Propagation of Gaussian beams through active layers

Verónica Vázquez[a]; Juan C. Fernández[a]; María T. Garea[a]; Claudia L. Matteo[b,c], Liliana I. Perez[*a,d]; Patricio A. Sorichetti[b]
[a] Grupo de Láser, Óptica de Materiales y Propiedades Electromagnéticas (*GLOmAe*) Dep.de Física, Facultad de Ingeniería, Univ. de Buenos Aires, Av. Paseo Colón 850, Buenos Aires, C1063ACV, Argentina; [b] Grupo de Sistemas Dispersos - Laboratorio de Sistemas Líquidos (GSD-LSL), Dep.de Física, Facultad de Ingeniería, Univ.de Buenos Aires, Av. Paseo Colón 850, Buenos Aires, C1063ACV, Argentina; [c] Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina; [d] INTECIN, Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET),Argentina

ABSTRACT

Knowledge of propagation, transmission and reflection properties of space- and time-limited beams is relevant to the classical description of electromagnetic field modes in laser and other optoelectronic devices. For many reasons, Gaussian beams have been the most widely studied; for instance, they correspond to the fundamental mode in cylindrical or rectangular resonators, and they are often desirable at the output of amplifiers. To describe the behavior of beams with a Gaussian amplitude profile, the usual method consists of making an approximation in Maxwell equations, such that the solution of the approximate equations is a Gaussian beam. In this work we propose a different method to study Gaussian beams in active media, describing the beam by a continuous spectrum (spatial or temporal) of plane waves. We consider active media far from saturation, i.e. the gain is independent of the electric field amplitude. As a first step in the study of propagation, transmission and reflection of pulses through thin layers of active media, we analyze the properties of the transmitted beam in the case of a thin slab with gain between two isotropic transparent semi-infinite media, assuming normal incidence of a two-dimensional, space- or time-limited gaussian beam.

Keywords: gaussian beams, active slabs, transmission

1. INTRODUCTION

In recent years, the large variety of new materials for scientific and technological applications, both natural and specifically designed, has led to an intensive study of their electromagnetic properties. This includes the propagation of space- or time-limited beams in linear and non-linear media (with gain or attenuation), anisotropic, heterogeneous and chiral materials. The theoretical analysis of these problems has been carried out by analytical means (through full or approximate solutions of Maxwell equations), or numerically (using software specifically designed for electromagnetic problems, or adapted from other applications).

Non-geometrical effects (NGEs) appear in the propagation, reflection or transmission of space-limited beams. These peculiarities, which cannot be predicted on the basis of geometrical optics, depend on the propagation media, and the characteristics of the incident beam (shape, mean angle of incidence, polarization and frequency). Although there are several approaches for the calculation of NGEs, we consider that the generalization of the method proposed by Tamir [1] gives the best results, due to its simplicity and ease of interpretation. Beam propagation effects may be obtained starting with a superposition of plane waves, taking into account the relative phase change of each component [2-4]. The method of Tamir consists of solving the problem analytically, approximating the integrand to the second order. In this way, NGEs may be separated: the zeroth-order effect corresponds to the difference between the directions of the propagating beam and the mean value of the Poynting vector, the first-order to the lateral shift (Goos-Hänchen effect) and angular

[*] lperez@fi.uba.ar; phone +54 11 43431418

8th Iberoamerican Optics Meeting and 11th Latin American Meeting on Optics, Lasers, and Applications, edited by Manuel Filipe P. C. Martins Costa, Proc. of SPIE Vol. 8785, 87852W

· doi: 10.1117/12.2025712

displacement of the beam center, and the second-order to a focal shift and changes in the beamwidth. These effects are closely related to the Fresnel coefficients, since a phase change leads to a lateral shift and modify the beamwidth; similarly, amplitude changes originate focal shifts and angular displacements. In two-dimensional (2D) beams, these effects are lateral, and for three-dimensional (3D) propagation the transversal effects also appear. Although Tamir applied his method to the reflection of a two-dimensional gaussian beam in isotropic transparent slabs, the application of the method was extended to the propagation, reflection and transmission of 2D and 3D beams in interfaces between different kinds of materials, [5-8], including anisotropic transparent media[9-12]. However, there are fewer references in the open literature concerning beam propagation in active media, even for isotropic materials far from saturation. Moreover, even the simple case of plane-wave reflection in the interface between a transparent dielectric and an isotropic active medium has lead to conceptual contradictions between the treatments of different authors [13-20]. Previous results for plane waves show that there are certain critical configurations (in terms of slab thickness, incidence angle or relative refractive index) which require a very careful analytical solution. During the last years we undertook the study, later extended to include slabs, of the influence of gain in the reflection and transmission of plane waves in interfaces between semi-infinite media where at least one of the materials has these properties [21-23].

Active media are intrinsically non-linear, and often anisotropic. However, given the difficulty in the treatment of their interaction with electromagnetic waves, the linear and isotropic approximations are often used. The linearity assumption is valid for small amplitudes, where the gain may be considered as independent of the incident electric field. The isotropic media approximation is acceptable when the waves are polarized in the eigenmodes corresponding to the problem geometry.

In this work we study the transmission properties of a space- or time-limited 2D gaussian beam, for normal incidence on an active slab immersed in an isotropic medium. Polarization is assumed perpendicular to the incidence plane (TE polarization). The simple geometry leads to a straightforward interpretation of results; TE polarization was chosen to make possible a scalar treatment. In this way, analytical solutions may be easily compared with numerical results.

2. TWO-DIMENSIONAL GAUSSIAN BEAMS

In this work we study the transmission of Gaussian beams through an isotropic active layer of width d and complex permittivity $\tilde{\varepsilon}$, immersed in a medium of permittivity ε (Figure 1) We analyze two cases, corresponding to space-limited and time-limited beams:

Case 1) A space-limited monochromatic beam of frequency ω_0 and spatial half-width σ_z . The mean wave direction of the incident beam is normal to the interfaces.

Case 2) A time-limited (non-monochromatic) beam with central frequency ω_0 and temporal half-width σ_t incident normally to the interfaces.

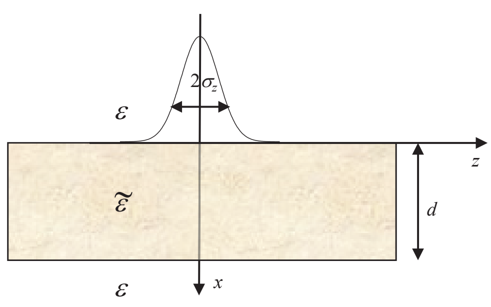

Figure 1.Representation of a time- or space-limited Gaussian beam impinging on an active slab

If the transmission coefficient of the structure is known, the properties of beams after traversing the slab may be calculated:

$$\mathrm{E}(x,z,t) = A \int_{-\infty}^{\infty} B(K) \quad e^{-\frac{K^2}{\Delta^2}} e^{i\,g(K,t)}\; dK \tag{1}$$

where A is a normalization factor, $B(K)$ is closely related to the transmission coefficient of the structure, Δ to the beamwidth, and $g(K,t)$ describes the spatial and temporal dependence of each plane wave component of the beam. For a monochromatic space-limited beam, K is the projection of the wave vector in the substrate at the interface and $\Delta = 2/\sigma_z$. In contrast, if we consider a time-limited (non-monochromatic) beam, K corresponds to $(\omega - \omega_0)$, where ω_0 is the center frequency and $\Delta = \sigma_t/2$. Applying to the transmission the method proposed by Tamir [1], that is, a second order approximation

$$B(K) = \exp(\ln B(K)) \cong \exp\left(M_0 + M_1\ K + M_2\ K^2\right) \tag{2}$$

and

$$g(K) \cong N_0 + N_1 K + N_2\ K^2 \tag{3}$$

Equation (1) can be solved analytically:

$$\mathrm{E}(x,z,t) = \frac{A\sqrt{\pi}\exp(iN_0 + M_0)}{\left(\frac{1}{\Delta^2} - iN_2 - M_2\right)^{1/2}} \exp\left(-\frac{(N_1 - iM_1)^2}{\left(\frac{1}{\Delta^2} - iN_2 - M_2\right)}\right) \tag{4}$$

Equation (4) corresponds to a field with a Gaussian distribution, but with amplitude, phase and profile modified by the transmission through the slab and the surrounding medium. To ascertain the validity of the second-order approximation it is necessary to study the characteristics of the transmission coefficient related to Equation (2).

3. TRANSMISSION COEFFICIENTS FOR ACTIVE SLABS

As it is well known, the transmission coefficient of a slab may be obtained from the boundary conditions in both interfaces. An equivalent calculation considers the superposition of reflected and transmitted waves, taking into account the reflection and transmission coefficients in each interface, and the phase change due to the propagation of each component wave [2, 3, 24]. The media may be transparent, absorbent, conductive or active, but must be linear. If these conditions are fulfilled, the transmission coefficient in the TE mode for a slab of thickness d and complex permittivity $\tilde{\varepsilon}$ immersed in an isotropic medium with permittivity ε is given by

$$T_s = \frac{4e^{ik_x' d} k_x k_x'}{\left(k_x + k_x'\right)^2 - e^{i2k_x' d}\left(k_x - k_x'\right)^2} \tag{5}$$

Therefore, the transmission coefficient depends on the incidence angle, the frequency, the permittivities (that in turn may depend on the frequency), and the slab thickness. When the material in the slab is active and modeled by a complex permittivity, the transmission coefficient may present anomalies (for instance, phase jumps). However, in this work we will consider slab under conditions far from any anomalies. To validate the approximation of the previous section, we analyze the transmission coefficient for the two cases, as a function of the mean incidence angle (case 1) or the center frequency (case 2). For the space-limited monochromatic beam (case1)

$$T_s(k_z) = \frac{4k_x k_x'\ e^{ik_x' d}}{\left(k_x + k_x'\right)^2 - e^{2ik_x' d}\left(k_x - k_x'\right)^2} \tag{6}$$

For each plane wave, the component of the wave vector in the direction parallel to the interface is $k_z = \sqrt{\mu\omega_0^2\varepsilon}\sin\alpha$, where α is the incidence angle. Therefore, the other components of the wave vectors (perpendicular to the interface) are $k_x = \sqrt{\mu\omega_0^2\varepsilon - k_z^2}$ and $k_x' = \sqrt{\mu\omega_0^2\tilde{\varepsilon} - k_z^2}$ in the surrounding medium and the slab, respectively.

For the time-limited non-monochromatic beam under normal incidence (case 2) we have

$$B(\omega) = \frac{4\sqrt{\varepsilon\tilde{\varepsilon}}e^{i\omega d\sqrt{\mu\tilde{\varepsilon}}}}{\left(\sqrt{\varepsilon}+\sqrt{\tilde{\varepsilon}}\right)^2 - e^{2i\omega d\sqrt{\mu\tilde{\varepsilon}}}\left(\sqrt{\varepsilon}-\sqrt{\tilde{\varepsilon}}\right)^2} \tag{7}$$

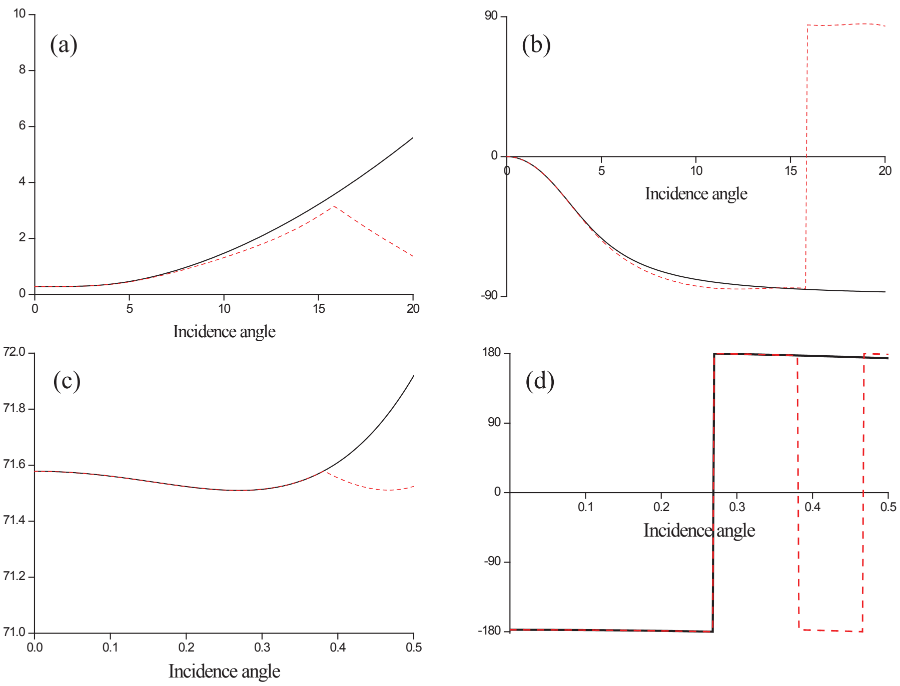

Figure 2. Comparison between approximate (solid lines) and non-approximate (dashed lines) values of the logarithms of the transmission coefficient (moduli and phases) as a function of the incidence angle. The slabs are in air (a) and (b) 1.5-i 0.002 and $d = 20\ \lambda_v$; (c) and (d) 1.33-i 0.0002 and $d = 60000\ \lambda_v$

For both cases, since the second-order approximations of $B(K)$ are non-oscillating, they may be valid only if restricted to those value ranges of K for which $B(K)$ has no oscillations. In Figure 2 we show the natural logarithm of the transmission coefficient, $\ln B(k_z)$, together with the second-order approximation, for slabs of active materials of different thicknesses. It may be seen that, for the same gain, the approximation is better in slabs of lower thickness. Moreover, for the slabs of Figure 2 the second order approximation is found to be valid if the width of the Gaussian beams (defined as $\arcsin(\lambda/\pi\sigma_z)$, the angular half-width far from the beam waist) is larger than 2 and 46 wavelengths, respectively. An equivalent result is found for the transmission coefficient of a time-limited (non monochromatic) beam under normal incidence. In this case, it is necessary for the half-width Gaussian pulse to be less than the oscillation interval in frequency for the transmission coefficient, that is $\sigma_t = \Delta\omega/2 = \omega_0\lambda_v\tilde{n}/2dn$ Under this condition, the exact transmission coefficient can be replaced by its approximation in the integral of the electric field, and the result will be satisfactory. Figure 3 shows different cases for this approximation method. As it can be seen in this figure, as the slab width grows, the approximated formula is more accurate in a wide frequency range, since the transmission coefficient tends to an exponential function. Moreover, the quadratic coefficient of the approximate method is close to zero and there is no limit for the σ_t value. The non-approximate transmission coefficient was calculated by numerical methods with a simulation program used for circuit modeling, among other applications [25].

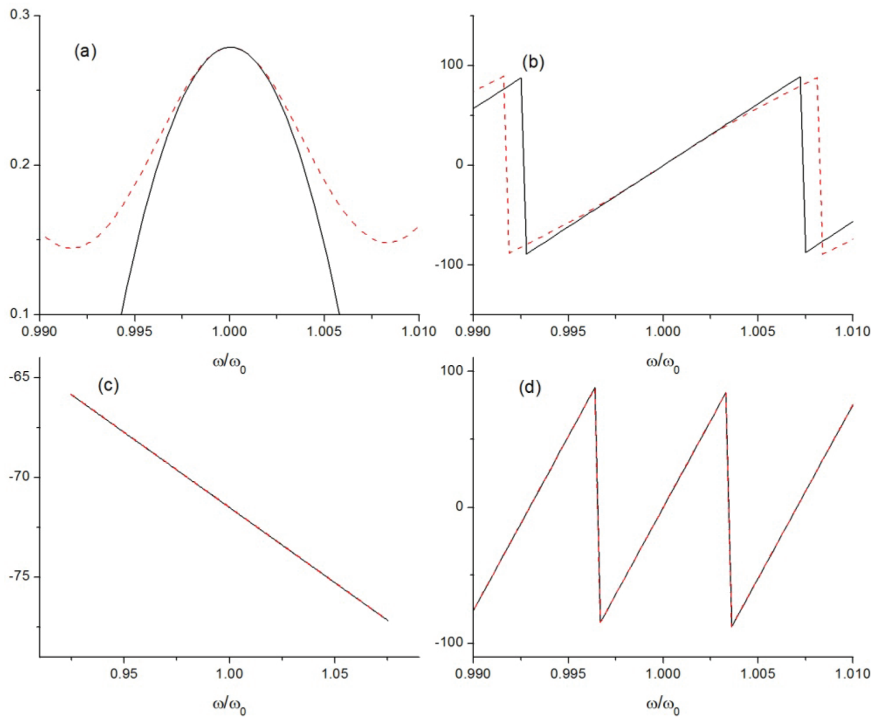

Figure 3: Comparison between approximate (solid lines) and non-approximate (dashed lines) values of the logarithms of the transmission coefficient (moduli and phases) as a function of the frequency. The slabs are in air (a) and (b) 1.5-i 0.002 and d = 20 λ_v; (c) and (d) 1.33-i 0.0002 and d = 60000 λ_v.

4. TRANSMISSION OF GAUSSIAN BEAMS

In this section we will analyze the characteristics of the transmitted beams, assuming that the half width of the incident beam is σ_z or σ_t at $x=0$ for the space-limited or time-limited cases, respectively (Figure 1).

4.1 Space-limited Gaussian beam

If we consider that the 2D Gaussian beam is polarized in the direction perpendicular to the incidence plane, the transmitted field through the slab is

$$\mathrm{E}(x,z,t)=\frac{E_s\sigma}{2\sqrt{\pi}}\int_{-\infty}^{\infty}T_s\left(k_x,k_x^{'},d\right)e^{-\frac{\sigma_z^2k_z^2}{4}}e^{i\left(k_x(x-d)+k_zz-\omega_0t\right)}dk_z \tag{8}$$

where E_s is the maximum amplitude of the incident beam and T_s is the transmission coefficient, Equation (6). In consequence, from Equation (4) the field amplitude at a distance x from the origin is given by

$$\mathrm{E}(x,z,t)=\frac{E_s\ \sigma_z\ T_s)_0}{2\sigma_m}e^{i\left[\sqrt{\mu\omega_0^2\varepsilon}(x-d)+\sqrt{\mu\omega_0^2\tilde{\varepsilon}}d-\omega_0t\right]}\exp\left[-\frac{z^2}{\sigma_m^2}\right] \tag{9}$$

where

$$T_s\big)_0 = \frac{4e^{i\sqrt{\mu\omega^2\tilde{\varepsilon}}d}\sqrt{\varepsilon\tilde{\varepsilon}}}{\left[\left(\sqrt{\varepsilon}+\sqrt{\tilde{\varepsilon}}\right)^2 - e^{i2\sqrt{\mu\omega^2\tilde{\varepsilon}}d}\left(\sqrt{\varepsilon}-\sqrt{\tilde{\varepsilon}}\right)^2\right]} \tag{10}$$

and σ_m , that in general is a complex number, is related to the modified Gaussian half-width:

$$\sigma_m^2 = \left(\sigma_z^2 - \frac{i2}{\sqrt{\mu\omega^2\varepsilon}}F\right) \tag{11}$$

As shown in [26], the effective beamwidth of the transmitted beam is given by

$$\sigma_{eff}^2 \approx \sigma^2\left(1+\frac{2\,Im(F)}{k\sigma^2}\right) \tag{12}$$

Where F is the complex focal shift. In terms of the refractive indexes, the slab thickness and the distance to the origin we have

$$F = \lambda_v\left(\frac{-2\pi\bar{d}n^2\tilde{n}\left(\left(n+\tilde{n}\right)^2 + e^{i4\pi\tilde{n}\bar{d}}\left(n-\tilde{n}\right)^2\right) + i\left(n^2-\tilde{n}^2\right)^2\left(1-e^{i4\pi\tilde{n}\bar{d}}\right)}{2\pi n\tilde{n}^2\left[\left(n+\tilde{n}\right)^2 - e^{i4\pi\tilde{n}\bar{d}}\left(n-\tilde{n}\right)^2\right]} - \bar{x} + \bar{d}\left(1+\frac{n}{\tilde{n}}\right)\right) \tag{13}$$

The real part of the complex focal beam shift also has a physical meaning: it corresponds to the change in the beam focus due to the refractions [1, 26]. Therefore, Equation (9) may be written as a function of the optical parameters: distances and refractive indexes:

$$\mathrm{E}\left(x,z,t\right) \cong E_s\,\frac{\overline{\sigma_z}}{\overline{\sigma_m}}\frac{2\,n\tilde{n}}{\left[\left(n+\tilde{n}\right)^2 - e^{i4\pi\tilde{n}\bar{d}}\left(n-\tilde{n}\right)^2\right]}e^{i\left[2\pi n\left(\bar{x}-\bar{d}\right)+2\pi\tilde{n}\bar{d}-\omega_0 t\right]}\exp\left[-\frac{\bar{z}^2}{\overline{\sigma_m}^2}\right] \tag{14}$$

where in equation (14) the variables indicated with an over bar are in expressed in wavelength units. The only NGEs are of second order: change of the beamwidth and focus shift. At the output of the slab the beamwidth change may increase, depending on the slab thickness (Figure 4) and gain, and the original beamwidth; the position of the focus may change in a few wavelengths. It must be noted that the beamwidth change and the focus position “oscillate” as the slab thickness increases.

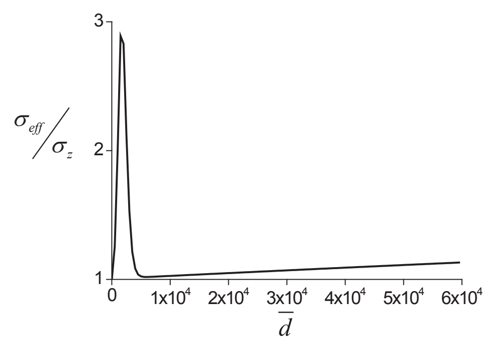

Figure 4.Variation of the effective waist σ_{eff} as a function of the slab width in units of the wavelength in vacuum, σ_z=100 λ_v and ñ=1.33-i 0.0002

4.2 Time-limited Gaussian beams

The field of a time-limited (non-monochromatic) beam under normal incidence conditions may be written as

$$\mathrm{E}\left(x,t\right) = \frac{E_s}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} T_s\left(\omega,d\right)e^{-\frac{\left(\omega-\omega_0\right)^2}{2\sigma_t^2}}e^{i\omega\left(\sqrt{\mu\varepsilon}\left(x-d\right)-t\right)}d\omega \tag{15}$$

Using the same procedure as in the previous section we get:

$$\mathrm{E}(x,t)=\frac{E_s\,T_s)_0\,\sigma_m}{\sigma_t}e^{i\left[\sqrt{\mu\omega_0^2\varepsilon}(x-d)-\omega_0 t\right]}\exp\left[-\frac{\sigma_m^2}{2}(t-\tau)^2\right] \tag{16}$$

where

$$\tau=-x\frac{n}{c}-\frac{d}{c}\left(\tilde{n}^2-n^2\right)\frac{(n+\tilde{n})-(n-\tilde{n})\exp\left(i4\pi\tilde{n}\overline{d}\right)}{(n+\tilde{n})^2-(n-\tilde{n})^2\exp\left(i4\pi\tilde{n}\overline{d}\right)} \tag{17}$$

$$\sigma_m=\frac{\sigma_t}{1-4\sigma_t^2\,\Sigma} \tag{18}$$

with

$$\Sigma=8\pi^2\frac{\overline{d}^2}{\omega_0^2}\frac{(n-\tilde{n})^4\exp\left(i8\pi\tilde{n}\overline{d}\right)}{(n+\tilde{n})^2-(n-\tilde{n})^2\exp\left(i8\pi\tilde{n}\overline{d}\right)} \tag{19}$$

Eq. (16) describes the electric field of a time-limited Gaussian beam, but with a different half-width σ_m (second order NGE) and delay τ (first order NGE). These parameters depend on the characteristics of the slab, as shown in Figure 5. The modified half-width, σ_m, tends to σ_t when the width slab grows, although both parameters oscillate in a similar fashion as the transmission coefficient.

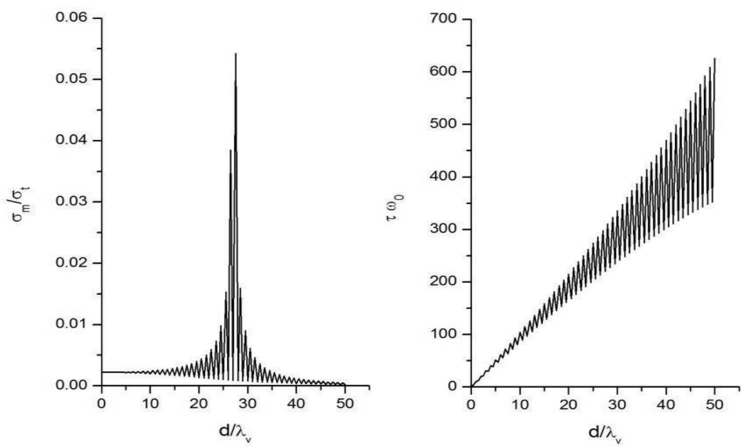

Figure 5: Modified half-width and delay for the time-limited Gaussian beam as a function of the normalized slab thickness. The slabs are in air (a) and (b) 1.5-i 0.002 and d = 20 λ_v.

5. DISCUSSION

In this work we have determined the main characteristics of the non-geometric effects (NGEs) of the transmission of space- and time-limited Gaussian beams through an active slab immersed in an isotropic medium. The validity of the results is limited to operating conditions far from saturation, since the active media is assumed to be linear. Starting with the superposition of plane waves (of the same frequency and different angles of incidence, or a given incidence direction and different frequencies), we have found that the space-limited gaussian beam only shows second-order NGEs. In contrast, in time-limited beams we find first- and second-order effects. The second order approximation of the transmission coefficient is shown to be valid for the analytical calculation of the electric field of Gaussian beam through a slab of active material, both for space- and time-limited cases. The comparison with the results of numerical integrations shows that the electric field calculated by using the approximate method has enough accuracy to be used with confidence practical applications. For slab of small thickness (a few wavelengths) the approximated method can be used only half-width of the incident Gaussian beam is less than the oscillation range of the transmission coefficient, for incident angle or frequency range in space- or time-limited Gaussian beams, respectively. However, this limitation disappears when the slab thickness is greater than about a hundred wavelengths. We have considered beams under normal incidence conditions; however, the extension to other angles of incidence is immediate. In this latter case the first- and second-order effects are expected to be complex instead of real. In both cases, the transmitted electric field results to be another Gaussian beam, but with modified half-width and delay. These parameters, that are function of the characteristic of the active media, can be used to analyze different configurations of optoelectronic devices, for instance, optical amplifiers in the linear regime.

REFERENCES

[1] Tamir T., "Nonspecular phenomena in beam fields reflected by multilayered media," J. Opt. Soc. Am. A **3,** 558-565 (1986).

[2] Hecht E., "*Optics,"* Addison-Wesley, (1986)

[3] Born M. and Wolf E., "*Principles of Optics*," Cambridge University Press, (1999)

[4] Goodman, J. W., "*Introduction to Fourier Optics,"* McGraw Hill, (1968)

[5] Chiu Chan C., Tamir T., "Beam phenomena at and near critical incidence upon a dielectric interface," J. Opt. Soc. Am. A 4, 655–663 (1987).

[6] Nasalski W., "Three dimensional beam reflection at dielectric interfaces," Opt. Commun. 197, 217–233 (2001).

[7] Aiello A., Woerdman J. P, "Role of beam propagation in Goos–Hänchen and Imbert–Fedorov shifts," Opt. Lett. 33, 1437–1439 (2008).

[8] Merano M., Aiello A., van Exter M. P., Woerdman J. P., "Observing angular deviations in the specular reflection of a light beam," Nat. Photon. 3, 337–340 (2009).

[9] Perez L. I., Ciocci F., "Nonspecular first-order effects in Kretschmann's configuration," J. Mod. Opt. 45, 2487–2502 (1998).

[10] Perez L. I., "Reflection and non-specular effects of 2D Gaussian beams in interfaces between isotropic and uniaxial anisotropic media," J. Mod. Opt. 47, 1645–1658 (2000)

[11] Perez L. I., Simon M. C., "Goos–Hänchen effect of an extraordinary refracted beam," J. Mod. Opt. 53, 1011–1021 (2006).

[12] Perez L.I., Echarri R.M., Garea M.T., Santiago G.D., "Determination of Non-geometric Effects: Equivalence between Artmann's and Tamir's Generalized Methods," J. Opt. Soc. Am. A **28,** 356-362 (2011)

[13] Romanov G. N., Shakhidzhanov S. S., "Amplification of electromagnetic field in total internal reflection from a region of inverted population," ZhETF Pis. Red. **16,** 298 – 301 (1972).

[14] Callary P.R.,. Carniglia C.K, "Internal reflection from an amplifying layer," J. Opt. Soc. Am. **66,** 775-779 (1976).

[15] Vainshtein L.A., "Propagation of pulses," Sov. Phys. Usp. **19,** 189-205 (1976)

[16] Cybulski Jr. R F , Silverman M P , "Investigation of light amplification by enhanced internal reflection. Part I. Theoretical reflectance and transmittance of an exponentially nonuniform gain region," J. Opt. Soc. Am. **73**, 1732-1738 (1983)

[17] Karapetyan R V, " Reflection of light from amplifying medium," Laser Physics **17,** 1053-1057 (2007)

[18] Willis K J, Schneider J B and Hagness S C, " Amplified total internal reflection: theory, analysis and demonstration of existence via FDTD," Opt. Exp. **16**, 1903-1914 (2008)

[19] Vinogradov A P, Zyablovsky A A, Dorofeenko A V, "Total internal reflection in gain medium slab," Appl. Phys. A **107**, 89-92 (2012)

[20] Siegman A, "Fresnel Reflection and Evanescent Gain," *Optics and Photonics News* **21**, 38-45 (2010)

[21] Duplaá M C, Perez L I, Matteo C L and Garea M T, "Reflection and refraction in active dielectric materials," *Journal of Physics: Conference Series (JPCS)* **274,** 012131 (2011).

[22] Duplaá M. C., Garea M. T., Matteo C. L., Perez L. I., Sorichetti P., "Reflectividad y transmisividad en monocapas activas," ANALES AFA (in press), (2012)

[23] Perez L. I., Matteo C. L., Etcheverry J., Duplaá M. C., "Active isotropic slabs: conditions for Amplified Reflection," Journal of the Optics 14, 125711 (2012)

[24] Azzam R.M., Bashara N. M., "Ellipsometry and Polarized Light," North-Holland Physics Publishing Elsevier Science Publishers B.V. , (1988)

[25] Ltspice 4.19e, "www.linear.com," Linear Technology Corp, (2012)

[26] Carniglia C.K. and Brownstein K. R., "Focal shift and ray model for total internal reflection," J. Opt. Soc. Am. 67, 121–122(1977).