3.2: Protocol used for interviews. Interviews were semi-structured and conversational: the interviewer occasionally asked follow-up questions or skipped questions in the protocol that the interviewee had already addressed.

each of our three research questions: whether students consider quantum mechanics intuitive; what students appear to mean by "intuitive"; and the role and importance that students attribute to intuition in quantum. We then applied the same codes to all open-ended preflight responses. We found evidence for all codes in the broader sample of student responses, and did not find that any new codes emerged from the broader sample.

The coding was primarily conducted by author GC, but all codes were refined in discussion with all three authors examining student quotations until consensus was reached. Coding was conducted for one research question at a time, and codes were applied to entire sentences (or fragments), but not all sentences received codes. Several complex student statements received multiple codes, but most received only one. The same process was applied to preflight responses.

We also identified themes related to a fourth question: do students initially expect quantum mechanics to be intuitive? Some discussion of this question arose in interviews, but it was also asked explicitly on the first preflight of both semesters. A broader range of more direct responses to the question arose on the preflights, hence our codes in related to this question were emergent from the preflight data rather than from the interviews.

The purpose of this thematic analysis was to draw out the range of ideas that are relevant for students when considering intuition in quantum mechanics. We do not focus on the relative frequencies of various themes because they may be sensitive to the student population, the instructor, and the specific survey and interview questions we asked. Nonetheless, all themes we report were observed in multiple student statements in both interviews and preflight responses. The identification and organization of themes presented below can guide other instructors and researchers who aim to discuss or investigate intuition.

3.4 Results

We present our findings beginning with survey data about whether students consider quantum mechanics intuitive, and discuss these results in light of student statements from the interviews (Section 3.4.1). Next, we consider what "intuitive" appears to mean to students. We describe *facets* of the word "intuitive" arising from a thematic analysis of interviews and free-response survey questions (Section 3.4.2) and we also report on the relationship between students' sense of intuition and their self-reported sense of struggle with the material (Section 3.4.3). Finally, we discuss what students describe as the role of intuition in their learning, and the expectations they have for intuition in quantum mechanics (Section 3.4.4). Overall, we set out to characterize the variety of ways students perceive and experience intuition in quantum mechanics, and we avoid making value judgements of student statements.

3.4.1 Is quantum mechanics intuitive for students?

This section addresses the first research question from the introduction: do students consider quantum mechanics as counterintuitive as the literature suggests they might? Our findings suggest that our students do not.

Approximately every other week during two semesters of undergraduate quantum mechanics, we asked students to characterize what they were learning as *intuitive, unintuitive,* or *counterintuitive* (Figure 3.1). Students were allowed to select multiple options, and could also indicate that they were *unconcerned* with their sense of intuition. Figure 3.3 shows the overall breakdown of responses between these four categories, aggregated over both semesters.

Nearly half of the responses indicated that "the ideas seem intuitive". About 6% of responses included the *unconcerned* option, while the remaining 45% indicated that the material was somehow nonintuitive. Notably, *unintuitive* accounted for over a quarter of responses, whereas *counterintuitive* accounted for slightly less than a fifth. Although the counterintuitive nature of quantum mechanics was relevant for these students, they more often reported simply having no intuition for quantum mechanical topics one way or another (i.e., *unintuitive*). Students also expressed this sentiment in interviews, as discussed in depth below (Section 3.4.4).

Before discussing interviews, we must clarify three things about Figure 3.3: the effect of allowing students to select multiple options; week-by-week shifts in the breakdown of responses; and how responses varied by student. On the preflight given during the first week of both semesters,

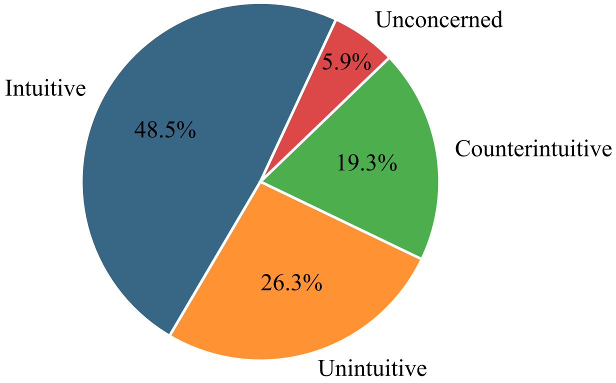

Figure 3.3: Overall breakdown of students' responses to a pre-lecture survey question about intuition (Figure 3.1), aggregated over both semesters of data collection. Includes 894 total responses from 163 students. Because students could select multiple options, the percentages are out of the total number of boxes checked.

	Intuitive	Unintuitive	Counterintuitive	Unconcerned
Intuitive	43.0%	—	—	—
Unintuitive	7.8%	18.9%	—	—
Counterintuitive	5.9%	3.6%	12.5%	—
Unconcerned	1.6%	0.5%	0.2%	4.2%

Table 3.2: Breakdown of students' responses to a pre-lecture survey question about intuition (Figure 3.1), aggregated over two semesters. The cells along the diagonal represent responses where students selected only one option. Off-diagonal cells represent responses where students selected two options (e.g., "The ideas seem intuitive" *and* "Some of the ideas seem UNintuitive"). Unlike in Figure 3.3, the percentages given in this table are out of the total number of responses (894) as opposed to the total number of boxes checked. Omitted from the table are the 2% of responses where students checked three or more boxes.

students selected 1.5 options on average, meaning approximately half selected two options (e.g., *intuitive* and *unintuitive*), whereas the rest selected only one. However, across the entire semester, only about one fifth of students selected two options in a given week (i.e., an average of 1.2 options). Only 2% of all responses included three or four options, which corresponds with about one student doing so each week.

Table 3.2 shows the breakdown of student responses aggregated over both semesters and accounting for the possibility that students selected two options. The students who selected multiple options chose *intuitive/unintuitive* and *intuitive/counterintuitive* at similar rates, and fairly few students selected *unintuitive/counterintuitive*. It was also rare for students to select *unconcerned* and another option in the same week.

There was minimal temporal variation in student responses, although some patterns did emerge. The *intuitive* option was less common on the first preflight of each semester—accounting for about 33% of boxes checked as opposed to 49% for the whole term (averaged over both semesters). In addition, *counterintuitive* and *unintuitive* were approximately equally common on the first preflight. After the first preflight, however, things roughly and rapidly leveled off to reflect the quantities reported in Figure 3.3 and Table 3.2.

We did observe an uptick in *counterintuitive* during the unit on entanglement and the Einstein-

Podolsky-Rosen paradox, which may be unsurprising to readers familiar with those topics. Conversely, we saw a decrease in *counterintuitive* in the week when the class transitioned from spin-$1/2$ systems to position-space wave functions. This could be related to the fact that students are more familiar with position than with spin, but overall we hesitate to over-interpret these weak temporal signals.

Whereas we did not observe strong temporal dependence in students' responses, we did see significant variation between students, as shown in Figure 3.4. Almost every student selected a unique combination of options during the span of their quantum mechanics class. Some students considered almost everything intuitive and others found almost everything nonintuitive, but most students were somewhere in the middle. Taken together with the lack of significant temporal variation, Figure 3.4 indicates that different students considered different weekly topics more or less intuitive.

Figure 3.4 strongly suggests that students are far from monolithic when it comes to their perceptions of intuition in quantum mechanics. It may be the case that this is true in physics more broadly as well. When asked if they thought physics was intuitive, three interviewees said the following three things:

> *Definitely very, very intuitive. . . that's what I love about it.*
> *Sometimes I feel that way. And then I'm proven wrong.*
> *I personally don't.*

The other interviewees offered a similar range of responses. As shown in Figure 3.4, interviewees' weekly preflight responses were equally varied.

One might expect that students' sense of intuition would correlate with their course performance, but we observed weak correlations. As shown in Figure 3.4, high-performing students varied significantly in how they responded to the preflight questions. The Spearman's rank correlation coefficient (ρ, which ranges in magnitude from 0, meaning no correlation, to 1, meaning perfect correlation) between students' exam averages and the rate at which they chose *intuitive* (i.e., the height of the *intuitive* bars in the figure) was 0.18 ($p < 0.05$). The correlation coefficient between students' exam averages and the rate at which they chose *counterintuitive* was -0.20 ($p < 0.05$; note that this correlation is negative). These both constitute weak correlations. There was also no

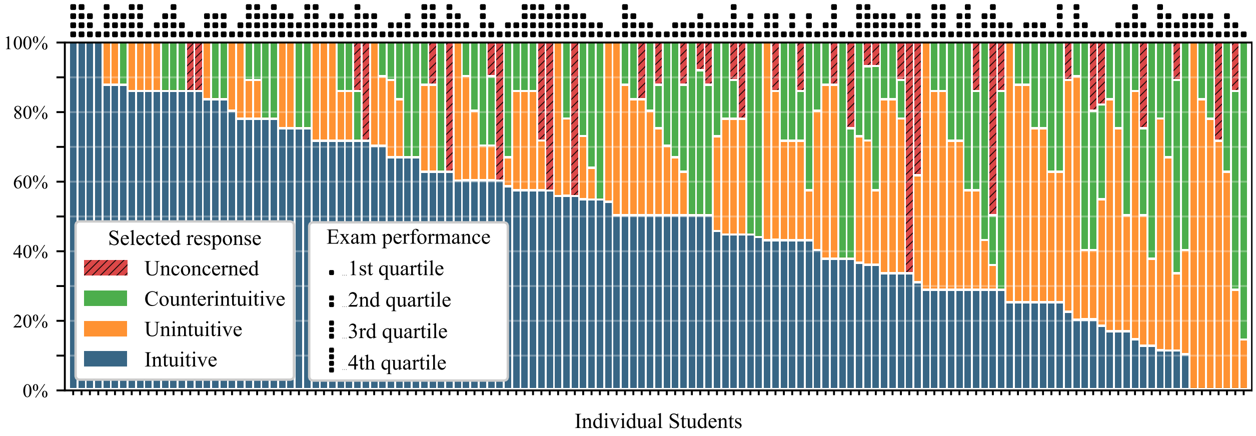

Figure 3.4: Student responses to a pre-lecture survey question about intuition (Figure 3.1), which was repeated approximately biweekly. The figure includes students' responses from two semesters. Each bar corresponds with a single student who answered the question on 4 or more pre-lecture surveys. 139 students are represented in the figure—24 students (15% of the 163 total) who only responded 3 or fewer times are omitted. Students could check multiple boxes. The percentages indicate how many times the student selected a given choice (e.g., "the ideas seem intuitive") out of the total number of boxes they checked aggregated over all the student's responses for the semester. The 10 columns marked "*" represent students who were also interviewed (the 11th interviewee is omitted because they dropped the class and did not respond to 4 or more surveys). Students are sorted horizontally based on the rate at which they chose the *intuitive* option. The vertically stacked squares above each bar represent the given student's overall exam average in their quantum mechanics course: one square indicates that the student's exam average fell in the first (lowest-performing) quartile, two squares indicates the second quartile, etc. As discussed in the text, there is little correlation between students' responses to the survey questions about intuition and their exam performance.

correlation between students' exam scores and the rate at which they chose *unintuitive* ($\rho = -0.03$, $p = 0.7$). We share speculations on these correlations (or lack thereof) in the discussion.

3.4.2 Facets of intuition

This section addresses the second research question from the introduction: what does "intuitive" mean to students? Our findings indicate that the word has a range of meanings for different students, as summarized in Table 3.3.

Students in quantum mechanics may consider the subject at least as unintuitive as they consider it counterintuitive, or possibly more so. Students also often reported that they found the ideas they were learning to be intuitive. But what does "intuitive" mean to students? Although we asked interviewees this question directly, we cannot paint a comprehensive picture of how any specific individual perceives the concept of intuition. We can, however, characterize the ways students used and describe the word, which we call *facets* of intuition.

We use the word facet because we consider intuition a multifaceted construct.[4] The "side" of intuition that surfaces in student statements likely depends on the context and content of the questions we ask them, but by collecting responses from many students we can begin to piece together the entire shape. Although each facet is distinct, some facets may overlap. Each facet generalizes ideas expressed by multiple students.

This section reports results of a thematic analysis of interviews and open-ended preflight responses, from which six facets of intuition emerged. The six facets, which we describe with examples below, are summarized in Table 3.3. Every facet arose in both interviews and preflight responses from multiple students. Each facet refers to a use of the word "intuitive" that we were able to distinguish from all other facets; however, the definitions of the facets are not intended to be mutually exclusive.

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[4] The use of the term facet is inspired by Minstrell (and Redish) [89, 109], but there are some differences in how we use the word. Minstrell uses facet to refer to distinct pieces of knowledge or strategies that students appear to employ when solving physics problems, but in this chapter facets are distinct ways that students discuss "intuition". Similar to Minstrell's approach, a facet of intuition generalizes several students' remarks.

Facet of intuition	**Something is intuitive when. . .**	**Example student quote**
Matching one's expectations	You can predict how a system will behave, or you can anticipate what the solution to a problem will be.	*I think intuition is, like, something you can predict.*
"Physical" or "real-world" observable	You encounter it in everyday life, or you can see it with your own eyes. For example, the behavior of a bouncing ball.	*My physics classes have been pretty intuitive. This is because usually, we have dealt with situations that I am familiar with and could encounter and observe.*
Related to prior intellectual experience	It is connected to concepts or techniques you have learned or practiced before, possibly in other classes. For example, the eigenfunctions of a "particle in a box" are related to standing sound waves in pipes.	*The Coulomb force being the same as, like, gravity...I think that automatically makes it way more intuitive because...I can connect those immediately.*
Something one experiences	It "just naturally makes sense to you" (interviewee quote), you can make leaps of logic or understanding, or you feel that the concepts you are learning build or fit together in a comprehensive way.	*If you have intuition, then you have an instinctive feel as to what is right and what is wrong [when solving a physics problem].*
Mathematical intuition	You can make sense of it using mathematical tools you feel you understand.	*Those points that were not intuitive became intuitive after I understood the math governing the laws.*
Visualization	You can visualize it in a helpful but abstract way (e.g., a graph).	*It has been...intuitive in the sense that I was able to utilize a lot of visualization to help me learn the material.*

Table 3.3: Facets of intuition: the ways students use the word "intuitive" or discuss "intuition" in interviews and open-ended preflight questions. All codes arose at least a dozen times (possibly including multiple times in the same interview) and in statements from multiple students. Something is unintuitive when none of these facets apply (e.g., you have no expectations for how a system should behave, or you have no directly related intellectual experience). Something is counterintuitive when a facet applies, but it leads to an incorrect conclusion (e.g., your expectation is inaccurate, or the connection you make to prior intellectual experience leads you astray). The facets are described in depth in the text.

3.4.2.1 Matching one's expectations

A physical problem may be intuitive if its behavior matches your expectations, counterintuitive if it doesn't, or unintuitive if you have few expectations for how it should behave. Two interviewees said,

> *For me, like intuitive is, you know, does it behave in the way I would expect it to?*
> *I would say to me, it would mean having a natural feeling as to what will happen.*

Whereas a third remarked that their quantum course,

> ...*didn't seem counterintuitive, but rather unintuitive, where I didn't know what to expect*

Facets of intuition can arise in negative statements. For example, this student indicates that "unintuitive" means *not* knowing "what to expect", implying that "intuitive" means the opposite (knowing what to expect).

In interviews and, commonly, in preflight responses, students said they rely on their intuition to check their answers. For example, one preflight response said, "I'll use my intuition to sanity check". This use of intuition aligns closely with this facet: checking the "sanity" of one's answers against one's intuition relies on having an intuitive expectation for what the answer ought to be.

3.4.2.2 "Physical" or observable in the "real world"

To quote one interviewee, "A lot of intuition comes from the physical world". Students said they gained intuition about physical systems by observing them, with their own senses, in everyday life. Commonly, this facet arose when students argued that classical mechanics is intuitive while quantum mechanics is not. One interviewee said:

> *Classical mechanics, I think that's very intuitive because it's basically dealing with things that you can witness happening.*

On the other hand, a preflight response read:

> *So many of the processes in quantum have little to no reference in our day-to-day lives; there's not much we can do to intuit it in [our] minds without a reference and so it becomes hard to conceptualize.*

This student argues that quantum mechanics is not intuitive for them because the systems under study do not relate to their everyday experiences.

3.4.2.3 Related to prior intellectual experience

In addition to relying on "real-world" experience, students can also gain intuition by relating new concepts to their academic experiences—that is, to things they learned previously, possibly in another class. For example, one interviewee said,

> *If there's something that I don't really understand, I often try to connect it to other things that I've learned in the past*

Others related the ability to make such a connection to the sense that something was intuitive. Along these lines, another interviewee said,

> *[if] I understand something as being intuitive, it's, can I look it at an equation or a sentence or something, a graph and can I feel like it's related to something else I know very well?*

Another interviewee said that, to them, intuitive "mostly means relatable. Like I have something to draw off of". Similarly, one preflight response described the week's quantum topics as "sort of intuitive because there are a lot of parallels here to classical mechanics". Connecting what they were learning to something they already knew granted this student some intuition for the new topics. Practicing theoretical physicists have also discussed how prior intellectual experience helps spark new ideas [46].

3.4.2.4 Something one experiences

This facet amalgamates three separate codes, each of which referred to a nebulous, elusive *sense* of intuition. The first code was for the idea that something is "intuitive" when "it just makes sense", as one interviewee described it. The second related to the ability to make leaps of logic or understanding: "something that you kind of subconsciously put together", as another interviewee said. The third was the feeling that concepts you are learning build or fit together in a satisfactory way. One interviewee, who considered quantum mechanics intuitive, said:

> *Everything aligns up and builds off of itself in a clear and intuitive way. That's sort of how physics works. And it's part of why I love it so much.*

Each of these codes describe intuition in an abstract fashion that is difficult to pin down. The idea that intuition can be felt or experienced is also described by Marton et al. in their work studying how Nobel laureates discuss scientific intuition [80].

An experience of intuition is potentially explained by other facets. For example, perhaps something "just makes sense" to you because it matches your expectation, or perhaps you can "subconsciously" put something together because of your prior intellectual experience. However, student statements assigned to this facet described a sense or experience of intuition without elaboration. One such statement read,

> *[Intuitive] would mean having a natural feeling as to what will happen or how things work, like, without being told the rules without being told the math and everything and you just know, based on a gut feeling.*

This student described intuition as a "gut feeling", which is consistent with the idea that intuition can be experienced.

3.4.2.5 Mathematical intuition

Students often discussed mathematics as playing a role in their intuitive understanding of physics, usually as an aid, but also as a hindrance. One interviewee said:

> *Intuition, for me is more in a mathematical sense. And physics is a fun way to apply that mathematical intuition, but it doesn't have to necessarily make, like real world sense for intuition for me to apply.*

This student's sense of intuition for math allowed them to feel that the physics they were learning was intuitive as well—even when that physics fails to make "real world sense". Similarly, a preflight response read:

> *As long as the math makes sense to me then it's 'intuitive' to my standards.*

For these students, their mathematical intuition was separable from—but played a key role in—their physical intuition.

Conversely, although less frequently, other students described the math as a barrier to their intuitive understanding. On one early preflight, one student justified their *counterintuitive* selection by saying:

> *The only thing that's really messing me up is the math and connecting it to the physics . . . I'm very intimidated by eigen stuff.*

This student also separated their mathematical understanding from their physical understanding (the math was the "only thing" messing them up), but they said their difficulty with the math caused them to select *counterintuitive.*

Many students said they considered math especially relevant when making sense of or developing intuition in quantum mechanics, in part because the theory is not directly observable in everyday life. On a preflight, one student chose the *counterintuitive* option but, in their explanation, said:

> *. . . because we are learning the math behind the strangeness, it is easier to comprehend*

This student felt that learning the underlying math was helping them overcome what they saw as the counterintuitive nature of quantum strangeness.

For another student, though, mathematical intuition wasn't enough to consider quantum mechanics intuitive. On a preflight, they said:

> *The math all feels intuitive but once you start thinking about what the math is saying happens it feels somewhat counterintuitive.*

This student considered the physical interpretation of the math to be counterintuitive, despite considering the math itself intuitive.

Finally, one student explicitly distinguished mathematical calculations from intuition, saying on a preflight:

> *The concepts in quantum are easier to understand because it relies more on calculations than intuitions.*

This student saw the need for intuitive understanding as a barrier to success, and found quantum mechanics easier because, to them, it relied more on mathematics that didn't require intuition. Although this student presents calculation as an alternative to intuition, most students appeared to consider math as relevant to their sense of intuition.

3.4.2.6 Visualization

The final facet that arose was the idea that intuition is related to one's ability to visualize a problem in one's head. Here we discuss abstract visualization as opposed to literally observing a physical scenario. Other research has pointed to the utility of abstract visualization in understanding quantum mechanics [9]. Abstract visualization might refer to a graph or other specific mathematical tool, but it includes any sense in which someone claims to "see" a physical concept or scenario in their head. The relation between visualization and intuition is bidirectional: visualization can help one gain intuition, and intuition can help one visualize a problem.

One preflight response read

> *For me, the most intuitive learning was... if graphs were involved that I could use to visualize what is physically going on.*

For this student, graphs (an abstract visualization) were a key feature of "intuitive learning". Meanwhile, another response read:

> *Intuition is somewhat important, as I have to fit ideas and concepts into my cognition as something I can visualize or otherwise manipulate in order to fully understand them.*

It is unclear whether this student sees intuition as an aid to visualization, visualization as an aid to intuition, or the two as intertwined. Regardless, abstract, "in one's own head" visualization arose as a distinct facet of intuition.

3.4.3 How does students' sense of intuition relate to their sense of struggle?

This section also addresses the second question from the introduction: what does "intuitive" mean to students? We consider the possibility that students' sense of intuition is entangled with their perception of how difficult the material is, and find some evidence in favor of this idea.

We have identified several meanings of "intuitive" in the form of facets that describe the varied ways students use the word. Zooming out from these nuances, interviewees seemed to agree that physics was *easier* when it felt intuitive. One interviewee said,

> *I just think it will be harder for you to learn the material if you're not having some sense of intuition for it.*

This idea also arose in multiple preflight responses, one of which read "I think that learning new things is easier with intuition". To what extent does students' sense of intuition relate to their sense of difficulty, or struggle, with the material?

Approximately every other week on preflights, we asked students to rank their struggle with six different categories of the material on a scale of 0–3 (*no struggle* to *severe struggle*). For each student, we calculated an overall struggle score by summing their rankings for all six categories over all preflights they responded to. We normalized the overall score by dividing by the maximum struggle the student could have reported: the number of preflights they responded to multiplied by 6 (categories) multiplied by 3 (maximum per-category ranking). To address our question, we computed the correlation between students' overall struggle score and their self-reported sense of intuition.

Spearman's rank correlation coefficient between students' overall sense of struggle and the frequency with which they selected *intuitive* was -0.54 ($p < 0.001$). We expected a negative coefficient because we expect these quantities to be inversely related: a student who reports more struggle will presumably report less intuition. Each of the six struggle categories independently exhibited a statistically significant correlation with intuition, with *math* having the weakest correlation, and *physics* and *weirdness* having the strongest. The correlation between overall struggle and intuition was stronger in the fall semester (-0.63, $p < 0.001$) than in the spring semester (-0.49, $p < 0.001$), possibly because the struggle and intuition questions were asked on the same weekly preflights in the fall, but on alternating weeks in the spring.

Overall, we did observe an anti-correlation between students' self-reported sense of struggle and their sense of intuition as characterized on preflights. However, we do not know if this correlation is due to a causal relationship and, if so, which is the causal direction: do students report more struggle because they find the material nonintuitive, do they report less intuition because they find themselves struggling, or were these two survey questions simply measuring similar things because students consider "intuitive" and "lack of struggle" to mean similar things? Independent of these questions, we see that when we discuss intuition with students, we also get a sense of how much they consider themselves to be struggling with the material.

3.4.4 Student reflections on intuition in quantum

This section addresses the third research question from the introduction: how do students' views on intuition affect their approach to quantum courses? Our findings indicate that students expected quantum mechanics to be nonintuitive, prompting some to intentionally drop or ignore their incoming intuitions.

On surveys and in interviews, students were asked to reflect on intuition in quantum mechanics beyond simply indicating whether they considered the subject matter intuitive. In this section, we report on students' expectations about intuition in quantum, and also briefly discuss the role that students consider intuition to play in their learning.

3.4.4.1 Expectations: cleaning the intuitive slate

On the first preflight each semester, which was due on the first day of class, we asked students whether they expected the material in quantum mechanics to be intuitive, and how they expected it to compare to previous classes. Over both semesters, 163 students responded. Of these responses, 127 gave a clear answer to the question and included some explanation. We coded students' responses

Students expect quantum mechanics to be...	**Percentage (%)**
Nonintuitive* (exclusively)	41
Less intuitive† than other courses (exclusively)	25
Both nonintuitive* *and* less intuitive†	9
Comparably intuitive‡ to other courses (exclusively)	9
Both nonintuitive* *and* comparably intuitive‡	3
Intuitive or more intuitive than other courses	6
Unsure or a mix of intuitive and nonintuitive	6

Table 3.4: Students' expectations for intuition in quantum mechanics, as described in responses to a pre-lecture survey question that was due on the first day of class. The percentages in the table are out of 127 student responses.

as presented in Table 3.4.

A significant majority of students (78%) expected quantum mechanics to be either nonintuitive or less intuitive than other courses (or possibly both). 9% of students expected quantum to be comparably intuitive to their other physics courses, but did not indicate whether they considered other courses intuitive. Only 6% of students expected quantum to be intuitive or more intuitive than other courses, and the remaining 6% were unsure what to expect.

Why do students expect quantum mechanics to be nonintuitive, and what effect does this expectation have on their approach to the course? We do not have a definitive answer to the first question, but we do present some speculation in the discussion. The interviews did, however, provide insight into the second question.

Most interviewees claimed to have intentionally let go of—or even ignored—their incoming intuitions at the start of their quantum mechanics class. One interviewee summarized:

> *Coming into this class, like, like, I knew, like, things are gonna be weird and things are gonna be counterintuitive. So I automatically dropped my assumptions before I started.*

Because this student expected quantum to be counterintuitive, they tried to approach the subject without making intuitive assumptions. Similar statements were made in most interviews, such as,

> *I came into the class, with this idea of, it's not going to behave in the way I think it's going to behave... I kind of just wiped my slate clean.*

For this student as well, the expectation that quantum mechanics would behave differently than expected prompted them to wipe their "intuitive slate" clean. Some interviewees also suggested that the need to drop one's intuitions at the start of a new physics course was not unique to quantum mechanics.

We can interpret students cleaning their intuitive slates in both positive and negative lights. The awareness that one's intuitions must be adjusted in light of new evidence could be considered evidence of intellectual maturity. A third interviewee considered this to be one of their strengths:

> *One of the things that makes me good at physics is I'm pretty good at shifting my thinking, to, to match the observations.*

A fourth interviewee felt strongly that, as a student, "the only thing we can do is to learn", saying it was their job to reconcile their intuitions with the new ideas and facts they learned in class.

Although the willingness to adjust one's intuitions is important, it is potentially concerning that students have so little faith in their existing physical intuitions when it comes to learning quantum mechanics. One student called this out on a preflight:

> *There is some notion of a quantum intuition forming in my head, though at the moment it mostly just consists of "don't listen to classical intuition." This is not great...*

The student was concerned that knowing to ignore their classical intuition, "only eliminates one wrong idea rather than pointing me toward the correct thought process". Although naively applied classical intuition may fail for quantum systems, quantum mechanics is not so hopelessly "weird" and disconnected from other subjects that classical intuition is never useful.

On a positive note, although most students expected quantum mechanics to be nonintuitive, and many may have dropped their incoming intuitions as a consequence, a number of students expressed faith that, with time, they would develop a new, "quantum" intuition.

3.4.4.2 The role of intuition in learning quantum

On surveys and in interviews, we asked students how important intuition was to them personally when learning physics. For some students, intuition is essential; for others, it barely matters (although they may still consider it nice to have). One preflight response said intuition plays a "very very very very large role" in doing physics; meanwhile, another student said in an interview,

> *At some point intuition doesn't really matter and you have to trust the theory.*

A second preflight response said intuition is "not very important. I just need to see the math", while a third considered intuition

> *The main part of physics to me. Without intuition, I'm just doing math.*

Preflights and interviews both contained a range of ideas about the importance of intuition in physics, although more students considered intuition important than unimportant.

Intuition can be useful to students in a variety of ways. Many students suggested that intuition could be used to check or make sense of their solutions to problems, which is related to the *matching one's expectations* facet of intuition. Some students made the point that their lack of intuition in quantum mechanics meant they couldn't tell if their solutions were reasonable. Intuition can also be relevant at the start of problems: some students suggested that intuitive insight was often required to identify a solution path. A handful of students said that intuition was not so important in the classroom, but that it was important when doing physics research.

As discussed above when considering the relationship between intuition and struggle, one common idea expressed by most students was that physics becomes easier when you have intuition. This could be localized to specific content (e.g., analyzing spin-$1/2$ systems is easier once you develop an intuition for them), but a small number of students expressed that physics was difficult for them in general because they tended to lack intuition for it. One interviewee who had previously made this point said,

> *I would obviously say I don't think [intuition] is that important...because without*

> *intuition, I can still usually try to understand what's happening anyway. I guess, like, it isn't a requirement...I think it really helps.*

This person and another interviewee both indicated that their success in physics to this point was *despite* their perceived lack of intuition, implying that their continued participation in physics required a degree of resiliency. These remarks suggest that students' sense of intuition may play a role in their self-efficacy or possibly in their sense of physics identity.

3.5 Discussion

In this section, we reflect on key findings and also discuss limitations of this work. Over two semesters of undergraduate quantum mechanics, we regularly asked students to characterize what they were learning as intuitive, unintuitive, or counterintuitive. We found that, half the time, students considered things intuitive, and, when they didn't, they were more likely to say things were unintuitive than counterintuitive. This may be surprising given quantum mechanics' popular reputation and given that quantum topics are often called counterintuitive in PER literature.

Before the study, we naively expected a strong correlation between sense of intuition and course performance. Further reflection led to some counterarguments; for example, we might expect strong students to recognize counterintuitive elements of quantum mechanics. It turned out that students' self-reported sense of intuition was, at most, weakly correlated with their exam scores. We can think of at least three possible explanations for this lack of correlation. First, students may be imperfect judges of whether or not they find material intuitive. Second, different students value intuition differently: where some may consider it crucial to their success, others may not. Third, students have different ideas about what intuition is and how it relates to answering questions correctly.

We asked students to describe their expectations for intuition in quantum mechanics, and conducted 11 interviews about this topic. We learned that a significant majority of students initially expect quantum mechanics to be somehow nonintuitive or less intuitive than other physics courses. Interviews provided evidence that, as a consequence of this expectation, many students may consciously drop their incoming intuitions at the start of their quantum class. This helps explain why *unintuitive* was more common than *counterintuitive* in preflight responses: something cannot disagree with your intuition—it cannot be counterintuitive—if you intentionally disregard your intuitions.

One possible reason why the surveyed students expected quantum mechanics to be nonintuitive is that they had all seen some quantum topics in their sophomore-level modern physics course. Some students mentioned this in their preflight response, saying they considered the subject nonintuitive then and expected it to be similar now. A second possible reason is that quantum mechanics is often described as "weird" both in popular culture and within the physics community. A third possible reason is that upper-division students may be noticing a trend of increasingly difficult and nonintuitive subject matter covered in their physics courses. Many preflight responses mentioned that upper-division electricity and magnetism had been mathematically intensive and, as a result, nonintuitive, and some said the same of middle-division classical mechanics. More study is required to discern the reasons behind students' expectations for intuition in quantum mechanics.

We identified six facets of intuition. The facets arose empirically from the data, but also relate to prior research in the field. For example, real-world observability and visualization were both previously found to be relevant for students' epistemologies in quantum mechanics [54]. The idea that intuition is something one experiences is similar to a theme identified in [80]. The idea that students find quantum mechanics less intuitive than classical mechanics, and as a consequence rely on mathematical understanding over conceptual understanding, has been reported elsewhere [31].

This study has several limitations, including the fact that students' perspectives on quantum mechanics were likely biased due to their prior exposure to the material. Although the study includes 163 students from two semesters, all students were in the same quantum mechanics class with the same professor. The professor considered helping students develop intuition to be an instructional goal, which may have biased students' perspectives on intuition. Moreover, interviewed students had been answering a preflight question about intuition on a biweekly basis leading up to their interviews, meaning they may have been thinking about intuition in quantum mechanics more than they otherwise would have been. Finally, the study was conducted during a period of emergency remote teaching prompted by the COVID-19 pandemic, which presented extraordinary and difficult circumstances for many students. Thus, although we have reported statistically meaningful results, their generalizability to other student populations remains open to investigation. We believe many of these limitations are mitigated by the qualitative focus of this study. For example, the six facets of intuition we identified may be useful to instructors and other researchers even if additional unidentified facets were to be relevant in other contexts.

3.6 Implications for instruction

We did not observe a strong correlation between students' self-reported sense of intuition and their grades. This suggests that, at least in our course, the instructional goal of helping students develop a sense of intuition is independent from the instructional goal of helping students achieve good grades. The professor of the course included in this study valued both goals, but the correlation was weak nonetheless. If instructors value intuition as an instructional goal, they likely need to address it explicitly.

It may be useful for instructors to know that students might expect quantum mechanics to be nonintuitive and furthermore that students may attempt to disregard their incoming intuitions. Distinguishing quantum and classical physics is a reasonable instructional goal. In our case, though, students already appeared to recognize this distinction, at least when it came to their sense of intuition. We have not investigated reasons for this, but it may have been due to our students' prior exposure to quantum topics in their academic career. We believe this leaves room for instruction to encourage students to refine their incoming intuitions instead of leaving them at the door. For example, instruction could point out cases where classical intuitions can be carefully and productively applied to quantum systems.

Instructors should first confirm that their students have similar incoming expectations. Instructors should also realize that students may have significantly varied perspectives on whether quantum topics are intuitive. Students may also have varied working definitions of "intuitive", and may or may not consider intuition important. Some considered intuition paramount, while others thought it was unnecessary. Nonetheless, most students seemed to agree that intuition (however defined) makes learning physics easier.

We emphasize that, despite initial expectations, students classified what they were learning as intuitive approximately half the time. Evidently, there is significant opportunity for students to develop intuition in quantum mechanics. We believe that the facets of intuition we identified point to ideas instructors can focus on when discussing intuition.

Although “real-world, everyday life” observability is more difficult in quantum physics as opposed to classical physics, instructors can recognize this difficulty explicitly and draw students’ attention to other facets. For example, abstract visualization can provide a useful alternative to direct observability. Students can also be encouraged to practice making qualitative predictions about quantum systems, which both emphasizes that one can have intuitive expectations for quantum mechanics and allows students to sharpen these intuitions. Familiar mathematics can also be used to guide students’ intuition, as can connections to physical examples or other knowledge that students have prior experience with. These elements probably already exist in most quantum courses, but it may be worth occasionally presenting them with the explicit, announced purpose of helping students form intuitions.

We have mentioned five facets of intuition, but the sixth—intuition as something one experiences—is harder to integrate directly into instruction: we do not have a prescription to make physics “just make sense”.

3.7 Conclusion

We used a mixed-methods approach to investigate students’ perceptions of intuition in two semesters of an upper-division undergraduate quantum mechanics course. We found that there is room for intuition in quantum mechanics, but that the majority of our students enter the class expecting the subject to be nonintuitive or less intuitive than other courses. Some students claim to intentionally disregard their incoming intuitions and, consistently, view quantum as more unintuitive than counterintuitive.

The present study could be extended several ways. We could repeat it in other quantum courses, ideally at other institutions, to determine whether the quantitative patterns hold true for other student populations taught by other instructors. For example, it could be interesting to compare the results between this class, which uses a “spins-first” instructional paradigm, and a “position-first” class. Future studies could also disentangle struggle and intuition: we observed a correlation between students’ self-reported senses of struggle and of intuition, but struggle or ease did not arise in the facets of intuition, suggesting that the correlation may be nontrivial.

The facets of intuition presented in this chapter can support future work. The mathematical intuition category was especially rich, and a future study could potentially identify sub-categories. A study of a broader population of quantum students could reveal how common each facet is relative to the others. Extended studies could also be conducted in non-quantum physics courses—or even in non-physics courses—which would allow for comparisons between these contexts, both in terms of the relative frequency of each facet, as well as to see if wholly new definitions of “intuitive” arise in other contexts. One might hypothesize, for example, that students would approach intuition differently in classical mechanics than they do in quantum mechanics (or in, say, a computer science class), or that their beliefs about intuition might evolve over their tenure in a physics major. A longitudinal study could track the evolution of students’ perspectives on intuition throughout their academic career (although longitudinal studies are logistically challenging to conduct). A future investigation could also examine the ways that and extent to which students use each facet of intuition when solving problems related to various topics.

Overall, student statements about intuition were rich, and they often touched on other PER topics, such as math-physics connections or student self-efficacy. We consider intuition a powerful lens for investigations into practitioners’ perspectives on learning, knowing, and doing physics. Within the context of this dissertation, we can consider the various facets when designing tutorials intended to help students develop intuition. As such, this chapter forms a research basis for a goal shared by all our tutorials: developing intuition. The next chapter presents a study of student understanding of basis within quantum mechanics, which informed the content learning goals for a specific tutorial.

Chapter 4

Exploring student ideas on change of basis in quantum mechanics[1]

4.1 Introduction

Basis is a fundamental concept in quantum mechanics, and converting between bases is a routine task when solving quantum mechanical problems. States and observables are typically expressed in terms of a single set of basis vectors. Physicists often choose to represent quantum states in terms of a basis that corresponds to an observable (measurable quantity) of interest. Depending on the physical system, the bases used may be continuous or discrete, but a basis is always present when writing the state of a quantum system. In this chapter, we explore student ideas related to basis, methods of changing basis, and coefficients in a basis expansion within the context of a spins-first quantum mechanics course and with a focus on Dirac notation. This work informed the creation of one of the ACE Physics tutorials discussed in Chapter 6.

Although all representations of a quantum state provide equal information in the abstract sense, the choice of basis determines which information is easily accessible. Changing basis involves performing a calculation that converts between such representations. It neither affects the mathematical identity of the represented state, nor does it alter the physical system that the state vector describes.

As an example, in spin-$^1/_2$ systems, there are two possible outcomes for measuring the spin angular momentum ($\mathbf{S}$) along a given axis: $+\hbar/2$ or $-\hbar/2$. Any such two-state system can be

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[1] This chapter was previously published in Physical Review [24], though it has been modified slightly. We also published a follow-up paper describing the development of an on-paper tutorial about basis and change of basis in quantum mechanics [115]. This tutorial became the ACE Physics Quantum Basis Tutorial discussed in later chapters.

described using two orthonormal basis states, corresponding to each of the possible outcomes. Consider the following two expressions for the same quantum mechanical state, $|\psi\rangle$:

$$|\psi\rangle = \frac{2}{\sqrt{5}}\,|+\rangle + \frac{1}{\sqrt{5}}\,|-\rangle \tag{4.1}$$

$$|\psi\rangle = \frac{3}{\sqrt{10}}\,|+\rangle_x + \frac{1}{\sqrt{10}}\,|-\rangle_x \tag{4.2}$$

where $|\pm\rangle$ are the eigenstates of the S_z operator, often referred to as the z-basis or S_z-basis, and $|\pm\rangle_x$ are the eigenstates of the S_x operator, often referred to as the x-basis or S_x-basis. The coefficients in Equation (4.1) are probability amplitudes related to measuring a particle to have spin $+\hbar/2$ or $-\hbar/2$ along the z-axis, corresponding to the eigenstates $|+\rangle$ and $|-\rangle$, respectively. Equation (4.2) displays different probability amplitudes because the values are related to measurements of the S_x operator corresponding with spin along the x-axis. It is convention to assume that the kets $|\pm\rangle$ are the eigenstates of S_z unless there is a subscript indicating a different direction.

Although Equations (4.1) and (4.2) are written in different bases, both represent the same quantum mechanical state. Any advantage of one expression over the other depends on the particular information that is desired or the calculation that is being performed. Converting between bases can be useful, e.g., for determining measurement probabilities for different operators or for determining how states evolve in time.

In recent years, there has been a growing body of research on students' difficulties with quantum mechanical concepts [69, 73, 75, 77, 91, 97, 98, 131, 134, 145] and on the resources students use when working with those concepts [30, 42, 96]. (See Section 2.2 for additional review.) Serbin et al. have investigated student and expert use of language related to basis in quantum mechanics [123], but little other research has specifically attended to student understanding of basis and change of basis in quantum mechanics.

There are multiple calculations that can be used to convert between bases. When working with orthonormal bases as is typically done in quantum mechanics, the method that often requires the fewest lines of algebra uses inner products of the given state with the new basis vectors to determine the coefficients in the new basis. For example, the coefficient $3/\sqrt{10}$ in Equation (4.2) is given by the inner product of the $|+\rangle_x$ basis state with $|\psi\rangle$: ${}_x\langle+|\psi\rangle$. This calculation (with a similar one for the other coefficient, $1/\sqrt{10}$) can be used to derive Equation (4.2) given Equation (4.1); i.e., to change $|\psi\rangle$ from being expressed in the S_z basis to being expressed in the S_x basis.

Previous research has explored how students connect the inner product with probabilities in a wave functions context [145]. Wan and colleagues found that students identified the coefficient associated with expansion in the energy basis, but did not know to solve for it using an inner product. This is similar to recognizing $2/\sqrt{5}$ as a coefficient in Equation (4.1), but not recognizing how to represent it as an inner product (e.g., $\langle+|\psi\rangle$). Note that although the two situations are analogous, they are not the same: the context of continuous wave functions includes additional concepts beyond those necessary for spins, such as connecting the inner product to an integral expression. As such, it is worth investigating this topic in the context of spins specifically.

In a separate study, Wan and colleagues noted that "many students do not recognize the measurable effects of relative phases" because they do not consider observables associated with bases other than the one provided [144]. Basis is also relevant in other studies of student reasoning about relative phases in spin-$^1/_2$ states [15]. Other work exploring students' methods for computing expectation values found that students may not think to change basis, even if doing so is required to solve the problem [116]. The present study complements prior work with a comprehensive examination of student ideas about basis and change of basis specifically in the discrete (spin-$^1/_2$) context.

In the mathematics education community, research has focused on student understanding of two properties associated with a general basis in linear algebra[2]: span and linear independence [2, 52, 106, 137, 154]. In quantum mechanics, basis states are typically further restricted to being orthogonal and normalized. Because physicists can exploit orthonormality, the methods for changing

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[2] A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ forms a basis for a vector space if and only if (a) the set is *linearly independent*, meaning if $c_1\mathbf{v}_1 + \ldots + c_n\mathbf{v}_n = 0$ then $c_1 = \ldots = c_n = 0$ for any set of coefficients $\{c_1, \ldots, c_n\}$; and, (b) the set *spans* the space, meaning all vectors in the space can we written as a linear combination of vectors in the set.

basis in physics classes are different from the more universal methods needed for general bases in linear algebra. Mathematics education research on basis has a broader scope and does not account for the specific use of basis in quantum mechanics.

Given the significant role that basis plays in a spins-first course, as well as the importance of changing basis when solving problems in quantum mechanics more generally, we extended prior research by administering several surveys, conducting interviews, and examining three years of students responses on quizzes and exams. Applying a phenomenographical perspective [5, 47, 78], we address the following research questions in a spin-$1/2$ context:

RQ1: How do students interpret a change of basis with respect to both the physical nature and the representation of the state?

RQ2: What methods do students use when performing a change of basis for a quantum state?

RQ3: How do students interpret coefficients in a basis expansion and/or relate them to inner products?

A state may be represented in a number of ways (e.g., with Dirac notation, with matrix notation, or graphically). Although this study focuses primarily on Dirac notation, we do discuss examples where other representations are relevant.

Using a variety of data sources, we identified the ways students change basis and the ideas related to different facets of understanding basis. This qualitative analysis will help inform the development of curricular materials.[3]

4.2 Methodology

Data were collected over three years from three public universities. Universities A and B are large, Hispanic-serving, primarily-undergraduate institutions with between 25–55 students enrolled in

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[3] Specifically, we have developed an instructional tutorial discussed in [115] as well as in Chapter 6 and available at [107] or, in an online format, at [23].

upper-division quantum mechanics in a given year. University C is an R1, PhD granting institution with 60+ students enrolled in quantum mechanics each semester.

The quantum mechanics courses from which data were collected used a spins-first instructional paradigm following McIntyre's "Quantum Mechanics" textbook [86]. Each course was taught by one of the authors, who used similar interactive instructional materials including multiple choice concept questions and tutorials to emphasize conceptual understanding of the material [107]. The authors met weekly to discuss the curriculum and shared select quiz and exam questions. Two of the courses were paced similarly and only covered material through the infinite and finite square well. Additional topics covered at University C included the free particle, orbital angular momentum, and the hydrogen atom. All courses met in person.

To address the three research questions, we analyzed a variety of data sources, including interviews, surveys, and for-credit assessments. Tables 4.1 to 4.3 show an abbreviated list of questions that were given to students for the purposes of this study. A standard change of basis question (Table 4.2) was given at different points over the three year span on exams at the three universities. An isomorphic version of the question was given to students in interviews conducted at Universities A and B—we refer to these as the "exploratory interviews". Several ungraded surveys (S1, S2, and S3) were later administered to provide insight into specific ideas related to research questions RQ1 and RQ3. Interviews probing these ideas were also conducted with students at University C—we refer to these as the "follow-up interviews". The surveys and interviews are described in more depth below.

We employed methods of phenomenography to investigate the range of student ideas. A phenomenographical approach is a qualitative approach that aims to identify and categorize the variations among individuals' perceptions and conceptualizations of phenomena [5, 47, 78]. Students responding to a question in similar ways may share a similar conceptualization of that phenomena regardless of correctness. In the present study, the categories used in analysis were emergent from the data and used as a description for procedures or lines of reasoning that arose multiple times. Because the majority of the data collected includes interviews and written explanations, we approach the data from a qualitative perspective to identify student ideas related to basis and change of basis. In this work, we catalogue both correct and incorrect ideas and procedures, as our goal is to uncover the breadth of ideas that students express.

4.2.1 Data and analysis: interpretations of changing basis (RQ1)

Of the three administered surveys, S1 and S2 included questions designed to probe students' interpretations of a change of basis. In addition to these surveys, both rounds of interviews included elements that targeted RQ1. In the exploratory interviews, which followed a semi-structured format, students were asked to carry out a change of basis, and then asked if the state at the end of calculation (in the new basis) was the same state as the one given in the problem (in the basis of S_z) or whether a new state had been created. The exploratory interviews preceded and informed the design of the surveys.

Survey S1 (Q1–Q3, Table 4.1; Q8, Table 4.3) was given as part of regular online pre-lecture assignments, which received participation credit only. The survey was completed by $N = 23$ students at University B after relevant course instruction. Survey S2 (Q4–Q6, Table 4.1) was given in the same class at the end of the semester as an optional survey ($N = 21$).

Survey S1 was also given at University C in a set of interviews carried out with 3 pairs of student volunteers ($N = 6$) who were currently enrolled in the quantum mechanics course. These "follow-up interviews" followed a semi-structured format. First, interviewees complete survey S1 working alone. Then the interviewed pairs completed an instructional activity related to basis and change of basis within the context of spin-$^1/_2$ systems, with limited guidance from the interviewer. Finally, the interviewed students also completed survey S2 on their own several weeks after their interview.

Questions Q1–Q3 are true/false questions given to probe ways that students might think a state has been physically altered by changing basis. These questions are presented in Table 4.1. Q1 asks whether changing basis affects measurement probabilities. Q2 asks whether measurement probabilities (not outcomes) are simultaneously knowable for multiple directions. Q3 asks whether changing basis creates a new state. These questions are primarily targeted towards students' interpretation of change of basis with respect to the physical nature of the state.

Questions Q4–Q6 probe student understanding of notation as it relates to basis (also in Table 4.1). Q4 asks whether a ket should be relabeled after changing basis. Relabeling a ket $|\phi\rangle$ means either adding a subscript (e.g., $|\phi\rangle_x$) or giving it a new name (e.g., $|\alpha\rangle$). Q5, like Q3, asks whether a new state has been created by the change of basis. Q5 was asked again to complement Q4, allowing the survey to distinguish students' use of notation for states in different bases from their interpretation of the process and consequences of change of basis. Q6 asks students to interpret the subscript on $|+\rangle_y$; we asked Q6 to probe student understanding of subscripts as they relate to basis to complement Q4 answer (b), which suggests applying a subscript to a ket after changing basis. These questions are primarily targeted towards students' interpretation of change of basis with respect to the representation of the state.

When coding students' responses to questions Q1–Q3, we identified specific *reasoning elements*. A reasoning element is an idea or line of argumentation a student uses to justify or explain their answer; for example, the "conflation of change of basis with measurement" element arose in students' explanations for why they expected measurement probabilities to differ after a change of basis was applied to a state. Reasoning elements are similar to *resources* [49], but are not necessarily as primitive or fundamental. In line with our phenomenographical approach [5, 47, 78], the reasoning elements we present generalize multiple students' statements, and we present all reasoning elements regardless of correctness.

We also identified *broad ideas*, which we define as possible answers to RQ1: according to students, what effect (if any) does changing basis have on the physical nature and/or the representation of a state? *A priori*, one might anticipate at least two broad ideas—changing basis *does* or *does not* have or result in a physical effect—as well as, possibly, others related to ways in which changing basis affects or is reflected in the *representation* of the state. We found that three distinct broad ideas emerged from student responses, outlined below (Section 4.3.1). Each reasoning element is consistent with only one broad idea, but a single broad idea may have multiple consistent reasoning elements.

Survey S1 $N = 29$	**Q1**	By writing the state $\lvert\psi\rangle$ in the x-basis, we've changed the probabilities for measuring along the z-direction. Choose one and *explain your response*: True **False**
	Q2	We can't know the probabilities for measurements along both the z-direction and the x-direction at the same time. Choose one and *explain your response*: True **False**.
	Q3	By representing the state $\lvert\psi\rangle$ in the x-basis, we've created a new quantum state. Choose one and *explain your response*: True **False**
Survey S2 $N = 27$	**Q4**	Consider a spin-1/2 state written in terms of the z-basis vectors: $\lvert\phi\rangle = \frac{3}{5}\lvert+\rangle - \frac{4}{5}\lvert-\rangle$. Suppose we rewrote this state in terms of the x-basis vectors. (The coefficients in all the answers are correct.) How would you choose to label the state now? *Explain your choice.* **a.** $\lvert\phi\rangle = -\frac{1}{5\sqrt{2}}\lvert+\rangle_x + \frac{7}{5\sqrt{2}}\lvert-\rangle_x$ **(Don't relabel the state)** b. $\lvert\phi\rangle_x = -\frac{1}{5\sqrt{2}}\lvert+\rangle_x + \frac{7}{5\sqrt{2}}\lvert-\rangle_x$ (Add an x-subscript on the state) c. $\lvert\alpha\rangle = -\frac{1}{5\sqrt{2}}\lvert+\rangle_x + \frac{7}{5\sqrt{2}}\lvert-\rangle_x$ (Give the state a new name)
	Q5	In the previous question, did rewriting the state in the x-basis create a new quantum state? *Explain your choice.* a. Yes **b. No**
	Q6	$\lvert+\rangle$ represents the spin-1/2 state with S_z eigenvalue $+\hbar/2$. What is $\lvert+\rangle_y$? *Explain your choice.* **a.** $\lvert+\rangle_y$ **is the state with** S_y **eigenvalue** $+\hbar/2$ b. $\lvert+\rangle_y$ represents $\lvert+\rangle$ in the y-basis. c. BOTH of the above.

Table 4.1: Survey questions targeting RQ1 regarding students' interpretation of a change of basis. Surveys S1 and S2 were both administered at at one institution throughout a quantum mechanics course. Identical versions of surveys S1 and S2 were administered to 6 student interviewees at a second institution. Correct answers to true/false and multiple-choice questions are given in bold.

For example, we label statements that changing basis alters the physical system as belonging to a single *broad idea* category, but students employed a variety of reasoning elements to justify answers consistent with this idea.

The broad ideas and reasoning elements were originally identified by the first author, and then refined in discussion with five of the authors. These codes, which were emergent from the Survey S1 data, were then applied to the analysis of related questions from survey S2. We also analyzed discussions from both the exploratory and follow-up interviews in light of these codes; in particular, interviews afforded us a closer look into student thinking related to the broad ideas and reasoning elements. Although we had *a priori* notions for how to categorize these data, using emergent coding allowed us to identify unexpected distinctions in students' ideas. Results from this analysis are discussed in Section 4.3.1.

4.2.2 Data and analysis: methods for changing basis (RQ2)

To determine which methods students use when changing basis, question Q7 was given in interviews and on for-credit assessments at all three institutions. Shown in Table 4.2, Q7 asks students to write a state in terms of a new set of basis states. In addition to being given on exams, a version of Q7 was part of a set of interviews across Universities A and B in the second year of data collection (the "exploratory interviews", $N = 22$). These think-aloud interviews were conducted with paid student volunteers who were enrolled in the quantum mechanics course. The interviews followed a semi-structured protocol, allowing room to probe students' conceptual understanding of basis. To address RQ2, we focus on how students went about changing basis in the interviews.

Beyond the interviews, separate variations of question Q7 were given in different semesters for a total of 215 written responses from the three universities ($N = 237$ including interviews). With the exception of one administration ($N = 37$), all variations included complex coefficients but used different numerical values for the probability amplitudes in the given state and basis states.

Analysis of the procedural change of basis question (Q7) produced categories describing the different approaches students used to calculate a change of basis. For example, student responses

Interviews & assessments $N = 237$	**Q7**	Consider a spin-$1/2$ particle prepared in the state $\lvert\psi\rangle$, written in the z-basis as $\lvert\psi\rangle = \frac{1}{\sqrt{2}}\lvert+\rangle + i\frac{1}{\sqrt{2}}\lvert-\rangle$ Write the state in the n-basis assuming $\lvert+\rangle_n = \frac{1}{\sqrt{3}}\lvert+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}\lvert-\rangle$ and $\lvert-\rangle_n = \frac{\sqrt{2}}{\sqrt{3}}\lvert+\rangle - i\frac{1}{\sqrt{3}}\lvert-\rangle$ *Hint: Solve for the values of* a *and* b *such that* $\lvert\psi\rangle = a\lvert+\rangle_n + b\lvert-\rangle_n$.

Table 4.2: Interview and exam question targeting RQ2 regarding methods students use when changing basis. Versions of this question were administered to students at each of the three institutions on several for-credit quizzes or exams in multiple semesters. Different versions omitted the hint and/or changed the numerical coefficients in the definitions of $\lvert\pm\rangle_n$.

that used inner products to calculate coefficients were labeled as using the "projection" method, while student responses that used algebraic manipulation and substitution were considered a separate category. The application of these categories required minimal interpretation beyond parsing students' handwritten work. All of the responses were coded by two authors, and the list of categories were agreed upon by all authors. Results from this analysis are discussed in Section 4.3.2

4.2.3 Data and analysis: coefficients and inner products (RQ3)

Survey questions were also administered to investigate students' interpretations of basis expansion coefficients and the ways they relate to inner products. Relevant questions are shown in Table 4.3. Survey S1, described above, also included Q8. In question Q8, students were given a state and asked what information is provided by the coefficients in the expansion of a state in a basis.

Survey S3 (Q9 and Q10) was given in-class ($N = 24$) after all relevant instruction. Question Q9 asked students to describe the meaning of an inner product written in Dirac notation, ${}_y\langle+\vert\psi\rangle$. Question Q10 probed whether students could generate the generic expression for the expansion of a state in a basis; e.g., $\lvert\psi\rangle = a\lvert+\rangle_n + b\lvert-\rangle_n$ in the n-basis. When previously asked to change basis (Q7), the question text included this expression in a hint.

When analyzing student responses to Q8–Q10, we focused on two steps that are involved when using projection inner products as a method for changing basis: recognizing that a state can be

Survey S1 $N = 29$	**Q8**	Consider a spin-$1/2$ electron prepared in the state $\lvert\psi\rangle = \frac{1}{\sqrt{3}}\lvert+\rangle + \frac{\sqrt{2}}{\sqrt{3}}\lvert-\rangle$ What do the coefficients in this expression (the $\frac{1}{\sqrt{3}}$ and the $\frac{\sqrt{2}}{\sqrt{3}}$) tell you about the state?
Survey S3 $N = 24$	**Q9**	Consider a quantum state $\lvert\psi\rangle$. What is the meaning of ${}_y\langle+\lvert\psi\rangle$? Describe what the symbols are as well as any physical meanings you associate with the expression as a whole.
	Q10	How can we represent a general state $\lvert\psi\rangle$ in the S_x basis? Write an expression for any unknown variables you use.

Table 4.3: Survey questions targeting RQ3 regarding students' interpretations of coefficients in a basis expansion. Survey S1 also included questions presented in Table 4.1. Like surveys S1 and S2, survey S3 was administered at one institution in the quantum mechanics course.

written as an superposition of other basis states; and connecting basis expansion coefficients with inner products. We identified the different ways students addressed these concepts in their responses, using a coding strategy similar to our strategy for RQ1: categories emerged from the data and were refined in discussion amongst the authors to increase confidence in our interpretation of students' written statements.

4.3 Results

In this section, we present a variety of student ideas on basis and change of basis in the context of a spin-$1/2$ system. First, addressing RQ1, we describe the ideas that students express when they are asked to interpret the meaning of a change of basis and its implications on the state (Section 4.3.1). Second, addressing RQ2, we identify the procedural methods that students use when they are asked to represent a state in a new basis (Section 4.3.2). Finally, addressing RQ3, we discuss student understanding of the structure of a basis expansion and students' interpretation of the coefficients, of inner products, and of the relationship between the two (Section 4.3.3).

4.3.1 Interpretations of changing basis (RQ1)

To explore student understanding of how changing basis affects the quantum state, interviews and written questions were designed to target specific conceptual and notational aspects of basis and changing basis. Three broad ideas arose in student responses to these survey questions, as well as in dialogue in interviews. These ideas are summarized in Table 4.4 and described in detail below.

A basis change is akin to expressing a vector in a different coordinate system. As such, it has no effect on the quantum state, and does not require relabeling the associated ket (quantum state vector). This is broad idea (1): the idea that changing basis does *not* affect the physical system or the state vector. In other words, changing basis does not affect the state.

Broad idea (2) is the idea that changing basis has a physical effect, such as modifying measurement probabilities or outcomes. If we ask "Does changing basis affect the state?", a student holding broad idea (2) might answer affirmatively. However, we must disambiguate the word "state",

Broad idea	Consistent expressed reasoning elements	Answers given
(1) Changing basis does not affect the physical system or the state vector (Section 4.3.1.1)	Choice of basis is strictly a means of representing a state vector. Change of basis is strictly a reversible calculation.	Stating that changing basis does not change measurement probabilities, and choosing to retain a ket's original label when representing it in a new basis. For example, answering "False" on Q1 or (a) on Q4.
(2) Changing basis alters the physical system (Section 4.3.1.2)	Changing basis is the same thing as making a measurement. One cannot know probabilities for spin measurements along different directions at the same time (misinterpretation of the uncertainty principle). $\|\psi\rangle$ is associated with just one pair of spin-up/down probabilities.	Stating that changing basis changes measurement probabilities. For example, answering "True" on Q1.
(3) Changing basis can involve relabeling the state vector (i.e., the ket) (Section 4.3.1.3)	Notation (e.g., a subscript on a ket) indicates choice of basis (by convention). It is useful to indicate choice of basis with notation (even if convention does not require it). A ket or state vector is mathematically distinct when represented in a new basis (even if it describes an unchanged physical system).	Choosing to relabel a ket when representing it in another basis (e.g., $\|\psi\rangle \rightarrow \|\alpha\rangle$), or choosing to add a subscript to a ket when representing it in another basis (e.g., $\|\psi\rangle \rightarrow \|\psi\rangle_x$). For example, answering (b) or (c) on Q4.

Table 4.4: Students expressed a range of reasoning elements on different questions probing their understanding of change of basis and its affect on both the physical system and the quantum state vector (ket) describing that system. Each reasoning element is consistent with one of three broad ideas, which represent possible interpretations of the effect of change of basis on a quantum state. Different questions (from Table 4.1) invoked different reasoning elements. The table also gives examples of answers that were consistent with the broad idea.

as students may use it to mean the state vector (the ket), or the physical system. Broad idea (2) specifically refers to students thinking that the physical system is changed as a result of changing basis.

A related question is, "Does changing basis involve relabeling the state vector (i.e., the ket)?" Broad idea (3) says yes: it is the idea that a state vector can or should be labeled differently when represented in a different basis. We saw no instances where students simultaneously argued that the physical system had been altered by a change of basis but that the notation for the state vector should remain unchanged. However, we did encounter cases where students opted to relabel the state vector following a change of basis, but nonetheless argued that the physical system was *un*altered. Therefore, broad idea (3) emerged as a distinct category from broad idea (2). As such, students opting to relabel after a change of basis certainly places their response under broad idea (3), but is insufficient evidence determine whether or not they also hold broad idea (2).

In Quantum Mechanics, there is (up to global phase) a one-to-one correspondence between the configuration of a physical system and the quantum state vector describing that configuration. When the distinction between physical system and quantum state vector is relevant to distinguishing student ideas in the discussion below, we use the terms "physical system" and "quantum state vector" (or "ket") explicitly. Otherwise, when we use the term "state,“ we refer to *both* the physical system it represents and the corresponding state vector. When students say "state," we cannot always be certain if they mean one or both of these things. For the purposes of this analysis, whenever possible, we infer what students mean by the word "state" from context.

Students' responses to the various questions included a range of reasoning elements consistent with one of the three broad ideas, as summarized in Table 4.4. Although individual responses largely fell under one of these three ideas, different responses from the same student sometimes aligned with multiple ideas, indicating that students' ideas about the meaning of basis and change of basis were fluid, and that different questions could draw out different reasoning elements for students.

In the following three subsections, we elaborate on the three broad ideas and provide relevant examples from student work. The survey questions referred to in this section can be found in

Table 4.1.

4.3.1.1 Idea that changing basis does not affect the physical system or state vector

Correct answers to questions Q1–Q5 are consistent with this broad idea. Students answering these questions correctly provided explanations along two lines: that basis is just a means of representing a state, and that change of basis is a reversible calculation.

First, some students argued that changing basis did not alter the physical system nor the state vector because a basis is just a means of representing the same state vector. In response to Q5, one student said:

> *It isn't creating a different state, it's writing the same state in a different representation.*

Some students drew an explicit analogy with coordinate systems, as in this response to Q5:

> *Our choice of basis does not change the state, just like our coordinate system does not change where something physically is.*

Another student made a similar analogy but instead with unit conversion, saying in response to Q1:

> *To me, I see doing a change in basis similar to doing unit conversions in a way. "Going from mass to moles does not change our initial mass."*

Whereas the first line of reasoning centers on the meaning of basis, the second focuses on the idea that change of basis is only a calculation—in particular, one that can be undone. One student presented this reversibility argument in response to Q3 as follows:

> *It is the same quantum state...converting back and forth will not change the original state.*

The argument that change of basis is a reversible calculation arose in multiple students' correct explanations for these five questions.

Students in both rounds of interviews cited both reversibility and the idea that a basis is a choice of representation as reasons why changing basis does not alter the state. No other lines of reasoning identified as completely correct were evident in our sample.

4.3.1.2 Idea that changing basis alters the physical system

Questions Q1, Q2, and Q3 targeted student thinking around how changing basis might affect the physical system—an answer of “True” to any of those question suggests student reasoning that is consistent with the broad idea that changing basis alters the physical system, and students’ explanations illuminate reasons why they may hold this belief. These findings confirm what we observed in the exploratory interviews, in which students made statements consistent with broad idea (2). We encountered three reasoning elements in explanations to answers consistent with the broad idea that changing basis has a physical effect.

The first is the notion that a state is associated with a specific basis and thus tied to a specific pair of (physical) spin-up/down probabilities. In reality, a single state encodes all possible pairs of probabilities at once—that is, probabilities for spin measurements along any axis. Among the reasons given in favor of “True” for Q1 and Q3 was the response that “the probabilities” would be different after the change of basis. By “probabilities,” students appear to mean the pair of probabilities for measuring spin-up/down along a specific direction. For example, in an explanation for Q1, one student wrote:

> *If you change the orientation, you will get different probabilities.*

While this statement would be true if the student were discussing changing the orientation of a *measurement*, they gave it as an explanation for why changing *basis* from the z-basis to the x-basis would alter the probabilities for measurement along the z-direction. This line of reasoning arose in responses to both Q1 and Q3. Similar reasoning arose in the exploratory interviews: some students argued that the probabilities for “spin-up” and “spin-down“ had changed with the change of basis. Again, students seemed to associate the state with just one pair of spin-up/down probabilities.

The second reasoning element that arose was misapplication of the uncertainty principle. Q2 asked whether it is possible to know the probabilities for measurements along multiple axes simultaneously. Because these probabilities are directly encoded in basis expansion coefficients, responses arguing that knowing this information simultaneously is physically impossible (answers of "True") are consistent with the idea that the state vector representing a physical system is tied to a single basis, and that therefore change of basis alters the physical system. Some students who answered Q2 in this way alluded to the uncertainty principle, incorrectly extending it to apply to measurement probabilities as well as measurement outcomes. For example, one student said:

> *We can only be certain of the probabilities for measurements in only one direction at a time.*

This interpretation of the uncertainty principle was also consistent with other students' justification for answering "True" for Q1:

> *By changing the z-basis to x-basis we...now only can calculate the outcome in the x-direction.*

If this interpretation of the uncertainty principle were accurate—meaning that probabilities for incompatible measurements could not be known simultaneously—it would imply that changing basis creates and destroys information and would suggest that change of basis is (or is associated with) a physical process.

The third reasoning element we observed demonstrated a conflation between the process of change of basis and the act of making a measurement. Unlike change of basis, measurement is a physical procedure that does alter probabilities. For example, measuring spin along the x-direction provides a definite value for that observable, but in general invalidates any previously known probabilities for, say, the z-component of spin.

If a student conflates measurement with change of basis, they may misapply a valid understanding of measurement to draw invalid conclusions about the consequences of a changing basis.

Some responses appeared consistent with this conflation. For example, some students referred to the effect of measurement to justify their answer for Q2. One student said:

> *The act of measuring in one direction will alter the system such that the other direction would be random.*

Arguments about measurement arose in responses to Q1 and Q3 as well. One student responded to Q1 with the argument that,

> *. . . measuring in another basis will alter the probability of the output that comes out in the z direction. I think this is because measuring a state alters it.*

The same student made a similar argument in answer to Q3:

> *By measuring it in that axis we have an altered quantum state. . .*

Responses like these suggest that some students conflate change of basis with the physical act of making a measurement. The same conflation was made by some students in the follow-up interviews. In a debrief at the end of their interview, one student named distinguishing basis-change from measurement as one of the key things they had learned during the interview. (The follow-up interviews included an instructional activity, which was completed after interviewees took survey S1.)

4.3.1.3 Idea that changing basis can involve relabeling the state vector (i.e., the ket)

In both interview and written contexts, students often elected to relabel a ket when representing it in a different basis. Options (b) $|\phi\rangle_x$ and (c) $|\alpha\rangle$ for Q4, which asked how best to label a ket after a change of basis, exemplify this idea (see Figure 4.1). We consider these choices incorrect because relabeling is not necessary and has the potential to lead to confusion (for example, relabeling could suggest that the Dirac notation expressions for the state in each basis cannot be related by an equals sign, whereas they are in fact equal).

Q4. Consider a spin-1/2 state written in terms of the z-basis vectors: $|\phi\rangle = \frac{3}{5}|+\rangle - \frac{4}{5}|-\rangle$.

Suppose we rewrote this state in terms of the x-basis vectors. (The coefficients in all the answers are correct.) How would you choose to label the state now?

a. $|\phi\rangle = -\frac{1}{5\sqrt{2}}|+\rangle_x + \frac{7}{5\sqrt{2}}|-\rangle_x$ **(Don't relabel)**
b. $|\phi\rangle_x = -\frac{1}{5\sqrt{2}}|+\rangle_x + \frac{7}{5\sqrt{2}}|-\rangle_x$ (Add an x-subscript)
c. $|\alpha\rangle = -\frac{1}{5\sqrt{2}}|+\rangle_x + \frac{7}{5\sqrt{2}}|-\rangle_x$ (Give it a new name)

Explain your choice.

Figure 4.1: Q4 from Table 4.1, repeated here for convenience. When considering whether changing basis involves relabeling the state vector, students may either: (a) not relabel; (b) relabel by adding a subscript to the ket; (c) relabel by using a new symbol inside the ket. The correct answer is (a).

Students' justifications for relabeling a ket after changing basis appeared to fall in two categories: one argument was that the notation for a ket plays a role in indicating choice of basis; the other was that a ket represented in a different basis is somehow a mathematically distinct object from the ket represented in the original basis.

The first argument that students gave in favor of relabeling a state vector after changing basis was that the notation for a ket either does, or should, indicate the basis used to represent the state. This reasoning element arose both in responses to Q4 and in the interviews. Students who choose to relabel a ket after changing basis may understand that neither the physical system nor the ket itself have changed, but they may still consider relabeling to be notationally useful.

Subscripts—specifically subscripts on the angle bracket of the ket, such as $|+\rangle_x$—arose as the notation of choice for some students who opted to relabel a ket after changing basis. Some did not see the addition of a subscript as equivalent to giving the ket a new name, and some saw subscripts as playing a special role in indicating choice of basis. This explanation for choosing $|\phi\rangle_x$ as the answer to Q4 demonstrates both of these ideas:

> *We can write states in different basis if we indicate it with a subscript on the state. The state itself doesn't change in a different basis so there's no need to give it a new name [referring to $|\alpha\rangle$].*

Q6 was designed to probe students' interpretation of subscripts on a ket. Some students opted for choice (b) alone, which argued that $|+\rangle_y$ is $|+\rangle$ in the y-basis. Other students argued that both (a) and (b) were true, with choice (a) correctly defining $|+\rangle_y$ as an eigenstate of S_y. Choices (a) and (b) are mutually inconsistent if you also recognize that $|+\rangle$ is *not* also an eigenstate of S_y. However, both choices are consistent if one believes that a subscript on a ket indicates the basis used to represent the state.

The idea that the subscript on a state vector denotes basis arose in the interviews as well. After concluding that a ket need not have a subscript added after changing basis, one student also concluded that the x subscript on $|+\rangle_x$ was optional, while another suggested that S_x and S_y were the same matrices expressed in different bases. Note that the notation for a ket does not indicate basis choice, but there are other contexts where subscripts are used to indicate basis, such as in matrix representations.[4]

The second argument students used for relabeling a state after changing basis was not that it was simply useful (or required) notation, but that the state vector written in the new basis was somehow distinct from the original state vector. Here we refer to the "state vector" as a distinct concept from the physical system it describes. Hence this line of reasoning belongs here, under broad idea (3), as opposed to under broad idea (2), because either we could not assume from context that the student believed the physical system had been altered, or alternatively, the student made it clear they did *not* believe anything had changed physically.

Students making the argument that the state vector somehow changed appeared to consider the choice of basis as part of the identity of the state vector. Interviews illuminated this reasoning element. Comparing the vector (temporarily labeled $|k\rangle$) resulting from a change of basis to the original vector (labeled $|u\rangle$), one student argued that:

> *They are the same vector when they are both expressed in the same basis...$|k\rangle$ in the i-basis is just $|u\rangle$.*

---

[4] When representing quantum states as matrices (i.e., column vectors), the courses included in this study always used the convention

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

for the spin-up state along the z-direction, meaning that matrices expressed in other bases would include a subscript.

This student identified being “expressed in the same basis” as part of what it means to be “the same vector.” Another made a similar claim:

> *It’s still different components even though it’s the same vector. It isn’t $|u\rangle$ [the original vector] because $|u\rangle$ is in a different basis*

This student argued that a vector written in two different bases can no longer be identified by the same symbol ($|u\rangle$) because of the different axes and values used to describe it, even if they both describe the same unaltered physical system. They focused on the difference in representation rather than the fact that both versions represented the same physical system.

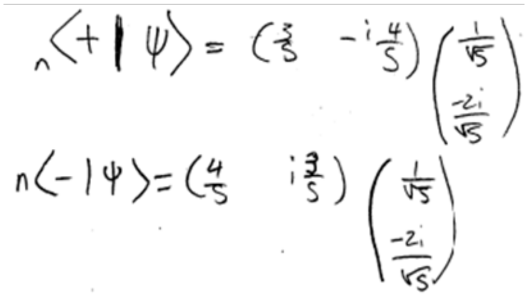
Broad idea (3)—that changing basis can involve relabeling the state vector—can be distinct from broad idea (2)—that the physical system has been altered. As an example, one student who opted to give the ket a new label in Q4 nonetheless argued that it described the original quantum state in response to Q5 (*not* consistent with broad idea (2)):

> *It’s the same state, but it is just represented in a different coordinate system. (it looks different but its the same.)*

This student argued convincingly that the physical system has not been altered by the change of basis. Their response is consistent with broad idea (3), because they elected to give the ket a new label (choice (c)), but *in*consistent with broad idea (2), because they argued the state was the same. For this student, the change in representation (the fact that the state vector “looks different”) was sufficient for them to choose to relabel the ket in Q4. (Because the student distinguished the “state” from its representation, we believe they used the word “state” to refer to the “physical system.”)

4.3.2 Methods for changing basis (RQ2)

Analysis of question Q7 from interviews and several years of quiz and exam data from three different institutions have shown that students use a variety of approaches to change the basis of a state in the spin-$1/2$ context. It is possible to correctly change basis using various methods, including



$${}_n\langle + | \psi\rangle = \left(\tfrac{3}{5} \quad -i\tfrac{4}{5}\right)\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{-2i}{\sqrt{5}}\end{pmatrix}$$

$${}_n\langle - | \psi\rangle = \left(\tfrac{4}{5} \quad i\tfrac{3}{5}\right)\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{-2i}{\sqrt{5}}\end{pmatrix}$$

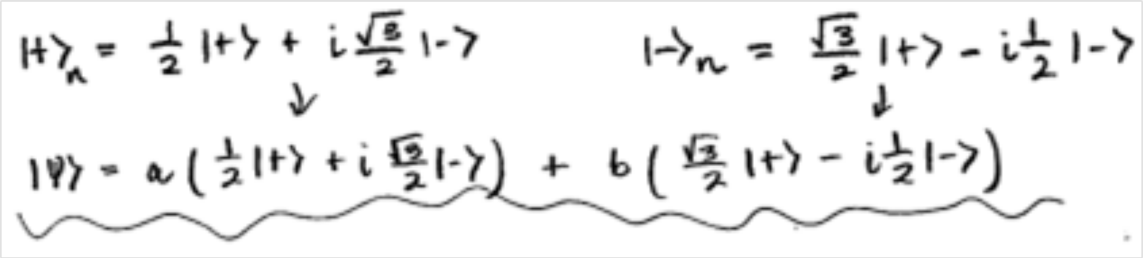
Figure 4.2: Student work showing the start of the projection method of finding a change of basis. The student began by writing the inner products and expanding into matrix notation.

projection (i.e., inner products), algebraic manipulation, or substitution. All three of these methods arose in the data.

Often, the most mathematically efficient method is to use orthogonal projection. This method produces the coefficients $a_\pm$ for the state in the new basis, $|\psi\rangle = a_+ |+\rangle_n + a_- |-\rangle_n$, by calculating the inner products $a_\pm = {}_n\langle \pm|\psi\rangle$ (Figure 4.2). More directly, the new representation can be found by operating with the identity $\mathbb{1} = \Sigma_i |a_i\rangle\langle a_i|$ on the state $|\psi\rangle$, where $|a_i\rangle$ are the eigenstates in the desired basis, but this first step is often skipped.

The remaining two methods are more algebraically intensive and were more subject to error for students. Both methods involve algebraic manipulation and substitution but are distinguished by which step is first in the process. The algebraic manipulation method involves treating the new basis states as a system of equations in order to solve for $|+\rangle$ and $|-\rangle$ in terms of $|+\rangle_n$ and $|-\rangle_n$. Finding these representations allowed students to insert them into the given state and then combine like terms.

Alternatively, students attempted calculation by substituting the new basis states (e.g., $|+\rangle_n = \frac{1}{\sqrt{3}}|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle$ and $|-\rangle_n = \frac{\sqrt{2}}{\sqrt{3}}|+\rangle - i\frac{1}{\sqrt{3}}|-\rangle$) directly into the generic representation ($|\psi\rangle = a|+\rangle_n + b|-\rangle_n$) in order to rewrite the state in terms of the basis of S_z (Figure 4.3). This step alone leaves the state in terms of unknown variables a and b. Almost all students using this method went on to compare the new coefficients for $|+\rangle$ and $|-\rangle$ with the original state to get a system of equations, which could be solved to find a and b.



$$|+\rangle_n = \frac{1}{2}|+\rangle + i\frac{\sqrt{3}}{2}|-\rangle \qquad |-\rangle_n = \frac{\sqrt{3}}{2}|+\rangle - i\frac{1}{2}|-\rangle$$

$$|\psi\rangle = a\left(\frac{1}{2}|+\rangle + i\frac{\sqrt{3}}{2}|-\rangle\right) + b\left(\frac{\sqrt{3}}{2}|+\rangle - i\frac{1}{2}|-\rangle\right)$$

Figure 4.3: Student work showing an alternative correct method for finding the representation of a state in a different basis. In this method, the student substitutes the new basis states into the generic representation. Subsequent combining of like terms allows the student to compare the expression to the original state and solve for a and b.

Neither the algebraic manipulation nor the substitution methods are generalizable to the context of continuous wave functions. Furthermore, in the context of spins, the amount of work required by these methods meant many students left their answer incomplete. Some got stuck after inserting the definition of the new basis kets into the generic representation. Others identified the system of equations but failed to solve it or attempted to solve it but made errors in the algebra.

The methods discussed so far will all lead to correct answers (when carried out correctly). The remaining "incorrect" methods included some that were only applicable for special cases, such as when there are no complex phase terms. Since Q7 included a complex phase in the relation between the basis states, the use of these methods resulted in incorrect answers for students. Two of the three incorrect methods involved finding the probabilities of measuring $\pm\hbar/2$ along the n-direction by calculating $|{}_n\langle\pm|\psi\rangle|^2$ instead of finding the probability amplitudes. Students then either (1) used the results of the probabilities as the values of the coefficients a and b or (2) took the square root of the result to use for the coefficients. Although taking the square root of the probabilities may happen to work in special cases, it does not account for any negative or complex phase associated with the coefficients.

The last incorrect method involved swapping the subscripts on the new basis states (e.g., rewriting $|+\rangle_n = \frac{1}{\sqrt{3}}|+\rangle + \sqrt{\frac{2}{3}}|-\rangle$ as $|+\rangle = \frac{1}{\sqrt{3}}|+\rangle_n + \sqrt{\frac{2}{3}}|-\rangle_n$). Students then substituted these for the $|\pm\rangle$ in the given state and then rearranged the result in terms of $|\pm\rangle_n$. Although this works in some special cases (e.g., for finding $|\pm\rangle$ in the basis of S_x), it does not work in general.

4.3.3 Representation of states, coefficients, and inner products (RQ3)

In the preceding sections, we categorized the ways students interpreted the effect of changing basis and the methods they used to do so. Several additional survey questions rounded out the study by probing student thinking about the mathematical structure of a state, and the notation and meaning associated with inner products. We summarize our findings from these questions here.

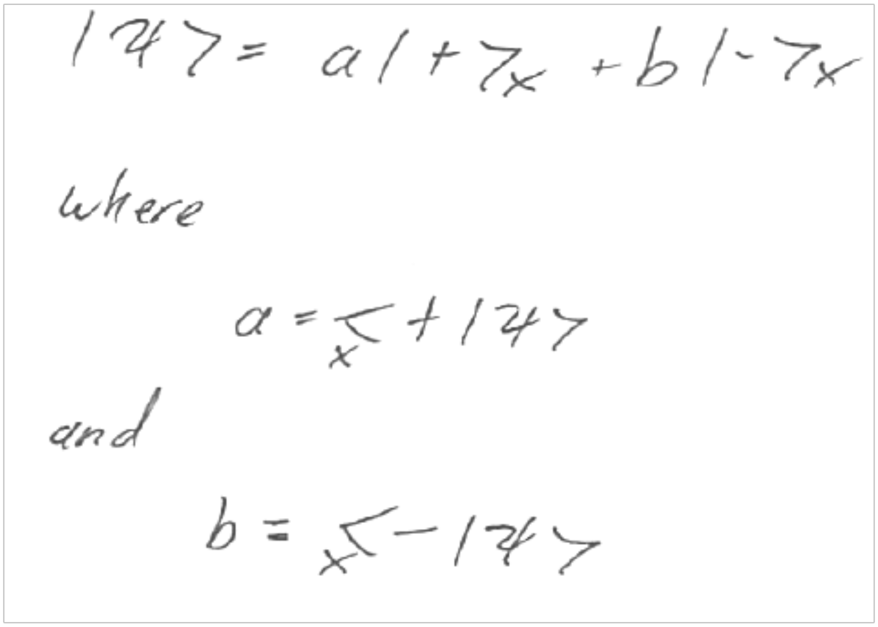
Q10 asked students to "represent a general state $|\psi\rangle$ in the S_x-basis". A correct answer to this question can be broken into two steps. Step one is to recognize that the state $|\psi\rangle$ can be written as a superposition of the S_x basis states,

$$|\psi\rangle = a\,|+\rangle_x + b\,|-\rangle_x \tag{4.3}$$

Step two is to connect the unknown coefficients (a and b) with inner products between the state and the basis states (${}_x\langle\pm|\psi\rangle$). Some students completed both steps correctly (e.g., Figure 4.4), some students only completed step one, and others gave other responses.

Only half of the students who completed step one by writing Equation (4.3) also completed step two by providing inner product expressions for a and b. Students who did not express the variables a and b as inner products either simply noted they were coefficients or invoked the normalization condition. Generating the form of a basis expansion (e.g., Equation (4.3)), or at least acknowledging that the current state can be connected to another basis representation, is an important element of changing basis using projection. However, our results show that students' production of a generic basis expansion does not guarantee an explicit connection of coefficients to inner products.

Students also did not connect coefficients to inner products on question Q8, which asked students to discuss the meaning of the coefficients for a state with numerical coefficients (as opposed to the generic basis expansion from Q10). Most students connected the coefficients explicitly to probabilities, with a subset specifying that the coefficients were probability amplitudes that needed



$|\psi\rangle = a|+\rangle_x + b|-\rangle_x$

where

$a = {}_x\langle +|\psi\rangle$

and

$b = {}_x\langle -|\psi\rangle$

Figure 4.4: A student's written response when asked to write the general form of a state in the S_x-basis. Here the student uses two unknown variables, a and b, and connects the coefficients to the appropriate inner products.

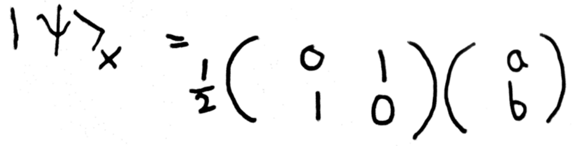
to be squared. Other students included that the coefficients showed that the state was normalized. No students spontaneously discussed the mathematical meaning of the coefficients as basis expansion coefficients.

To further probe the connection between the coefficients and projection, students were asked to discuss the meaning of the expression ${}_y\langle +|\psi\rangle$ and the symbols that make it up (Q9, from the same survey as Q10 above). Responses about the expression as a whole included connections to probability or specifically to the coefficient or probability amplitude of the $|+\rangle_y$ state, as shown in the two examples below.

> *This picks out the probability amplitude of measuring spin up in the y direction of the state ψ*
>
> *I associate the probability of finding $+\hbar/2$ in the y direction with this since I can absolute value and square.*

Other responses to the survey identified the expression as an inner product, or one that represented a projection coefficient, but did not provide any interpretation. For example, one response simply read "${}_y\langle +|\psi\rangle$ is the inner product of $|+\rangle_y$ with $|\psi\rangle$". Students who identified the bra and ket individually labeled $\langle +|_y$ as related to a "spin up in the y direction" or as the "bra in the y basis" and labeled $|\psi\rangle$ as the "initial quantum state" or "general state ket."

Finally, returning to Q10, a small number of incorrect student responses included an application of the S_x operator. For example, writing an expectation value of S_x or writing the S_x operator acting on the state (Figure 4.5). Course observations at our institutions and prior literature have shown that some students connect measurement to acting an operator on a state $S_x|\psi\rangle$ (Figure 4.5) [73, 134]. This response to Q10 could potentially be due to a conflation between measurement and changing basis representation, or simply a belief that the S_x operator should "do something" and a knowledge that it is connected to the basis states $|\pm\rangle_x$.



$$|\psi\rangle_x = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}$$

Figure 4.5: A second student's written response when asked to write the general form of a state in the S_x-basis. Here the student equates a state $|\psi\rangle_x$ to the S_x matrix acting on a generic column vector.

4.4 Discussion and conclusion

Basis and change of basis are key concepts in quantum mechanics with which students associate a variety of ideas and procedures. Our research has examined a broad range of data to extract a qualitative picture of student understanding of these ideas. Conceptual questions were asked on free response surveys while procedural questions were given on quizzes and exams. Interviews included both kinds of questions to provide additional insight into student thinking. By categorizing the different methods students use to change basis and the different ideas they associate with basis representation, we establish a research base for the the creation of instructional materials.

Investigating students' ideas about the state following a change of basis revealed several categories of reasoning. First is the recognition that changing basis does not affect the physical system or state vector, possibly because the student recognizes that basis is merely a choice of representation, or change of basis is merely a reversible calculation. Second is the notion that changing basis alters the physical system (e.g., changing measurement probabilities), potentially due to confusion between change of basis and measurement. Third is the idea that changing basis can involve changing the label of a ket to reflect the basis it is represented in. These lines of reasoning are not mutually exclusive: we found that students may draw on two or even all three depending on context. That is, students' conceptualizations of the meaning of change of basis are not always robust.

Although when using bra-ket notation we do not add subscripts to a ket's label after changing basis, we can imagine why students might do so. There are instances in QM where subscripts *are* used to indicate choice of basis, namely when using matrix notation. Moreover, subscripts on kets are typically only used to distinguish related basis states (e.g., $|+\rangle_x$ from $|+\rangle_y$); they are related to basis. Students also learn that, by convention, the z-subscript may be omitted from the z-basis states $|\pm\rangle$, so it is not unreasonable that they might conclude that any ket labeled without a subscript is implicitly in the z-basis. Subscripts are also used for a similar purpose in classical physics contexts such as electricity and magnetism, where subscripts label the components of a vector.

Probing students' thinking about basis expansion coefficients revealed that many students connected the coefficients to measurement probabilities, but that the connection from coefficients to inner products can be more subtle. When asked about the expression, ${}_y\langle+|\psi\rangle$, most students described it as an inner product, but only some connected it to the expansion coefficient for the $|+\rangle_y$ basis state. Meanwhile, when asked for a generic representation of a state in a new basis, few students expressed the components as inner products, even if they did generate the generic representation (e.g., Equation (4.3)). It is possible that a more targeted survey question would have been more likely to prompt students to include inner product expressions in their answer, but previous research does support a disconnect between inner products and coefficients [134], including in a wave functions context [145].

Moreover, we found that the labeling of coefficients as inner products is insufficient to determine whether or not a student would use projection to carry out a change of basis. A student might connect ${}_x\langle+|\psi\rangle$ to the basis expansion coefficient a in Equation (4.3), which presents $|\psi\rangle$ in the x-basis. However, the same student might *not* recognize that the same relationship holds if the state $|\psi\rangle$ was initially expressed in the z-basis, possibly because of a belief that a change of basis changes the state physically.

Analysis of quiz and exam data given as part of regular coursework revealed the ways in which students found the representation of a given state in a different basis. An efficient method for finding the new basis expansion coefficients involved calculating inner products between the state and the new basis states. We refer to this method as "projection" (or "orthogonal projection"). Some students also treated the new eigenstates as a system of equations and attempted to rewrite them, or substituted the new eigenstates into the generic equation, $|\psi\rangle = a\,|+\rangle_n + b\,|-\rangle_n$, and compared to the given state. Both of these latter solutions involved extended algebra, which led to various calculation errors or was sometimes abandoned by students. Nonetheless, these algebraic approaches may be appealing for students who are more comfortable working with systems of equations than with projection.

The projection, substitution, and system of equations methods all lead to the correct answer when executed properly. One incorrect method that students used was to calculate the probabilities associated with measurements in the new basis (i.e., of the S_n observable) and use the square-roots of these probabilities as the new coefficients. We also found that students articulated the connection between coefficients and measurement probabilities more often than describing coefficients in terms of basis expansion or connecting them with inner products, which may explain why some students employed this incorrect method for changing basis.

Inconsistency in student responses over several questions exemplifies a larger pattern: students may demonstrate the distinct procedural and conceptual elements of basis and change of basis in independent contexts without connecting them together to form a comprehensive understanding. For example, a student might successfully implement a basis change calculation and still argue that the state has changed physically. Another student might articulate the connection between coefficients in a basis expansion and projection in one context, but not recognize projection as a method for changing basis in another. The projection method is supported both by connecting coefficients and inner products and by identifying the state as the same regardless of which basis it is represented in. The latter is what allows the inner product to be calculated regardless of the basis used for the calculation. As discussed in the next section, instruction could explicitly encourage students to make these connections towards the goal of forming a more robust understanding.

4.5 Instructional implications and future work

Our investigation uncovered a range of student thinking on the meaning of basis and change of basis, and identified the variety of methods—varying in efficiency, correctness, and generality—that students employ when changing basis. There are a number of ideas employed by students that proved productive. On the other hand, our investigation also revealed some confusion about notation or physical interpretation.

Our knowledge of students' ideas informs a set of learning goals for instruction targeted at basis and change of basis, especially in the context of spin-$1/2$ systems. These learning goals are not a set of observed misconceptions that we believe instruction should correct; in fact, these goals encourage productive lines of reasoning observed in our research. By studying the span of student thinking on these topics, we have identified areas on which instruction should focus. After instruction, students should be able to:

Recognize that changing basis does not change the state or any associated measurement probabilities. Some students described basis as a means of representation akin to unit or coordinate systems when explaining their correct answers to questions about change of basis; instruction can promote these productive analogies. Instruction can also clarify the distinction between changing basis and making a measurement, as a conflation between these two concepts arose in student responses. Viewing change of basis as a reversible calculation is another useful idea that arose in student responses, which instruction can emphasize.

Use projection as a method for changing basis. In order to implement projection for this purpose, and in order to fully use basis representation as a tool for tackling quantum mechanical problems, students must connect the inner products they take for computing measurement probabilities with the coefficients in a basis expansion. Moreover, they must recognize that the same state may be represented in any basis. Our results show a variety of challenges related to these aspects of change of basis, indicating that instruction should attend directly to these ideas. Projection is the most general and usually most efficient method of changing basis, but students employed a range of procedural approaches to changing basis. After instruction, students should be able to identify projection as a preferred method for changing basis.

Identify the coefficients in a basis expansion (a) physically as probability amplitudes and (b) mathematically as inner products. Our results suggest that the connections between these ideas are important: some students produced a generic expression for a basis expansion but did not produce the projection inner products for computing the expansion coefficients. Instruction should emphasize these connections, which can also help students recognize that a state encodes all probabilities at once, contrary to the idea that a state is associated with just one pair of spin-up/down probabilities, as arose in some student responses. Instruction should also clarify the meaning of subscripts in Dirac notation—students should recognize that $|+\rangle$ and $|+\rangle_y$ are physically different states.

Future work can expand our investigation to student understanding of basis and change of basis in the context of continuous vector spaces (such as the position basis for a quantum mechanical particle). Additional research is also needed to examine how students connect the notion of a basis with its corresponding measurement operator (i.e., observables). Future work can also investigate student use of Hilbert space representations other than algebraic basis, such as the Bloch Sphere.

As part of this project, our research team developed an instructional tutorial for quantum mechanics targeted at change of basis in the context of spin-$1/2$ systems, as mentioned in Section 4.2. This tutorial draws on our findings to improve student understanding of change of basis via analogy to two-dimensional Cartesian space [23, 107]. We describe the process of developing the activity in [115] and evaluate the ACE Physics version of the tutorial, as assigned on homework, in Chapter 6. Before moving ahead to assigning tutorials for completion outside class, we now turn our focus to an investigation of the in-class tutorial environment

Chapter 5

Student engagement with tutorials in in-class, group-work settings[1]

5.1 Introduction

Tutorials have been implemented in a variety of classroom contexts throughout the physics curriculum [33, 37, 62, 67, 84, 105, 107, 132, 140]. As described in previous chapters, students typically work on tutorial worksheets in groups of 2–5 while instructors walk around the room and lend assistance, often (but not always) in the style of Socratic questioning. One distinguishing feature of tutorial worksheets is their focus on debate and conceptual reasoning. Tutorial activities have proven to be effective learning tools in multiple contexts [33, 37, 60, 62, 70, 73–76, 83, 124, 125], but less education research has studied student behaviors during tutorials themselves.[2] This chapter is one such study and builds on similar work from Scherr *et al.* [117, 118].

There are multiple reasons to study student behavior during tutorial sessions. Student conversations during tutorials provide a lens into their reasoning about specific physics concepts as well as into group dynamics. Student epistemologies (beliefs about knowledge and learning) may also be evident in these discussions. We know that tutorials *work* (i.e., they often produce measurable learning gains), but studying students during tutorials might help us understand *why* they work. Tutorials provide a rich context for research because they encourage students to discuss conceptual

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[1] This chapter builds on work presented in [10], on which Bianca Cervantes, an undergraduate at California State University, Fullerton, is the first author. I mentored Bianca throughout the project that led to the publication, which I co-authored. Bianca plans to expand the research in other directions, but the focus of this chapter (applying the analysis to support development of ACE Physics) is novel. Some figures and small portions of the text are reproduced from [10]. Bianca conducted the majority of the coding of videos.

[2] Some PER has investigated tutorial *instructors,* such as [45].

physics problems and articulate their reasoning aloud.

This chapter will attend to some of the reasons listed above, but will focus on a different objective. The ACE Physics project aims to export some of the benefits of in-class tutorial activities to an online environment in which students will (likely) work alone and without instructor support. This requires modeling some of the interactions students have with one another and with instructors in the form of interactive guidance elements. To support this undertaking, we will examine the ways students engage with each other, instructors, and the worksheets during in-class tutorials. Such an examination can guide changes to tutorial questions as well as the creation of interactive feedback pathways in the ACE Physics versions.

Our dataset consists of video recordings of students working on tutorials in groups. We have a large volume of video data, which presents a methodological challenge—even if only studying videos of one tutorial at a time. We must address this challenge before using the video data to improve the tutorials. Therefore this chapter has two parts. The first part introduces a methodology for analyzing videos of student groups working on tutorials—a framework we call "modified color frames" [10]—and demonstrates its application to videos of students working on two different tutorials (EPR and Entanglement[3] [22] and Time Dependence [19]). The second part explores how this process can inform changes to ACE Physics, using the Time Dependence tutorial as an example.

5.2 Overview of data collection

During two semesters of an upper-division quantum mechanics course at a large, R1, predominantly white university, ACE Physics tutorials were used during synchronous tutorial recitation sections, which were conducted virtually via Zoom (video conferencing software). The tutorial sessions, like all other class time during those two semesters, were conducted virtually because of the COVID-19 pandemic. ACE Physics was conceived of before the COVID-19 crisis, and the online tutorials are intended to complement in-person classes. Their *raison d'être* is to enable the use of

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[3] We are grateful to A. Heckler for providing us his version of the EPR and Entanglement tutorial, on which the ACE Physics version is based. Heckler's version is derived from one of C. Singh's QuILTs [132].

interactive, guided-inquiry activities outside of the classroom, thus extending their benefits to more quantum mechanics students at more times. Naturally, however, the sudden and long-lasting shift to fully- or partially-remote teaching precipitated by the pandemic created an obvious and immediate need for remote-friendly instructional strategies and materials.

The early versions of the ACE Physics tutorials used during these semesters were typically quite similar to the original, on-paper versions, and did not include many interactive guidance elements. Our investigation here is of in-class tutorials that happened to be distributed via ACE Physics; had the courses been taught in person, the tutorials would have been administered on paper instead. We did find, however, that ACE Physics was an effective way to administer tutorials in a virtual classroom.

In the virtual tutorial sessions, students worked on the tutorials in groups in Zoom breakout rooms while instructors moved between these breakout rooms. If all group members consented, one student in each breakout room was asked to record their Zoom session and share the recording with the research team at the end of the session. Overall, we collected a roughly 45-minute long video for most groups for each of the tutorials completed during the semester. This included 4 ACE Physics tutorials in the first semester (Fall 2020), and 9 in the second semester (Spring 2021). During the recording, one student typically shared their screen showing the ACE Physics website.

Groups usually consisted of 3 or 4 students, but very occasionally had 2 or 5 students. The instructional team included the professor (PER faculty), a graduate teaching assistant (author GC), and an undergraduate learning assistant who had taken the course in a previous semester. Students were allowed to form their own groups. New groups were formed each week, although many students worked with the same partners during most weeks. The tutorial sessions were optional and were attended by approximately 30% of the class (many but not all students who did attend did so consistently week after week). This fraction indicates a selection bias in our dataset, one likely exacerbated by pandemic-related difficulties that may have made class attendance more difficult for some students. Nonetheless, tutorial attendees were fairly representative of the class as a whole in terms of their course performance.

5.3 Methodology for video analysis

There is extensive precedent in physics education research (PER) for the analysis of group work in various environments, such as [4, 55, 95, 119, 138]. Videos of students working in groups on conceptual quantum mechanics activities are rich and could be studied through a variety of lenses. In this chapter we describe an analysis strategy intended to inform improvements to the ACE Physics tutorials. Before describing the strategy, we will outline what we require from it.

Our strategy should allow us to efficiently analyze an entire tutorial session for multiple groups (i.e., hours of video). Often, video analysis in PER focuses on short episodes selected from a longer recording [117], because focusing on small episodes allows for in-depth qualitative study. However, using a coarser coding strategy applied to entire videos will enable us to characterize student engagement during the course of an entire tutorial session, and also can motivate the selection of shorter episodes for detailed examination.

Our strategy should also be applicable to multiple tutorials, each of which covers a different physics topic. Therefore, we will not code for specific student ideas (e.g., student ideas about basis) or resources, because these will likely vary between tutorials. Instead of coding for what students are thinking—how they are reasoning about the specific tutorial content—we will code for what they are doing—with whom they are interacting, and how.

We will not, however, focus on group dynamics. As discussed in Section 2.5, social factors play a large role in the in-class tutorial environment, and another project should study the quality of group interactions in our video data. There are, however, two reasons for us to focus elsewhere. First, we expect that group dynamics are influenced more by the classroom and cultural context of the tutorial sessions than by the tutorial worksheets themselves, but our goal is to improve the tutorial worksheets. Second, group dynamics will not be a large factor for ACE Physics tutorials assigned as homework, on which students will likely work alone.[4] Group dynamics are inevitably in play during in-class tutorials, so they will play a role in our analysis, but we will not set out to

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[4] As reported in Chapter 7, students do typically work alone on ACE Physics tutorials assigned as homework.

characterize their nuances.

5.3.1 Coding videos using modified color frames

We have developed a framework for coding videos of student group work on Zoom based on work by Scherr *et al.* [118]. Codes were assigned for every minute of video, and we additionally tracked which page and question students were working on during each minute, as well as whether the instructor was present in the Zoom breakout room. If students moved between pages or questions within the same minute, that minute was labeled with whichever page or question they worked on for the longest. As a result, in some cases, the coding for quick-moving groups might indicate that they never worked on a question although they spent about 30 seconds on it. Similarly, instructor presence was only indicated when the instructor was present for the majority of the minute. Beyond these facts, we also coded each minute for the ways students interacted with each other, instructors, and the worksheet.

Scherr *et al.* created a framework called *color frames* that defines four categories ("frames") for students behaviors during in-class activities [118]. The four original frames are: "worksheet frame" (blue); "discussion frame" (green); "TA frame" (red); "joking frame" (yellow). In the original framework these categories are mutually exclusive: a student group will be in exactly one frame for each unit of time coded. Each frame is associated with a set of behaviors that can be easily identified in a video (e.g., "hands quiet, face neutral" for the worksheet frame) [118]. We set out to apply the color frames framework to the recordings of virtual tutorial sessions, but during the process we made some modifications to the framework, hence our new version: *modified color frames.* We describe the development of the modified framework in detail in [10], but we summarize the framework below.

Table 5.1 presents the four modified color frames and associated student behaviors, as well as differences from the original framework. Many, but not all, of the changes were made to account for the virtual setting. For example, the blue frame was renamed from "worksheet" to "individual" because in our recordings of Zoom sessions we cannot always distinguish students working quietly on the tutorial worksheet from students distracted by other things on their computer screen or in

Discussion frame — Green	**Individual frame — Blue**
(1) Prolific gesturing. (2) Animated tone, face, *clarity of speech.* (3) Sit up straight, *focus on screen* (4) *Disagreement between students not immediately resolved with clarification.* (5) *Substantial modification of ideas exchanged.*	(1) Eyes on paper, *screen, or downward.* Brief glances to peers *or screen.* (2) Muttering, *trail-off sentences, vague language.* (3) *"Echo-back" talk without disagreement or substantial modification.* (4) Hands quiet, face neutral
Friendly frame — Yellow	**Authority frame — Red**
(1) Giggle, smile, self-touch, fidget, unsettled gaze, *glance away from screen, shielding/dismissive gestures.* (2) Laughter, casual language, friendly/familiar demeanor	(1) Sit up straight, *attention on authority figure.* (2) *Acceptance of authority's claim with minimal modification or disagreement.* (3) Reduced gestures.

Table 5.1: Modified color frames and the behaviors that were coded for each frame. The original color frame definitions were presented by Scherr *et al.* in [118]. *Emphasized text* indicates behaviors that have been added or modified for the purposes of this study. Table reproduced from [10].

their physical environment. Additionally, a variety of behaviors were added to each frame to allow them to be distinguished in the virtual setting. The four renamed frames are: “individual frame” (blue); “discussion frame” (green); “authority frame” (red); “friendly frame” (yellow).

The modified color frames are *not* mutually exclusive; every minute of video was assigned *at least* one frame, but possibly more, as long as the group spent at least 20 seconds in a given frame. In some cases this meant groups switched quickly between frames, but we also found that it was possible for groups to inhabit multiple frames at the same time. For example, whereas friendly frame alone typically indicated off-topic conversation, a group having an overtly casual discussion that was nonetheless on-topic would be coded as being in both the discussion frame and the friendly frame simultaneously.

The modified framework also differs from the original because the red frame (now “authority”, previously “TA”) no longer requires instructor presence. We observed cases where an individual student temporarily assumed an authoritative role in a discussion. It is possible that this phenomenon was more apparent in the virtual setting, because it may be easier for an individual to dominate a video conference as opposed to an in-person conversation. Whatever the reason, we found this phenomenon worth capturing as distinct from green-frame discussion. Moreover, instructor presence no longer automatically triggers the red frame, because we observed moments where instructors visited Zoom breakout rooms and either remained silent or engaged with students without adopting an authoritative tone.

5.4 Applying the modified color frames framework

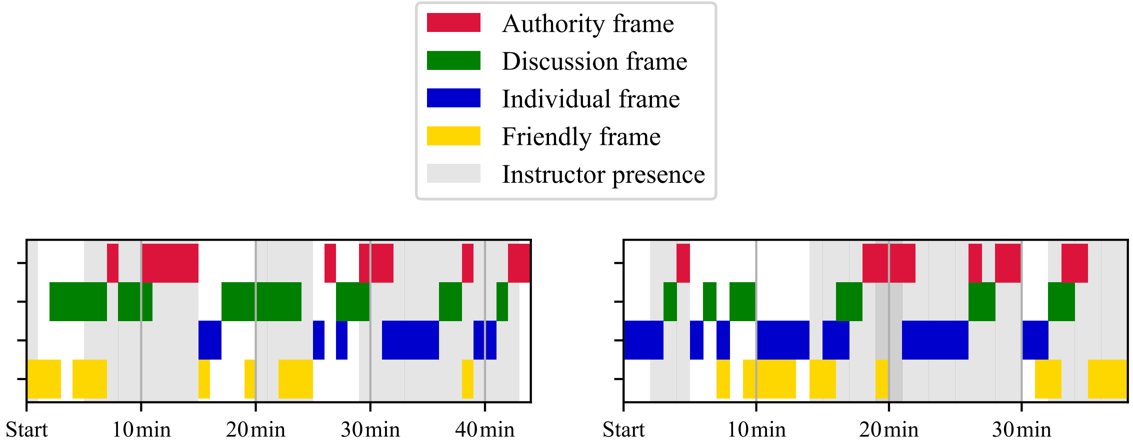
We have applied our coding scheme to recordings of students completing two different tutorials: the EPR and Entanglement Tutorial (EPRT) [22] and the Time Dependence Tutorial (TDT) [19]. The TDT is intended to help students visualize the time-dependence of one-dimensional spatial wave functions and includes a simulation [141]. Meanwhile, the EPRT covers quantum entanglement and develops a simple quantum key distribution scheme. The EPRT is distinct from our other tutorials because most of its questions are multiple choice, and students are provided with the correct answers to every question at the end of each page. (We employ this technique when administering the EPRT on paper, in person, as well as on ACE Physics; see Section 8.3.6.) The application of the modified color frames framework to the EPRT data was previously reported in [10], but the application to the TDT has not been reported elsewhere. As reported in [10], we found an inter-rater reliability rate of 87% for the portions of the EPRT data that were coded independently by two researchers. This section demonstrates what the modified color frames framework tells us about student behavior during tutorials generally; the next section will explore how these results can inform possible changes to ACE Physics.

Video data for the EPRT includes 3 groups from the first semester of data collection (fall semester), and 5 groups from the second semester (spring). Video data for the TDT only includes 5 groups from the spring semester, because the ACE Physics version of the TDT was not yet complete in the preceding fall. Although there was some overlap, the 5 spring groups for the EPRT and the 5 groups for the TDT did not consist of the same students and were considered to be independent groups for the purpose of analysis.

Figures 5.1 and 5.2 present timelines showing which frame(s) each student group was in at each minute over the course the session for the EPRT and TDT, respectively. The timelines also show when an instructor was present in the given group's Zoom breakout room. There are some commonalities between timelines—for example, every group spent some time in each frame—but each one is idiosyncratic and creates a unique fingerprint of each group's tutorial session.

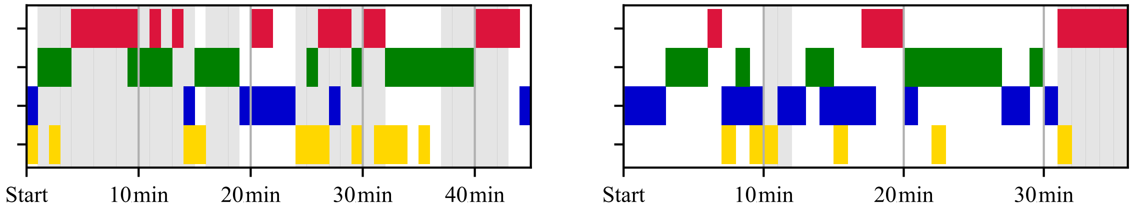
As apparent in the timelines, there are many instances where multiple codes were given to a single minute. The average number of codes assigned for each minute was 1.3. A total of 385 minutes (67%) had one color frame identified, 183 (32%) had two colors, and 3 (1%) had three color frames coded. No minute was coded with all four frames. For those minutes coded with two frames, one of them was the friendly frame (yellow) approximately two thirds of the time. In addition, only 20 total minutes (4%) were coded exclusively with the friendly frame.

Most groups spent most time in either individual frame or discussion frame, both of which sometimes overlapped with the friendly frame. Some groups biased towards individual frame (e.g.,



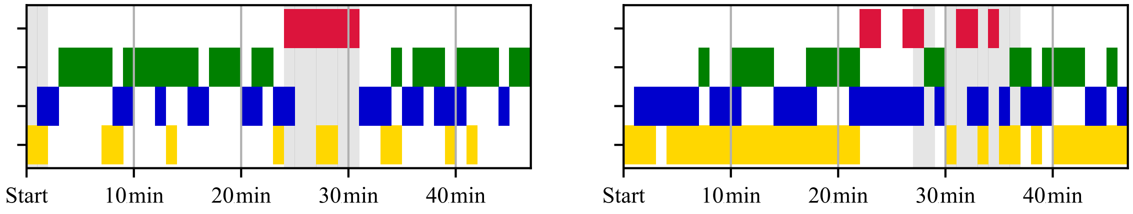
(a) EPRT Fall Group 1

(b) EPRT Fall Group 2



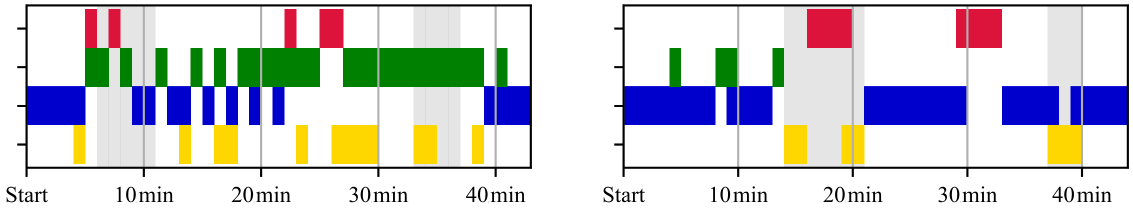
(c) EPRT Fall Group 3

(d) EPRT Spring Group 1



(e) EPRT Spring Group 2

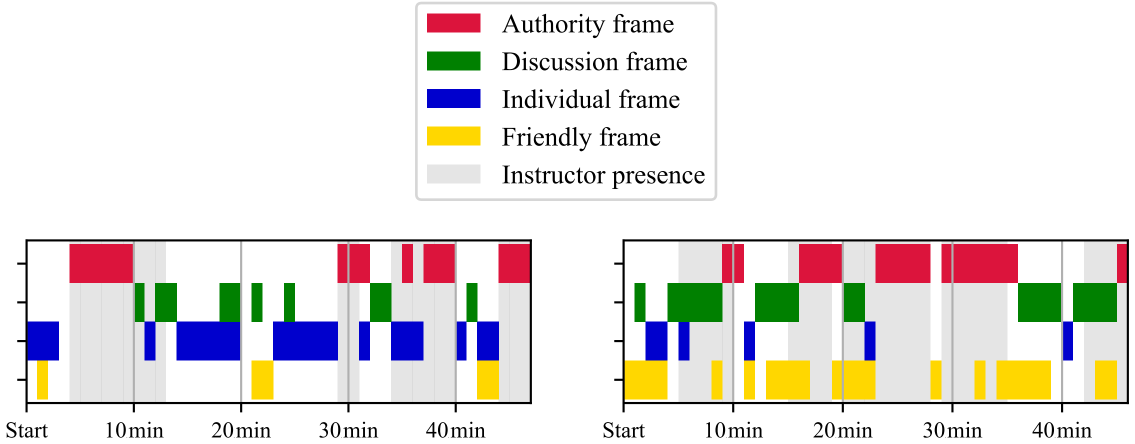
(f) EPRT Spring Group 3



(g) EPRT Spring Group 4

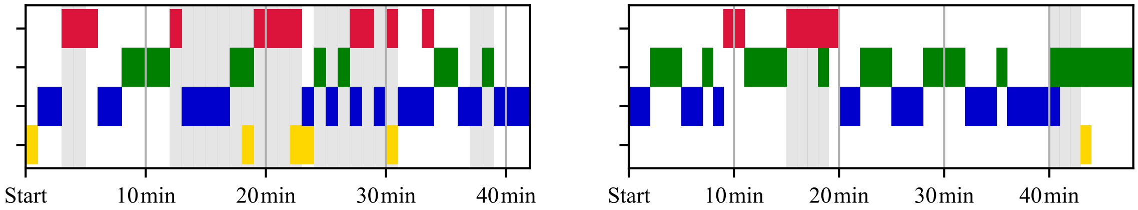
(h) EPRT Spring Group 5

Figure 5.1: Color frame timelines for eight groups completing the ACE Physics EPRT. Instructor presence was more common in the fall semester because of a higher instructor-student ratio. The timeline for Spring Group 3 (f) was previously reported in [10].



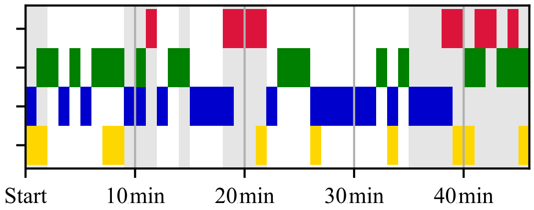
(a) TDT Group 1

(b) TDT Group 2



(c) TDT Group 3

(d) TDT Group 4



(e) TDT Group 5

Figure 5.2: Color frame timelines for five groups completing the ACE Physics TDT.

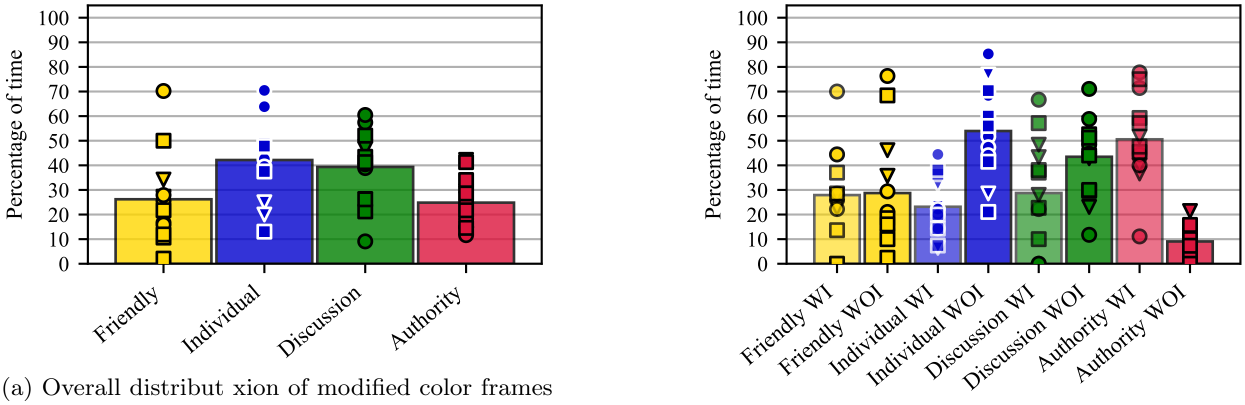
Figure 5.1h) while others biased towards discussion (e.g., Figure 5.2b), but most groups had a fairly even mix of both. In some cases (e.g., Figure 5.2e), groups spent only a few minutes in discussion or doing individual work before switching to the other frame, but some timelines do show prolonged periods of either individual work or discussion (e.g., Figure 5.1g).

While some groups spent little time in the friendly frame, others were in that frame for the majority of the time—most notably EPRT Spring Group 3 (Figure 5.1f). A significant majority of time spent in the friendly frame overlapped with time spent in other frames, especially individual or discussion. This indicates that, although students were interacting in an overtly casual or jovial manner, they were nonetheless staying on task and discussing the tutorial content.

Although the individual timelines are idiosyncratic, we can identify some trends by aggregating the data from each group. The bars in Figure 5.3a show the average fraction of the tutorial time that groups spent in each color frame. Additionally, we have separated the data into times when the instructor is present and times when the instructor is not present (Figure 5.3b). The averages do not vary significantly between the EPRT and the TDT, so the figures include data from both tutorials. The individual points in the figure represent individual groups. These points are included because, for each color frame, there is a high variance across groups for the fraction of time they spent in the frame. This means the average values are of limited utility; we cannot make strong statistical statements with only 13 groups.

As shown in Figure 5.3a, the individual (blue) and discussion (green) frames are most common, both arising in roughly 40% of tutorial time on average. Meanwhile, the authority (red) and friendly (yellow) frames arise in roughly 25% of tutorial time on average. All groups spent some time in every frame, and all frames account for at least ∼10% of tutorial time in all groups, with the exception of one group that only spent one minute in the friendly frame (Figure 5.2d). The most pronounced outlier is the group that spent over 70% of the time in the friendly frame. The timeline for this group is shown in Figure 5.1f, which shows that most of the time the group spent in the friendly frame overlapped with time spent in other frames.

Figure 5.3b shows the effect of instructor presence on the breakdown of modified color frames.



(a) Overall distribut xion of modified color frames for the 13 coded videos. Each bar represents the aggretate of times with and without instructor presence.

(b) Distribution of color frames with/without instructor presence (WI/WOI). Percentages are of the time with/without instructors—not total time. Instructors were present 40% of the time on average.

Figure 5.3: The points show the time spent in each frame as a fraction of the total time each group spent working on the given tutorial. Triangles indicate EPRT groups from the fall semester, circles indicate EPRT groups from the spring semester, and squares indicate TDT groups. The bars indicate the averages over all 13 groups. Because multiple codes could be assigned to each coded minute, the percentages add to more than 100%.

Unsurprisingly, instructor presence increases the authority frame significantly: groups spent only 10% of the time without instructors in the red frame, but this increased to about 50% of the time when instructors were present. On the other hand, no groups spent 100% of instructor time in the red frame, indicating that instructors can be present without dominating the discussion.

On average, instructor presence reduced the discussion frame by approximately a third (from 42% to 29%). For one group, instructor presence completely eliminated the discussion frame, but other groups tended to have active discussions regardless of instructor presence. This was more common in the fall semester, which had a higher instructor-student ratio, meaning instructors were present for a larger percent of the time. Perhaps surprisingly, instructor presence did not significantly impact the time spent in the friendly frame. This may reflect the instructors' intentions of cultivating a friendly, informal tutorial environment.

Instructor presence significantly reduced time spent in the individual frame for every group (from 54% on average without instructors to 22% on average with instructors). With instructors in the Zoom breakout room, students tended to not work individually. To some extent, instructor presence tended to replace time spent in the individual (blue) frame with time spent in the authority (red) frame. It is possible that instructors tended to leave breakout rooms when students began to return to individual work, which would amplify this trend.

5.4.1 Discussion

We found that all four color frames arose in every group.[5] The individual and discussion frames were the most common, both arising in an average of 40% of coded minutes for each group. The time spent in the individual frame was much lower when an instructor was present in the room.

We do not know whether we would observe similar patterns during in-person tutorial sessions. We expect that the virtual environment may skew group dynamics in favor of more individual work interspersed with sporadic discussions, as students do not share a physical environment and thus may be less inclined to engage with each other the same way they did in person. Further study is

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[5] Although TDT Group 4 (Figure 5.2d) only had one minute of friendly frame.

required to illuminate differences between group work online and in person.

Modifying the color frames framework allowed us to categorize student behaviors throughout a single virtual tutorial to gain insights into the varied ways students interact with each other and instructors. By broadening the frames and allowing them to overlap, we were able to describe some group behaviors not otherwise captured. For example, we distinguished overtly casual, but nonetheless on-topic discussion (friendly and discussion frames simultaneously), from distracted or unfocused discussions irrelevant to the tutorial (friendly frame only).

We also discovered that the presence of an instructor does not necessarily place students in the authority frame, but that individual frames and authority frames rarely overlapped. We believe it is possible that the lack of overlap between individual and authority frames results from instructors "pulling students out" of the individual frame, where they can then engage in any of the other frames. Because not every instance of the authority frame is initiated by instructors, we believe it is beneficial for both instructors and students to notice when authority frames are repeatedly initiated by students without an instructor. Although we did not focus on student content understanding in our analysis, we did notice that students who act as authorities do not always relay correct information to their peers.

Finally, we were pleased to discover that the friendly frame did not eliminate the possibility of productive discussion between students. Not every instance of friendly and familiar behavior is inherently off-topic. In fact, we observed individual episodes where friendly, casual interactions appeared to encourage all students to participate in conversation and lead to productive discussions. While we cannot use these results to make generalized claims regarding "typical" group behaviors, it does appear that all frames likely serve important and necessary roles in group completion of tutorials (given that all frames arose in all groups).

5.5 Implications for changes to ACE Physics

In the previous section, we presented results of applying the modified color frames framework to video data of students working on the EPRT and the TDT. In this section, we will explore how we can apply these results to inform changes to the ACE Physics, which is intended to be completed by students working alone and without instructor support. We will use the example of the TDT to demonstrate the types of changes our analysis can inform.

Before looking at color frames, we can consider timing data available from the videos. For example, the videos show us that although the version of the TDT completed by these students included 5 total pages, groups finished at most 3 pages during the in-class tutorial session (in contrast with the EPRT, which many groups finished during the session). On average, groups spent 25 minutes on the first page, 16 minutes on the second page, and 8 minutes on the third page (note that not every group finished the third page). These data are useful as tutorial designers: the first page is intended as scaffolding for the rest of the tutorial; if anything, we would hope students spend the most time on the second page, which encourages them to explore an interactive simulation. This suggests we need to revisit and likely shorten the first page of the TDT, although it might also imply that the students doing the tutorial were under-prepared for the content of this page.

It is useful to know that the first page of the TDT is too long, but that doesn't paint a complete picture. Although in some cases the content of an entire page (or even an entire tutorial) must be reconsidered holistically, we typically adjust individual tutorial questions independently. To inform changes to individual questions, we can examine the per-question timing and color frame breakdown presented in Figure 5.4.

The figure shows the time each of the five groups spent on each question in the TDT (questions that no group attempted are omitted), as well as the average time across all five groups. Individual groups are represented as points in the figure. Because precision was limited to single minutes, some groups were coded as taking the same amount of time on some questions, meaning the points representing some groups overlap. In addition, not all groups got to all the questions on page 3. For these reasons, and because there are only five groups, the average values are not statistically meaningful, but they are useful for visualizing the breakdown of time the groups spent in each color frame during each question.

It can be useful to attend to outlying groups (e.g., the group that took 10 minutes to complete

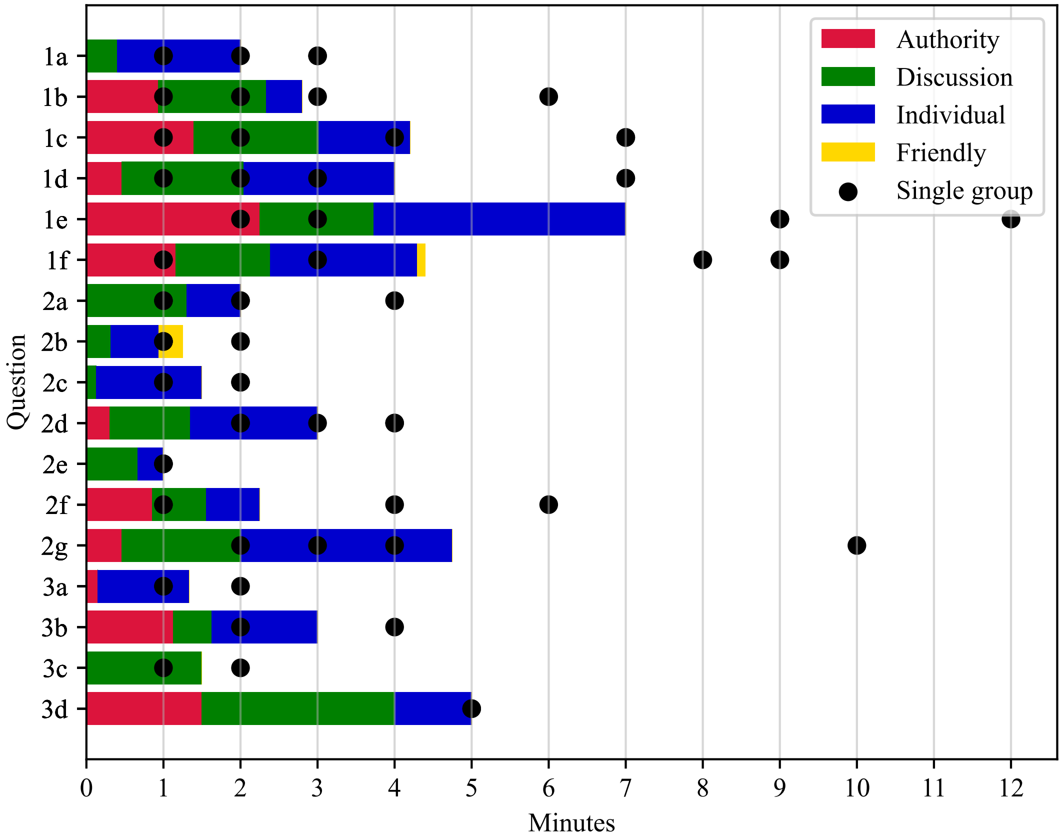

Figure 5.4: Color frame breakdown of time spent on each question in the TDT, reflecting the 5 groups included in Figure 5.2. The width of each bar shows the average amount of time the groups spent on the given question in the TDT. The bars are divided into colored sections showing the average fraction of the time that groups spent in each color frame on the given question. As discussed in the text, friendly frame was disregarded unless it did not overlap with other frames. The points represent the amount of time that individual groups spent on each question. In some cases, multiple groups were coded as working on the same question for the same number of minutes, so the points for those groups overlap in the figure.

question 2g) when considering changes to the tutorial. ACE Physics tutorials are intended for every student, not just the "average" student. If four groups breeze through a question but a fifth struggles with it, it is reasonable to imagine that a subset of students working alone will also struggle with that question. ACE Physics should support these students; in some cases, it may even be possible to do so without changing the way other students experience the question (e.g., via feedback that only becomes visible if the question is answered incorrectly). We can use the different color frames to motivate different types of changes to the ACE Physics tutorial.

5.5.1 Implications of each frame for the solo tutorial environment

The bars in Figure 5.4 show the average time that groups spent on each question in the TDT. The bars are broken into colored sections showing the average fraction of the time groups spent in each color frame on the given question. Each frame suggests certain considerations when modifying tutorial questions for the solo environment.

Friendly frame: For the purposes of this analysis, we disregarded the friendly (yellow) frame whenever it overlapped with another frame. (As shown in the figure, minutes coded as exclusively friendly frame were rare.) The friendly frame, which involves casual or joking behavior, is not especially relevant for the outside-class ACE Physics tutorial environment in which students will work (primarily) alone. As discussed above, some tutorial groups (e.g., Figures 5.1c and 5.2b) spent a significant amount of time—in some cases, the majority of the time—in the friendly frame plus another frame. That this frame is not a factor for the solo ACE Physics environment reflects a loss of this environment compared with the in-class tutorial environment. The other three frames—individual (blue), authority (red), and discussion (green)—do, however, have analogs in the solo environment.

Individual frame: The individual frame translates most directly to the solo environment; arguably, students working in the solo environment spend 100% of their time in the individual frame. Questions dominated by this frame, such as 1a and 2c in Figure 5.4, therefore require the least attention when adjusting the tutorial for the solo environment—for the most part, students seem able to solve these questions alone. This does not, however, mean they require no attention.

There are three things to consider for the individual frame. First, one behavior present in the individual frame was "echo-back talk"—typically, this meant students worked out the questions individually but briefly checked in with one another to validate their answers. To mimic this in ACE Physics, we can check students' answers on these questions without necessarily providing elaborate feedback or explanations. Second, extended periods of individual frame can suggest that students need time to solve a question, such as by working out algebra. This can be done in the solo environment as well, but it may be worth adding hints to such questions to allow students to confirm that they are on the right track. Finally, extended stretches of individual frame—especially for multiple questions in a row—may indicate that the questions are easy for students, meaning they could possibly be shortened or removed.

Authority frame: In the solo environment, the tutorial itself—specifically, the built-in dynamic feedback—takes on the "authority" role. The presence of the authority frame during a question may indicate that students required instructor assistance when answering it. For a given question, the tutorial can assert its authority to provide assistance at two points: either before students have answered, or after. To provide guidance before students answer, we can add clear hints or possibly modify the language of the question to be more direct. Once they answer the question, the tutorial can check students' answers and provide direct feedback including detailed explanations—instructors will, at times, provide similarly heavy-handed guidance to students during in-class tutorials, and these moments will be captured in the authority frame. In rarer cases, the authority frame may also indicate that an instructor is leading students in a discussion that extends beyond the scope of the specific question; it may be possible to model some of this discussion in ACE Physics, but exactly how will depend on the question.

Discussion frame: The discussion frame is likely the most difficult to capture in the solo tutorial environment. In the in-class environment, discussion can indicate that students are able to collectively answer a question, but that they may not have been able to do so individually. Discussion often arises when students have differing opinions on the answer to a question. In some cases, one student has the answer exactly right while their peers do not, but often students work together to reach the correct answer. Without peers available for discussion in the solo tutorial environment, students may struggle to answer some questions and may resort to their initial impulse for an answer, which may be incorrect.

The mechanisms for modeling discussion available in ACE Physics are the same as those for modeling authority: hints and feedback. When modeling discussion, however, we can use less heavy-handed language, such as employing Socratic questioning. For example, if a student submits an incorrect answer, we might encourage them to consider a specific physical scenario that makes their mistake obvious as opposed to simply telling them the correct answer. The choice of language will depend on the specific question and can be informed by watching the minutes coded as discussion in the original video. Watching these portions of the video can also reveal whether the discussion is indeed a debate about the answer to the question or if it is instead a conversation making sense of that answer. The former case can be modeled in ACE Physics by asking students to evaluate various fictional student statements, whereas the latter case is better modeled through hints and feedback.

5.5.2 Summary of resulting changes to the ACE Physics TDT

As discussed above, we found that students spent longer than expected on the first page of the TDT. Furthermore, as shown in Figure 5.4, much of this time was spent in the authority or discussion frames. Because this page is intended as review meant to scaffold the rest of the tutorial, we decided to shorten it. In terms of color frames, we essentially set out to modify the questions so that they were more amenable to individual work. In some cases, this meant converting open-ended questions to close-ended, multiple choice versions. In addition, we added answer-checking and feedback to each of these questions, so that students who were uncertain could be brought up to speed before continuing with the rest of the tutorial. As shown in the figure, question 1e took students a particularly long time to work through. We broke this question down into multiple smaller questions, which we felt drew attention to the key ideas of the original question and were more amenable to targeted feedback.

The data for the second page of the tutorial did not suggest that such significant changes were required. Question 2g took students the longest on this page, but this was consistent with our expectations and intentions. Although not every group reached the third page, we did observe that question 3d prompted a lot of discussion for the group working on it. Similar to question 1e, we broke this question into multiple parts and added a variety of hints and feedback that escalated from being discussion-oriented to authority-oriented. When revising the tutorial for ACE Physics, we did make some changes to the fourth and fifth pages based on our intuition as instructors and experience running the on-paper tutorial in previous semesters, but these modifications were more limited because we did not have student data to motivate them.

5.6 Discussion and conclusion

We presented a framework—modified color frames—for analyzing videos of virtual tutorial group work. The framework allows us to characterize student behavior over the duration of a tutorial session. All of the student groups we observed spent time in every frame indicating that each one—*friendly, individual, discussion, and authority*—plays an important role in the in-class tutorial environment, but there was significant variation in the breakdown of frames between groups. The individual and discussion frames were the most prevalent, and we observed that the friendly frame often overlapped with other frames. We also found that instructor presence tended to correspond with increased time spent in authority frame and reduced time spent in individual frame, but had less impact on the discussion and friendly frames.

We also explored how this framework can inform changes to ACE Physics tutorials to support their use by students working on tutorials alone, outside of class. The individual, discussion, and authority frames can each be modeled to some extent in the solo tutorial environment, but the friendly frame does not have a direct analog in that environment. Considering each of these frames allows us to better articulate the goals of individual guidance elements added to ACE Physics; for example, when writing a specific piece of feedback, it is useful to consider whether we intend for it to model discussion versus authority.

Finally, we examined the breakdown of time groups spent in each frame for each question on the Time Dependence Tutorial. Doing so allowed us to target our efforts when revising that tutorial for individual student work. Our analysis helped us decide which questions to focus on, but modifications to the activity depended on the specifics of each question and were best informed by re-watching episodes of discussion or authority in the videos. In some cases, we could use the dialogue in the videos to motivate feedback elements intended to mimic that dialogue, but in other cases the types of changes we made were idiosyncratic to the specific question and episode. The framework nonetheless served as a useful starting point for informing these changes. In the next chapter, we will measure content learning gains associated with the revised ACE Physics Time Dependence Tutorial, which was assigned to students as homework.

Chapter 6

Measuring content learning gains of two ACE Physics tutorials[1]

6.1 Introduction

In this chapter we evaluate the effectiveness of two ACE Physics tutorials by investigating whether the activities produced content learning gains. We focus on two tutorials: the Quantum Basis Tutorial (QBT) and the Time Dependence Tutorial (TDT). As discussed in Chapter 1, content learning gains are a necessary but incomplete metric for the success of instructional activities. In Chapter 7 we will discuss student engagement with the ACE Physics tutorials they did as part of homework assignments, which serves as another measure of the success of the tutorials.

The first tutorial, the Quantum Basis Tutorial, is about basis and changing basis. Quantum mechanical problems often require converting between bases. Chapter 4 explored student ideas about basis and change of basis and identified conceptual and procedural challenges that students encounter when learning these topics in a spins-first quantum mechanics course. We developed an on-paper version of the QBT to support student learning about basis.[2] Our prior work described the development process for the on-paper QBT and presented preliminary evidence of its effectiveness [115]. We have since developed an ACE Physics version of the QBT.

The QBT discusses basis in the context of spin-$1/2$ quantum systems. In this context, changing

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[1] This chapter expands on work previously published as [18], which evaluated the QBT, but not the TDT.

[2] The on-paper Quantum Basis Tutorial is available at [107].

basis refers, for example, to the calculation required to derive Equation (6.2) from Equation (6.1),

$$|\psi\rangle = \frac{2}{\sqrt{5}}|+\rangle + \frac{1}{\sqrt{5}}|-\rangle \tag{6.1}$$

$$|\psi\rangle = \frac{3}{\sqrt{10}}|+\rangle_x + \frac{1}{\sqrt{10}}|-\rangle_x \tag{6.2}$$

One method for performing this calculation involves projection inner products. For example, the coefficient in front of $|+\rangle_x$ is equal to the inner product ${}_x\langle+|\psi\rangle$. Other methods also work, but projection typically requires fewer algebraic steps and generalizes to higher dimensional Hilbert spaces.

As presented in [115], the QBT has two primary learning goals, which are motivated by the research presented in Chapter 4. After completing the QBT, students should be able to:

LG1. Recognize that changing basis does not change the state or the probabilities of any measurement on a state;

LG2. Use projection as a method to change basis with and without prompting.

The tutorial uses an analogy to two-dimensional Cartesian space. It asks students to represent a Cartesian vector using Dirac notation, matrix notation, and graphically all in two different bases (i.e., two sets of rotated Cartesian axes) and to compare these representations.

The second tutorial, the Time Dependence Tutorial, allows students to explore the time evolution of energy eigenstates and simple superpositions in position space. The paper version of the tutorial was developed as part of work described in [99]. The tutorial makes use of an interactive simulation (also described in [99]) to help students visualize the time evolution of a wave function as a rotation in complex space. The tutorial aims to support student understanding of energy and position measurements of both stationary states and superposition states accounting for time evolution. It also encourages visualization as a tool for making sense of these states.

Both the ACE Physics QBT and the ACE Physics TDT were assigned on homework in two semesters of quantum mechanics. The QBT was unchanged between semesters, and the QBT analysis in this chapter uses data only from the first semester. Meanwhile, the TDT was modified significantly (see Chapter 5) between semesters, and the TDT analysis in this chapter uses data from only the second semester.

As discussed in Chapter 2, we consider the classroom to be the ideal tutorial environment, and we encourage faculty to consider running in-class tutorials for quantum mechanics using the variety of available resources (e.g., [33, 62, 67, 105, 107, 132, 140]), including possibly ACE Physics itself (the guidance elements may be useful in classes with high student-instructor ratios). However, we recognize that in-class tutorials are not always an option, and we hypothesize that students will benefit more from an ACE Physics tutorial than from no tutorial at all. This chapter presents a preliminary test of that hypothesis for the QBT and the TDT.

6.2 Research context

This study includes data from five semesters—Fall 2016, Fall 2018, Fall 2021, Spring 2022, and Fall 2022—of the upper-division quantum mechanics course at a large, R1 university. All five semesters were taught by PER faculty using similar curricular materials [107] and the text by McIntyre [86], which adopts a *spins-first* approach to introductory quantum mechanics. A spins-first course presents the postulates of quantum mechanics in the context of spin-$1/2$ systems before discussing position-space wave functions in the second half of the semester.

All five semesters included three hours a week of in-person lecture (which included clicker questions), a weekly homework problem set, and three exams. The 2016, 2018, and 2021 semesters were taught by Steven Pollock and both included an optional, weekly, in-person recitation section during which on-paper ACEQM tutorials were run. The tutorial recitations were attended by roughly 30% of the class in each semester. The on-paper QBT was never administered during these recitations. Other than the QBT, SP used nearly identical instructional materials in all three semesters. The two semesters in 2022, taught by Bethany Wilcox, did not include a recitation section, but on-paper tutorials (excluding both the QBT and the TDT) were occasionally facilitated during lecture periods.

6.3 Investigation of the Quantum Basis Tutorial (QBT)

The ACE Physics QBT [21] was assigned as a homework problem in the sixth week of both the Fall 2021 and Spring 2022 semesters. The version of the ACE Physics QBT used in these semesters consists of only three of the five pages of the full QBT described in [115]. In both semesters, students' tutorial submissions were not graded for correctness. Collaboration on homework assignments—including ACE Physics—was encouraged in both semesters.

In Fall 2021, the QBT was the only ACE Physics tutorial assigned. To receive credit for completing the tutorial, students provided answers to two tutorial questions that were repeated verbatim on the homework assignment, one of which was Pre-test Question 1 (Figure 6.1), described below.

In Spring 2022, the QBT was the third ACE Physics tutorial assigned as homework. Additional ACE Physics tutorials were also assigned later in the semester. A new feature in the platform provided the instructor with an estimate of how much of the tutorial each student had completed, which was used to assign credit for the homework problem (about 15% of the given homework assignment). Of all the students who accessed the QBT, only one did not reach the completion threshold required to receive full credit.

The QBT was not administered in any form during the Fall 2018 semester, so we use data collected during that semester as a baseline for comparison.

6.3.1 Data collection related to the QBT

Data relevant to the QBT were collected in two settings: directly via the ACE Physics website, and on exams. The ACE Physics QBT and TDT both begin with a page entitled "Before You Start", which includes several pre-test questions. Once a student moves on from the pre-test to the remainder of the tutorial, the ACE Physics system disallows changing pre-test answers. The pre-test page instructs students to work alone, but we cannot confirm that students did not collaborate. We might expect collaboration to inflate pre-test scores.

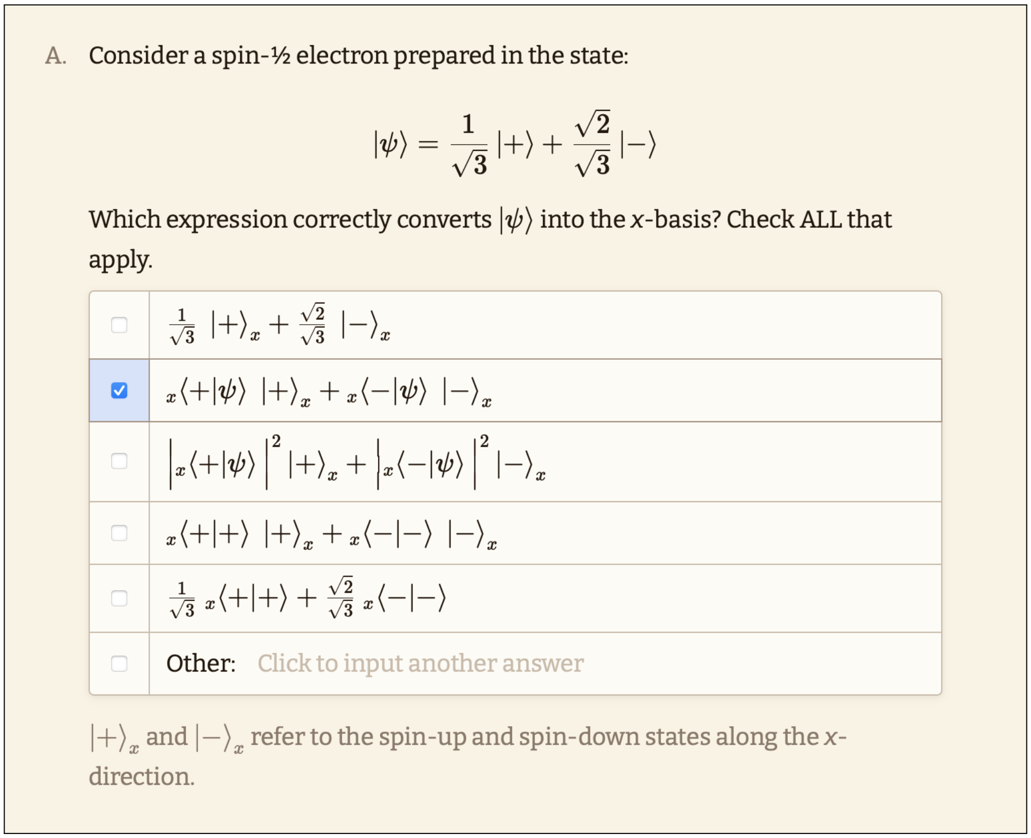
The QBT pre-test includes two questions relevant to this study. Pre-test Question 1 (Figure 6.1), which targets LG2, asks students to identify the expression that converts a ket written in the z-basis to the x-basis from a list of expressions including a variety of inner products. Students were allowed to select multiple expressions, but a student's response was only considered correct if they exclusively selected the lone correct expression.

Pre-test Question 2 (Figure 6.2), which targets LG1, asks students to consider whether changing basis affects measurement probabilities. A student's response was considered correct if they selected "False" and also provided a correct explanation. This question was repeated on the exams in Fall 2021 and Spring 2022, allowing for direct pre-/post-tutorial comparison.

One exam in each of the three semesters also included the exam question shown in Figure 6.3. The exam question, which targets LG2, asks students to convert a state expressed in the z-basis to the n-basis, defined by a pair of basis states provided in the question text. Although students' exam grades included partial credit, for the purpose of this study we only considered a student's response correct if they provided completely correct coefficients for the state in the new basis. We also coded students' exam responses according to the method they applied to change basis, specifically tracking whether they used projection inner products (indicated by expressions such as $a = {}_n\langle +|\psi\rangle$) versus any another method. (These results were previously presented in [115] for the Fall 2018 exam data.)

Due to various curricular constraints, the exam question was given on the final exam in Fall 2018, on the first midterm in Fall 2021, and on the second midterm in Spring 2022. Because of the continued relevance of basis change and projection inner products throughout the semester, we would not expect performance on this question to worsen as the semester progresses (we might expect performance to improve). The Fall 2021 and Spring 2022 exams were held after the homework assignment that included the ACE Physics QBT was due.

Other than for Fall 2018, we will only report data for students who both completed the ACE Physics QBT and took the exam. We have matched data for $N = 66$ students from Fall 2021 and $N = 92$ students from Spring 2022. These account for the vast majority of students in both semesters. In Fall 2018, $N = 63$ students took the final exam, and none did the QBT.



A. Consider a spin-½ electron prepared in the state:

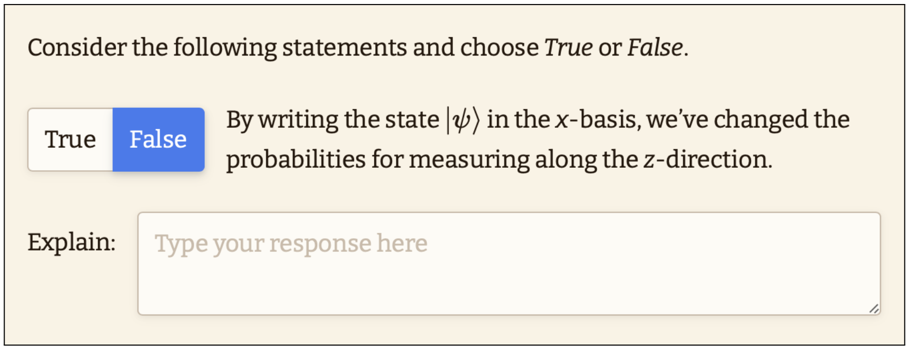
$$|\psi\rangle = \frac{1}{\sqrt{3}}|+\rangle + \frac{\sqrt{2}}{\sqrt{3}}|-\rangle$$

Which expression correctly converts $|\psi\rangle$ into the x-basis? Check ALL that apply.

- ☐ $\frac{1}{\sqrt{3}}\ |+\rangle_x + \frac{\sqrt{2}}{\sqrt{3}}\ |-\rangle_x$
- ☑ ${}_x\langle+|\psi\rangle\ |+\rangle_x + {}_x\langle-|\psi\rangle\ |-\rangle_x$
- ☐ $\left|{}_x\langle+|\psi\rangle\right|^2 |+\rangle_x + \left|{}_x\langle-|\psi\rangle\right|^2 |-\rangle_x$
- ☐ ${}_x\langle+|+\rangle\ |+\rangle_x + {}_x\langle-|-\rangle\ |-\rangle_x$
- ☐ $\frac{1}{\sqrt{3}}\ {}_x\langle+|+\rangle + \frac{\sqrt{2}}{\sqrt{3}}\ {}_x\langle-|-\rangle$
- ☐ Other: Click to input another answer

$|+\rangle_x$ and $|-\rangle_x$ refer to the spin-up and spin-down states along the x-direction.

Figure 6.1: A screenshot of Pre-test Question 1 from the ACE Physics QBT. The sole correct answer is checked. The pre-test is included as the first page of the tutorial on the ACE Physics website. The user interface in the rest of the tutorial looks similar to this screenshot.



Consider the following statements and choose *True* or *False*.

True [✓ False] By writing the state $|\psi\rangle$ in the x-basis, we've changed the probabilities for measuring along the z-direction.

Explain: Type your response here

Figure 6.2: A screenshot of Pre-test Question 2 from the ACE Physics QBT. The correct answer (False) is selected. Students were only marked as correct if their response included a valid explanation.

Consider a spin-1/2 particle prepared in the state $|\psi\rangle$, written in the z-basis as $|\psi\rangle = \frac{1}{\sqrt{2}}\,|+\rangle + i\frac{1}{\sqrt{2}}\,|-\rangle$. Write the state in the n-basis assuming

$$|+\rangle_n = \frac{1}{\sqrt{3}}\,|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}\,|-\rangle \text{ and } |-\rangle_n = \frac{\sqrt{2}}{\sqrt{3}}\,|+\rangle - i\frac{1}{\sqrt{3}}\,|-\rangle$$

Hint: Solve for the values of a *and* b *such that*
$|\psi\rangle = a\,|+\rangle_n + b\,|-\rangle_n$.

Figure 6.3: One version of the QBT Exam Question, which asked students to convert a spin-1/2 state to a different basis. Versions of this question were asked on exams in three semesters: Fall 2018, Fall 2021, and Spring 2022. Different semesters provided different coefficients for the $|\pm\rangle_n$ states, but all semesters included an imaginary coefficient. The exams in Fall 2021 and Spring 2022 also repeated Pre-test Question 2 (Figure 6.2) immediately after this change of basis question. Figure reproduced from [115].

	Without tutorial	With tutorial	
	Instructor A	Instructor A	Instructor B
	$N = 63$	$N = 66$	$N = 92$
Applied method	**Fall 2018**	**Fall 2021**	**Spring 2022**
Projection (correct/N)	57% (42%)	71% (61%)	87% (70%)
Other (correct/N)	43% (8%)	29% (6%)	13% (3%)

Table 6.1: The methods students used to change basis on the exam question shown in Figure 6.3 from all three semesters. In each row, the numbers in parentheses give the percentage of all students who both applied the given method and also produced the correct answer. The Fall 2018 data were previously reported in [115].

6.3.2 Results for the QBT

Table 6.1 reports student performance on the change of basis exam question from three similar semesters of upper-division quantum mechanics. Comparison of results across semesters will allow us to evaluate the effect of the tutorial (with caveats discussed below). We will compare (a) the number of students using projection; and (b) the number of students who answered the question correctly. We will report statistical significance (p, computed with a chi-squared test) and effect size (V, computed via Cramér's V [122]).

A goal of the QBT is for students to apply the projection method when changing basis (LG2). In Fall 2018, without the tutorial, 57% of students used projection, whereas 71% and 88% of students used projection in Fall 2021 and Spring 2022, respectively (both later semesters included the QBT). The most direct comparison can be made between Fall 2018 and Fall 2021, because both were taught by the same instructor. The increase in the fraction of students using projection from Fall 2018 to Fall 2021 is not statistically significant given this sample size ($p = 0.095$) and shows a weak effect size ($V = 0.15$). The increase from Fall 2018 to Spring 2022 is significant ($p < 0.01$) with a moderate effect size ($V = 0.34$). The difference in the fraction of students using projection between the two semesters with the ACE Physics QBT (Fall 2021 and Spring 2022) is also significant ($p < 0.05$) with a moderate effect size ($V = 0.20$), suggesting that factors beyond the QBT account for the more common use of projection in Spring 2022.

Overall, 50% of students answered the change of basis exam question correctly in Fall 2018, 67% in Fall 2021, and 73% in Spring 2022. The improvement from Fall 2018 to the other two semesters is significant in both cases ($p < 0.05$), but we cannot necessarily attribute these improvements exclusively to the QBT. The QBT focuses on the projection method for changing basis. While the correctness rate for the projection method was higher in both Fall 2021 and Spring 2022 as compared with Fall 2018, the correctness rate for students using other methods was also marginally higher. (The correctness rate is the percentage of students who applied a method correctly divided by the percentage who applied it at all.)

In addition to comparisons across semesters, we can also compare student performance on questions asked before and after they completed the ACE Physics QBT. Pre-test Question 1 (Figure 6.1) was administered to students immediately before completing the tutorial and asked students to identify the expression with the correct inner products to convert a state from the z-basis to the x-basis. Because identification of the correct inner products is a necessary step in applying the projection method to change basis, we consider these percentages an upper bound on the fraction of students who could successfully apply the projection method at the time of the pre-test.

We did not administer Pre-test Question 1 a second time in either Fall 2021 or Spring 2022; however, we consider the change of basis exam question as a post test. Specifically, we can compare the number of students answering Pre-test Question 1 correctly against the number of students who used the projection method on the change of basis exam question and, at most, made a minor error. Minor errors are limited to sign errors or simple arithmetic errors; all students who made at most a minor error used the correct inner product expressions. It is possible that students using other methods on the exam might also have been able to apply projection, so these numbers are a lower bound on the fraction of students who could apply the projection method at the time of the exam.

Before completing the ACE Physics QBT, 46% and 47% of students answered Pre-test Question 1 correctly in the Fall 2021 and Spring 2022 semesters, respectively (Table 6.2). After completing the QBT—and other relevant instruction—68% and 83% of students applied the projection method

	Pre-test	**Direct post-test**	**Indirect post-test**
Fall 2021 ($N = 66$)			
Question 1	46%	—	68%
Question 2	65%	97%	—
Spring 2022 ($N = 92$)			
Question 1	47%	—	83%
Question 2	78%	97%	—

Table 6.2: Percentage of students answering Question 1 (Figure 6.1) and Question 2 (Figure 6.2) correctly on the pre-test (given immediately before the tutorial) and on the exam (given weeks after the tutorial) from the two semesters in which the tutorial was assigned. Question 1 was not repeated directly on the exam; as the post-test percentage, we report the fraction of students that applied the projection method with at most minor errors on the change of basis exam question.

correctly or with minor errors on the exam question in Fall 2021 and Spring 2022, respectively. The improvements in both semesters are significant ($p < 0.01$) with moderate effect sizes ($V = 0.23$ in Fall 2021 and $V = 0.35$ in Spring 2022).

Table 6.2 also presents the percentages of students who answered Pre-test Questions 2 (Figure 6.2) correctly both on the tutorial pre-test and on the exam from both semesters. The improvements between the pre-test and exam are significant in both semesters ($p < 0.01$) with moderate effect sizes ($V = 0.39$ in Fall 2021 and $V = 0.21$ in Spring 2022). The pre-test scores were already fairly high—65% in Fall 2021 and 78% in Spring 2022. Nonetheless, following the ACE Physics QBT—and other relevant instruction—nearly every student demonstrated an understanding in line with LG1.

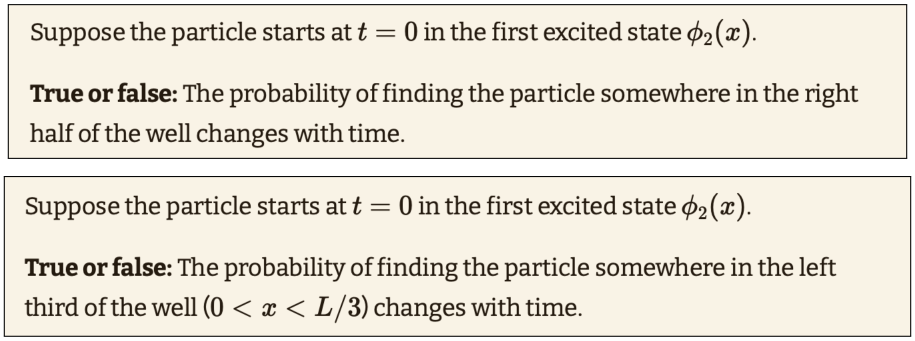
6.4 Investigation of the Time Dependence Tutorial (TDT)

The ACE Physics TDT [19] was assigned as a homework problem in the 12th week of the Fall 2022 semester. As noted in the introduction, an older version of the ACE Physics TDT was assigned in Spring 2022, but we did not collect data relevant to the TDT in that semester. The version of the TDT evaluated in this chapter includes the changes we made based on our findings in Chapter 5. As with the QBT, students received credit for completing the tutorial, but their submissions were not graded for correctness.

In Fall 2016, an on-paper version of the TDT was administered in the optional tutorial recitation section. Only 9 students attended this session, 21% of students in the class. The remaining 33 students in the Fall 2016 class did not do the TDT in any form.

6.4.1 Data collection related to the TDT

As with the QBT, data relevant to the TDT were collected in two settings: on the ACE Physics website, and on exams. Similar to the QBT, the ACE Physics TDT included a set of pre-test questions on the “Before you start” page. In addition, slightly modified versions of two of the pre-test questions were included on the final page of the tutorial (entitled “Review”), which served as an



Suppose the particle starts at $t = 0$ in the first excited state $\phi_2(x)$.

True or false: The probability of finding the particle somewhere in the right half of the well changes with time.

Suppose the particle starts at $t = 0$ in the first excited state $\phi_2(x)$.

True or false: The probability of finding the particle somewhere in the left third of the well $(0 < x < L/3)$ changes with time.

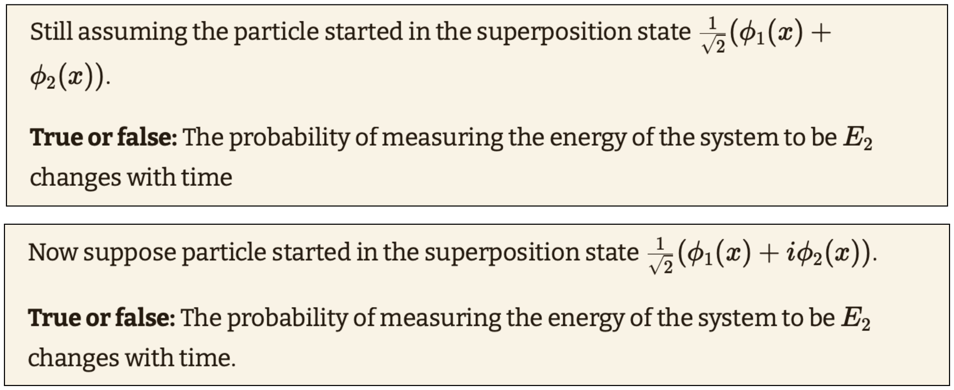
Figure 6.4: Screenshots of Pre-test (top) and Post-test (bottom) Question 1 from the ACE Physics TDT. The correct answer to both questions is false.

immediate post-test for the tutorial.

The first pre-test question and the corresponding post-test question (Figure 6.4) ask whether the probabilities of position measurement outcomes change with time for a stationary state. The second pre-test question and the corresponding post-test question (Figure 6.5) ask whether the probabilities of energy measurement outcomes change with time for a state that is a superposition of energy. As can be seen in the figures, the post-test questions are slightly modified versions of the pre-test questions. The pre- and post-test data were only collected in the Fall 2022 semester in which the ACE Physics TDT was administered.

To allow comparison between students who did and did not do the TDT, a relevant exam question that was asked in Fall 2016 was repeated in Fall 2022. The question was administered on the final exam in both semesters. The question, shown in Figure 6.6, presents students with graphical representations of two energy eigenstates of an unspecified quantum system. Students are not expected to recognize these states (they are not from a physical system previously introduced in the class).

The question prompts students to consider a superposition of the two pictured states. The five parts of the exam question are similar to questions asked on the TDT, which also starts with the graphical representation of two energy eigenstates and prompts students to consider a superposition. In the TDT, the states in question are energy eigenstates of the one-dimensional particle in a box,



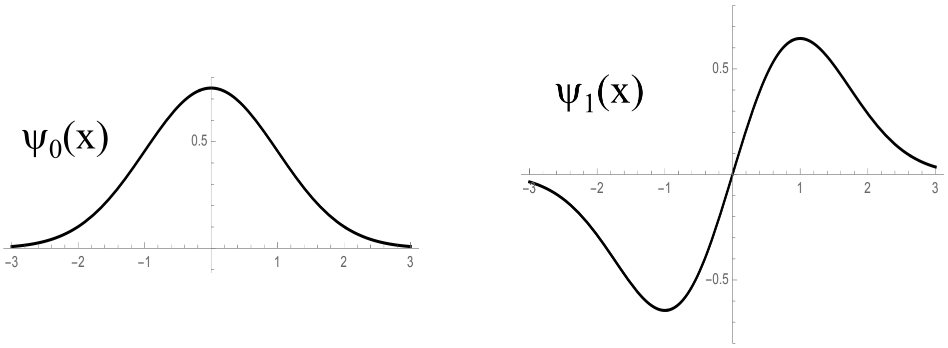
Still assuming the particle started in the superposition state $\frac{1}{\sqrt{2}}(\phi_1(x) + \phi_2(x))$.

True or false: The probability of measuring the energy of the system to be E_2 changes with time

Now suppose particle started in the superposition state $\frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$.

True or false: The probability of measuring the energy of the system to be E_2 changes with time.

Figure 6.5: Screenshots of Pre-test (top) and Post-test (bottom) Question 2 from the ACE Physics TDT. The correct answer to both questions is false.



Imagine we solve Schrödinger's equation for a new potential (one we have not yet worked on in class) and find that the (symmetric) ground state wave function looks like $\psi_0(x)$ (see above).

Assume that the energy E_1 of the first excited state is four times the energy of the ground state E_0, i.e. $E_1 = 4E_0$.

The (antisymmetric) first excited state wave function $\psi_1(x)$ at time $t = 0$ is also shown above. (Both functions are real, normalized, drawn to the same scale.)

The system is initially (at time $t = 0$) in a state given by,

$$\psi(x, t = 0) = \frac{1}{\sqrt{2}} (\psi_0(x) - \psi_1(x))$$

(A) Sketch the wave function $\psi(x)$ for this state at $t = 0$. (A crude sketch is fine)

(B) Sketch the probability density $|\psi(x)|^2$ for this state at time $t = 0$.

(C) Is this state a stationary state? Explain your reasoning.

(D) Is there ever a time when this spatial wave function is *exactly equal* to the ground state wave function $\psi_0(x)$? If so, give such a time. If not, explain why not.

(E) Recall we said $E_1 = 4E_0$. (Please note that factor of 4).
At time $T = \frac{\pi}{2}\frac{\hbar}{E_0}$, sketch the position probability density, i.e. $|\psi(x, T)|^2$.
Please show/explain your work so I understand where your sketch is coming from.

Figure 6.6: The TDT Exam Question, which included five parts and was asked on the final exam in both Fall 2016 and Fall 2022.

with which students are expected to be familiar. Hence, the exam question pushes students to extend what they learned in the tutorial to a new context. Other than minor formatting differences, the TDT exam question was identical in Fall 2016 and Fall 2022.

6.4.2 Results for the TDT

Table 6.3 shows student performance on the TDT exam question (Figure 6.6) from the Fall 2016 and Fall 2022 semester. Performance of students in the Fall 2016 semester who did the on-paper version of the TDT is reported separately from performance of those students in that semester who did not do any version of the TDT. The percentages reported in the table indicate the fraction of students that got the given question completely correct. Students were coded as either correct or incorrect, without the possibility of partial credit (students *did* receive partial credit when the exams were graded). Allowances were made for the following two minor errors: students were coded as correct for parts A and B if their graphs both represented the *sum* of $\psi_0(x)$ and $\psi_1(x)$ instead of the difference; and students were coded as correct for part E despite minor errors in the shape of their graph. Students were only coded as correct for parts C and D if they both gave the correct answer and also provided a valid explanation.

There are no statistically significant differences between student performance without the tutorial (Fall 2016) and with the ACE Physics TDT (Fall 2022). These results suggest that the ACE Physics TDT had no significant impact on student performance Because only 9 students did the on-paper version of the TDT, statistical comparisons between this and the other two groups are not meaningful, but these 9 students did appear to perform better on questions C and D. There is a selection bias introduced by the fact that the on-paper TDT was optional, so it is difficult to infer anything about the tutorial's effectiveness from these data.

In addition to comparing across semesters, we can compare student performance on the ACE Physics TDT pre-test, administered immediately before the students completed the tutorial, against student performance on the post-test, which was administered immediately after students completed the tutorial. Table 6.4 shows student performance on these questions. There are no statistical

	Fall 2016		**Fall 2022**
	Without TDT	On-paper TDT	ACE Physics TDT
Question	$N = 33$	$N = 9$	$N = 67$
A	85%	89%	84%
B	58%	67%	69%
C	82%	100%	76%
D	70%	89%	66%
E	39%	33%	42%

Table 6.3: Student performance on the five parts of the TDT exam question (Figure 6.6).

	Pre-test $N = 60$	**Post-test** $N = 57$
Question 1	67%	67%
Question 2	29%	27%

Table 6.4: Student performance on the ACE Physics TDT pre- and post-test questions (Figures 6.4 and 6.5). Three students who started the tutorial did not take the post-test. These data were collected in Fall 2022.

differences between student performance on the pre- and post-test questions, indicating that the tutorial had no significant impact. Only a handful of students changed their pre-test responses on the post-test.

It is notable that student performance on Question 2 is especially poor. We suspect that students are jumping to the conclusion that the probabilities for energy measurements change with time because, in general, a superposition of energy eigenstates will demonstrate nontrivial time dependence. It is unclear whether this error reflects an underlying student difficulty or if it indicates that students were rushing to complete the pre- and post-test questions. A future study might include a similar question in a higher stakes context, such as an exam, or in interviews.

6.5 Discussion and conclusion

We assigned two different ACE Physics tutorials as homework in upper-division quantum mechanics, administered various pre- and post-tests targeting the learning goals of these tutorials, and compared student performance on exam questions against semesters in which students did not do the tutorials. One tutorial, the ACE Physics TDT, did not appear to impact student performance on post-test or exam questions. The comparison of exam performance across semesters is imperfect because the two semesters had different instructors. Nonetheless, these results suggest that the ACE Physics TDT needs to be revised and, possibly, that we need to reassess the learning goals for the tutorial.

One concerning aspect of the lack of improvement on TDT Pre-/Post-test Question 1 is that, in a written explanation of their incorrect answer on the post-test question, a small number of students pointed out that energy eigenstates do exhibit time dependence—which is technically true, a time-evolved energy eigenstate picks up a global phase. The tutorial emphasizes this point to scaffold discussion of time evolution of a superposition state (when these phase factors become physically relevant). It is possible that this emphasis misled students and caused them to conclude that energy eigenstates in fact have nontrivial time dependence.

Meanwhile, for the ACE Physics QBT, we observed statistically significant improvements in student performance on the post-tests versus the pre-tests; however, we cannot isolate the effect of the QBT because other relevant instruction occurred between the pre- and post-tests. We also compared student performance on an exam question targeting LG2 against exam data collected in a semester that did not include the QBT but was otherwise quite similar. We observed a significant increase in the number of students using projection to change basis between the semester without the QBT and one of the semesters with the QBT—Spring 2022, which was taught by a different professor. We did also observe an increase in the number of students using projection between the non-QBT semester and the other semester with the QBT—Fall 2021, which was taught by the same professor—but the difference was significant only at the $p < 0.1$ level.

What explains the discrepancy between the Fall 2021 and Spring 2022 semesters? It is impossible to fully characterize the effects of different instructors, but one notable difference is that the Spring 2022 professor emphasized the derivation of the projection method for changing basis via the insertion of the identity operator $|+\rangle\langle+| + |-\rangle\langle-|$, whereas the Fall 2021 professor did not. This form of the projection method appeared in over a quarter of Spring 2022 exam responses, whereas it only appeared twice in the Fall 2021 exam data.

How we administered the ACE Physics QBT also differed between semesters: the QBT was the only ACE Physics tutorial used in Fall 2021, whereas it was the third assigned in Spring 2022. Moreover, a more robust mechanism was used to assign homework credit for tutorial completion in Spring 2022. Students may have been better incentivized to engage with the ACE Physics QBT in

Spring 2022, as well as better prepared to do so thanks to increased familiarity with the platform. Student buy-in is relevant for any tutorial implementation, and our ongoing work investigates student engagement in the online, outside-class tutorial setting.

Overall, we consider our results sufficient to justify the assignment of the ACE Physics QBT as a single homework problem. The cost-benefit analysis would differ if administering the QBT required an entire lecture period—indeed, given the relatively high pre-test scores, we have not elected to use the on-paper version of the QBT at our institution. This is the exact goal of ACE Physics: to reduce the costs of administering tutorials while preserving some benefits. Significant future research is required to assess the extent to which ACE Physics achieves this goal more broadly (including multiple tutorials and with different student populations), and we also continue to improve the platform. Nonetheless—from the three courses considered in this study—the winning combination for teaching change of basis in a spins-first quantum mechanics class included assigning the ACE Physics QBT as homework and also emphasizing the insertion of the identity operator as a tool for understanding the projection method for basis change.

The different results for the ACE Physics QBT and TDT indicate, perhaps unsurprisingly, that ACE Physics should not be evaluated only as a monolithic platform. Although (as discussed in the previous chapter as well as in the next chapter), some student behaviors may be consistent across tutorials, different tutorials can differ in learning goals, design, guidance strategies, and also in effectiveness. One key difference between the QBT and TDT is that the QBT resulted from a dedicated research study (Chapter 4, and [115]), whereas the original TDT did not receive as much dedicated research attention.[3] Additional work is needed to refine the TDT.

Content learning gains are clearly an essential metric for the success of instructional activities, but they are not the only important metric. An activity such as the TDT that has no impact (either positive or negative) on content learning as measured by a narrow set of post-test questions may still have some pedagogical utility. The next chapter presents a complimentary evaluation of several ACE Physics tutorials assigned as homework that focuses on student engagement and feedback.

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[3] The simulation used within the TDT, though, is the subject of a dedicated study [99].

Chapter 7

Student engagement with ACE Physics outside of class

7.1 Introduction

There are multiple ways to measure the success of instructional activities. One obvious and important metric is whether the activities help students learn the intended physics content. We investigated this question for select ACE Physics tutorials in Chapter 6. However, such an investigation faces various challenges, especially for tutorials. First, because tutorials are designed to complement other instructional techniques (such as a lecture on the same topic), it can be difficult to isolate the effect of the tutorial on student learning from the effect of other instruction. Second, in addition to specific content learning goals, tutorials (including ours) often have secondary goals, such as encouraging students to reflect on new concepts and to articulate their understanding in ways they might not be prompted to do by a traditional problem set. These secondary goals cannot be assessed by evaluating student content knowledge, especially because it is expected for students to occasionally articulate incorrect or incomplete ideas while completing a tutorial.

When running tutorials in the classroom, the instructor can directly observe the ways that students engage with the activities. For example: do students tend to get stuck on a specific page, or does a particular question prompt the intended type of discussion among students? Instructors in the room can also pay attention to simpler questions: How long does the activity take students? Or, do the students stay on task and engage with the tutorial in good faith, or do they just guess answers for the sake of completing it? By paying attention to these and similar questions in the moment, an instructor will usually leave the classroom with a sense of whether (and in what ways) they consider the tutorial successful. Moreover, the presence of the instructor in the room often encourages students to stay on task.

By assigning ACE Physics for completion outside the classroom as part of homework, we lose the opportunity to directly observe student interactions with the activity. Moreover, because there is no instructor present, it is easier to imagine that some students might disengage from the activities and merely “click through” them—inputting the bare minimum to qualify for participation credit. We must, then, find other strategies to investigate the ways that students engage with ACE Physics tutorials when completing them on homework.

In this chapter, we use both quantitative and qualitative data to explore student engagement with ACE Physics outside of class. The data were collected during two semesters of undergraduate quantum mechanics in which ACE Physics tutorials were regularly assigned on homework sets. The results in this chapter complement the studies reported in Chapter 6 regarding the effectiveness of two ACE Physics tutorials as measured by student learning gains.

7.2 Methodology

7.2.1 Quantum mechanics course and administration of ACE Physics tutorials

A variety of data were collected during two, back-to-back semesters of upper-division, undergraduate quantum mechanics offered at a large, predominantly-white, public R1 university. The course is required for physics majors; a minority of students in the class come from physics-adjacent majors (such as astrophysics). Both semesters were taught by the same professor (PER faculty), meaning the class was nearly identical (three weekly lectures, use of interactive engagement such as clicker questions, weekly pre-lecture surveys, etc.) across both semesters. The primary difference between semesters was class size. The first semester (Spring 2022) had 106 students while the second semester (Fall 2022) only had 72 students. Physics majors on a typical schedule take this quantum mechanics course in the spring as opposed to the fall. The course followed a spins-first approach using the text by McIntyre [86].

The courses lasted 16 weeks and one homework set was due every week except the first week, exam weeks, and spring/fall breaks. Homework sets consisted of around 5–8 problems, similar to typical end-of-chapter textbook problems. In addition, an ACE Physics tutorial was included as an extra problem on 5 of 12 assignments in the spring and 4 of 12 in the fall. Table 7.1 shows which tutorials were assigned during which weeks. The Quantum Mouse Tutorial was completed partially in class and partially on homework in the spring, but was done completely in class in the fall.

Students were awarded 15 points for completing the assigned ACE Physics tutorial, 15% of the total 100 points available on each homework assignment. Students were told

> *These activities will be graded for participation only—just work them through and you will get the full points on this question.*

A feature of the ACE Physics platform provided the instructor with an estimate of how much of the tutorial each student had completed. The vast majority of students who began a tutorial as part of homework completed enough to receive credit (about 15% of the given homework assignment).

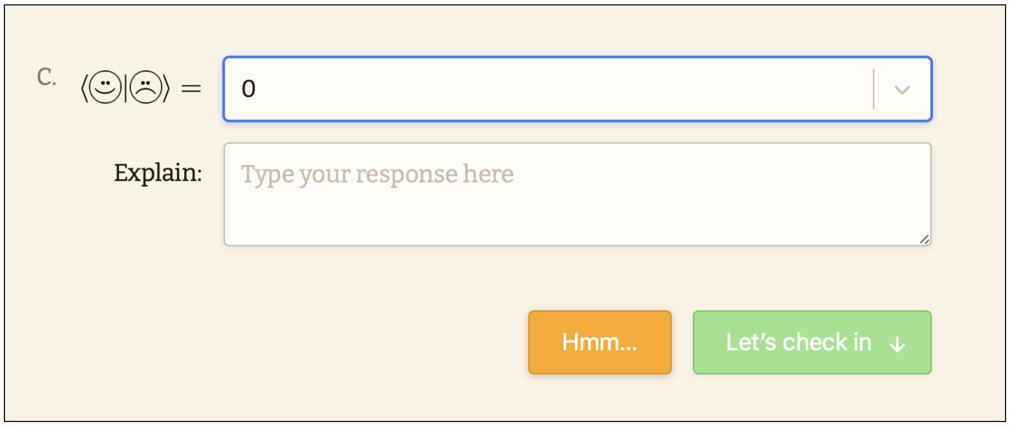
7.2.2 ACE Physics interaction data

The answers that students input on the ACE Physics website are saved in a database, meaning that they can be analyzed after the fact. In addition, some other interactions with the website are also saved, such as which pages a student completed and which hints they used (see Figure 7.1 for an example hint button). Finally, the timestamps of each student's very first and very last interaction with a specific tutorial were also saved. The difference between these timestamps gives a limited indication of how much time a student spent on a given tutorial. However, there is a significant flaw with this methodology: there is no way to know if students worked on the tutorial continuously between the first and last timestamps. Nonetheless, this timing data does provide a useful upper bound for how long each student spent on the given activity.

It is possible, with a significant coding effort, to do a better job tracking user interactions with a piece of software, although there are limitations even in the best case because of constraints imposed by web browsers. We prioritized other goals, such as improving the tutorials themselves,

	ACE Physics tutorial assigned on homework	
Week	**Spring**	**Fall**
1	—	—
2	Spin Lab 1	Spin Lab 1
3	—	—
4	Quantum Mouse Lab (pp. 5–8)	—
5	—	—
6	Visualizing a Vector in Different Basis (Lite Edition)	Visualizing a Vector in Different Basis (Lite Edition)
7	—	—
8	—	EPR and Entanglement
9	EPR and Entanglement	—
10	—	—
11	—	—
12	(Spring break)	Time Dependence
13	Time Dependence	—
14	—	(Fall break)
15	—	—
16	—	—

Table 7.1: Timeline of administration of ACE Physics tutorials. No homework was due in weeks 1, 5, 11, or 12 (spring) or 14 (fall). An ACE Physics tutorial was included on 5 of 12 assignments. Only the last four pages of the Quantum Mouse Lab tutorial were assigned outside of class, because the first four pages were completed as an in-class activity. More of that activity was completed in-class in the fall, so none of it was assigned as homework in that semester.



C. ⟨☺|☹⟩ = 0

Explain: Type your response here

Figure 7.1: A screenshot of an ACE Physics tutorial question (from the Quantum Mouse tutorial) that includes a button labeled “Hmm. . . ”. Clicking this button reveals a hint: a small suggestion or clarification to help the student answer the question. Not every question included a hint button, and some hint buttons had more specific labels. Students were not penalized for using hints.

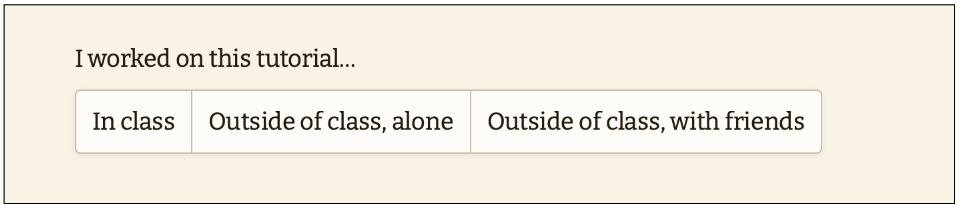

I worked on this tutorial...

Figure 7.2: A screenshot of the first question from the end-of-tutorial feedback survey, which was included in every ACE Physics tutorial.

instead of implementing more complex interaction tracking. Below we report what we can learn from our limited data.

7.2.3 End-of-tutorial feedback surveys

At the end of each ACE Physics tutorial, students were asked to share their feedback on a brief, optional survey built-in to the ACE Physics website. The survey included three questions. The first question (Figure 7.2) asked students to indicate whether they had worked on the activity alone or with peers. The second question (Figure 7.3) asked students to select whether they found the tutorial easy or challenging as well as useful or frustrating. The final question offered students a text box to input any other feedback they may have.

7.2.4 End-of-semester survey

In addition to surveys at the end of each tutorial, a more extensive survey was administered to students during the last full week of each semester. This end-of-semester survey consisted primarily of

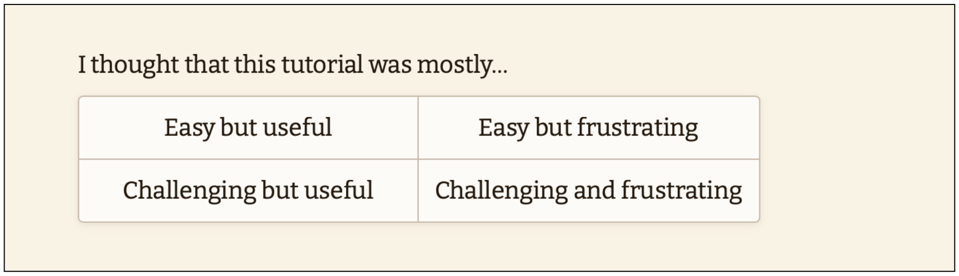

I thought that this tutorial was mostly...

Figure 7.3: A screenshot of the second question from the end-of-tutorial feedback survey.

a sequence of Likert-scale questions asking students to reflect on various aspects of their experiences doing ACE Physics tutorials over the semester. The full set of questions can be seen in Figures 7.9 to 7.12. To encourage answering questions honestly, students were informed that their survey responses would be anonymized. Because this survey was optional, not every student completed it: 76 students who consented to participate in our research completed the survey in the spring, and 58 students in the fall. Four questions were added to the survey in the fall semester.

7.3 Results

We will first discuss quantitative results, and then turn to qualitative analysis of videos of students using ACE Physics tutorials.

7.3.1 Quantitative summary of interactions with ACE Physics

This section will focus on quantitative interaction data from the four ACE Physics tutorials assigned for completion outside of class in both the spring and fall semesters (see Table 7.1). We exclude data from the Quantum Mouse Lab tutorial, because it was completed partially in class in the spring and fully in class in the fall. For the four included tutorials, there were 652 total submissions across the two semesters (these submissions are distributed approximately evenly between the four tutorials). There were an additional 108 submissions for the Quantum Mouse Lab tutorial, and also 48 submissions for Quantum Mouse Lab 2, which the instructor suggested students complete as review for the second midterm exam.

The first question we can answer with these data is: how much of each tutorial did students complete? Figure 7.4 shows the page completion rate for the 652 submissions from the two semesters. The vast majority of students completed every page, meaning they clicked the “move on to the next page button”, which is only visible once every question has been answered (possibly incorrectly). Excluding the pre-test page, Spin Lab 1 has 6 pages, Visualizing a Vector in a Different Basis has 3, EPR & Entanglement has 5, and Time Dependence has 4. Given these numbers, every submission included in the 90–100 bin in the histogram completed exactly 100% of the pages.

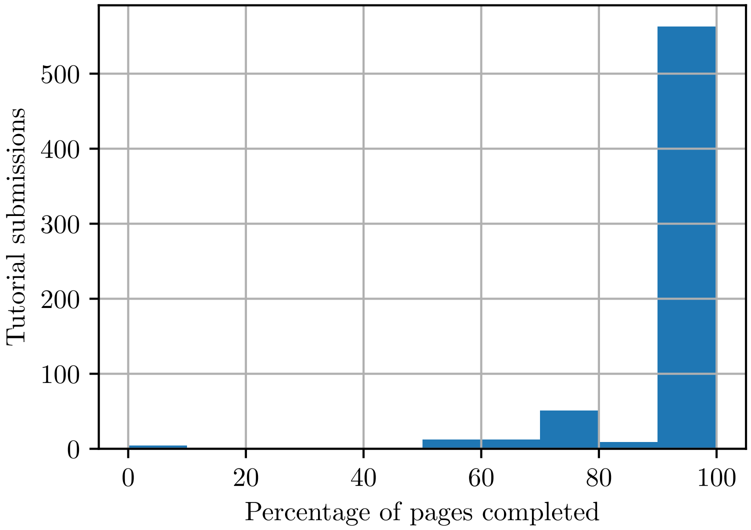


Figure 7.4: Page completion rate for the four ACE Physics tutorials assigned as homework in both the spring and fall semesters. A page is marked as "completed" when a student clicks the "move on to the next page" button, which is only revealed once all questions on the current page have been completed. The vast majority of students completed every page of the assigned tutorials, which were 4.5 pages long on average.

Page completion only tells us that students at least did the bare minimum to get through the tutorial: provide *an* answer for every question. We can learn more about how engaged students were with the activities by looking at how much they typed into free-response text boxes on the tutorials. Figure 7.5 presents a histogram showing the number of characters students typed into every text box included in the four tutorials under study in this section. Nearly 6,000 individual text box responses are represented in total.

The raw data includes text box responses with exactly 0 characters, but these are excluded from the figure. The ACE Physics software enforces a 1-character minimum for any required text box question. Therefore, 0-length responses correspond with one of the following scenarios: (a) the student did not see the question because they did not complete that page of the tutorial; (b) the student did not see the question because of conditional logic in the tutorial (e.g., they answered a previous question correctly and so were not prompted with a follow-up question); or, (c) the student saw the question, but it was flagged as "optional" (this is rare). In addition, students are sometimes asked to input equations in text boxes on ACE Physics, but these questions were also excluded from this analysis, which aims to assess how much students wrote when they were asked to articulate their reasoning in prose.

Students wrote about 86 characters on average in text boxes. For reference, this sentence has 86 characters, including all punctuation plus spaces. An 86-character sentence is enough to express a reasonably complete thought, and this result suggests strongly that most students engaged with the tutorials in good faith. Although we have not collected similar data for in-person tutorials, we do believe 86 characters is on the long end of written responses for the in-person setting. When working on tutorials in groups, students typically come to consensus verbally and then write just a few words on their worksheet summarizing the key ideas from the discussion.

Despite the overall positive results regarding text box response length, however, we can see that the histogram in Figure 7.5 has a peak in the 0–10 character bin. (Although this is a peak, a significant majority of responses were in fact longer than 10 characters.) Responses in this bin represent at best single-word answers, and 145 of these responses were exactly 1 character. It is likely that the 1-character answers reflect cases where students "clicked through" the tutorial question, either simply to receive participation credit for reaching the end, or perhaps hoping that the system would check their answer to a multiple choice question coupled with the free-response question. Thus there is evidence of “click-through” behavior in our data, but fortunately it only occurred in a minority of cases (∼2.5%).

Another proxy indicator for the quality of student engagement with ACE Physics is the amount of time students spent on each tutorial. As discussed in the methodology, the timing data we have can only provide an upper bound on how long each student spent on a given tutorial. These data are presented in Figure 7.6. The figure is truncated at 150 minutes, but there is a long tail of 22% of all the submissions that almost certainly represent cases where students worked on the tutorial in multiple sittings. Nonetheless, the histogram has a clear peak in the 20–30 minute range, which is a reasonable, if slightly short, amount of time for a student to spend completing an ACE Physics tutorial. Students in the 10–20 minute range likely rushed through the activity, and students in the 0–10 minute range either clicked through the tutorial as fast as possible, or did not finish.

We can readily extract one other fact about the ways students interacted with ACE Physics from these quantitative data: how often did students use the available hints (see Figure 7.1 for an example)? As seen in Figure 7.7, most students do not use the hints. We are unsure why this is the case, and hint usage does not necessarily tell us anything about whether the student was engaged in the tutorial in good faith. Based on discussions with some students in interviews, we do suspect that some students are accustomed to online homework systems that attach a penalty to hint usage. We discuss hints further in the next chapter.

7.3.2 End-of-tutorial feedback surveys

Each ACE Physics activity ended with an optional feedback survey including the two questions shown in Figures 7.2 and 7.3. As in the last section, we narrow our focus to student responses from the four tutorials that were assigned for completion outside of class in both semesters. The first question asked whether students worked alone or with peers. There were 456 answers to this

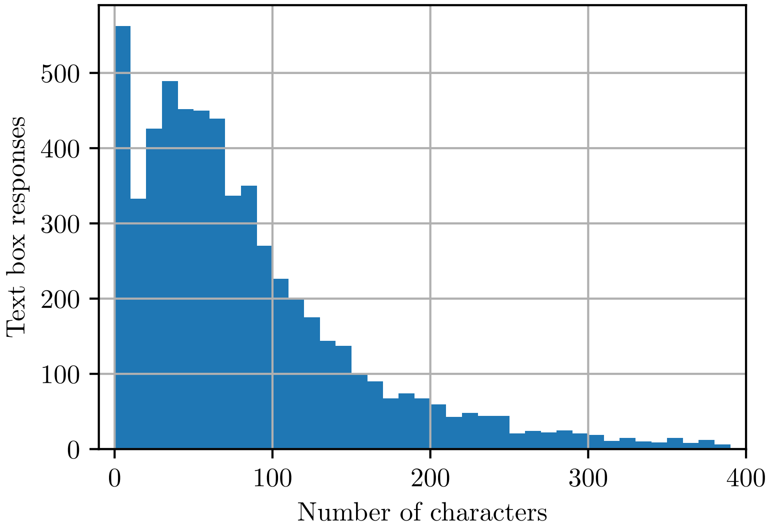


Figure 7.5: Histogram of text box response lengths for the four ACE Physics tutorials assigned as homework in both the spring and fall semesters. The average response length was about 86 characters (including punctuation). The four tutorials included about 14 open-response text box questions on average. No responses with exactly 0 characters are included, because the ACE Physics software enforces a 1-character minimum length for any required text box question.

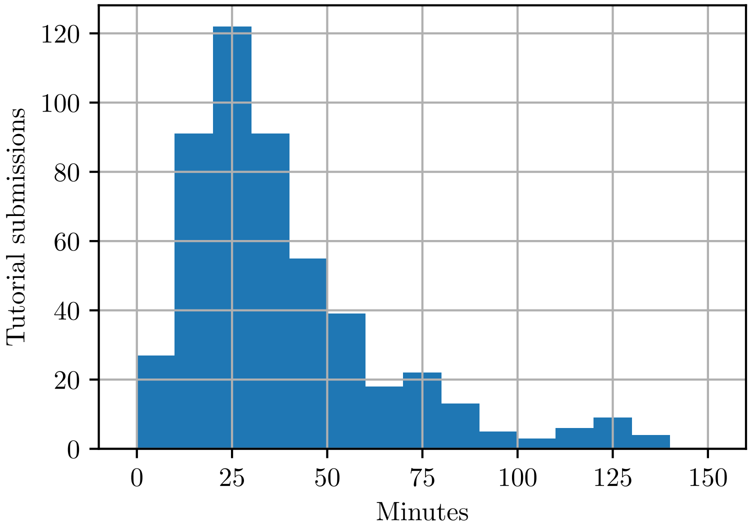


Figure 7.6: Time between a student's first and last interaction on the four ACE Physics tutorials assigned as homework in both the spring and fall semesters. This value provides an upper bound on how long each student spent working on the tutorial. 142 of the 652 (about 22%) of submissions are not included in the figure because they exceeded 150 minutes, which strongly suggests that the student completed the activity in multiple sittings. The average value of the submissions included in the figure is about 39 minutes, but the modal value is between 20–30 minutes.

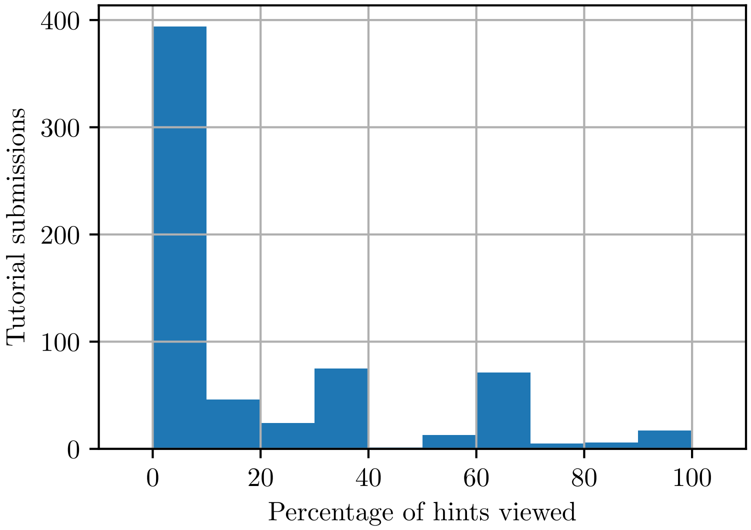


Figure 7.7: Hint usage rate for the four ACE Physics tutorials assigned as homework in both the spring and fall semesters. Most students used zero hints. The four tutorials each had about 6 hints available on average. The Time Dependence tutorial included zero hints in the spring but was updated to include 13 hints in the fall.

question (about 70% response rate out of all tutorial submissions) across both semesters. Students reported working alone in 97% of responses ($N = 442$). Students were in general encouraged to collaborate on homework assignments, so we do not suspect that students were hesitant to admit to having done so.

The second end-of-tutorial feedback question asked students to characterize the activity they had just completed as easy or challenging and as useful or frustrating. Results are shown in Figure 7.8. This question also had a 70% response rate. Overall, a significant majority of students indicated that they considered the tutorial "useful" as opposed to "frustrating", but the difficulty level was more evenly split between "easy" and "challenging".

Of the responses that said the tutorial was "frustrating", half as many considered it "challenging" as considered it "easy". As with in-class tutorials, we do anticipate that there will be a subset of strong students for whom the activity is easy, possibly frustratingly so. In the in-class setting, these students have the chance to help teach their peers, and the instructor can also engage them with challenge questions, but these opportunities are lost in the online setting. Thus it is not surprising to see this contingent represented in these data (assuming that indeed the "easy/frustrating" bar represents these students).

The responses in the "challenging/frustrating" bar are the most concerning, but fortunately these reflect a minority of all responses. The most common complaint submitted in the open-ended feedback box included in the end-of-tutorial surveys was that the tutorials did not provide an answer for every question, meaning that if you were confused you didn't always have the opportunity to check your answer. We suspect that this describes the "challenging" and "frustrating" experience some of these students were referring to. It is an obvious and unfortunate consequence of assigning tutorials without instructors present that those students who are already behind in the class will have a more difficult time benefiting from the activities. We discuss considerations related to when and how we provide answers and feedback in ACE Physics tutorials in the next chapter, but as described in Section 2.3, directly revealing the answers to every question conflicts with the goals of tutorials, which are meant to foster active engagement and reasoning.

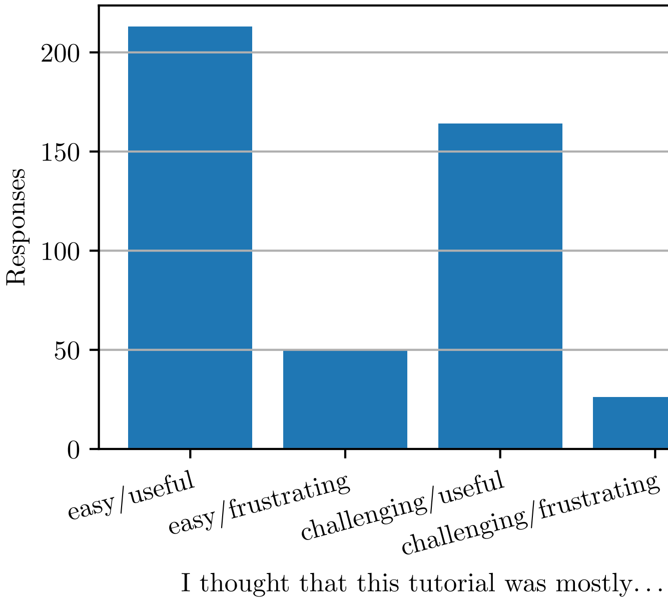


Figure 7.8: Student responses to the question in Figure 7.8 from the four tutorials assigned on homework in both the spring and fall semesters. The figure includes 452 responses, about a 70% response rate.

7.3.3 End-of-semester reflections on ACE Physics

At the end of both semesters, an optional survey was administered to the class asking students to reflect on their experience with ACE Physics tutorials. The survey included a sequence of Likert-scale questions, the responses to which are given in Figures 7.9 to 7.12. Responses did not vary significantly across the two semesters, so the figures aggregate data from both semesters. This consistency across semesters is unsurprising considering that neither the tutorials nor the way they were assigned or framed on homeworks were changed significantly between semesters (with the exception of the Time Dependence Tutorial—in the fall semester we assigned a version of this tutorial modified per our findings in Chapter 5).

The first sequence of Likert-scale questions (Figure 7.9) asked students to reflect on the utility of ACE Physics. To establish a baseline, we asked students each question in reference to both ACE Physics tutorials as well as to "other homework problems" (i.e., the typical end-of-chapter-style problems assigned on homework sets). Overall, a strong majority of students indicated that they considered ACE Physics useful for their learning—nearly as useful as other homework problems.

Differences between ACE Physics and other homework problems arose for two of the three questions. Roughly 90% of students agreed other homework problems helped strengthen their problem-solving skills, only 55% of students agreed with this statement for ACE Physics. Meanwhile, more than 85% of students agreed that both ACE Physics and other homework problems strengthened their conceptual understanding. This discrepancy is unsurprising because our tutorials do emphasize conceptual understanding, whereas other homework problems often included a mix of conceptual and calculational questions.

The second set of questions (Figure 7.10) asked students to reflect on various aspects of their experience answering questions on ACE Physics tutorials. First, we asked if students believe their instructor assigns ACE Physics to test them, and also whether students find that ACE Physics allows them to test themselves. Most students do not believe ACE Physics is assigned to test them, but the majority do find that it allows them to test themselves. These results align with our goals

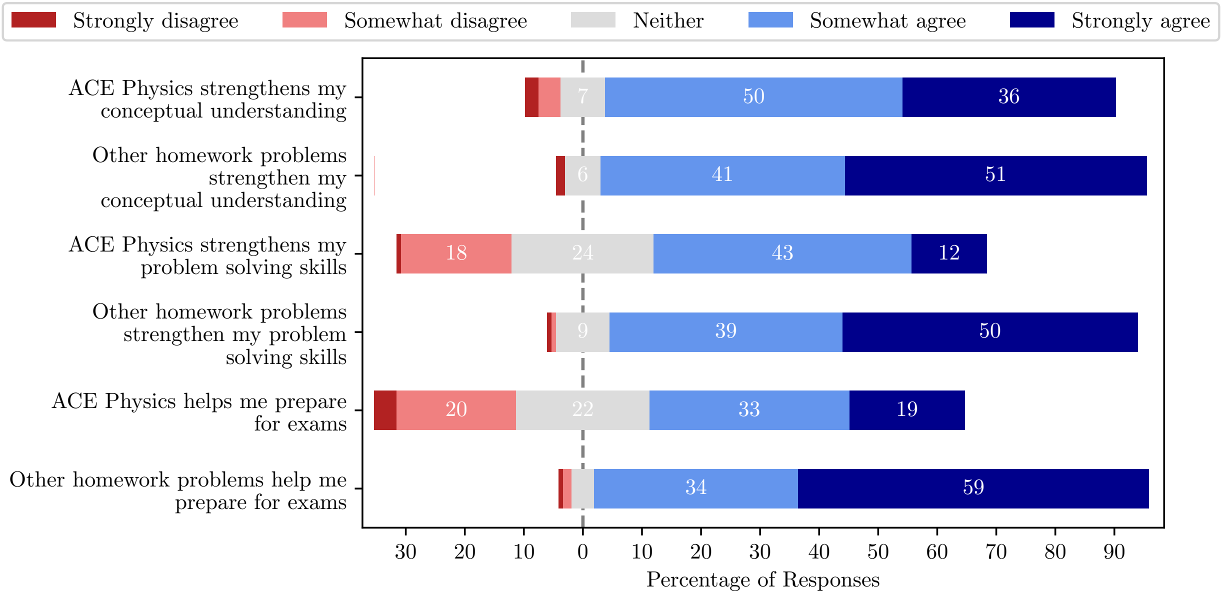


Figure 7.9: End-of-semester survey responses about students' sense of the utility of ACE Physics. Responses are aggregated over two semesters; each question had 134 student responses.

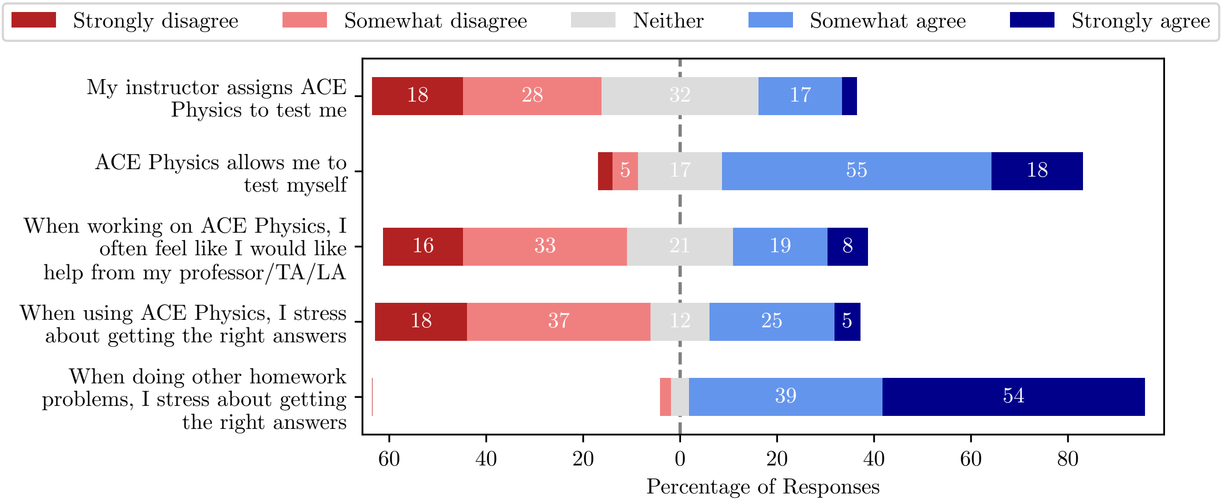


Figure 7.10: End-of-semester survey responses about students' experience doing ACE Physics tutorials. Responses are aggregated over two semesters; each question had 134 student responses.

for assigning ACE Physics and suggest that the tutorials were framed successfully by the instructor.

The last two questions in this sequence asked students to indicate whether they "stressed" about getting the right answers on ACE Physics and on other homework problems. The vast majority of students indicated that they stress about the right answers on other homework problems, but most students said they do not stress about the answers on ACE Physics. These results are unsurprising considering that, unlike other homework problems, students' responses on ACE Physics were not graded for correctness.

This question about stress suggests that assigning ACE Physics tutorials on homework, where students do not have support from instructors or group mates, is not detrimental in terms of student affect. Twenty-seven percent of students reported often wanting help from instructors when doing ACE Physics tutorials. Tutorials are intended to challenge students, and there was a risk that students would have a negative experience completing them in an unsupported environment, but this risk was not borne out for the majority of students. On the other hand, although tutorials are not intended to be stressful for students, arguably a small degree of stress may cause students to attend more carefully to the activities and, as a result, learn more. Students' stress level is only one dimension of students' approach to the activities; the next sequence of questions interrogated other aspects of students approach to doing ACE Physics tutorials.

The questions in Figure 7.11 asked students to report on their behavior when doing ACE Physics tutorials on homework. Despite not stressing about the correct answers, the majority of students report trying to answer tutorial questions correctly, trying to do so before using hints or receiving automatic feedback, and taking time to think about their answers, although most students disagreed with the statement that they put roughly the same effort into ACE Physics as other homework problems (we assume this means those students put *less* effort into ACE Physics), and one third of students said they tried to finish the tutorials as fast as possible.

The final sequence of questions on the survey (Figure 7.12) asked about students' opinion of ACE Physics. Most students said they considered ACE Physics an opportunity to learn, with only 7% disagreeing with this statement, although a quarter of students did say they viewed ACE Physics as "busy work" required to finish their homework. This is consistent with roughly a quarter of students saying they tried to finish as fast as possible.

The last three questions on the survey give us an overall sense of how students felt about the tutorials: overall, students had a positive opinion. Sixty-eight percent of students said they liked doing ACE Physics tutorials, with another 21% neutral on the question. Over 70% of students said that ACE Physics should continue to be assigned in future semesters of the quantum mechanics class ("PHYS 3220"), and over 60% said they would like ACE Physics activities to be assigned in other physics classes.

7.4 Discussion and conclusion

We find that our quantitative results paint an encouraging picture when it comes to student engagement with ACE Physics tutorials assigned for completion outside of class. For the most part, students spend a reasonable amount of time on the activities (20–30 minutes), they tend to finish every page, and they typically write about one sentence per free-response text box, suggesting that they are taking time to articulate their reasoning, if briefly. We also learned that the vast majority

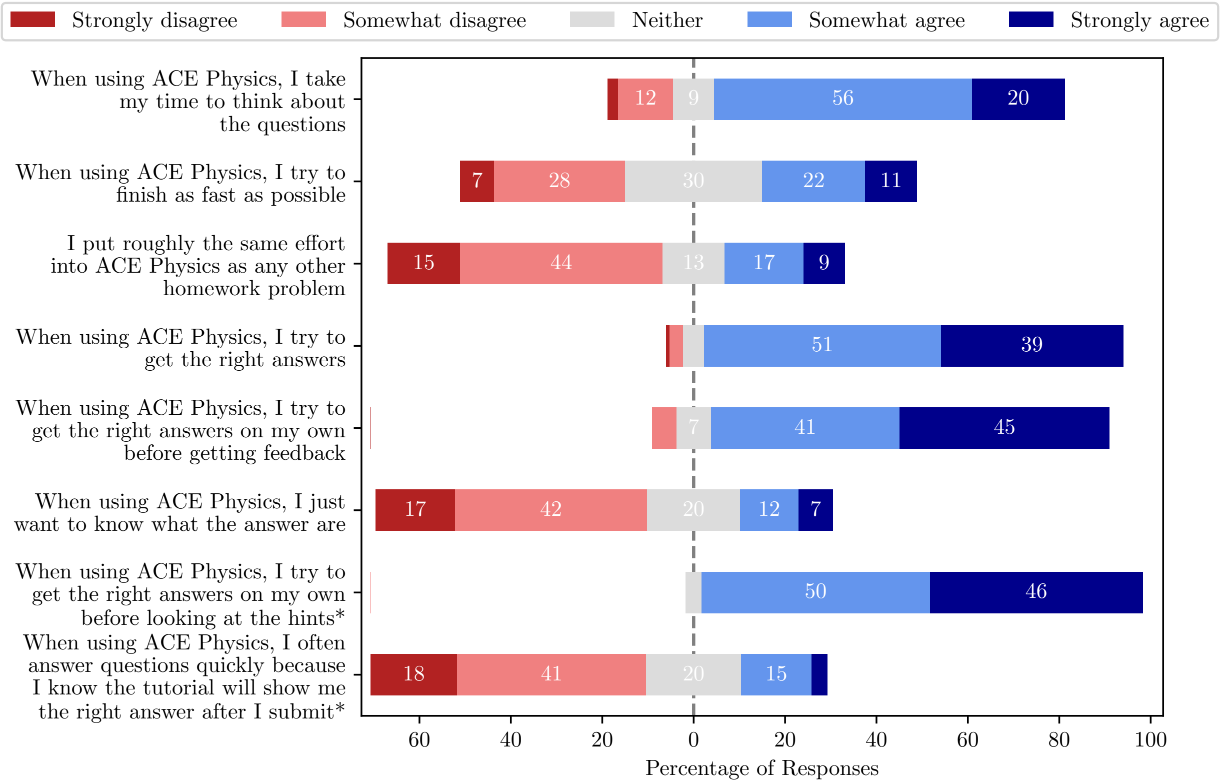


Figure 7.11: End-of-semester survey responses about students' approach to doing ACE Physics tutorials. Responses are aggregated over two semesters; each question had 134 student responses. *These questions were only asked in the fall semester and had only 58 student responses.

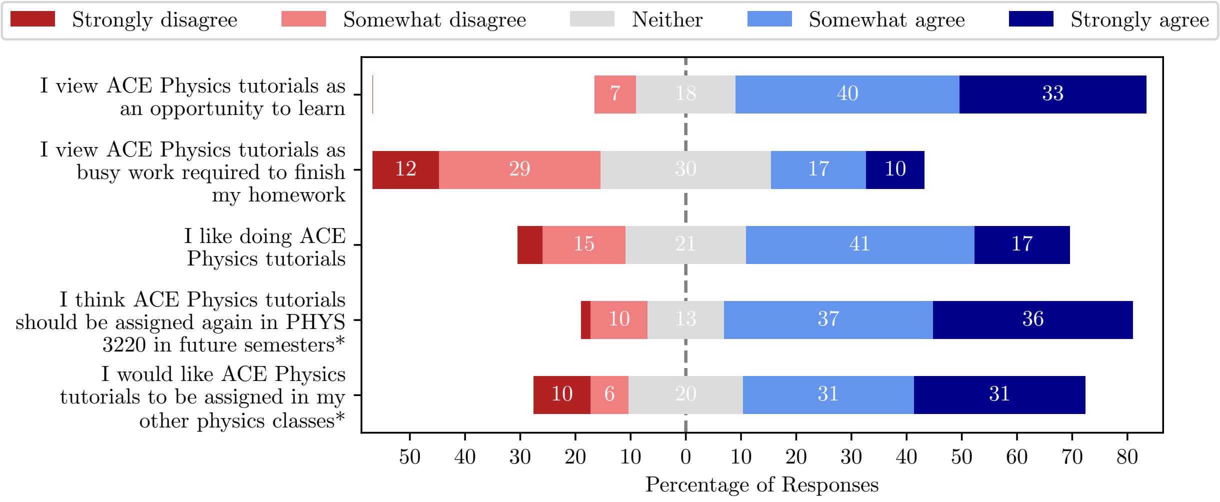


Figure 7.12: End-of-semester survey responses about students' overall opinion on ACE Physics. Responses are aggregated over two semesters; each question had 134 student responses. *These questions were only asked in the fall semester and had only 58 student responses.

of students worked on the activities alone. These data align with the self-reported behavior patterns students described in the end-of-semester surveys: unlike other homework problems, students do not stress about the answers on ACE Physics tutorials, and they typically spend less effort on ACE Physics, but they do try to get the right answers (before getting feedback) and they view the activities as an opportunity to learn.

In addition, students, for the most part, consider the tutorials useful, as indicated by both the end-of-tutorial surveys as well as the end-of-semester surveys. Students found the tutorials especially useful for improving their conceptual understanding, which aligns with the goals of the activities. Most students agreed that ACE Physics activities should be included again in future semesters of quantum mechanics, and many also said they would like to do similar online tutorials in other classes as well. Given that the tutorials were assigned as *homework*—not something students necessarily default to having a positive opinion of—we consider these results to be quite positive.

We may have expected significantly more negative results. The data might have shown that most students rushed through the activities, that they found them frustrating, or that they did not consider them useful for their learning. Students are already burdened with traditional homework problems, three lectures a week, and weekly reading assignments and pre-lecture surveys, not to mention classwork from other courses. Fortunately, students responded positively to the addition of ACE Physics to this workload.

This study is limited to only one student population and one instructor. The class also included some in-class tutorials, which may have influenced how students interacted with tutorials in the outside class environment as well. Moreover, our data provide limited direct insight into how students interacted with the tutorials outside of class, and it is reasonable to be skeptical that students' self-reported behavior may misrepresent their actual behavior. In future work, we could collect additional student interaction data on the online platform and also observe students working alone on ACE Physics activities in structured interviews. Nonetheless, we are encouraged that students reacted positively to ACE Physics and appeared to engage with the tutorials in good faith. We consider these to be preliminary successes of ACE Physics' pilot deployment, which we attribute to two factors: the instructor's framing of the activities (which likely affected how students perceived them); and the design of the ACE Physics tutorials themselves. The next chapter explores the design of the activities.

Chapter 8

Strategies for designing online physics tutorials

8.1 Introduction

What makes an instructional activity “research-based”? In PER, this term implies some combination of the following criteria. First, the activity’s goals are motivated by research—studies of learning broadly or of student ideas about specific topics. Second, the activity’s effectiveness has been evaluated using research techniques (e.g., pre- and post-testing): does it achieve its goals? Finally, research has informed the activity’s design—*how* it achieves its goals. These criteria enable the community to systematically improve instruction with new or refined activities.

So far, this dissertation has engaged primarily with the first two criteria. Chapter 3 investigated one common but ambiguous instructional goal—intuition—and identified specific facets to which instruction (and instructional activities) can attend; meanwhile, Chapter 4 studied student ideas about basis in quantum mechanics, which motivated the learning goals for the Quantum Basis Tutorial. Chapters 6 and 7, on the other hand, evaluated different aspects of ACE Physics’ effectiveness. Only Chapter 5, which examined student behaviors during in-class tutorials, discussed implications for the design of ACE Physics tutorials (Section 5.5), but that discussion was fairly general.

This chapter engages wholly with questions of activity design, but instead of presenting research to motivate our design choices, we will examine the design of ACE Physics directly. A variety of research (both ours and others’) influenced our choices (see Sections 2.2 to 2.4), but, like many instructional activities, our “instructor’s intuition” shaped much of ACE Physics as well, especially small-scale design decisions such as the content of a specific question or design of a specific feedback mechanism. This chapter sets out to catalog the types of small-scale choices we made. In so doing, we will illuminate our process for making these decisions and characterize various design patterns present in ACE Physics.

Whereas the preceding chapters form a research basis for the current version of ACE Physics, this chapter aims to support future activity development (on ACE Physics or otherwise). Its purpose is twofold. First, future activity designers can adopt—and possibly modify—any of the design patterns or processes documented here (especially when creating interactive online activities). Second, future research should investigate the pedagogical effectiveness of the various design patterns we have employed in ACE Physics, and cataloging these patterns is a necessary first step for such work.

We believe more curriculum development projects in PER should document their design goals, strategies, and process in greater detail, but the ACE Physics project is especially ripe for such a discussion. As described in Section 2.5, the loss of the classroom context (and, with it, instructor facilitation) emphasizes the role of ACE Physics' design in shaping the outside class tutorial environment. In addition, because we adapted the ACE Physics tutorials from on-paper versions used in class, we were forced to carefully consider the goals and features of the original tutorials while navigating the obstacles present in the new environment. This chapter is organized around two of these obstacles: Section 8.4 addresses the limits of the mouse and keyboard as input devices (as compared with a pen) while Section 8.3 discusses the broader challenge of providing students with automated guidance without sacrificing the essence of a *tutorial* (as described in Section 2.3). Before diving into these issues, we outline our process for adapting tutorials for ACE Physics in Section 8.2. This chapter does not discuss the technical implementation of the ACE Physics website, but technical documentation, as well as the source code, is available upon request.

8.2 Overview of our process for developing ACE Physics tutorials

As described in Chapter 2, the initial set of ACE Physics tutorials originated as on-paper activities intended for use in the classroom [107]. More recently, we have created several ACE

Physics tutorials about quantum computing topics from scratch, but early drafts of these also began as static documents. This chapter focuses on the process of developing an ACE Physics tutorial from an existing, static version, as opposed to the process of writing a tutorial from scratch. We have described the process of creating a brand new, on-paper tutorial (specifically the original Quantum Basis Tutorial) in our previous work [115], which takes the study of student ideas presented in Chapter 4 as its research base informing the choice of learning goals for the activity.

When adapting a static tutorial for ACE Physics, the first challenge entails modifying questions to fit the on-screen, online medium. We discuss strategies for overcoming this challenge in Section 8.4. Once we have completed this step, we have a functional draft of an ACE Physics tutorial, but one that contains no guidance elements, meaning the draft is not yet suitable for students working alone outside of class. The second, more significant challenge involves adding elements that provide students with guidance or feedback based on their answers to tutorial questions. We discuss different patterns for guidance elements in Section 8.3, but honing the guidance for an ACE Physics tutorial involves an iterative process. Developing effective guidance for tutorials requires anticipating the ways that students may become confused, stuck, or misled as they work through the questions.[1] We endeavor to inform our anticipation from empirical evidence or classroom experience whenever possible.

We have used three sources when developing guidance for ACE Physics. First, we refer to research into student understanding within the topic of the given tutorial. For example, our exploration of student ideas about basis (Chapter 4) helped us anticipate common mistakes on the Quantum Basis Tutorial, which we added dynamic feedback to address. Second, we rely on our experience running the on-paper tutorials in-class. Chapter 5 presents a study of the in-class tutorial environment that has influenced the design of ACE Physics. In addition, we have maintained notes about each tutorial after running them in class, which we also referred to when developing ACE

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[1] Tutorial questions are designed to challenge students; a student able to answer every question easily likely does not need the tutorial (but they may still appreciate the practice). Although guidance features necessarily address topics where students struggle, we maintain our strengths-based approach (Section 2.4) in the character of the feedback we give students.

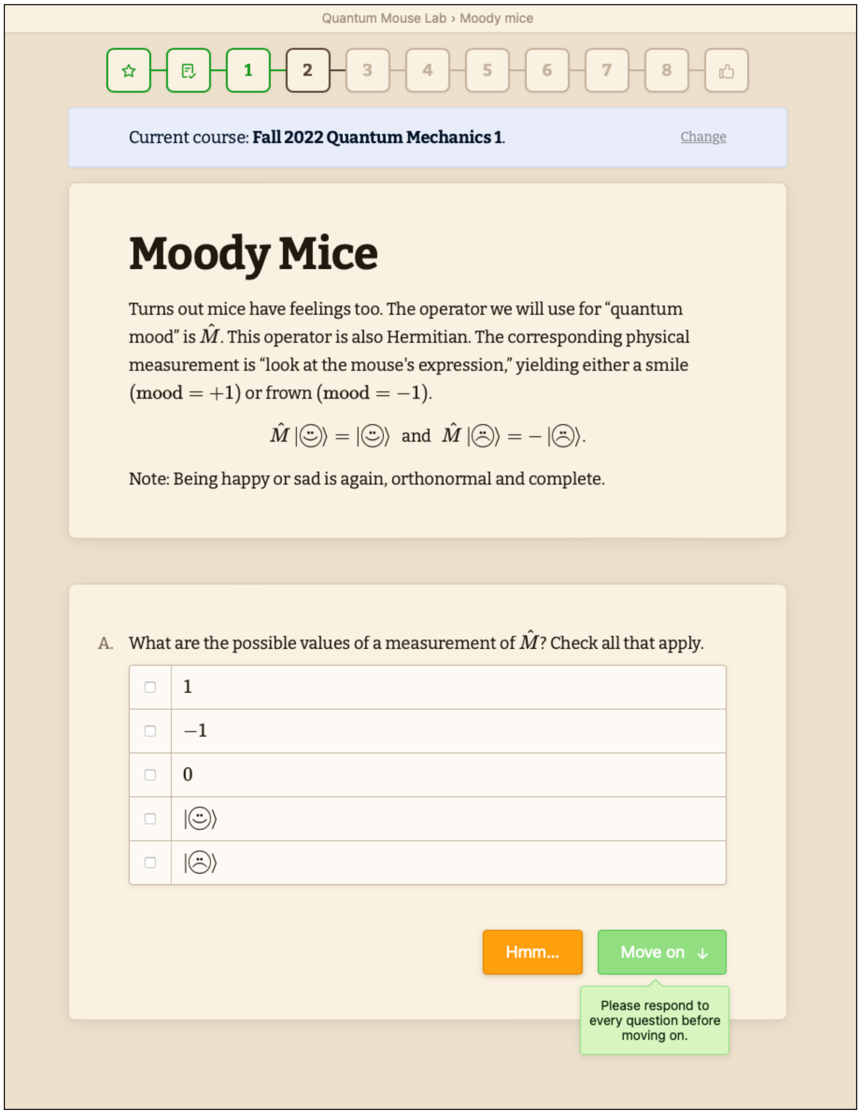
Physics feedback elements. Finally, we reflect on the success of the ACE Physics versions after either assigning them to students on homework (such as the data presented in Chapters 6 and 7) or administering them to students in recorded, semi-structured interviews, which we have done for two different ACE Physics tutorials. Although we have not yet performed a rigorous analysis of these interview data, we will occasionally reference the recordings in our discussion below.

This process is iterative because we can continually refine a tutorial after gathering more data when administering a version of it to students. Other curriculum development projects in PER have employed a similar iterative process [33, 57, 82, 115, 124]. We plan to refine all of the existing ACE Physics tutorials in this way. In some cases (such as for the Time Dependence Tutorial), the ACE Physics version may begin to look quite different from the original on-paper version.

8.2.1 The structure of an ACE Physics tutorial

Before proceeding, it will be useful to describe the basic structure of an ACE Physics tutorial, exemplified by Figure 8.1. Tutorials are broken into pages, which are broken into sections, which contain zero or more questions. Each tutorial targets a handful of content learning goals specific to one unit in the quantum mechanics class (in addition to some common goals described in Section 2.3). Pages (the numbered boxes across the top of Figure 8.1) divide the tutorial into conceptual chunks. Every page begins with a section of introductory text, and subsequent sections typically include one or more related questions. Students must provide an answer to every question in a section before the software allows them to move on to the next section (their answers do not need to be correct); subsequent sections are hidden until the student explicitly clicks “Move on”, after which we may (or may not) provide feedback on their answers. We require answers to every question both because this helps regulate student behavior (i.e., students cannot zip through a tutorial by clicking “Move on” repeatedly) and because, as in in-class tutorials, we want students to engage with the question and at least guess an answer before they receive guidance.

Dynamic elements—content that changes depending on student input—exist in various forms throughout ACE Physics (e.g. Figure 8.4), but most conditional logic occurs when students click



Quantum Mouse Lab › Moody mice

Current course: **Fall 2022 Quantum Mechanics 1**. Change

Moody Mice

Turns out mice have feelings too. The operator we will use for "quantum mood" is $\hat{M}$. This operator is also Hermitian. The corresponding physical measurement is "look at the mouse's expression," yielding either a smile (mood $= +1$) or frown (mood $= -1$).

$$\hat{M}\,|\text{☺}\rangle = |\text{☺}\rangle \quad \text{and} \quad \hat{M}\,|\text{☹}\rangle = -\,|\text{☹}\rangle .$$

Note: Being happy or sad is again, orthonormal and complete.

A. What are the possible values of a measurement of $\hat{M}$? Check all that apply.

- ☐ 1
- ☐ -1
- ☐ 0
- ☐ $|\text{☺}\rangle$
- ☐ $|\text{☹}\rangle$

Please respond to every question before moving on.

Figure 8.1: A typical ACE Physics tutorial. The numbers across the top allow students to navigate between pages. The boxed sections are revealed one at a time, and students cannot move on to the next section until they have answered every question (possibly incorrectly) in the current section. The "Hmm…" button reveals a hint.

the "Move on" button at the end of a section. The next element that is revealed (either another section or a feedback message) when students click that button will in general depend on their answers to previous questions; different students working on the same tutorial may see some different sections depending on their answers. Sections may include multiple related questions, so delaying feedback until students "submit" their answers allows for the possibility that they will catch their own mistakes after considering multiple questions or attempting to articulate their reasoning in a text box. Students may return to any question to change their answer at any point (in fact, we occasionally ask them to do so explicitly), which is useful for three reasons: first, in some cases students will receive additional feedback on updated answers; second, students may use their completed tutorial as a study tool at later date (when having correct answers in place is preferable); and third, updating their answers—especially written explanations—can help students actively engage with the feedback (instead of just reading it). Students in interviews did tend to update their graded answers after receiving feedback, although some did not update their written explanations accordingly.

Once they have completed every section on a page, students will see a button that takes them to the next page. Because sections are introduced one at a time, it was not strictly necessary to also break tutorials up into pages, but page divisions help students keep track of their progress through a tutorial, and also enable instructors to assign only a portion of a tutorial.[2] Students can always navigate between pages using the links at the top of the screen; we do not gate subsequent pages on the completion of previous pages because instructors may assign only a subset of the available pages in a tutorial, so we must allow students to start from any page. The software tracks which pages a student completes, which can be used to assign participation credit.

8.3 Implementing tutorial-like guidance in online activities

Section 2.3 describes our broad goals for tutorials, all of which factor into our design strategies for ACE Physics. However, the obvious key tutorial feature lacking in the online environment

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[2] As shown in Table 7.1, only a portion of the Quantum Mouse Lab Tutorial was assigned to students on homework; the first half of the tutorial was completed in class instead.

is instructors. Tutorial instructors use Socratic dialogue to guide students through challenging questions, and they can adapt their guidance in the moment based on nuanced information about student understanding. In addition, students working on tutorials in class will do so in groups, with whom they will discuss and debate their answers. These features of in-class tutorials keep students actively engaged in the learning process—including when they are struggling. This section describes strategies we have developed to approximate portions of this using conditional logic and other design features possible in online tutorials. Additional research is required to determine the extent to which each of these strategies is effective, although we do discuss evidence from videos of students using ACE Physics in semi-structured interview settings where relevant.

8.3.1 Affordances of not scoring tutorials

ACE Physics tutorials are not graded for correctness: we never intend for student work on ACE Physics to be scored, either by instructors or automatically. Tutorials are not designed to be assessment tools but instead are meant to provide a space where students feel comfortable articulating their reasoning—even when it may be incomplete or incorrect, because doing so helps them refine it. Similar to in-class tutorials, we encourage instructors to grant students credit for completing ACE Physics tutorials but discourage them from penalizing incorrect answers. In the remainder of this chapter we do use the word “grade” as shorthand for “parse student input and provide feedback or guidance”, but ACE Physics never computes or displays scores for students.

Many ACE Physics questions are, in fact, ungraded in either sense of the word: they are neither scored nor does the system evaluate student input to provide dynamic feedback. Notably, we never attempt to parse answers to free-response questions, yet the tutorials include many such questions (e.g., asking students to explain their reasoning). We could not include ungraded or possibly ungradable questions were ACE Physics designed as a scored, online homework system. Instead, our dynamic guidance features are merely intended to complement students’ self-regulated learning. We also have minimal concerns about revealing the answers to tutorial questions within the tutorials; students have little incentive to cheat on an un-scored activity. Chapter 7 reports evidence that students generally appear to engage with ungraded tutorial questions in good faith.

8.3.2 Making questions “auto-gradable”

To provide students with automatic feedback based on their answers to previous questions, the ACE Physics software must be able to parse students’ responses to those questions. This means the user interface must accept answers in a structured format, such as multiple choice questions or numerical inputs. (When grading decimal values we accept answers within ± 0.02.) More complex structures also work: ACE Physics allows students to input matrices or tables in some locations, and hypothetically we could integrate an equation editor and computer algebra system to parse, simplify, and compare mathematical expressions (see Section 8.4.2 below). In technical terms, we must be able to store the answer in a dedicated data structure that allows us to distinguish student’s answers using simple logic.

We never parse free-response inputs; arbitrary strings do not have the necessary structure. One might imagine any number of tricks to programmatically extract meaning from a student’s written response (e.g., searching for keywords), but we have found this direction unproductive for ACE Physics. ACE Physics’ guidance features are a stand-in for a tutorial instructor, meaning that—in the language of Chapter 5—students are in the authority frame when reading the feedback that ACE Physics provides. Feedback must therefore be 100% reliable, otherwise we risk misleading, confusing, or frustrating students. Two students should not receive different feedback just because they used different words to express the same idea, but employing natural language parsing risks this possibility. Perhaps the advent of sufficiently advanced language learning models or other artificial intelligence will one day cause us to reconsider this stance. For now, though, we believe ACE Physics’ success will depend on thoughtful, research-based design, not clever programming.

Most on-paper tutorial questions are free-response in that they provide students with empty space to write their answer, regardless of whether the question demands a mathematical expression, a sketch, a written explanation, or some combination of the three. We must convert these to a structured format whenever we wish for ACE Physics to provide students with conditional feedback.

Sometimes this means asking for a numerical input (and perhaps an ungraded explanation). Often, though, it means converting the question to a multiple-choice version, but this comes with a cost: recognizing the correct answer from a list of possibilities is in general easier than generating the answer on your own. We do strive to introduce compelling distractors based on our knowledge of common mistakes, but nonetheless we consider this cost every time we make a question multiple-choice, and have developed various techniques to account for it.

When the set of plausible answers to a question is already small, a multiple-choice version is not much different from the free-response version, but we will still typically provide an ungraded text box asking students to explain their choice (requiring students to articulate their reasoning is a key principle of tutorials). In other cases, the answer to a question can be composed from a structured set of simple choices; for example, a column vector where the student can independently select the value of each component from a short list of options. Input like this can be auto-graded without difficulty, but this format has some benefits. First, it is difficult to simply guess the answer because of the combinatorial explosion in the set of possible answers. Second, it breaks the mold of a traditional multiple choice question, which students are used to from standardized exams. In general, we have attempt to vary the input format across questions in ACE Physics. We hypothesize that this helps students stay engaged with the tutorial, because it makes the separation between tasks more obvious.

One benefit of free-response questions during group work is that different students may generate different answers that are both correct, either because they represented the same answer differently or because there are indeed multiple correct answers. We can capture this to some extent by using multiple-choice multi-response questions (i.e., “Choose all that apply”). We do also sometimes include an “Other” option that allows students to input an arbitrary answer to multiple-choice questions, but we have found that students rarely choose that option (and their input cannot be auto-graded).

The final technique we use to circumvent the cost of multiple-choice questions is to introduce “litmus test” questions. A litmus test question is a multiple-choice version of a free-response question that provides some indication of whether the student understands the relevant concept. We will ask a litmus test question *in addition to* the free response version. For example, we might ask a student to articulate their reasoning in a text box and then, only once they move on to the next section, ask them to select the option from a list that most closely matches what they typed. This gives students the opportunity to generate an answer on their own but also allows us to provide targeted feedback. It has the added benefit of having students engage with multiple lines of reasoning, which happens naturally during group work.

It is still possible to provide students with feedback for ungradable questions: we can simply present our answer to a question after a student has submitted theirs. This technique, while crude, can be useful, especially when we want to be sure a student has the opportunity to read the "correct" reasoning regardless of what they wrote themselves. We typically preface these answers with a message like, "This software is not designed to analyze what you've written, but our answer would read. . . ", which encourages students to decide for themselves the extent to which their answer matches ours. This pattern essentially implements the idea of "preparation for further learning" at a mini scale: a student may engage more actively while reading our answer if they have already considered—by writing down—the answer for themselves [120, 121].

8.3.3 Hints

Many sections in ACE Physics tutorials will include one (or occasionally two) hints, which students can reveal by clicking a button. Hints do not reveal the answers, but instead attempt to encode the first thing an instructor might say to a student who is struggling with the question. Hints may include reminders of useful equations or other relevant information, but often (in the style of Socratic dialogue) they pose a suggestive question. Some hints simply clarify the meaning of the original question; as mentioned in Section 2.3, we often use varied language in tutorials, and students sometimes need to a see a question written with words they are more familiar with. When we expect a question to be difficult for students, we may add multiple hints, which can be revealed one at a time, which attempts to model an ongoing Socratic dialogue (albeit a one-sided one). By revealing hints one at a time and only when students request them, we enable students to decide how much assistance they require, and when they are ready to receive it.

As shown in Figure 7.7, students tend not to use hints. We hypothesize two reasons for this. First, we suspect some online homework platforms may have trained students to believe that they will be penalized for accessing hints. Second, the default label for hint buttons is “Hmm…”, but students may not know what this means. (The original, more whimsical language was intended to impart a friendly tone.) We will likely change the default label to “Hint”, but have also begun labelling hint buttons to indicate the specific information they will reveal, such as “Remind me about…”, for maximum clarity. Otherwise, we have not yet determined how to best encourage hint usage or dispel the idea that it imposes a penalty.

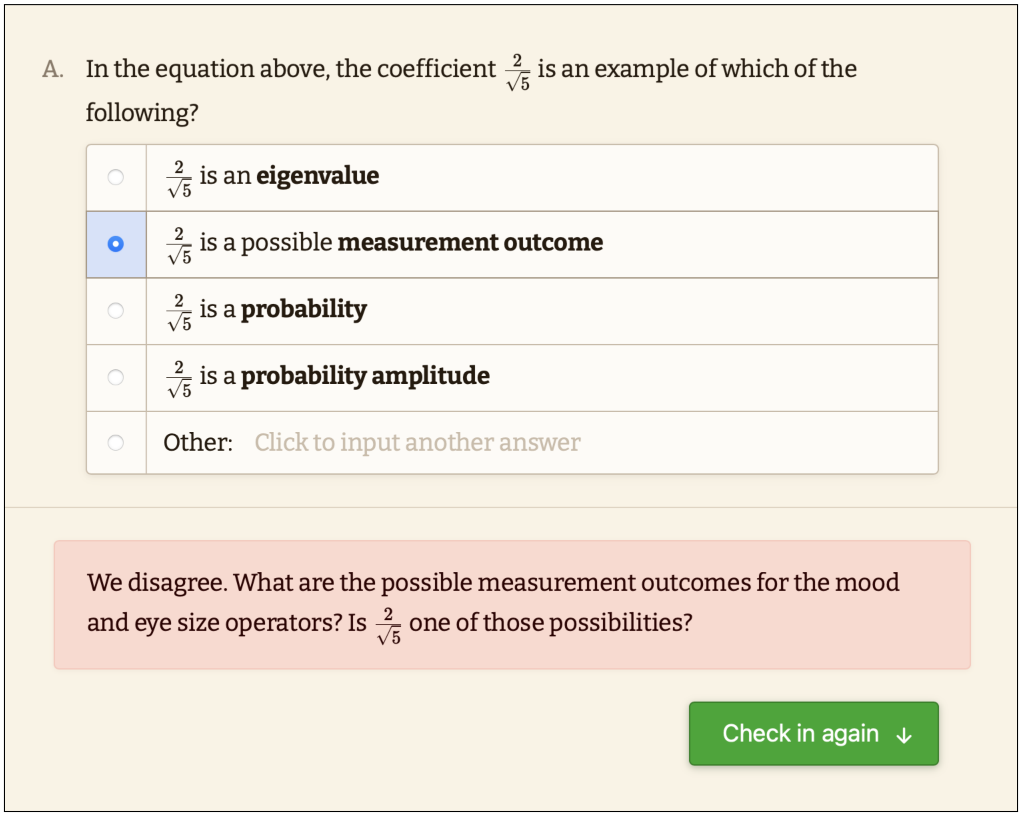
We observed one student in interviews who spent a long time struggling with a question but still never opened the hint. To gently encourage students to read hints, hint buttons now wiggle after a student has been working on the same section for several minutes. Conversely, we also observed a student who opened every single hint, typically *after* inputting their answer; they used the hints to help decide if their answers were correct. Hints remain available even after a section has been completed to enable this behavior, which we consider productive.

8.3.4 Immediate feedback

When a question is auto-gradable, we can provide students with targeted feedback immediately after they submit their answer. For example, in Figure 8.2, the student has submitted an incorrect answer, so they see a feedback message that is specific to the incorrect answer they chose. They can change their answer and click “Check in again” to receive more feedback. As shown in the figure, the feedback message is phrased as a Socratic question, which matches how we would expect a tutorial instructor to respond. We can also provide immediate feedback for other types of auto-graded answers, such as numerical inputs. In these cases we attempt to anticipate common errors and provide targeted feedback messages for those, but we also include a generic message for unanticipated answers. If students check in multiple times and see the same message twice, we allow them to “Move on anyway”, because a student stuck on one question may wish to attempt the remainder of the tutorial and revisit the offending question later, such as in office hours. In addition, this rule makes it impossible for a bug in our code to trap a student without the option to move on to the next question.

Immediate feedback is relatively straightforward to implement and easy for students to interact with, because it is localized to one question. We write targeted feedback messages by imagining what we might say to a student in an in-class tutorial who selected the same incorrect answer. Doing so essentially requires guessing at what mistake the student may have made, but research into student reasoning about the topic helps limit the guesswork. Immediate feedback is especially useful when upcoming questions in the tutorial depend on the question for which we are providing feedback. In these cases, it is important to get a struggling student back on track, otherwise they may be unable to complete the tutorial. It is also useful when the question will not be addressed again by the tutorial, meaning this is our final opportunity to point out a student's error.

We hypothesize that providing immediate feedback too frequently (e.g., after every question) may compromise some of the goals of the tutorials. Specifically, we fear that if students expect feedback after every question, they may spend less time considering their answers and instead rely on the software to immediately tell them if they are correct or incorrect. We have not yet conducted sufficient research to test this hypothesis. Figure 7.11 shows that many students self-report *not* behaving this way (although some did), but their self-reported behavior may not match their actual behavior. Even students who generally engage with tutorials in ways we consider productive may, in a given week where they are pressed for time, be tempted to resort to less productive behavior. Students in interviews also did not exhibit this behavior, but we suspect that interviewees were generally more likely to engage fully with the tutorial because they knew they were being observed. To evaluate this hypothesis more robustly, we must implement more advanced interaction-tracking features in the ACE Physics software.



A. In the equation above, the coefficient $\frac{2}{\sqrt{5}}$ is an example of which of the following?

○ $\frac{2}{\sqrt{5}}$ is an **eigenvalue**

● $\frac{2}{\sqrt{5}}$ is a possible **measurement outcome**

○ $\frac{2}{\sqrt{5}}$ is a **probability**

○ $\frac{2}{\sqrt{5}}$ is a **probability amplitude**

○ Other: Click to input another answer

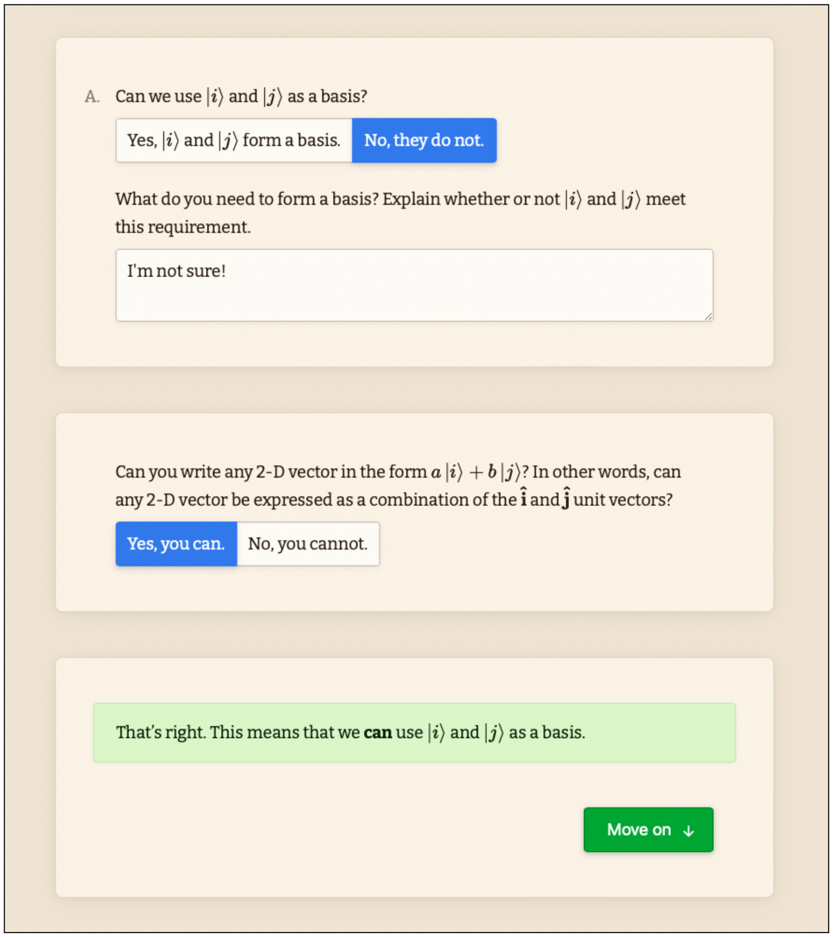
We disagree. What are the possible measurement outcomes for the mood and eye size operators? Is $\frac{2}{\sqrt{5}}$ one of those possibilities?

Figure 8.2: A tutorial question with immediate feedback. The student will see a new feedback message if they change their answer and click "Check in again". The referenced equation is $|\text{small eyes}\rangle = \frac{1}{\sqrt{5}}|\text{happy}\rangle + \frac{2}{\sqrt{5}}|\text{sad}\rangle$, which represents the state of a "quantum mouse" with two associated observables, eye size and mood, so the correct answer is "probability amplitude".

8.3.5 Delayed feedback

As described in Section 2.3, tutorials will typically present a sequence of questions related to the same concept, and will also repeat similar questions in slightly different contexts. By asking students multiple related questions or similar questions in different contexts, we can help students identify specific places where they may be confused. These cases provide opportunities for delayed feedback: feedback that does not appear until the student has answered multiple questions spanning multiple sections. Figure 8.3 shows a simple example of delayed feedback that involves two questions asked back-to-back, but delayed feedback logic might encompass more than two questions or questions asked further apart on the tutorial (even on separate pages). Delayed feedback messages will often point out discrepancies in a student's answers and ask them to reconcile these inconsistencies. If delayed feedback tells students which of their answers is correct (e.g., "We agree with your answer to B, but we think it means you should reconsider your answer to D."), the student has the opportunity to build a more robust understanding atop correct ideas they already have, which aligns with our strengths-based approach to tutorial design.

In Figure 8.3, students will not see the second question (or the feedback message) unless they get the first question wrong. We have *manufactured* an opportunity for delayed feedback by introducing a new question for the explicit purpose of contrasting it with the original question. This technique mimics Socratic dialogue more faithfully than the questions posed in the feedback message in Figure 8.2, because in this case students must answer the Socratic question before moving on, and we then have the opportunity to present follow-up feedback. Arguably most immediate feedback should be implemented in this fashion, but extended Socratic dialogue chains are difficult to build and run the risk of making the tutorials too long. We continue to explore the tradeoffs between immediate and delayed feedback.



A. Can we use $|i\rangle$ and $|j\rangle$ as a basis?

Yes, $|i\rangle$ and $|j\rangle$ form a basis. [✓ No, they do not.]

What do you need to form a basis? Explain whether or not $|i\rangle$ and $|j\rangle$ meet this requirement.

I'm not sure!

Can you write any 2-D vector in the form $a\,|i\rangle + b\,|j\rangle$? In other words, can any 2-D vector be expressed as a combination of the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ unit vectors?

[✓ Yes, you can.] No, you cannot.

That's right. This means that we **can** use $|i\rangle$ and $|j\rangle$ as a basis.

Figure 8.3: A sequence of questions from the Quantum Basis Tutorial that demonstrates a delayed feedback structure. The second question only appears for students who get the first question wrong. The content of the feedback message will vary based on the student's answer to the second question. On this page of the tutorial, we use Dirac notation ($|i\rangle$ and $|j\rangle$) for the cartesian unit vectors.

8.3.6 Providing the answers

The EPR and Entanglement Tutorial includes a special feature: once students complete a page, they are shown the answers (with explanations) to every question on that page. To encourage students to engage with our answers beyond simply determining if theirs were right or wrong (which most students eagerly do), we ask students to summarize what they have learned from instances where their answers disagreed with ours. This feature predates ACE Physics: the in-class implementation of the EPR and Entanglement Tutorial involves handouts with the answers for each page. In our experience, this pattern works well for this activity, but it is unusual and arguably antithetical to the principles of tutorials, which suggest that students should be carefully guided to the correct answers, not simply told them. The example of the EPR and Entanglement Tutorial demonstrates how we view the principles of tutorials as guidelines, not laws, but we are nonetheless wary of using this strategy often.

One benefit of this strategy is that it is significantly easier to implement than immediate or delayed feedback intended to model Socratic dialogue. We do not need to anticipate common errors nor consider how we should respond to them; we can simply write an answer key for the tutorial. As such, we have now employed this strategy as a temporary measure in several new ACE Physics tutorials targeted at quantum computing topics. We plan to administer drafts of these tutorials to students in interview settings and use these data to implement more robust feedback mechanisms.

8.3.7 Student expectations about automatic feedback

We have observed that students exhibit certain expectations for automatic feedback. First, some interviewees appeared to assume that, if they did not receive immediate feedback for a question, their answer must be correct—including free-response answers. This expectation is concerning because it may leave students with undue confidence in incorrect answers. In an effort to address this potential issue, we tell students how much answer-checking has been performed at the bottom of each page. For example, if some questions have feedback but others do not, we say, “We only checked some of your answers, so you may want to check in with an instructor".

We have also found that students typically want more, and clearer cut, feedback than the ACE Physics tutorials provide. This is unsurprising: even in in-class tutorials, students sometimes express frustration with the Socratic method—it can feel satisfying to just be told the answers. However, some students have pointed out in surveys that they struggle to use the ACE Physics tutorials as a resource for studying because they are never told what the correct answers are. In addition, a tutorial instructor will, eventually, ensure that students know the answers, but there is no such instructor for ACE Physics (realistically many will not visit office hours to ask about tutorials). We continue to search for the balance between providing limited, tutorial-like guidance and providing students with access to the answer key, but we generally bias away from providing answers whenever we believe doing so risks the goals of the tutorial .

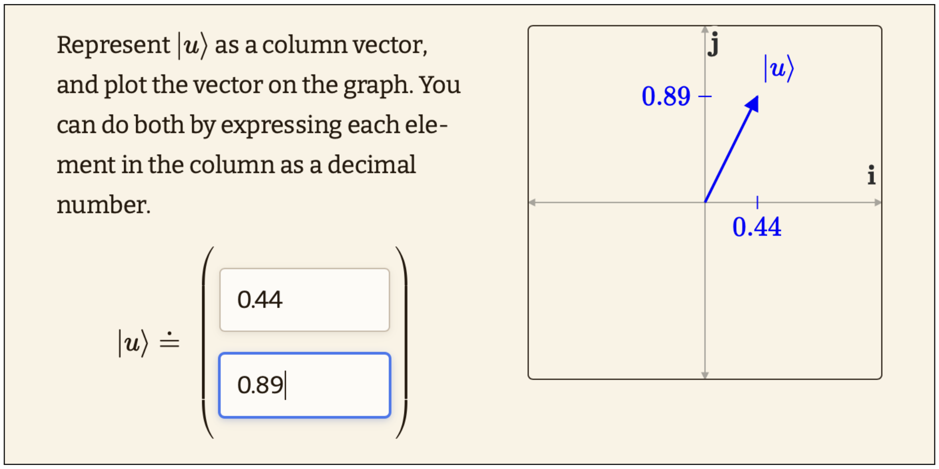
8.4 Overcoming constraints of the online medium

Whereas the previous section dealt with challenges introduced by moving tutorials from in-class to outside class, this section discusses implications of converting tutorials from paper documents to on-screen computer software. As mentioned in Section 2.6, software has the benefit of dynamism (it can display different content based on student input) but the tradeoff of a less flexible input mechanism (i.e., keyboard and mouse versus pen and paper). Input constraints affect three cases necessary for physics tutorials: sketching; mathematical notation; and scratch work.

8.4.1 Sketching

We never ask students to input sketches into the ACE Physics website because drawing graphs or diagrams using a mouse or trackpad is awkward at best. However, multi-representational competency is a key goal of many of our tutorials and many questions asks students for sketches, so we must find ways to adapt these questions for ACE Physics. How we do so depends on the specific question and its goals.

In some cases it is less important for students to generate a sketch than it is for them to notice



Represent $|u\rangle$ as a column vector, and plot the vector on the graph. You can do both by expressing each element in the column as a decimal number.

$$|u\rangle \doteq \begin{pmatrix} 0.44 \\ 0.89 \end{pmatrix}$$

Figure 8.4: A question from the Quantum Basis Tutorial. The graph updates dynamically when students type numerical components into the column vector. Students add more detail to the graph and analyze its features in subsequent questions.

certain features apparent in the visual representation. In these cases, we can embed a figure in ACE Physics and ask students to analyze it. Sometimes, such as in Figure 8.4, we can generate the figure dynamically based on student input. This affords students a degree of control over the figure and enables them to explore relationships between representations (e.g., changing one component of the column vector changes the coordinates of the vector in the graph in Figure 8.4), but does not require them to do any sketching directly.

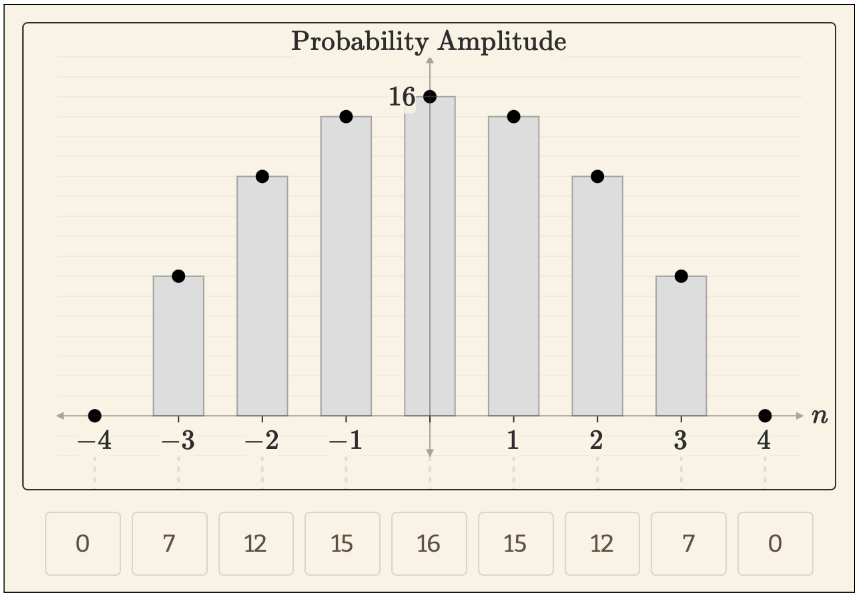
We have two options when we do consider it essential for students to generate a sketch on their own. Occasionally we can make figures directly editable by students on the screen, but this only works well when the representation is discrete (as opposed to continuous). Figure 8.5 shows an example where students can manipulate the heights of the bars in the histogram using their mouse . More often, though, we must convert questions that require sketching to a multiple-choice format wherein students select the correct figure from a set of static, predefined options. We may precede such a question by prompting students to sketch the figure for themselves on paper, and only reveal the options once they indicate they have completed the sketch. We ask students to, “Select the figure that looks most like your sketch”. This approach affords students the opportunity to generate the sketch from scratch but also allows the software to grade their response. The figures in the set of multiple-choice options typically appear similar but differ in key features we wish to draw attention to (e.g., asymptotic behavior).

8.4.2 Mathematical notation

Software can accept mathematical notation in three formats: simple ASCII (i.e., characters on typical American computer keyboards); TEX syntax (or another special syntax); or via a dedicated equation editor widget like that found in Microsoft Word. TEX is onerous and we could not find an existing, embeddable equation editor that met our standards for usability and also supported bra-ket notation, so ACE Physics only supports simple ASCII input. As such, we generally avoid asking students to input mathematical notation, especially complex expressions. When we do, we provide guidance, such as "Type kets like '`|+>`'", or provide unicode characters for students to copy-paste (e.g., "Copy-paste: ψ"). In addition, numeric inputs on ACE Physics only accept decimal notation—you cannot, for example, type $1/\sqrt{2}$.

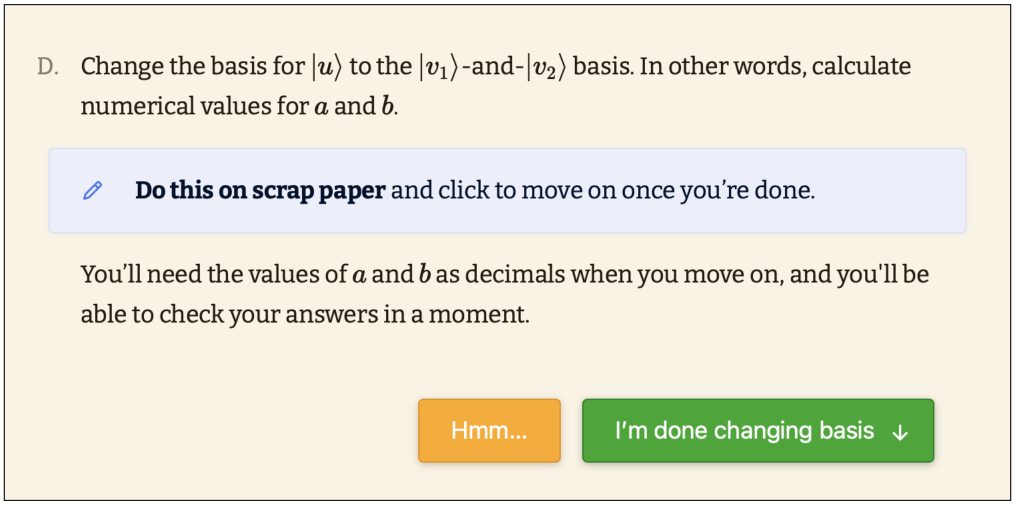
Obviously communicating about physics requires mathematical notation, but in practice ASCII-only input has proven less restrictive for ACE Physics than we expected. Because our quantum tutorials are concept-focused instead of calculation-focused, they generally do not call for extended algebra or complicated mathematical expressions, and students appear comfortable typing simple bra-ket notation using ASCII. We imagine this challenge might be more salient for tutorials about other topics, such as upper-division electricity and magnetism, which focuses more on mathematical methods.

Although ASCII-only input introduced fewer challenges than expected, we nonetheless must deal with questions that do require students to input more complicated expressions. The considerations and solutions are similar to those for sketching. We can always convert free-response questions to multiple-choice versions that render static, correctly typeset equations. When necessary, we will first prompt students to answer the question either on scrap paper or in a text input box before presenting them with the multiple-choice options. Finally, we can compose multiple simple inputs into the shape of a more involved mathematical expression, such as the column vector in Figure 8.4,



0	7	12	15	16	15	12	7	0

Figure 8.5: A question from the Vectors to Functions Tutorial [20]. Students can manipulate the height of each bar in the histogram by dragging the dots up and down.



D. Change the basis for $|u\rangle$ to the $|v_1\rangle$-and-$|v_2\rangle$ basis. In other words, calculate numerical values for a and b.

Do this on scrap paper and click to move on once you're done.

You'll need the values of a and b as decimals when you move on, and you'll be able to check your answers in a moment.

Figure 8.6: A question from the Quantum Basis Tutorial. The question emphasizes that students should do this step on scratch paper and prevents them from seeing the next question until they indicate that they have done so.

although we must take care to not provide unintended extra information when doing so (e.g., the column vector input indicates to the student that the vector has only two real-valued components).

8.4.3 Scratch work

Physicists often use scratch work when reasoning through problems [92, 142], and students do the same when working on paper tutorials in person. ACE Physics in no way prohibits scratch work, but in interviews we observed that students tend to avoid reaching for scrap paper when working on their computer screen. In the language of Section 2.5, this behavior appears to be a feature of the idioculture represented by students' devices. This can hinder students when tutorial questions are better solved on scrap paper than in one's head. To encourage students, we call out questions that may benefit from scratch work, and in some cases dedicate an entire question (e.g., Figure 8.6) to emphasize that using scrap paper is a normal part of doing an ACE Physics tutorial.

8.4.4 Opportunities for improvement

ACE Physics would benefit from a user-friendly equation editor for ACE Physics with high-quality support for quantum mechanics notation. This editor should support at least three modes:

numeric input, including fractions and square roots; complex numeric input ($a + bi$); and algebraic expressions including integrals, bra-ket notation, etc. Ideally students would also be able to insert equations inline in free response text boxes. We could integrate a computer algebra system[3] to support grading student responses. Such an editor would be an asset to the development of any future online physics activities, but implementing one well is nontrivial.

As large touchscreen devices (e.g., tablets) become increasingly common, ACE Physics could add features for these devices. For example, the website could provide spaces for students to draw graphs or write equations using a stylus. Handwriting recognition could even be used to parse student input. Such devices combine the flexibility of pen and paper with the dynamism of software to make for a "best of both worlds" medium. This technology exists today, but many students do not have access to such devices, and implementing features across various proprietary platforms would present a significant challenge. At the present time, optimizing for touchscreen devices would compromise ACE Physics' goal of making tutorials accessible to as many students as possible. We do not consider small touchscreen devices (e.g., smartphones) optimal for using ACE Physics because they are designed for short-form communication and content consumption as opposed to extended, focused use.

8.5 Conclusion

We have presented various design patterns we employ in ACE Physics to emulate tutorial-like guidance in the online environment and to overcome constraints of the on-screen medium. We have developed these patterns over several years of iteratively refining the tutorials, which has involved multiple stages of research (i.e., Chapters 5 to 7 as well as the interviews mentioned above). In addition, the design of ACE Physics activities is informed by the well-established tradition of tutorials in PER, as described in Section 2.3. However, we have yet not conducted sufficient research to rigorously evaluate each of these design patterns, and we have indicated some open questions, such as the effect of immediate feedback on student behavior. We believe this chapter can be useful

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[3] For example, Khan Academy's open-source "KAS" library [61].

to anyone seeking to build online physics activities, but it is not prescriptive. Instead, we present it in the interest of supporting future research and of empowering others working on similar projects to build on—and reevaluate—what we have developed.

Chapter 9

Conclusion

We have presented a variety of research surrounding the design, implementation, and administration of ACE Physics, a new platform for online instructional activities about quantum mechanics. The findings from each study also have implications for research and instruction that extend beyond the scope of ACE Physics. This dissertation builds on a strong tradition in PER that combines research, curriculum development, and instruction in an iterative process wherein each component strengthens the other two [82].

By investigating intuition in quantum mechanics, we found that students often expect the subject to be nonintuitive and moreover that students may attempt to disregard their incoming intuitions at the start of their quantum class, yet nonetheless students did consider portions of the class intuitive. We also identified six facets of intuition—matching one's expectations; physical observability; relation to prior experience; mathematical intuition; visualization; and intuition as something that one experiences—which can be attended to by instructors and can inform further education research about the role of intuition in doing and knowing physics. Students' sense of intuition also appears to relate to their self-efficacy and sense of belonging in physics classes.

We studied student reasoning about basis and change of basis in quantum mechanics and discovered that although projection is typically the most general and algebraically efficient method for changing basis, students often use other methods, such as solving a system of equations, which tend to be more error-prone. We also found that students may misinterpret a change of basis as having an affect on the quantum state, whereas it is only a change in representation. Instructors should be aware that their students may have these ideas, and future research might look for these ideas in other contexts, such as that of continuous-space wave functions. Based on our results, we designed the Quantum Basis Tutorial, for which we also created an ACE Physics version.

We characterized the ways upper-division quantum students behave during virtual, in-class tutorials using our modified color frames framework. We found that all four frames—individual work; discussion; authority; and casual, friendly behavior—play a role in tutorial sessions, as every frame arose in each group we studied. Instructors need not always trigger the authority frame when visiting student groups, but they do tend to pull students away from individual work and into the conversation. By allowing multiple frames to be coded for simultaneously, we could identify more nuance in student behavior, and found that the friendly frame often coexisted with the other three frames. The framework can be applied to characterize student behavior during other instances of in-class group work, and we also used it to motivate changes to ACE Physics tutorials intended for completion outside the classroom.

We administered and evaluated ACE Physics tutorials assigned as homework during two semesters of upper-division quantum mechanics using two different metrics. We measured content learning gains associated with two tutorials and found that the Quantum Basis Tutorial led to a significant improvement in students' post-test scores, but that the Time Dependence Tutorial did not have a significant impact. We additionally assessed the ways that students engaged with all the ACE Physics tutorials they had been assigned and generally found that students appeared or claimed to engage with the activities in good faith and considered them useful for their learning. These studies present preliminary tests of the efficacy of ACE Physics and also of the viability of administering tutorials outside of class generally. We believe more research-based evaluations of instructional activities should include student feedback surveys similar to the one we used in this study.

Finally, we presented an overview of our goals and process for developing online tutorial and documented various design patterns we used for dynamic guidance elements and to overcome constraints of the on-screen medium. By documenting these patterns we enable future research to investigate them systematically, or to compare ACE Physics tutorials with other instructional physics activities. This documentation will also serve any future projects developing online physics activities, including future ACE Physics tutorials.

We plan to continue the iterative process of research and activity design, and have begun developing a new batch of ACE Physics tutorials targeting introductory topics in quantum computing, such as basic quantum circuits. While we do so, we will continue investigating ACE Physics' efficacy, including by studying videos of students working on ACE Physics tutorials alone in semi-structured interviews. We believe carefully designed, research-based activities implemented as high-quality software and accessible online for free can serve as a powerful asset to instructors seeking to actively engage their students.

Students are at the center of each of these research studies. To understand the role of intuition in the quantum mechanics classroom, we asked students what it meant to them, and received a rich range of answers. To improve instruction related to change of basis, we identified student ideas about basis without only searching for incorrect ideas, and found some ideas that could be productively fostered by our instruction. To better understand the in-class tutorial environment, we examined the ways students behave during these activities and used that behavior as a guide for improving an ACE Physics tutorial. Finally, instead of exclusively defining the success of ACE Physics in terms of content learning gains, we also solicited student opinions on the activities and their utility.

We have found this student-centered approach to physics education research both productive and enriching. Students are not only our research subjects but also our partners in the collective and inherently social task of physics education. We believe the most successful instructional activities derive from research that captures the needs and ideas of students as expressed in their own words, and we consider attention to student voices doubly important when designing online activities.

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