Eur. Phys. J. D **64**, 593–599 (2011)
DOI: 10.1140/epjd/e2011-20212-3

THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

Gauge independent theory applied to a model of atomic ionization by an intense laser pulse

V.D. Rodríguez[1,a] and R.O. Barrachina[2]

[1] Departamento de Física and Instituto de Física de Buenos Aires, FCEyN, Universidad de Buenos Aires, 1428 Buenos Aires, Argentina
[2] Centro Atómico Bariloche – Instituto Balseiro (Comisión Nacional de Energía Atómica y Universidad Nacional de Cuyo), 8400 S. C. Bariloche, Río Negro, Argentina

Received 11 April 2011 / Received in final form 28 June 2011
Published online 6 October 2011 –

Abstract. An explicitly gauge invariant strong field theory is introduced and tested using a model laser-atom interaction. The theory relies on a power series in the target potential. Transitions amplitudes are obtained by using a corresponding series in the momentum space wave function. We demonstrate that this approach is explicitly gauge invariant to all orders. A well know 1D delta function potential model is used to test the convergence of the series in the evaluation of total ionization probabilities and ionization spectra. Actually, the convergence is verified when both, the perturbation as well as the order of the expansion, are increased.

1 Introduction

In recent works the subject of the gauge dependence (or independence) of the theoretical descriptions for the ionization of atoms and molecules by laser pulses has captured a great deal of attention [1–3]. It is widely known that generally, not only the first Born approximation but also the so called strong field approximations SFA [4–6] are strongly gauge dependent [7]. Therefore the question arises on which gauge should be used for a given problem. Until recently there was a kind of agreement on the superiority of the length form of the SFA [8]. However, the SFA in velocity gauge has been gaining consensus as the proper theory for quantitatively accounting for recent experimental results of fluorine negative ions detachment [9]. Furthermore, even when a numerical treatment can be carried out in any of both gauges, it is well know that the velocity gauge is actually preferred due to the faster convergence of the angular momentum expansion of the electronic wave function [10].

Among the efforts to clarify this controversy, explicitly gauge-independent theories have been proposed [11–13], as – for instance – the gauge-independent strong field model, developed by Antunes Neto and Davidovich [12]. In order to test their theory at work, they apply it to a one-dimensional system with an electron bounded by a delta-function potential and found a rather good agreement with full time dependent Schrödinger equation (TDSE) simulations. However, they only calculated the zero order term in the series expansion. Furthermore, no electron spectra, but only survival probabilities, were reported. In particular, they use the unitary condition to deduce the total ionization probability. Quite recently a new derivation of this theory has been presented for a model detachment of H^- along with some limited calculations up to the first order [14]. Unfortunately, no comparison with experiments and/or TDSE calculations was presented, and the convergence issue of the strong field series remains as an open question.

The purpose of this communication is twofold. First, a new simpler derivation based on an expansion of the momentum-space wavefunction on the “weak” target potential is presented. Second, up to three orders of the expansion are calculated for a simple model, consisting of a 1D delta function potential under the action of a strong laser pulse field [15–19]. Let us point out that, even for this simple model, some controversy arises on the reliability of the corresponding TDSE numerical calculations [20–22]. In this communication we avoid this issue by using a borderless approach based on a solution of the integral equation for the coordinate wave function at the origin. This equation defines the momentum wave function without any requirement on a particular box size or, equivalently, basis set size.

In the following section, the theory is presented in the context of the atom-laser interaction. We focus in the two main gauges, namely the velocity gauge and the length gauge.

[a] e-mail: vladimir@df.uba.ar

In Section 3 we apply the model to the well known simple 1D delta function potential model subject to a laser pulse. Our results for highly non-perturbative conditions are presented and compared with a numerical "exact" TDSE solution in Section 4. Finally, conclusions and perspectives are discussed in Section 5.

Atomic units are used throughout unless otherwise stated.

2 Theory

2.1 Derivation of the transition amplitudes in velocity gauge

Under non-relativistic conditions, the Hamiltonian for a minimal coupling atomic - laser field interaction is written as

$$H(t) = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}(\mathbf{r},t)\right)^2 + e\phi(\mathbf{r},t) + V_T(r). \quad (1)$$

Here $\mathbf{A}(\mathbf{r},t)$ and $\phi(\mathbf{r},t)$ are, respectively, the vector and scalar potentials, and $V_T(r)$ is the electron-atom interaction.

By using the Coulomb gauge ($\phi(\mathbf{r},t) = 0$ and $\nabla\cdot\mathbf{A}(\mathbf{r},t) = 0$) and the dipolar approximation ($\lambda \gg \langle\mathbf{r}\rangle \Rightarrow \mathbf{A}(\mathbf{r},t) \approx \mathbf{A}(t)$), and by rewriting $\mathbf{A}(t)/c \rightarrow \mathbf{A}(t)$, we obtain the following expression for the laser-atom interaction Hamiltonian:

$$H\left(t\right) = \frac{1}{2}(\mathbf{p} + \mathbf{A}(t))^2 + V_T(r). \quad (2)$$

The corresponding time dependent Schrödinger equation (TDSE) reads:

$$i\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = H(t)\left|\Psi(t)\right\rangle \quad (3)$$

with the initial condition: $|\Psi(0)\rangle = |\varphi_i\rangle$.

By noting that the first term of the Hamiltonian operator in (2) is diagonal in the momentum space, we project the TDSE in order to obtain:

$$\begin{cases} i\frac{\partial}{\partial t}\tilde{\psi}(\mathbf{p},t) = \frac{1}{2}(\mathbf{p}+\mathbf{A}(t))^2\tilde{\psi}(\mathbf{p},t) \\ \qquad + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}(\mathbf{p}',t)d\mathbf{p}' \\ \tilde{\psi}(\mathbf{p},0) = \tilde{\varphi}_i(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}}\int d\mathbf{r}\,e^{-i\mathbf{p}\cdot\mathbf{r}}\tilde{\varphi}_i(\mathbf{r}). \end{cases} \quad (4)$$

Following Lamb [23], the time dependent transition amplitude is calculated in the velocity gauge (2) by:

$$a_{fi}(t) = \langle\varphi_f|\; e^{i\,\mathbf{A}(t)\cdot\mathbf{r}}\;|\Psi(t)\rangle = \int d\mathbf{p}'\,\tilde{\varphi}_i^*(\mathbf{p}')\tilde{\psi}(\mathbf{p}'-\mathbf{A}(t),t). \quad (5)$$

As usual we will consider $\mathbf{A}(0) = \mathbf{A}(\tau) = 0$, being $t = 0$ and $t = \tau$, the initial and final time of the laser pulse. However, equation (5) is useful in order to compare transitions amplitudes calculated at intermediate times.

Let's define a solution series for TDSE (4) in powers of the target potential V_T:

$$\tilde{\psi}^{[n]}(\mathbf{p},t) = \tilde{\psi}^{(0)}(\mathbf{p},t) + \tilde{\psi}^{(0)}(\mathbf{p},t) + \ldots + \tilde{\psi}^{(n)}(\mathbf{p},t). \quad (6)$$

By replacing (6) into (4) and identifying equal powers of the target potential V_T we obtain:

$$\begin{aligned} i\frac{\partial}{\partial t}\tilde{\psi}^{(0)}(\mathbf{p},t) &= \frac{1}{2}(\mathbf{p}+\mathbf{A}(t))^2\tilde{\psi}^{(0)}(\mathbf{p},t) \\ i\frac{\partial}{\partial t}\tilde{\psi}^{(1)}(\mathbf{p},t) &= \frac{1}{2}(\mathbf{p}+\mathbf{A}(t))^2\tilde{\psi^{(1)}}(\mathbf{p},t) \\ &\quad + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}^{(0)}(\mathbf{p}',t)d\mathbf{p}' \\ &\ldots \\ i\frac{\partial}{\partial t}\tilde{\psi}^{(n)}(\mathbf{p},t) &= \frac{1}{2}(\mathbf{p}+\mathbf{A}(t))^2\tilde{\psi}^{(n)}(\mathbf{p},t) \\ &\quad + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}^{(n-1)}(\mathbf{p}',t)d\mathbf{p}'. \end{aligned} \quad (7)$$

The iterative equations (7) can be exactly solved, obtaining:

$$\begin{aligned} \tilde{\psi}^{(0)}(\mathbf{p},t) &= e^{-i\int_0^t dt'\frac{1}{2}(\mathbf{p}+\mathbf{A}(t'))^2}\tilde{\varphi}_i(\mathbf{p}) \\ \tilde{\psi}^{(1)}(\mathbf{p},t) &= -i\int_0^t dt' e^{-i\int_{t'}^t dt''\frac{1}{2}(\mathbf{p}+\mathbf{A}(t''))^2} \\ &\quad \times \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}^{(0)}(\mathbf{p}',t')d\mathbf{p}' \\ &\ldots \\ \tilde{\psi}^{(n)}(\mathbf{p},t) &= -i\int_0^t dt' e^{-i\int_{t'}^t dt''\frac{1}{2}(\mathbf{p}+\mathbf{A}(t''))^2} \\ &\quad \times \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}^{(n-1)}(\mathbf{p}',t')d\mathbf{p}'. \end{aligned} \quad (8)$$

Having defined the series in the momentum space wave function (6), we can use (5) to obtain the corresponding series for the transition amplitudes:

$$\begin{aligned} a_{fi}^{[n]}(t) &= a_{fi}^0(t) + a_{fi}^1(t) + \ldots + a_{fi}^n(t), \\ a_{fi}^{(n)}(t) &= \int d\mathbf{p}'\,\tilde{\varphi}_f^*(\mathbf{p}')\tilde{\psi}^{(n)}(\mathbf{p}'-\mathbf{A}(t),t). \end{aligned} \quad (9)$$

We would like to discuss the validity limit of expansion (9). From equation (8) it is evident that the functions $\tilde{\psi}^{(n)}$, and therefore the factors $a_{fi}^{(n)}$, are roughly proportional to the n power of the time duration and the strength of the target potential. Furthermore, the overlap in equation (9) diminishes for larger values of the vector potential. Thus, the convergence of expansion (9) would improve for shorter times, weaker target potentials and stronger fields.

Let us point out that the role of the field strength in the convergence of equation (9) is not limited to the overlap, but is also present in the time integrals in (8) through the Volkov phase $e^{-i\int_{t'}^{t} dt'' \frac{1}{2}(\mathbf{p}+\mathbf{A}(t''))^2}$. Therefore we can crudely estimate a parameter for the series in terms of the pulse duration τ, and vector potential amplitude $A_0 \approx E_0/\omega$: $\eta = \frac{E_{Z_T}}{E_0}\omega\tau$, where E_{Z_T} is the mean electric field over the electron by the target, E_0 the electric field amplitude of the laser field, and ω is the laser frequency. We note that this parameter is dimensionless.

2.2 Derivation of transition amplitudes in length gauge

The length gauge is derived as usual by the phase change:

$$|\psi(t)\rangle = e^{-i\mathbf{A}(t)\cdot\mathbf{r}}\,|\psi_L(t)\rangle\,. \qquad (10)$$

The Hamiltonian changes to:

$$H_L = \frac{1}{2}\mathbf{p}^2 - \dot{\mathbf{A}}(t)\cdot\mathbf{r} + V_T(r), \quad \mathbf{E}(t) = -\dot{\mathbf{A}}(t) \qquad (11)$$

and therefore the momentum space TDSE reads

$$i\frac{\partial}{\partial t}\tilde{\psi}_L(\mathbf{p},t) = \frac{1}{2}\mathbf{p}^2\tilde{\psi}_L(\mathbf{p},t) - i\dot{\mathbf{A}}\cdot\nabla_{\mathbf{p}}\tilde{\psi}_L(\mathbf{p},t) + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}_L(\mathbf{p}',t)d\mathbf{p}'$$

$$\tilde{\psi}_{[L]}(\mathbf{p},0) = \tilde{\varphi}_i(\mathbf{p}). \qquad (12)$$

The transition amplitude can now be calculated as

$$a_{fiL}(t) = \langle\varphi_f|\,\psi_L(t)\rangle = \int d\mathbf{p}'\,\tilde{\varphi}_f^*(\mathbf{p}')\tilde{\psi}_L(\mathbf{p}',t). \qquad (13)$$

As we did in the previous section, let's define a solution series for the TDSE (12) in powers of the target potential V_T:

$$\tilde{\psi}^{[n]}(\mathbf{p},t) = \tilde{\psi}_L^{(0)}(\mathbf{p},t) + \tilde{\psi}_L^{(1)}(\mathbf{p},t) + \ldots + \tilde{\psi}_L^{(n)}(\mathbf{p},t). \quad (14)$$

By replacing (13) into (12) and identifying equal powers of the target potential we obtain:

$$i\frac{\partial}{\partial t}\tilde{\psi}_L^{(0)}(\mathbf{p},t) = \frac{1}{2}p^2\tilde{\psi}_L^{(0)}(\mathbf{p},t) - iA(t)\cdot\nabla_{\mathbf{p}}\tilde{\psi}_L^{(0)}(\mathbf{p},t)$$

$$i\frac{\partial}{\partial t}\tilde{\psi}_L^{(1)}(\mathbf{p},t) = \frac{1}{2}p^2\tilde{\psi}_L^{(1)}(\mathbf{p},t) + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}_L^{(0)}(\mathbf{p}',t)d\mathbf{p}'$$

$$\ldots$$

$$i\frac{\partial}{\partial t}\tilde{\psi}_L^{(n)}(\mathbf{p},t) = \frac{1}{2}p^2\tilde{\psi}_L^{(n)}(\mathbf{p},t) + \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}_L^{(n-1)}(\mathbf{p}',t)d\mathbf{p}'. \qquad (15)$$

These iterative equations can be exactly solved:

$$\tilde{\psi}_L^{(0)}(\mathbf{p},t) = e^{-i\int_0^t dt' \frac{1}{2}(\mathbf{p}-\mathbf{A}(t)+\mathbf{A}(t'))^2}\,\tilde{\varphi}_i(\mathbf{p}-\mathbf{A}(t),0)$$

$$\tilde{\psi}_L^{(1)}(\mathbf{p},t) = -i\int_0^t dt' e^{-i\int_{t'}^{t} dt'' \frac{1}{2}(\mathbf{p}-\mathbf{A}(t)+\mathbf{A}(t''))^2} \times \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}_L^{(0)}(\mathbf{p}',t')d\mathbf{p}'$$

$$\ldots$$

$$\tilde{\psi}_L^{(n)}(\mathbf{p},t) = -i\int_0^t dt' e^{-i\int_{t'}^{t} dt'' \frac{1}{2}(\mathbf{p}-\mathbf{A}(t)+\mathbf{A}(t''))^2} \times \int \tilde{V}_T(\mathbf{p}-\mathbf{p}')\,\tilde{\psi}_L^{(n-1)}(\mathbf{p}',t')d\mathbf{p}'. \qquad (16)$$

By comparing (16) with (7) we obtain the following relationship:

$$\tilde{\psi}_L^{(n)}(\mathbf{p},t) = \tilde{\psi}^{(n)}(\mathbf{p}-\mathbf{A}(t),t) \Rightarrow \tilde{\psi}_L^{[n]}(\mathbf{p},t) = \tilde{\psi}^{[n]}(\mathbf{p}-\mathbf{A}(t),t). \quad (17)$$

Having obtained the series for the momentum-space wave function (13), we can use (15) to calculate the corresponding series for the transition amplitudes:

$$a_{fi\,L}^{[n]}(t) = a_{fi\,L}^{(0)}(t) + a_{fi\,L}^{(1)}(t) + \ldots + a_{fi\,L}^{(n)}(t), \qquad (18)$$

$$a_{fi\,L}^{(n)}(t) = \int d\mathbf{p}'\,\tilde{\varphi}_f^*(\mathbf{p}')\tilde{\psi}_L^{(n)}(\mathbf{p}',t) = \int d\mathbf{p}'\,\tilde{\varphi}_f^*(\mathbf{p}')\tilde{\psi}^{(n)}(\mathbf{p}'-\mathbf{A}(t),t) = a_{fi}^{(n)}(t). \quad (19)$$

Note that by using (17) in the last term of equation (19), we have demonstrated the gauge invariance of every truncated series in powers of the target potential, namely

$$a_{fi\,L}^{[n]}(t) = a_{fi}^{[n]}(t). \qquad (20)$$

Quite obviously, the gauge invariance would arise by replacing (10) into equation (5). However, this would be valid as far as the relationship between the exact or approximated wave functions does satisfy (10). For instance, this is the relationship between the exact numerical solutions of the TDSE in both gauges [24]. The main point here is that we have demonstrated its validity at every order of the series (6).

Not every perturbation series satisfies (19) or (20). For instance, the Born series does not. We would like to remark that the derived strong field series is closely related to that obtained by Antunes and Davidovich [12]. These authors used a propagator in the laser field in order to obtain the perturbative series in the target potential. By expressing their transitions amplitudes in the momentum space, a series in power of the target potential arises naturally as in equations (9) and (19). We believe that this form of presenting the theory is illuminating. As a matter of fact, the gauge invariance is due to gauge invariance of the target potential, different from the manifestly gauge dependence of the perturbation in the standard Born series.

3 The model

Here we briefly present the 1D delta function potential model [15–19] used to test the strong field series (9) and (18). We choose this rather simple, widely used model in order to calculate the series at least up to the second order. The Hamiltonian

$$H(t) = \frac{1}{2m_e}(p + A(t))^2 - Z\delta(x) \tag{21}$$

leads, in the momentum space, to the following TDSE

$$i\frac{\partial}{\partial t}\tilde{\psi}(p,t) = \frac{1}{2}(p + A(t))^2\tilde{\psi}(p,t) - Z\,\psi(0,t). \tag{22}$$

Note that this equation can be explicitly solved in terms of the time dependent coordinate wave function at the origin $\psi(0,t)$, namely

$$\tilde{\psi}(p,t) = e^{-i\int_0^t dt''\frac{(p+A(t''))^2}{2}}\tilde{\varphi}_i(p) + \frac{i\,Z}{\sqrt{2\pi}} \times \int dt' e^{-i\int_{t'}^t dt''\frac{(p+A(t''))^2}{2}}\psi(0,t'). \tag{23}$$

By integrating (23) over p we obtain a simple integral equation for $\psi(0,t)$

$$\psi(0,t) = \phi(0,t) + i\,Z\int_0^t dt' G(t',t)\,\psi(0,t') \tag{24}$$

with the functions $\phi(0,t)$ and $G(t',t)$ defined by

$$\phi(0,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dpe^{-i\int_0^t dt''\frac{(p+A(t''))^2}{2}}\tilde{\varphi}_i(p)$$

$$G(t',t) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi(t-t')}}e^{-i\beta(t',t)}e^{-i\frac{\alpha(t',t)^2}{2(t-t')}}$$

$$\beta(t',t) = \int_{t'}^{t} dt''\frac{A(t'')^2}{2}$$

$$\alpha(t',t) = -\int_{t'}^{t} dt'' A(t'')$$

$$\tilde{\varphi}_i(p) = \frac{1}{(2\pi)^{1/2}}\int dxe^{-ipx}\varphi_i(x) = \frac{2Z^{3/2}}{(2\pi)^{1/2}}\frac{1}{(Z^2+p^2)}. \tag{25}$$

This integral equation is numerically solved by means of a simple advanced discretization scheme, and the time dependent wave function in the momentum space is obtained by using either (23) or (22). Finally, equation (5) provides all the required transition amplitudes, both for the initial and the continuum states.

We use a laser pulse given by the electric field:

$$E\,(t) = -\frac{dA(t)}{dt} = E_0\ \sin\left(\omega\, t + \varphi\right)\ \sin\left(\frac{\pi t}{\tau}\right)^2 \tag{26}$$

with

$$Z = 1, \quad \omega = 1.5, \quad \tau = N_{cycles}\frac{2\pi}{\omega}, \quad \varphi = 0.$$

These parameters correspond to a deep bound state with ionization potential 0.5 a.u. ∼13.6 eV, and a laser photon energy three times larger. We should mention that appropriate scaling rules, should be used if this ultra short range potential is intended to simulate negative ions detachment, as in these cases ionization potentials are considerably smaller.

4 Results

In this section some calculations are performed in order to test the convergence of the strong field series presented in Section 2. The parameters have been taken from the paper by Geltman [19]. We point out that Geltman calls "Volkov first order", to what we here called zero-order in the expressions (9) or (18), consistently with the actual order in the weak target potential expansion. Our approach also differs from Geltman's in that we evaluate higher orders in the expansion, and calculate the TDSE for all the cases here considered. We focus our attention on a target with $Z = 1$, and a fixed laser frequency $\omega = 1.5$.

4.1 Above threshold ionization (ATI) structure in the strong field regime

In Figure 1, the ejected electron distribution as a function of the energy is shown for a strong field regimen, $E_0 = 1$. The breakdown of the simple first Born approximation (dotted curve), but also of the SFA (full curve) is clearly observed, when compared with the full TDSE (down triangles). Both the first Born and strong field approximations overestimate the one photon ionization peak. This failure can be understood as due to a strong depletion that is not accounted for by none of these two theories. On the other hand, SFA underestimates the subsequent ATI peaks, which only slightly emerges from the background. We should mention that ATI peaks in SFA become sharper if the number of cycles is increased. TDSE results show a clear ATI structure well above the background even for this short pulse.

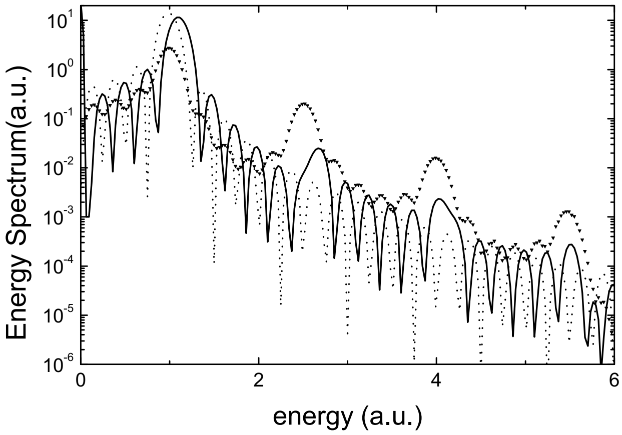

Fig. 1. Ejected electron energy distribution as a function of the energy for a model atom ($Z = 1$, $\omega = 1.5$), a laser field amplitude $E_0 = 1$ and a pulse duration of 6 cycles. Down triangles: exact TDSE, full and dotted curve, are SFA and first Born approximation, respectively.

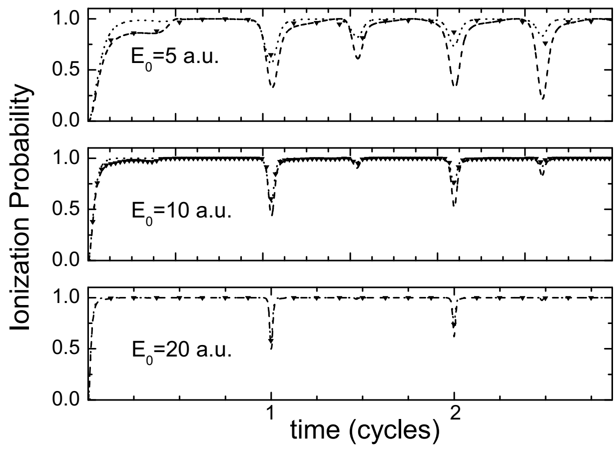

Fig. 2. Ionization probabilities in a model atom ($Z = 1$, $\omega = 1.5$) at various ultra-strong E_0 (5, 10 and 20) over 3.9 cycles of the applied fields. The Zero-order (dotted curve), first-order (down triangles) and TDSE solution (full curve) are shown.

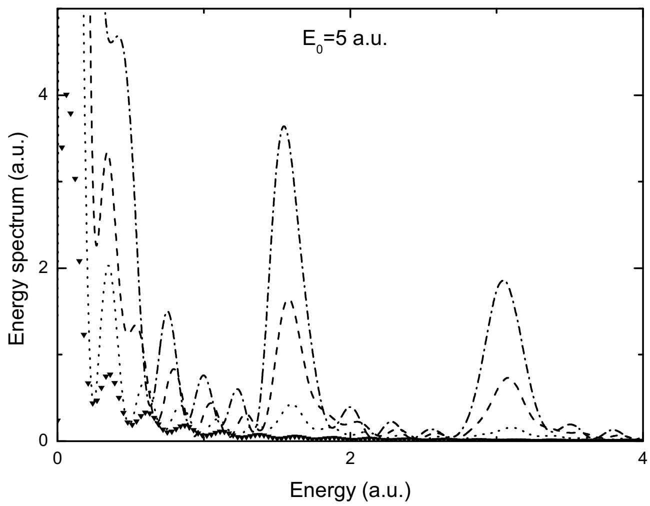

Fig. 3. Ejected electron energy distribution as a function of the energy for a model atom ($Z = 1$, $\omega = 1.5$) and a laser field amplitude $E_0 = 5$ at the end of a pulse with a duration of 6 cycles. The Down triangles correspond to the exact TDSE, while the dotted, dashed and dot-dashed curves represent the zero, first and second orders, in expansion (9), respectively.

4.2 Ultra-strong field regime

We have not used the present expansion in the calculation shown in the previous figure, since the perturbation is not strong enough for assuring the convergence of equation (9). Let us now consider the ultra-strong regimen, exemplified by $E_0 = 5$, 10 and 20. First of all we displayed in Figure 2 the ionization probability (evaluated as one minus the elastic probability) as a function of time. Only zero- and first-order calculations are displayed (dotted and dashed curves) along with the TDSE results (down triangles). Let us remark that only the zero order verifies unitarity. For higher orders this property arises only from the overall convergence of both, the electron energy distributions and elastic probabilities. As can be seen in Figure 2, the convergence improves as the field amplitude increases. In particular for $E_0 = 20$, we can hardly distinguish the different curves. Further discussion about the results shown in this figure is referred to the paper by Geltman [19], as we are more interested in the convergence of series (9).

More detailed information about the convergence properties of the strong field expansion can be obtained from the analysis of the energy electron spectrum. In Figure 3 we show this spectrum for $E_0 = 5$ a.u. and a pulse duration of 6 cycles. The zero (dotted line) and first (dashes line) and second-order (dot-dashed) are displayed, as well as the TDSE (down triangles). We clearly see that the series is still not convergent, since the results differ from the TDSE and this departure is larger for larger orders.

In Figure 4, we show the spectrum for $E_0 = 10$. Now we display the results up to third-order (solid line). All the orders show the same structure in a linear scale plot. The apparent divergence from TDSE as the order increases is reverted at the third order for which pretty accurate results are obtained as compared with TDSE. Furthermore, in the inset we compare the TDSE and third-order spectra on a logarithmic scale over a larger energy range. The agreement is rather good even for quite small electron distributions. An ATI peak structure, separated by the photon energy can be clearly appreciated.

Figure 5 shows the same results of Figure 4 but for $E_0 = 20$. The overall structure of the spectra is similar, but agreement between the TDSE and third-order results is much better. We would like to state that even for this simple model, the numerical calculation of higher orders becomes complex and subject to a certain amount of numerical uncertainties.

Finally, we show in Figure 6 the absolute value of the TDSE exact wave function at the origin that satisfies the integral equation (24) for different electric fields amplitudes. For the particular model we deal with in the present

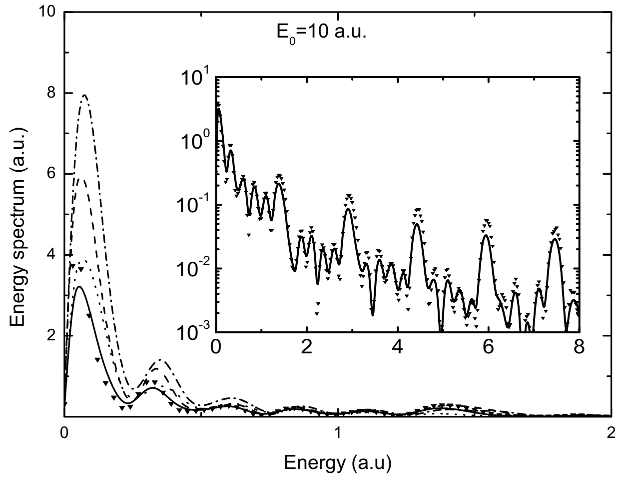

Fig. 4. Same as Figure 3, but for $E_0 = 10$, and with the additional third-order result show as a solid line. The inset is an expanded range figure in logarithmic scale.

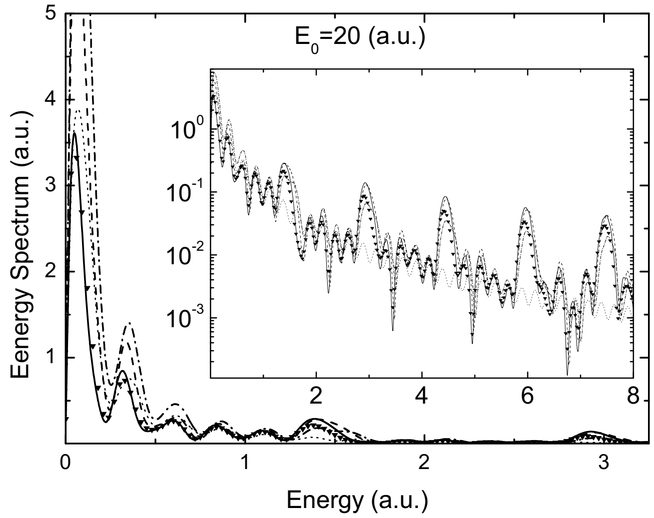

Fig. 5. Same as Figure 4, but for $E_0 = 20$.

work, the series in (14) can be obtained by iteration of (24) and the subsequent use of (22). We display the results for the zero- (dotted line) and first-order (dashed line). We can observe that the zero-order decreases faster than the first-order for small times. This can be understood as due to the fact that, since the zero-order does not account for any target potential, the wave function propagates only under the influence of the laser field. As the electric field amplitude increases, overall convergence is achieved. However, as mentioned before, only when the target potential is accounted for the description is good even for small times. The convergence obtained is in agreement with our estimated parameter, $\eta = \frac{E_{Z_T}}{E_0}\omega\tau$. In the present case, estimating $E_{Z_T} \approx Z/2$ we obtain $\eta \sim 0.3$ ($Z = 1$, $\tau = 8$, $E_0 = 20$, $\omega = 1.5$).

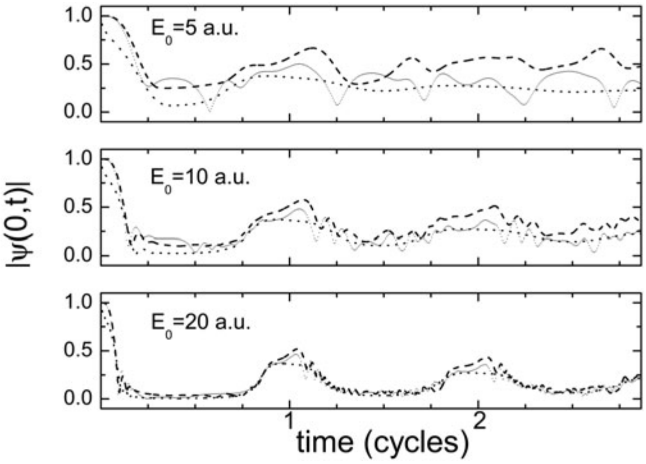

Fig. 6. Absolute value of the wave function at the origin as a function of time for $E_0 = 5$, 10 and 20 a.u. Down triangles: exact TDSE. The dotted and dashed curves correspond to the zero and first order iteration of (24), respectively.

5 Conclusions

A theory for calculating transition amplitudes for the atomic ionization of atoms by intense laser pulses as a series in the weak target potential has been derived. An attempt to delimit the validity of the expansion has been done by defining a convergence parameter $\eta = \frac{E_{Z_T}}{E_0}\omega\tau$. We demonstrated that both the velocity and length forms of the theory give identical results at any given truncation order of the expansion series. We tested the theory up to third order for a simple 1D delta function potential model. We showed that our theory works better for strong and short pulses of the laser field. We conclude that convergence can be achieved for higher order, accounting for rescattering in the target potential.

The authors are also members of the Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), Argentina. This work was partially supported by, the Consejo Nacional de Investigaciones Cientificas y Tecnicas (Grants PIP 112-200801-01269 and PIP 112-2009-0100166), Universidad de Buenos Aires (Grant X483), Universidad Nacional de Cuyo (Grant 06/C340) and Fundación Balseiro, Argentina.

References

1. Y.C. Han, L.B. Madsen, Phys. Rev. A **81**, 063430 (2010)
2. Y.V. Vanne, A. Saenz, Phys. Rev. A **79**, 023421 (2009)
3. H.R. Reiss, Phys. Rev. A **77**, 067401 (2008)
4. L.V. Keldysh, Sov. Phys. JETP **20**, 1307 (1965) (Engl. Transl.)
5. F.H.M Faisal, J. Phys. B **6**, L89 (1973)
6. H.R. Reiss, Phys. Rev. A **22**, 1786 (1980)
7. D. Bauer, D.B. Milošević, W. Becker, Phys. Rev. A **72**, 023415 (2005)

8. J. Bauer, Phys. Rev. A **73**, 023421 (2006)
9. H.R. Reiss, Phys. Rev. A **76**, 033404 (2007)
10. E. Cormier, P. Lambropoulos, J. Phys. B At. Mol. Opt. Phys. **29**, 1667 (1996)
11. W. Becker, D.B. Milosevic, Laser Physics **19**, 1621 (2009)
12. H.S. Antunes Neto, L. Davidovich, Phys. Rev. Lett. **53**, 2238 (1984)
13. F.H.M. Faisal, J. Phys. B **40**, F145 (2007)
14. A. Bechler, M. Ślęczka, Phys. Lett. A **375**, 1579 (2011)
15. E.J. Austin, J. Phys. B **12**, 4045 (1979)
16. K.J. LaGattuta, Phys. Rev. A **40**, 683 (1989)
17. A. Sanpero, L. Roso-Franco, Phys. Rev. A **41**, 6515 (1990)
18. S.M. Susskind, S.C. Cowley, E.J. Valeo, Phys. Rev. A **42**, 3090 (1990)
19. S. Geltman, Phys. Rev. A **45**, 5293 (1992)
20. Q. Su, B.P. Irving, C.W. Johnson, J.H. Eberly, J. Phys. B At. Mol. Opt. Phys. **29**, 5755 (1996)
21. T. Mercouris, C.A. Nicolaides, J. Phys. B At. Mol. Opt. Phys. **32**, 2371 (1999)
22. M. Dörr, R.M. Potvliege, J. Phys. B At. Mol. Opt. Phys. **33**, L233 (2000).
23. W.E. Lamb Jr,. R.R.S., M.O. Scully, Phys. Rev. A **36**, 2763 (1987)
24. D. Bauer, P. Koval, Comput. Phys. Commun. **174**, 396 (2006)